Numerical characterization of the Kähler cone of a compact Kähler manifold

Joint work by
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The goal of this work is give a precise numerical description of the Kähler cone of a compact Kähler manifold. Our main result states that the Kähler cone depends only on the intersection form of the cohomology ring, the Hodge structure and the homology classes of analytic cycles: if X is a compact Kähler manifold, the Kähler cone $\mathcal{K}$ of X is one of the connected components of the set $\mathcal{P}$ of real $(1,1)$ cohomology classes $\{\alpha\}$ which are numerically positive on analytic cycles, i.e. $\int_Y \alpha^p > 0$ for every irreducible analytic set Y in X, $p = \dim Y$. This result is new even in the case of projective manifolds, where it can be seen as a generalization of the well-known Nakai-Moishezon criterion, and it also extends previous results by Campana-Peternell and Eyssidieux. Our main result admits a number of variants and corollaries, including a description of the cone of numerically effective $(1,1)$ classes and their dual cone. We are aware of a useful application by D. Huybrechts, which calculates the Kähler cone of an arbitrary hyperkähler manifold.

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