SPHERICAL CHARACTERS ON CERTAIN P-ADIC SYMMETRIC SPACES

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Abstract. Let E be a quadratic extension field of a p-adic local field F. We prove that spherical characters on the symmetric space $GL(2n,F)/GL(n,E)$ are locally integrable functions which are locally constant on the set of regular elements.

1. Introduction

Suppose F is a nonarchimedean local field of characteristic zero and $\mathbf{G}$ is a connected reductive algebraic group defined over F. Let $G = \mathbf{G}(F)$. Harish-Chandra proved in [HC1] that the character of an irreducible admissible representation π of G is a locally integrable distribution on G in the sense that there is a locally integrable function χ_π on G such that

$$\mathrm{trace}\pi(f) = \int_G f(g)\chi_\pi(g)dg, \quad \text{for any} f \in C_c^\infty(G).$$

Furthermore the function χ_π is locally constant on the set of regular elements of G, and has asymptotic expansions around singular elements in terms of Fourier transforms of nilpotent distributions on the tangent spaces of the centralizers of these elements.

Now let θ be an involution of G and let $H \subset G$ be the subgroup of fixed elements of θ. Then G/H is a symmetric space over F. An irreducible admissible representation π of G is called class one if there exists a nonzero H-invariant linear form l_π on π as well as a nonzero H-invariant linear form $l_{\tilde{\pi}}$ on the representation $\tilde{\pi}$ contragredient to π. Then the bi-H-invariant distribution Θ_π on G defined by

$$\Theta_\pi(f) =< l_\pi, \pi(f)l_{\hat{\pi}} >, \quad f \in C_c^\infty(G)$$

is called a spherical character for the class one representation π. These distributions come up in the spectral decomposition of global trace formulas on the symmetric spaces. It is one of the main object in the study of the theory of harmonic analysis on the p-adic symmetric space G/H. Due to the work of Jacquet and his coworkers on trace formulas on symmetric spaces, this theory is currently of considerable interest. Here a natural question is whether or not a spherical character has the properties of the characters in the group case described above. This question was largely answered in a recent work of Rader and Rallis ([RR]). By extending certain techniques of Harish-Chandra ([HC1]) as well as some results of Howe ([Ho1], [Ho2]) from the group case to the symmetric space case, they proved that a spherical character is a locally constant function on the set of θ-regular elements and has an asymptotic expansion around the origin similar to Harish-Chandra's description of the characters in the group case. But they also showed by examples that, unlike the characters in the group case,spherical characters are generally not locally integrable functions.

However in certain cases we can still expect the local integrability of the spherical characters, a result which usually has important applications in the theory of automorphic forms. An example for this is provided by the symmetric space $\mathbf{G}(E)/\mathbf{G}(F)$ where E is a quadratic extension field of F, as proved by Hakim in [Ha1]. (In fact this is the only example of a symmetric space of general rank, other than the group case, we know so far where the spherical characters are locally integrable.)

In this paper we will investigate the case where $G = GL(2n, F)$ and $H \subset G$ is a subgroup which is isomorphic to $GL(n, E)$ over F. We also assume that the residual characteristic of F is odd. In this case (G, H) is a Gelfand pair ([G3]) in the sense that the dimensions of both the space of the H-invariant linear forms on π and the space of the H-invariant linear forms on $\tilde{\pi}$ are at most one. So the spherical character for a class one representation is unique up to a scalar. We will prove the following theorem.

Theorem 1.1. *Let G and H be the groups as above. Then the spherical character Θ_π for a class one representation π of G is locally integrable on G which is locally constant on the set of θ-regular elements.*

Since the set of θ-regular elements in the symmetric space G/H is open and dense with complement in G/H of measure zero, the next corollary follows immediately from the above theorem.

Corollary 1.2. *The spherical character Θ_π for a class one representation π of G is nonzero when restricted to the set of θ-regular elements.*

To explain the motivation of the work, we go to a global setting. Let F be a number field and let E be a quadratic extension field of F. Let $G = GL(2n, F)$ and $H \subset G$ be a subgroup which is isomorphic to $GL(n, E)$. We denote by Z the center of G. For an automorphic cuspidal representation π of G, it is conjectured by Jacquet that if there is a cusp form ϕ in the space of π such that the period integral

$$P(\phi) = \int_{H(F_A)/H(F)Z(F_A)} \phi(h)dh$$

is nonzero, then the exterior square L-function of π has a pole at 1 and $|P(\phi)|^2$ is essentially the product $L(1/2, \pi)L(1/2, \pi \otimes \eta)$ (where F_A is the adele ring of F and η is the quadratic idele class character of F attached to E, see [G2] for details). When $n = 1$, this is a fundamental result of Waldspurger as proved by him by using the theory of Weil representations ([W]). Jacquet reproved this result of Waldspurger later by comparing the trace formula for the symmetric space $GL(2, F)/E^*$ and the trace formula with weight factor η for the symmetric space $GL(2, F)/F^* \times F^*$ ([J1], [J2], see also [G1] for further applications). In extending the method of Jacquet to the $GL(2n)$ case, it is expected that the result in this paper will play the same role as the one played by Harish-Chandra's result in the problems concerning the comparison of Arthur-Selberg trace formula.

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2. The spherical characters

Suppose F is a p-adic local field of characteristic zero and odd residual characteristic and suppose $\mathbf{G}$ is a connected reductive algebraic group defined over F. Let $G=\mathbf{G}(F)$ and let θ be an involution in G. We denote by H the subgroup of fixed points of θ in G. Let S be the subset of G of elements of the form $g\theta(g^{-1})$ for $g \in G$. The group G acts on S by twisted conjugation

$$(g, s) \mapsto gs\theta(g^{-1}), \quad g \in G, \quad s \in S.$$

Because $\theta(h) = h$ for any $h \in H$, the subgroup H acts on S by conjugation

$$(h, s) \mapsto hsh^{-1}.$$

Then the surjective map $\rho : G \to S$ defined by $\rho(g) = g\theta(g)^{-1}$ for $g \in G$ induces an isomorphism between the symmetric space G/H and the space S as G-spaces.

Let π be an irreducible admissible representation of G on the complex vector space V. We say π is H-distinguished if there is a nonzero H-invariant linear form l_π on V. Let $\tilde{V}$ be the space of smooth linear forms on V and let $\tilde{\pi}$ be the representation of G on $\tilde{V}$ contragredient to π. Then the representation π is called class one if both π and $\tilde{\pi}$ are H-distinguished. Now let π be a class one representation of G and let l_π and $l_{\tilde{\pi}}$ be nonzero H-invariant linear forms on π and $\tilde{\pi}$ respectively. For any $f \in C_c^\infty(G)$ then $\pi(f)l_{\tilde{\pi}}$ can be regarded as a vector in V. A spherical character of π is a distribution on G defined by

$$\Theta_\pi(f) =< l_\pi, \pi(f)l_{\tilde{\pi}} >, \quad f \in C_c^\infty(G).$$

Clearly Θ_π is bi-H-invariant.

An element $s \in S$ is called θ-regular or simply regular if it is semisimple in G and has maximal dimensional H-orbit([R],[RR]). We denote by S' the set of the regular elements in S. It is a dense open subset of S with the complement of measure 0. It is proved in [RR] that the spherical character Θ_π is represented by a locally constant function on S'.

We now consider the behavior of Θ_π around the singular elements in S. Let $\mathfrak{g}$ be the Lie algebra of G. Then G acts on $\mathfrak{g}$ by adjoint representation:

$$\mathrm{Ad}g(X) = gXg^{-1}, \quad g \in G, \quad X \in \mathfrak{g}.$$

We fix a nondegenerate G-invariant bilinear form B on $\mathfrak{g}$ as well as a nontrivial additive character ψ on F. Let $\mathfrak{h}$ and $\mathfrak{s}$ be the 1 and -1 eigenspaces of θ in $\mathfrak{g}$. Then $\mathfrak{h}$ is the Lie algebra of H and $\mathfrak{s}$ is the tangent space for S at the coset $\bar{1}$. We have $\mathfrak{g} = \mathfrak{h} + \mathfrak{s}$. Given a semisimple element s in S, we denote by $\mathbf{M}_G$ the identity component of the centralizer of s in $\mathbf{G}$ and we let

$$M_G = \mathbf{M}_G(F), \quad M_H = M_G \cap H.$$

Then M_G is invariant under θ and $M_H = \{m \in M_G | \theta(m) = m\}$. Let $\mathfrak{M}_\mathfrak{g}$ be the centralizer of s in $\mathfrak{g}$ and let

$$\mathfrak{M}_\mathfrak{s} = \mathfrak{s} \cap \mathfrak{M}_\mathfrak{g}, \quad \mathfrak{M}_\mathfrak{h} = \mathfrak{h} \cap \mathfrak{M}_\mathfrak{g}.$$

Then $\mathfrak{M}_\mathfrak{g}$ is the Lie algebra of M_G, $\mathfrak{M}_\mathfrak{h}$ is the Lie algebra of M_H and $\mathfrak{M}_\mathfrak{s}$ is the tangent space at $\bar{1}$ of the induced symmetric space

$$M_G/M_H \backsimeq M_S = \{m\theta(m^{-1}) | m \in M_G\}$$

Let x be a fixed element in G such that $\rho(x) = s$. We consider the map

$$H \times M_G \times H \to G, \quad (h_1, g, h_2) \mapsto h_1 x g h_2.$$

Let U_x be the set of $g \in M_G$ such that the above map is submersive at $(1, g, 1)$ (hence at (h_1, g, h_2) for any $h_1, h_2 \in H$) and let Ω_x be the subset HxU_xH of G. Then U_x is an open and bi-M_H-invariant neighborhood of 1 in M_G, and Ω_x is an open and bi-H-invariant neighborhood of x in G (§3, [RR]). By the standard theory of Harish-Chandra ([HC2]), there exists a surjective map $\alpha \mapsto f_\alpha$ from $C_c^\infty(H \times U_x \times H)$ to $C_c^\infty(\Omega_x)$ such that

$$\int_{\Omega_x} f_\alpha(t)F(t)dt = \int_{H\times U_x \times H} \alpha(h_1, g, h_2)F(h_1 x g h_2) dh_1 dg dh_2$$

for all $F \in C_c^\infty(\Omega_x)$. Then there is a unique bi-M_H-invariant distribution Θ_M on U_x (see, for example, p 103 of [JR]) such that

$$\Theta_\pi(f_\alpha) = \Theta_M(\beta_\alpha)$$

for all $\alpha \in C^\infty(H \times U_x \times H)$ where

$$\beta_\alpha(g) = \int_H \int_H \alpha(h_1, g, h_2) dh_1 dh_2, \quad g \in U_x.$$

To state the germ expansion theorem, we need to fix some further notation. We first note that the map ρ induces a surjective map $f \mapsto f^\rho$ from $C_c^\infty(M_G)$ to $C_c^\infty(M_S)$ by the formula

$$f^\rho(m\theta(m^{-1})) = \int_{M_H} f(mh)dh, \quad f \in C_c^\infty(M_G).$$

We denote by Θ_M^ρ the M_H-invariant distribution on M_S defined by $\Theta_M^\rho(f^\rho) = \Theta_M(f)$ for $f \in C_c^\infty(U_x)$. Next we recall that the exponential map **exp** induces a diffeomorphism from a sufficiently small neighborhood of 0 in $\mathfrak{M}_\mathfrak{s}$ onto a neighborhood of $\bar{1}$ in M_S (Lemma 7.1, [RR]). Let $\mathfrak{N}_\mathfrak{s}$ be the set of nilpotent elements in $\mathfrak{s}$ and let $\mathfrak{N}_{\mathfrak{M}_\mathfrak{s}} = \mathfrak{N}_\mathfrak{s} \cap \mathfrak{M}_\mathfrak{s}$. Then the Fourier transform of a function $f \in C_c^\infty(\mathfrak{M}_\mathfrak{s})$ is a function $\hat{f}$ in $C_c^\infty(\mathfrak{M}_\mathfrak{s})$ defined by

$$\hat{f}(Y) = \int_{\mathfrak{M}_\mathfrak{s}} f(X)\psi(B(Y, X))dX, \quad X, Y \in \mathfrak{M}_\mathfrak{s}.$$

The Fourier transform of a distribution D on $\mathfrak{M}_\mathfrak{s}$ is defined by

$$\hat{D}(f) = D(\hat{f}), \quad f \in C_c^\infty(\mathfrak{M}_\mathfrak{s}).$$

We have:

Theorem 2.1. *Let Λ_1, ..., Λ_l be a basis for the space of the M_H-invariant distributions on $\mathfrak{M}_\mathfrak{s}$ supported in $\mathfrak{N}_{\mathfrak{M}_\mathfrak{s}}$. Then for a class one representation π of G, there exist unique complex numbers c_1,..., c_l such that*

$$\Theta_\pi(f_\alpha) = \Theta_M(\beta_\alpha) = \Theta_M^\rho(\beta_\alpha^\rho) = \sum_{i=1}^{l} c_i \hat{\Lambda}_i(\beta_\alpha^\rho \circ exp)$$

for all $\alpha \in C_c^\infty(H \times U_x \times H)$ with β_α supported in a sufficiently small bi-M_H-invariant neighborhood of 1 in U_x.

This result was proved by Rader-Rallis ([RR]) for $x = 1$ for a general symmetric space, and by Hakim ([Ha1]) for any semisimple elements for the symmetric space

$\mathbf{G}(E)/\mathbf{G}(F)$ where E is a quadratic extension field of F. For completeness, we will formulate a proof of this theorem in this general setting based on their work in the next section. Then it is easily seen that Θ_π is locally integrable for any class one representation π of G if the distributions $\hat{\Lambda}_i$ are locally integrable for each pair $(M_H, \mathfrak{M}_\mathfrak{s})$ (§2, [Ha1]). But unfortunately this is not the case generally (see [RR] for examples).

The classical case where these distributions $\hat{\Lambda}_i$ are known to be locally integrable is for the pair $(G, \mathfrak{g})$ with a reductive group G acting on its Lie algebra $\mathfrak{g}$ by the adjoint map. This result is stated in the following theorem of Harish-Chandra ([HC1]).

Theorem 2.2. *Consider the pair $(G, \mathfrak{g})$ with the adjoint action of a reductive group G on its Lie algebra $\mathfrak{g}$. Then each nilpotent orbit in $\mathfrak{g}$ carries an invariant measure with locally integrable Fourier transform. Furthermore these measures form a basis for the space of G invariant distributions on $\mathfrak{g}$ supported in the nilpotent subset of $\mathfrak{g}$.*

To put this theorem in the setting of this paper, we consider the symmetric space $G \times G/G$ where the involution θ on $G \times G$ is defined by $\theta(g_1, g_2) = (g_2, g_1)$ and G is embedded in $G \times G$ diagonally. Then it is easily seen that $G \times G/G$ is isomorphic to G as G-spaces. So $\mathfrak{g}$ is the tangent space for $G \times G/G$ at $(1,1)^-$. In this case any class one representation of $G \times G/G$ is of the form $\pi \times \tilde{\pi}$ where π is a representation of G. Then the spherical character for $\pi \times \tilde{\pi}$ coincides with the usual character for π which is locally integrable by theorem 2.1 and theorem 2.2 as already proved by Harish-Chandra ([HC1]). On the other hand it is not hard to see that the tangent space for each of the induced symmetric spaces for the symmetric space $\mathbf{G}(E)/\mathbf{G}(F)$ is also of the type as in theorem 2.2. So it follows again from this theorem that the spherical characters for $\mathbf{G}(E)/\mathbf{G}(F)$ are locally integrable as proved by Hakim ([Ha1]).

The case of interest in this paper has the following setting. Let E be a quadratic extension field of F. Let $G = GL(2n, F)$ and let $H \subset G$ be a subgroup isomorphic to $GL(n, E)$ over F (see section 4 for a precise description of the situation). Then we can show that the pair $(H, \mathfrak{s})$ is isomorphic to the pair

$$(GL(n, E), M(n, E))$$

with $GL(n, E)$ acts on $M(n, E)$ by twisted conjugation

$$(g, \alpha) \mapsto g\alpha\bar{g}^{-1}, \quad g \in GL(n, E), \quad \alpha \in M(n, E).$$

So theorem 2.2 does not apply in this case. In fact one of the main tasks of this paper is to establish a result similar to theorem 2.2 for this new pair. More precisely we will prove

Theorem 2.3. *Consider the symmetric space G/H with the groups G, H as above. Then each nilpotent H-orbit of $\mathfrak{s}$ carries an H-invariant measure with locally integrable Fourier transform.*

It follows then easily (see, for example, proposition 1.2 in [RR]) that these measures corresponding to the nilpotent orbits form a basis for the space of H-invariant distributions on $\mathfrak{s}$ supported in the nilpotent set. In section 4 we will prove that for each semisimple element $s \in S$ the pair $(M_H, \mathfrak{M}_{\mathfrak{s}})$ is isomorphic to a product of pairs of the two types dealt with in theorem 2.2 and 2.3. It follows then from theorem 2.1 that in this case the spherical characters are locally integrable. This is the assertion of theorem 1.1 in the introduction.

3. The germ expansion

We keep to the notation described above theorem 2.1 in the last section. As we noted before when $s = 1$ the assertion of theorem 2.1 is proved by Rader and Rallis in [RR]. Their proof can be extended to the case of a general θ-semisimple element s when combined with certain results in [Ha1]. For completeness, we summarize here the main steps of a proof of theorem 2.1 and refer the reader to the papers cited above for details.

We first recall Howe's Kirillov-orbit method as adapted by Rader-Rallis and Hakim in the case of symmetric spaces. Let L be a θ-stable lattice in $\mathfrak{M}_{\mathfrak{g}}$ with $L = L_+ + L_-$ where $L_+ = L \cap \mathfrak{M}_{\mathfrak{h}}$ and $L_- = L \cap \mathfrak{M}_{\mathfrak{s}}$. Suppose L is *ee* (see p.158 of [RR] for the definition). Then exp maps L homeomorphically to a compact open subgroup K of M_G. We denote by $\varepsilon(K)$ the set of equivalence classes of irreducible unitary representations of K. Let $K_+ = \exp L_+$. Let $\varepsilon(K/K_+)$ be the subset of $\varepsilon(K)$ consisting of representations which possess nonzero K_+ invariant vectors. For $\mu \in \varepsilon(K/K_+)$, we denote by $d(\mu)$ the degree of μ. The spherical function ϕ_μ of μ is a function ϕ_μ on K defined by $\phi_\mu(k) =< \mu(k)e_1, e_1 >$ for $k \in K$ where e_1 is the K_+-invariant unit vector in the space of μ. This unit vector is unique since (K, K_+) is a Gelfand pair ([Ha2]). Let $L_-^\perp$ be the annihilator of L_- in $\mathfrak{M}_{\mathfrak{s}}$ with respect to $\psi \circ B$. It is invariant under K_+. So K_+ acts on $\mathfrak{M}_{\mathfrak{s}}/L_-^\perp$ by conjugation (Here we remind the reader who is familiar with Howe's work that we assume the residual characteristic of F is odd, so we have $\frac{1}{2}L = L$ and $\exp\frac{1}{2}L = K^{1/2} = K$).

Lemma 3.1. *There is a bijection $\mu \mapsto \mathcal{O}_\mu$ from $\varepsilon(K/K_+)$ to the set of K_+-orbits in $\mathfrak{M}_{\mathfrak{s}}/L_-^\perp$ such that*

$$d(\mu)\phi_\mu(exp\ X) = \sum_{Z \in \mathcal{O}_\mu/L_-^\perp} \psi(B(Z, X)), \quad X \in L_-.$$

Here we regard $\mathcal{O}_\mu$ as a $L_-^\perp$- invariant subset of $\mathfrak{M}_{\mathfrak{s}}$. This lemma is the symmetric space version of Howe's parameterization of representations of small compact subgroups ([Ho2]). See proposition 7.8 of [RR] and theorem 3 of [Ha2] for the proof of this lemma. Here we note that if μ corresponds to the K-orbit $\mathcal{O}'_\mu$ in $\mathfrak{M}_{\mathfrak{g}}/L^\perp$ by Howe's parameterization of $\varepsilon(K)$ as in [Ho2], then $\mathcal{O}_\mu = \mathcal{O}'_\mu \cap \mathfrak{M}_{\mathfrak{s}}$.

We now fix a norm $|| \quad ||$ on $\mathfrak{M}_{\mathfrak{g}}$ as in [RR]. Let

$$S_1 = \{X \in \mathfrak{M}_{\mathfrak{g}} \quad | \quad ||X|| = 1\}$$

and for $\epsilon > 0$, let

$$V(\epsilon) = \{X \in S_1 \quad | \quad ||X - n|| < \epsilon \text{ for some } n \in \mathfrak{N}_{\mathfrak{M}_{\mathfrak{g}}} \cap S_1\}.$$

So $V(\epsilon)$ is a neighborhood of $\mathfrak{N}_{\mathfrak{M}_{\mathfrak{g}}} \cap S_1$ in S_1. Then we have

Lemma 3.2. *We choose $\epsilon > 0$ sufficiently small and $R > 0$ sufficiently large. Let μ be a representation in $\varepsilon(K/K_+)$ such that $\Theta^{\rho}_M(\bar{\phi}_\mu) \neq 0$ and $||Z|| > R$ for some $Z \in \mathcal{O}_\mu$. Then we have $\mathcal{O}_\mu \subset FV(\epsilon)$.*

Here $\bar{\phi}_\mu$ is the complex conjugate of ϕ_μ. This is theorem 4 in section 7 in [Ha1]. We remark that although the main purpose of [Ha1] is to prove the local integrability of the spherical characters for a certain symmetric space, but the results in §2 –§7 in [Ha1] are worked out for a general symmetric space and are generalizations of the corresponding results of Harish-Chandra in the group case ([HC1], [C]).

Next we recall a formula for the spherical functions as given by Rader-Rallis.

Lemma 3.3. *Assume $\mu \in \varepsilon(K/K_+)$. Then we have*

$$d(\mu)\phi_\mu(exp\ X) = \hat{\kappa}_{\mathcal{O}_\mu}(X)/vol(L_-^{\perp}), \quad X \in L_-,$$

where $\kappa_{\mathcal{O}_\mu}$ is the characteristic function of $\mathcal{O}_\mu$ in $\mathfrak{M}_{\mathfrak{s}}$ and $vol(L_-^{\perp})$ is the volumn of $L_-^{\perp}$.

This is proposition 7.10 (2) of [RR].

We now choose an M_H-invariant open and closed subset $\mathfrak{M}_0$ of $\mathfrak{M}_{\mathfrak{s}}$ as in page 168 in [RR] such that exp defines an analytic isomorphism from $\mathfrak{M}_0$ to an M_H-invariant open and closed subset U_0 of $\rho(U_x) \subset M_S$. We restrict Θ^{ρ}_M to U_0, then lift it to $\mathfrak{M}_0$ by the exponential map, and extend it to $\mathfrak{M}_{\mathfrak{s}}$ by 0. In this way we obtain a distribution Θ_0 on $\mathfrak{M}_{\mathfrak{s}}$. For $Z \in \mathfrak{M}_{\mathfrak{s}}$, let f_Z be the characteristic function of $Z + L_-^{\perp}$ in $\mathfrak{M}_{\mathfrak{s}}$.

Lemma 3.4. *If ϵ is small enough and R is large enough, then for $Z \in \mathfrak{M}_{\mathfrak{s}}$ such that $||Z|| > R$ and $\hat{\Theta}_0(f_Z) \neq 0$, we have $||Z - n|| \leq \epsilon||n||$ for some $n \in \mathfrak{N}_{\mathfrak{M}_{\mathfrak{s}}}$.*

Proof. Let $\mathcal{O}_\mu$ be the K_+-orbit of $-Z$ in $\mathfrak{M}_{\mathfrak{s}}/L_-^{\perp}$ which is associated to a class one representation $\mu \in \varepsilon(K/K_+)$ by lemma 3.1. We have $\kappa_{\mathcal{O}_\mu} = \sum_{Z' \in \mathcal{O}_\mu/L_-^{\perp}} f_{Z'}$. Then by Lemma 3.3 we obtain

$$d(\mu)\bar{\phi}_\mu(\exp\ X) = \sum_{Z' \in \mathcal{O}_\mu/L_-^{\perp}} \hat{f}_{-Z'}(X)/\mathrm{vol}(L_-^{\perp}), \quad X \in L_-.$$

Thus we have

$$\Theta^{\rho}_{M}(\bar{\phi}_{\mu}) = \mathrm{card}(\mathcal{O}_{\mu}/L^{\perp}_{-})\hat{\Theta}_0(f_Z)/\mathrm{vol}(L^{\perp}_{-}).$$

Here we use the fact that $\hat{\Theta}_0$ is M_H-invariant. It follows that if $\hat{\Theta}_0(f_Z) \neq 0$, then $\Theta^{\rho}_{M}(\bar{\phi}_{\mu}) \neq 0$. Since $||-Z|| = ||Z|| > R$, we obtain $-Z \in \mathcal{O}_{\mu} \subset FV(\epsilon)$ by Lemma 3.2. Thus we have $Z \in FV(\epsilon)$. In other words there exists $a \in F^*$ and $n_1 \in \mathfrak{N}_{\mathfrak{M}_{\mathfrak{g}}}$ such that $||a^{-1}Z - n_1|| \leq \epsilon$. Let $n = an_1 \in \mathfrak{N}_{\mathfrak{M}_{\mathfrak{g}}}$. Then $||Z - n|| \leq \epsilon ||n||$. Following the same argument as in the second paragraph of the proof of theorem 7.11 of [RR], we can even choose $n \in \mathfrak{N}_{\mathfrak{M}_{\mathfrak{s}}}$. This proves the lemma.

It follows then from a fundamental result of Howe (see theorem 6.8 of [RR]) that

$$\hat{\Theta}_0 * \kappa_{L^{\perp}_{-}}/volL^{\perp}_{-} \in \mathcal{J}(\omega) * \kappa_{L^{\perp}_{-}}/volL^{\perp}_{-}.$$

Here ω is a compact set in $\mathfrak{M}_{\mathfrak{s}}$ and $\mathcal{J}(\omega)$ is the set of M_H invariant distributions on $\mathfrak{M}_{\mathfrak{s}}$ supported in $\mathrm{Ad}M_H(\omega)$. By another result of Howe (see theorem 6.9 of [RR]), if L is sufficiently large, then the space $\mathcal{J}(\omega) * \kappa_{L^{\perp}_{-}}/volL^{\perp}_{-}$ is spanned by the projections $\Lambda_i * \kappa_{L^{\perp}_{-}}/volL^{\perp}_{-}$ for $i = 1, ..., l$. Thus theorem 2.1 follows by taking the inverse Fourier transform. For details see the last paragraph in the proof of theorem 7.11 in [RR].

4. Induced infinitesimal symmetric spaces

Let F be a p-adic local field with odd residual characteristic, and let $E(\sqrt{\tau})$ be a quadratic extension field of F. We denote by $\sigma : z \mapsto \bar{z}$ the Galois conjugation in E. The norm map N from E to F is defined by $N(z) = z\bar{z}$ for $z \in E$. For any positive integer m, we use G_m to denote the linear group $GL(m)$ of $m \times m$ invertible matrices. Suppose G is the subgroup of $G_{2n}(E)$ consisting of the elements of the form

$$\begin{pmatrix} \alpha & \beta \\ \bar{\beta} & \bar{\alpha} \end{pmatrix} \in G_{2n}(E).$$

Regarded as an algebraic group over F, it is isomorphic to $GL_{2n}(F)$. We consider an involution θ of G defined by $\theta(g) = \epsilon g \epsilon^{-1}$ where

$$\epsilon = \begin{pmatrix} \sqrt{\tau} & 0 \\ 0 & -\sqrt{\tau} \end{pmatrix}.$$

Then the subgroup $H = \{g \in G | \theta(g) = g\}$ of G consists of the elements of the form

$$h_g = \begin{pmatrix} g & 0 \\ 0 & \bar{g} \end{pmatrix}, \quad g \in G_n(E).$$

It is clear that H is isomorphic to $G_n(E)$. Now we consider the variety

$$S=\{g\theta(g)^{-1}|g\in G\}\subset G.$$

Then for $g\in G$ we have that $g\in S$ if and only if $(g\epsilon)(g\epsilon)=\tau I_{2n}$ (for a proof of this fact see page 281 in [G3]). This condition becomes

$$\alpha^2=I_n+\beta\bar{\beta},\quad \alpha\beta=\beta\bar{\alpha} \tag{4.1}$$

if we write

$$g=\begin{pmatrix}\alpha & \beta\\ \bar{\beta} & \bar{\alpha}\end{pmatrix}.$$

The Lie algebra of G is

$$\mathfrak{g}=\{\begin{pmatrix}\alpha & \beta\\ \bar{\beta} & \bar{\alpha}\end{pmatrix}|\alpha,\beta\in M_n(E)\}.$$

The Lie subalgebra of H in $\mathfrak{g}$ is

$$\mathfrak{h}=\{\begin{pmatrix}\alpha & 0\\ 0 & \bar{\alpha}\end{pmatrix}|\alpha\in M_n(E)\}$$

and the -1 eigenspace of $\mathrm{Ad}(\epsilon)$ in $\mathfrak{g}$ is the subspace $\mathfrak{s}$ of $\mathfrak{g}$ of the element of the form

$$s_\alpha=\begin{pmatrix}0 & \alpha\\ \bar{\alpha} & 0\end{pmatrix},\quad \alpha\in M(n,E).$$

As in the last section, for a semisimple element $s\in\mathfrak{s}$, let $\mathfrak{M}_\mathfrak{g}$ be the centralizer of s in $\mathfrak{g}$. Let $M_H=M_G\cap H$ and $\mathfrak{M}_\mathfrak{s}=\mathfrak{M}_g\cap\mathfrak{s}$. The task in this section is to determine the pair $(M_H,\mathfrak{M}_\mathfrak{s})$ for each semisimple element s of S. If necessary we sometimes write H and $\mathfrak{s}$ as H_n and $\mathfrak{s}_n$ to indicate they are related to the group G_{2n}.

Up to a conjugation by H, we may assume that s is the matrix

$$\begin{pmatrix}\alpha & 0 & \beta & 0\\ 0 & \mu_{n_1,n_2} & 0 & 0\\ \bar{\beta} & 0 & \alpha & 0\\ 0 & 0 & 0 & \mu_{n_1,n_2}\end{pmatrix} \tag{4.2}$$

where

$$\mu_{n_1,n_2}=\begin{pmatrix}I_{n_1} & 0\\ 0 & -I_{n_2}\end{pmatrix}$$

and α is a matrix in $M_{n-n_1-n_2}(F)$ without eigenvalues $\{1,-1\}$ (see [G2]). To continue, we first consider two special cases.

Case I Suppose we have $n_1 = n_2 = 0$. Then

$$s = \begin{pmatrix} \alpha & \beta \\ \bar{\beta} & \alpha \end{pmatrix}$$

where α is in $M_n(F)$ such that $\alpha^2 - I_n = \beta\bar{\beta}$ is nonsingular and $\alpha\beta = \beta\alpha$. Let $\gamma = \beta(I_n+\alpha)^{-1}$. Then it is an element in $G_n(E)$. We have

$$I_n - \gamma\bar{\gamma} = I_n - \beta\bar{\beta}(I_n+\alpha)^{-2}$$

$$= I_n + (I_n-\alpha)(I_n+\alpha)^{-1} = 2(I_n+\alpha)^{-1}$$

which is also nonsingular. Then by a simple computation we get

$$\alpha = (I_n+\gamma\bar{\gamma})(I_n-\gamma\bar{\gamma})^{-1}, \quad \beta = 2\gamma(I_n-\gamma\bar{\gamma})^{-1}. \tag{4.3}$$

It follows immediately from these relations that if $g \in G_n(E)$, then $g\gamma\bar{g}^{-1} = \gamma$ if and only if $g\alpha = \alpha g, g\beta = \beta\bar{g}$, which in turn are equivalent to the condition

$$h_g s = \begin{pmatrix} g & 0 \\ 0 & \bar{g} \end{pmatrix} s = s \begin{pmatrix} g & 0 \\ 0 & \bar{g} \end{pmatrix} = s h_g.$$

Let $M_n(E)^\sigma_\gamma$ be the set of the element ν in $M_n(E)$ where $\nu\gamma = \gamma\bar{\nu}$ and let $G_n(E)^\sigma_\gamma = M_n(E)^\sigma_\gamma \cap G_n(E)$. Then we may regard $G_n(E)^\sigma_\gamma$ as the set of F-points of a reductive algebraic group over F, with $M_n(E)^\sigma_\gamma$ as its Lie algebra ([AC]). It is now clear that the centralizer of s in H is the subgroup

$$M_H = \{h_g \in H | g \in G_n(E)^\sigma_\gamma\}.$$

Next we determine the centralizer $\mathfrak{M}_\mathfrak{s}$ of s in $\mathfrak{s}$. Let

$$x = s_\lambda = \begin{pmatrix} 0 & \lambda \\ \bar{\lambda} & 0 \end{pmatrix}$$

be an element in $\mathfrak{s}$. Then $xs = sx$ if and only if $\lambda\alpha = \alpha\lambda, \lambda\bar{\beta} = \beta\bar{\lambda}$. Let $\lambda\beta^{-1} = \nu$. We will show that the above conditions for λ is equivalent to the condition $\nu\gamma = \gamma\bar{\nu}$.

First of all if $\lambda\alpha = \alpha\lambda$ and $\lambda\bar{\beta} = \beta\bar{\lambda}$, then we have

$$\nu\alpha\beta = \nu\beta\alpha = \lambda\alpha = \alpha\lambda = \alpha\nu\beta.$$

Thus $\nu\alpha = \alpha\nu$. We also have that

$$\nu\beta\bar{\beta} = \lambda\bar{\beta} = \beta\bar{\lambda} = \beta\bar{\nu}\bar{\beta}.$$

So we have $\nu\beta = \beta\bar{\nu}$. This implies that

$$\nu\gamma = \nu\beta(I_n + \alpha)^{-1} = \beta\bar{\nu}(I_n + \alpha)^{-1} = \beta(I_n + \alpha)^{-1}\bar{\nu} = \gamma\bar{\nu}.$$

On the other hand, if $\nu\gamma = \gamma\bar{\nu}$, then

$$\nu\gamma\bar{\gamma} = \gamma\bar{\nu}\bar{\gamma} = \gamma\bar{\gamma}\nu.$$

It follows from (4.3) that

$$\nu\alpha = \alpha\nu, \quad \nu\beta = \beta\bar{\nu}.$$

This implies

$$\lambda\alpha = \nu\beta\alpha = \nu\alpha\beta = \alpha\nu\beta = \alpha\lambda, \quad \lambda\bar{\beta} = \nu\beta\bar{\beta} = \beta\bar{\nu}\bar{\beta} = \beta\bar{\lambda}.$$

So we obtain

$$\mathfrak{M}_{\mathfrak{s}} = \{s_{\nu\beta} = \begin{pmatrix} 0 & \nu\beta \\ \nu\bar{\beta} & 0 \end{pmatrix} \in \mathfrak{s} | \nu \in M_n(E)^{\sigma}_{\gamma}\}.$$

Note that for $h_g \in M$ and $s_{\nu\beta} \in \mathfrak{M}_{\mathfrak{s}}$ where $g \in G_n(E)^{\sigma}_{\gamma}$ and $\nu \in M_n(E)^{\sigma}_{\gamma}$, we have

$$h_g s_{\nu\beta} h_g^{-1} = \begin{pmatrix} g & 0 \\ 0 & \bar{g} \end{pmatrix} \begin{pmatrix} 0 & \nu\beta \\ \bar{\nu}\bar{\beta} & 0 \end{pmatrix} \begin{pmatrix} g^{-1} & 0 \\ 0 & \bar{g}^{-1} \end{pmatrix} = s_{g\nu\beta\bar{g}^{-1}} = s_{g\nu g^{-1}\beta}.$$

So if we identify $\mathfrak{M}_{\mathfrak{s}}$ with $M_n(E)^{\sigma}_{\gamma}$ by sending $s_{\nu\beta}$ to ν and identify M_H with $G_n(E)^{\sigma}_{\gamma}$ in the obvious way, then the pair $(M_H, \mathfrak{M}_{\mathfrak{s}})$ is the pair $(G_n(E)^{\sigma}_{\gamma}, M_n(E)^{\sigma}_{\gamma})$ with $G_n(E)^{\sigma}_{\gamma}$ acting on its Lie algebra $M_n(E)^{\sigma}_{\gamma}$ by the adjoint representation.

Case II Suppose n_1 or n_2 is n. Then s is either I_{2n} or $-I_{2n}$. In both cases we have $M_H = H$ and $\mathfrak{M}_{\mathfrak{s}} = s$.

We now consider a general semisimple element s of S in the form of (4.2). Then for an element $h_g \in H$, we have that $h_g s = s h_g$ if and only if

$$g\text{diag}(\alpha, I_{n_1}, -I_{n_2})g^{-1} = \text{diag}(\alpha, I_{n_1}, -I_{n_2})$$

and

$$g\text{diag}(\beta, 0, 0)\bar{g}^{-1} = \text{diag}(\beta, 0, 0).$$

Set $n_0 = n - n_1 - n_2$. Since α has no eigenvalues 1 and -1, the matrix g must be of the form

$$\text{diag}(g_0, g_1, g_2), \quad g_0 \in G_{n_0}(E), \quad g_1 \in G_{n_1}(E), \quad g_2 \in G_{n_2}(E)$$

such that $g_0\alpha g_0^{-1} = \alpha$ and $g_0\beta\bar{g}_0^{-1} = \beta$. Thus if we set $\gamma = \beta(I_{n_0} + \alpha)^{-1}$ as before, these conditions for g_0 mean $g_0 \in G_{n_0}(E)^\sigma_\gamma$.

So the centralizer of s in H is

$$M_H = G_{n_0}(E)^\sigma_\gamma \times H_{n_1} \times H_{n_2}. \tag{4.4}$$

Similarly we can show that the centralizer $\mathfrak{M}_\mathfrak{s}$ of s in $\mathfrak{s}$ consists of the elements of the form s_λ where

$$\lambda = \mathrm{diag}(\lambda_0\beta, \lambda_1, \lambda_2), \quad \lambda_0 \in M_{n_0}(E)^\sigma_\gamma, \quad \lambda_1 \in M_{n_1}(E), \quad \lambda_2 \in M_{n_2}(E).$$

So we obtain

$$\mathfrak{M}_\mathfrak{s} = M_{n_0}(E)^\sigma_\gamma \times \mathfrak{s}_{n_1} \times \mathfrak{s}_{n_2}. \tag{4.5}$$

Thus we have proved the following proposition.

Proposition 4. *Suppose s is a semisimple element in $\mathfrak{s}$. Let M_H and $\mathfrak{M}_\mathfrak{s}$ be the centralizers of s in H and $\mathfrak{s}$ respectively. Then there is a partition $n = n_0 + n_1 + n_2$ of n such that $(M_H, \mathfrak{M}_\mathfrak{s})$ is isomorphic to*

$$(G_{n_0}(E)^\sigma_\gamma, M_{n_0}(E)^\sigma_\gamma) \times (H_{n_1}, \mathfrak{s}_{n_1}) \times (H_{n_2}, \mathfrak{s}_{n_2})$$

where γ is an element in $G_{n_0}(E)$ related to s.

5. The nilpotent orbits (I)

We continue with the notation of the last section. By proposition 4 and theorem 2.2, we are reduced to study the pair $(H, \mathfrak{s})$. From now on in this paper, we will concentrate on the proof of theorem 2.3. In this section we will show that each nilpotent orbit of $\mathfrak{s}$ carries an H-invariant measure by deriving an explicit formula for this measure . We will also give an explicit formula for its Fourier transform.

Consider the G-invariant F-bilinear form B on $\mathfrak{g}$ defined by $B(X, Y) = \mathrm{tr}(XY)$ for $X, Y \in \mathfrak{g}$. Then the restriction of B on $\mathfrak{s}$ is

$$B(s_\alpha, s_\beta) = \mathrm{tr}(s_\alpha s_\beta) = \mathrm{tr}(\alpha\bar{\beta} + \bar{\alpha}\beta) \quad \alpha, \beta \in M_n(E)$$

which is nondegenerate. We fix a nontrivial additive character ψ of F of conductor O_F. The Fourier transform of a function $f \in C_c^\infty(\mathfrak{s})$ is a function $\hat{f} \in C_c^\infty(\mathfrak{s})$ defined by

$$\hat{f}(s_\alpha) = \int_\mathfrak{s} f(s)\psi(B(s, s_\alpha))ds, \quad s_\alpha \in \mathfrak{s} \tag{5.1}$$

and the Fourier transform $\hat{D}$ of a distribution D on $\mathfrak{s}$ is defined by $\hat{D}(f) = D(\hat{f})$.

We now consider the nilpotent orbits in $\mathfrak{s}$. In [G3] we proved that each nilpotent orbit of $\mathfrak{s}$ contains an element s_{α_0} where α_0 is a nilpotent matrix in $M_n(F)$. Let V be the n-dimensional vector space E^n of column vectors over E. The matrix α_0 acts on V as well as on the F-vector space F^n which is a subset of V. Let $V_i = \mathrm{ker}\alpha_0^i$. Then we have a flag

$$\{0\} = V_0 \subseteq V_1 \subseteq ... \subseteq V_l = E^n$$

of E^n where l is the smallest positive integer such that $\alpha_0^l = 0$. Let

$$\mathfrak{P} = \{p \in M_n(E) | p(V_i) \subset V_i, i = 0, 1, ..., l\}$$

and

$$\mathfrak{N} = \{x \in M_n(E) | x(V_i) = V_{i-1}, i = 1, ..., l\}.$$

Then $\mathfrak{P}$ is a parabolic subalgebra of $M_n(E)$ and $\mathfrak{N}$ is the nilpotent radical of $\mathfrak{P}$. Since α_0 is an element in $M_n(F)$, we have that $\bar{V}_i = V_i$ for any i. This implies that $\bar{\alpha} \in \mathfrak{P}$ (resp. $\mathfrak{N}$) if and only if $\alpha \in \mathfrak{P}$ (resp. $\mathfrak{N}$). Let $P = \mathfrak{P} \cap G_n(E)$ be the corresponding parabolic subgroup in $G_n(E)$ and let N be the unipotent radical of P. We set

$$P_H = \{h_g \in H | g \in P\}, \quad N_H = \{h_g \in H | g \in N\}.$$

Then we let

$$\mathfrak{P}_{\mathfrak{s}} = \{s_\alpha \in \mathfrak{s} | \alpha \in \mathfrak{P}\}, \quad \mathfrak{N}_{\mathfrak{s}} = \{s_\alpha \in \mathfrak{s} | \alpha \in \mathfrak{N}\}.$$

Finally let $K_E = G_n(O_E)$ and let $K_H = \{h_k | k \in K_E\}$.

Suppose Λ is the distribution on $\mathfrak{s}$ determined by the orbital integral on the H-orbit of s_{α_0}. Then for $f \in c_c^\infty(\mathfrak{s})$, we have

$$\Lambda(f) = \int_{H/H_{s_{\alpha_0}}} f(h s_{\alpha_0} h^{-1}) dh$$

where $H_{s_{\alpha_0}}$ is the centralizer of s_{α_0} in H (for some normalization of the quotient measure dh).

Proposition 5.1. *For $f \in C_c^\infty(\mathfrak{s})$, the above integral for $\Lambda(f)$ is convergent. In fact we have*

$$\Lambda(f) = \int_{\mathfrak{N}_{\mathfrak{s}}} (\int_{K_H} f(kuk^{-1}) dk) du \tag{5.2}$$

and

$$\hat{\Lambda}(f) = \int_{\mathfrak{P}_{\mathfrak{s}}} (\int_{K_H} f(kpk^{-1}) dk) dp \tag{5.3}$$

with suitably normalized Haar measures on $\mathfrak{N}$ *and* $\mathfrak{P}$.

For convenience in the proof, we identify H with $G_n(E)$ and $\mathfrak{s}$ with $M_n(E)$. Then the action of H on $\mathfrak{s}$ is equivalent to the action of $G_n(E)$ on $M_n(E)$ by twisted conjugation. If we denote this action by Tc, then we have $\mathrm{Tc}g(\alpha) = g\alpha\bar{g}^{-1}$ for $g \in G_n(E)$ and $\alpha \in M_n(E)$. From now on, unless otherwise specified, the action of $G_n(E)$ on $M_n(E)$ always means the twisted conjugation. We may regard B as a F-bilinear form on $M_n(E)$. Then for $\alpha, \beta \in M_n(E)$, we have $B(\alpha, \beta) = \mathrm{tr}(\alpha\bar{\beta} + \bar{\alpha}\beta)$. We use the same notation Λ to denote the $G_n(E)$ invariant distribution on $M_n(E)$ defined by the orbital integral on the twisted conjugacy class of α_0 under $G_n(E)$. Then in this setting the formulas (5.2) and (5.3) become

$$\Lambda(f) = \int_{\mathfrak{N}} (\int_K f(ku\bar{k}^{-1})dk)du \tag{5.4}$$

and

$$\hat{\Lambda}(f) = \int_{\mathfrak{P}} (\int_K f(ku\bar{k}^{-1})dk)dp \tag{5.5}$$

for $f \in C_c^\infty(M_n(E))$. The rest of this section is devoted to the proof of these two formulas. We will follow closely the arguments of Howe in [Ho1].

We start the proof with two lemmas. Given $p_1, p_2 \in P$, and $\alpha \in \mathfrak{N}$, we have

$$p_1\alpha p_2 V_i \subseteq p_1\alpha V_i \subseteq p_1 V_{i-1} \subseteq V_{i-1}$$

for any i. So $p_1\mathfrak{N}p_2 \subseteq \mathfrak{N}$. In particular we have $p_1\alpha_0 p_2 \in \mathfrak{N}$. It is proved in lemma 2 of [Ho1] that the intersection of $\mathfrak{N}$ with the set

$$\mathrm{Ad}G_n(E)(\alpha_0) = \{g\alpha_0 g^{-1} | g \in G_n(E)\}$$

is precisely $\mathrm{Ad}P(\alpha_0)$ which is open and dense in $\mathfrak{N}$. We first prove a twisted version of this result.

Lemma 5.2. *If* $g\alpha_0\bar{g}^{-1} \in \mathfrak{N}$ *for* $g \in G_n(E)$, *then* $g \in P$. *Furthermore the set* $TcP(\alpha_0)$ *is open and dense in* $\mathfrak{N}$.

Proof. Let $g \in G_n(E)$ such that $\alpha = g\alpha_0\bar{g}^{-1} \in \mathfrak{N}$. We want to prove that $g(V_i) \subset V_i$ for $i = 0, 1, ..., l$. We distinguish two cases.

If $i = 2j$, we have $(\alpha\bar{\alpha})^j = g(\alpha_0)^i g^{-1}$. So $\ker(\alpha\bar{\alpha})^j = g(\ker\alpha_0^i) = g(V_i)$. On the other hand since $\alpha \in \mathfrak{N}$, so is $\bar{\alpha} \in \mathcal{N}$. Thus $\alpha\bar{\alpha}V_m \subset \alpha V_{m-1} \subset V_{m-2}$ for any m. We have

$$(\alpha\bar{\alpha})^j V_i \subset (\alpha\bar{\alpha})^{j-1} V_{i-2} \subset ... \subset V_0 = \{0\}.$$

Thus $V_i \subset \ker(\alpha\bar{\alpha})^j = g(V_i)$. But $\dim V_i = \dim\,(g(V_i))$. So $g(V_i) = V_i$ for $i = 2j$.

If $i = 2j+1$, we can prove similarly that

$$\bar{g}(V_i) = \ker[(\alpha\bar{\alpha})^j\alpha] = V_i.$$

Since $\bar{V_i} = V_i$, So we have

$$\overline{g(V_i)} = \bar{g}(\bar{V_i}) = \bar{g}(V_i) = V_i = \bar{V_i}.$$

Thus $gV_i = (V_i)$. Therefore $g \in P$. This proves the first assertion of the lemma.

To continue we may assume that

$$\alpha_0 = \begin{pmatrix} 0 & I_{n_1n_2} & 0 & \dots & 0 \\ & 0 & I_{n_2n_3} & \dots & 0 \\ & & \ddots & \ddots & \vdots \\ & & & \ddots & I_{n_{l-1}n_l} \\ & & & & 0 \end{pmatrix}, \quad n_1 \le n_2 \le \dots \le n_l \tag{5.6}$$

where we use $I_{n_in_{i+1}}$ to denote the $n_i \times n_{i+1}$ matrix $\begin{pmatrix} I_{n_i} & 0 \end{pmatrix}$.

Then $\mathfrak{N}$ consists of the matrices of the form

$$\alpha = \begin{pmatrix} 0 & A_1 & A_{13} & \dots & A_{1l} \\ & 0 & A_2 & \dots & A_{2l} \\ & & \ddots & \ddots & \vdots \\ & & & \ddots & A_{l-1} \\ & & & & 0 \end{pmatrix}. \tag{5.7}$$

Let M be the subgroup of $G_n(E)$ of the matrices of the form $\mathrm{diag}(B_1, B_2, \dots, B_l)$ with $B_i \in G_{n_i}(E)$. Then it is a Levi subgroup of P. We claim that $\mathrm{Tc}P(\alpha_0)$ is precisely the subset $\mathfrak{N}_1 = \{\alpha \in \mathfrak{N} | rank(A_i) = n_i, i = 1, 2, ..., l-1\}$ of $\mathfrak{N}$.

Clearly $\mathrm{Tc}P(\alpha_0)$ is contained in $\mathfrak{N}_1$. Suppose α is an element in $\mathfrak{N}_1$ in the form of (5.7). We need to show that α is twisted conjugate to α_0. Because the rank of A_1 is n_1, we can find matrices $B_i, i = 1, 2, ..., l$ such that $A_1B_i = A_{1i}$. Then twisted-conjugating α by the matrix

$$\begin{pmatrix} I & & & & \\ & I & \bar{B}_3 & \dots & \bar{B}_l \\ & & I & & \\ & & & \ddots & \\ & & & & I \end{pmatrix} \in P,$$

we make $A_{13}, \ldots, A_{1l}$ vanish. So, continuing in this way, we have $A_{ij} = 0$ for all i, j in the expression (5.7) of α. Then for such α it is easily seen that we can find matrices $D_i \in G_{n_i}(E), i = 2, ..., l$ inductively such that $A_1 D_2 = I_{n_1 n_2}$ and $\bar{D}_i^{-1} A_{i+1} D_{i+1} = I_{n_i n_{i+1}}$ for $i \geq 2$. Let $D = \mathrm{diag}(I, \bar{D}_2^{-1}, ..., \bar{D}_l^{-1})$ Then we have $D\alpha\bar{D}^{-1} = \alpha_0$. Hence $\mathrm{Tc}P(\alpha_0) = \mathfrak{N}_1$ which is open and dense in $\mathfrak{N}$.

Remark: It is easily seen that the set $\mathrm{Ad}P(\alpha_0)$ is also $\mathfrak{N}_1$ by the similar argument. So we have $\mathrm{Ad}P(\alpha_0) = \mathrm{Tc}P(\alpha_0)$.

Next let $M_{n_i n_j}(E)$ be the set of all the $n_i \times n_j$ matrices over E. We may regard it as a vector space over E. Then for $A \in G_{n_i}(E)$ and $B \in G_{n_j}(E)$, the Jacobian of the linear map from $M_{n_i n_j}$ to itself defined by $X \mapsto AXB^{-1}$ is

$$\frac{|\mathrm{det}A|^{n_j}}{|\mathrm{det}B|^{n_i}} = \frac{|\mathrm{det}A|^{n_j}}{|\mathrm{det}\ \bar{B}|^{n_i}}$$

which is also the Jacobian of the map $X \mapsto AX\bar{B}^{-1}$. Here we use $|\quad|$ to denote the absolute value on E. It follows immediately from this simple fact that $|\det(\mathrm{Tc}m|_{\mathfrak{N}})| = |\det(\mathrm{Ad}m|_{\mathfrak{N}})|$. So for $p = mn \in P$ where $m \in M$ and $n \in N$, we have

$$|\det(\mathrm{Tc}p|_{\mathfrak{N}})| = |\det(\mathrm{Tc}m \circ \mathrm{Tc}n|_{\mathfrak{N}})| = |\det(\mathrm{Tc}m|_{\mathfrak{N}})| = |\det(\mathrm{Ad}m|_{\mathfrak{N}})|.$$

Recall that if we denote by δ the modular function on P, then $\delta(p) = |\det(\mathrm{Ad}m|_{\mathfrak{N}})|$. Thus we have proved:

Lemma 5.3. *For $p \in P$, we have $|det(\mathit{Tc}p|_{\mathfrak{N}})| = \delta(p)$.*

We now prove the formula (5.4). Here the the argument of Howe in the proof of proposition 5 in [Ho1] goes through largely unchanged. Let ρ be the representation of $G_n(E)$ on the vector space $W = C_c^\infty(M_n(E))$ defined by $\rho(g)f(\alpha) = f(g\alpha\bar{g}^{-1})$ for $g \in G_n(E)$, $f \in W$ and $\alpha \in M_n(E)$. Let ρ' be the contragredient representation of G on the linear dual of W. Suppose w' is a linear functional on W defined by

$$w'(f) = \int_{\mathfrak{N}} f(\alpha) d\alpha.$$

Then for any $p \in P$, we have

$$\rho'(p)w'(f) = \int_{\mathfrak{N}} f(p\alpha\bar{p}^{-1}) d\alpha = |\det(\mathrm{Tc}p^{-1}|_{\mathfrak{N}})| w'(f) = \delta(p)^{-1} w'(f).$$

It follows from proposition 5 in [Ho1] that $\rho'(\chi_K)w'$ is $G_n(E)$ invariant, where χ_K is the characteristic function of K. If we denote by D the distribution defined by $\rho'(\chi_K)w'$, then We have

$$D(f) = \int_{\mathfrak{N}} \int_K f(k\alpha\bar{k}^{-1}) dk d\alpha.$$

Clearly D is supported on the closure of $\mathrm{Tc}K(\mathfrak{N})$. Recall that we have $G_n(E) = PK$ and $\mathrm{Tc}P(\alpha_0)$ is dense in $\mathfrak{N}$. Hence D is a $G_n(E)$ invariant distribution which is supported on the closure of $\mathrm{Tc}G_n(E)\alpha_0$. So we must have $\Lambda = D$.

It remains to compute the Fourier transform $\hat{\Lambda}$ of Λ. Let ω be an uniformizer of E. For any positive integer m, let L_m be the lattice of $M_n(E)$ consisting of matrices with entries in $\omega^m O_E$, and let $L_m^{\perp} = \{\alpha \in M_n(E) | \mathrm{tr}(\alpha\bar{\beta} + \bar{\alpha}\beta) \in O_E\}$. Then by an easy computation we find $L_m^{\perp} = L_{-m}$. We normalize the Haar measure on E such that $\mathrm{measure}(O_E) = 1$. Given $f \in C_c^{\infty}(M_n(F))$, there is an m such that $f(\alpha_1+\alpha_2) = f(\alpha_1)$ for $\alpha_1 \in M_n(E)$ and $\alpha_2 \in L_m$. Then we have

$$\hat{f}(\beta) = \int_{M_n(E)} f(\alpha)\psi(\mathrm{tr}(\alpha\bar{\beta} + \bar{\alpha}\beta))d\alpha$$

$$= \int_{M_n(E)/L_m} f(\alpha_1)[\int_{L_m} \psi(\mathrm{tr}(\alpha_1\bar{\beta} + \bar{\alpha}_1\beta))\psi(\mathrm{tr}(\alpha_2\bar{\beta} + \bar{\alpha}_2\beta))d\alpha_2]d\alpha_1.$$

It follows immediately from this expression that $\hat{f}$ is supported in $L_m^{\perp}$. So we have

$$\hat{\Lambda}(f) = \Lambda(\hat{f}) = \int_{\mathfrak{N}}\int_K \hat{f}(k\alpha\bar{k}^{-1})dkd\alpha = \int_{\mathfrak{N}\cap L_m^{\perp}}\int_K \hat{f}(k\alpha\bar{k}^{-1})dkd\alpha$$

$$= \int_{\mathfrak{N}\cap L_m^{\perp}}\int_K\int_{M_n(E)} f(\beta)\psi(\mathrm{tr}(\bar{\beta}k\alpha\bar{k}^{-1} + \beta\bar{k}\bar{\alpha}k^{-1}))d\beta dkd\alpha$$

$$= \int_{\mathfrak{N}\cap L_m^{\perp}}\int_K\int_{M_n(E)} f(\beta)\psi(\mathrm{tr}[\bar{k}(\bar{k}^{-1}\bar{\beta}k)\alpha\bar{k}^{-1} + k(k^{-1}\beta\bar{k})\bar{\alpha}k^{-1}])d\beta dkd\alpha$$

$$= \int_{\mathfrak{N}\cap L_m^{\perp}}\int_K\int_{M_n(E)} f(k\beta\bar{k}^{-1})\psi(\mathrm{tr}(\bar{\beta}\alpha + \beta\bar{\alpha}))d\beta dkd\alpha.$$

Now let $\mathfrak{N}^-$ be the nilpotent radical of the parabolic subalgebra $\mathfrak{P}^-$of $M_n(E)$ which is opposite to $\mathfrak{P}$. Then we have

$$M_n(E) = \mathfrak{P} + \mathfrak{N}^- = \mathfrak{N} + \mathfrak{P}^-.$$

It is easily seen that $\mathrm{tr}(\beta\bar{\alpha} + \bar{\beta}\alpha) = 0$ for $\beta \in \mathfrak{N}, \alpha \in \mathfrak{P}$, and that

$$\mathfrak{N}^- \cap L_m = \{\alpha \in \mathfrak{N}^- | \mathrm{tr}(\beta\bar{\alpha} + \bar{\beta}\alpha) \in O_E \text{ for any } \beta \in \mathfrak{N} \cap L_m^{\perp}\}.$$

By our normalization of the Haar measure, we have

$$\mathrm{measure}(\mathfrak{N}^- \cap L_m)\mathrm{measure}(\mathfrak{N} \cap L_m^{\perp}) = 1.$$

So the above integral becomes

$$\int_{\mathfrak{P}}\int_K\int_{\mathfrak{N}^-} f(k\beta_1\bar{k}^{-1})[\int_{\mathfrak{N}\cap L_m^{\perp}} \psi(\mathrm{tr}(\bar{\beta}_2\alpha+\beta_2\bar{\alpha}))d\alpha]d\beta_2 dk d\beta_1$$

$$=\int_{\mathfrak{P}}\int_K f(k\beta\bar{k}^{-1})dkdp.$$

Thus we obtain the formula (5.5). This ends the proof of proposition 5.1.

So we obtain the first assertion of theorem 2.3. To finish the proof of this theorem, we still need to show that the distribution $\hat{\Lambda}$ on $\mathfrak{s}$ as expressed by formula (5.5) is represented by a locally integrable function. The basic tools for doing this will be certain type of Weyl integration formula as well as a descent formula for the regular orbital integrals on $\mathfrak{s}$. These are the subjects of the next section.

6. Cartan subspaces and orbital integrals

A Cartan subspace of $\mathfrak{s}$ is a maximal abelian subspace consisting of semisimple elements. An element $s\in\mathfrak{s}$ is called θ-regular or simply regular if it is semisimple and its centralizer in $\mathfrak{s}$ is a Cartan subspace. We denote by $\mathfrak{s}'$ the set of θ-regular elements in $\mathfrak{s}$. We first give a description of the θ-regular elements and the Cartan subspaces.

Lemma 6.1. *Let $s=s_\alpha$ be an element in $\mathfrak{s}$ with $\alpha\in M_n(E)$. Then the following statements are equivalent.*

(1) s_α is θ-regular.

(2) $\alpha\bar{\alpha}$ is a regular element in $G_n(E)$.

(3) s_α is a regular element in $\mathfrak{g}$.

Proof. We first note that if two elements s_α and s_β of $\mathfrak{s}$ are H-conjugate, then $\alpha\bar{\alpha}$ and $\beta\bar{\beta}$ are $G_n(E)$ conjugate. According to a result in [G3], we may assume, up to a H conjugation, that

$$\alpha=\begin{pmatrix}\alpha_1 & 0\\ 0 & \alpha_2\end{pmatrix}$$

where $\alpha_1\in G_{m_1}(E)$ with $\alpha_1\bar{\alpha}_1=\bar{\alpha}_1\alpha_1\in G_n(F)$ and α_2 is a nilpotent element in $M_{m_2}(F)$. Here $m_1+m_2=n$. We note that in this case

$$s_\alpha=\begin{pmatrix}0&0&\alpha_1&0\\0&0&0&0\\\bar{\alpha}_1&0&0&0\\0&0&0&0\end{pmatrix}+\begin{pmatrix}0&0&0&0\\0&0&0&\alpha_2\\0&0&0&0\\0&\bar{\alpha}_2&0&0\end{pmatrix}$$

is the Jordan Decomposition for s_α in $\mathfrak{s}$. Now given an element $s_\beta \in \mathfrak{s}$, then it commutes with s_α if and only $\beta\bar{\alpha} = \alpha\bar{\beta}$. This is equivalent to

$$\beta = \begin{pmatrix} \beta_1 & 0 \\ 0 & \beta_2 \end{pmatrix}, \quad \alpha_1\bar{\beta}_1 = \beta_1\bar{\alpha}_1, \quad \beta_2\alpha_2 = \alpha_2\bar{\beta}_2.$$

We first consider the subspace $\mathfrak{t}_{\mathfrak{s}_{m_1}}$ of $\mathfrak{s}_{m_1}$ consists of elements $s_{\beta_1} \in \mathfrak{s}_{m_1}$ with $\alpha_1\bar{\beta}_1 = \beta_1\bar{\alpha}_1$. Let $\beta_1\alpha_1^{-1} = \gamma$. Then it is easily seen that the above condition for β_1 is equivalent to $\gamma\alpha_1 = \alpha_1\bar{\gamma}$. By an easy computation we find that $\mathfrak{t}_{\mathfrak{s}_{m_1}}$ is an abelian subspace if and only if the subspace

$$M_{m_1}(E)^\sigma_{\alpha_1} = \{\gamma \in M_{n_1}(E) | \gamma\alpha_1 = \alpha_1\bar{\gamma}\}$$

of $M_{n_1}(E)$ is abelian. By [AC], this in turn is equivalent to the fact that $\alpha_1\bar{\alpha}_1$ is regular in $G_{m_1}(F)$, and if this is the case then $M_{m_1}(E)^\sigma_{\alpha_1} = M_{m_1}(F)_{\alpha_1\bar{\alpha}_1}$- the centralizer of $\alpha_1\bar{\alpha}_1$ in $M_{m_1}(F)$. Then we have

$$\mathfrak{t}_{\mathfrak{s}_{m_1}} = \{s_{\gamma\alpha_1} | \gamma \in M_{m_1}(F)_{\alpha_1\bar{\alpha}_1}\}$$

which is a Cartan subspace of $\mathfrak{s}_{m_1}$.

As for the subspace $\mathfrak{t}_2 = \{s_{\beta_2} | \beta_2\alpha_2 = \alpha_2\bar{\beta}_2\}$ of $\mathfrak{s}_{m_2}$, it is easily seen that it is not abelian unless $m_2 = 0$. In fact if we assume that α_2 is in the form of (5.6), then both $s_{I_{m_2}}$ and s_{β_2} with $\beta_2 = \mathrm{diag}(\sqrt{\tau}I_{n_1}, -\sqrt{\tau}I_{n_2}, \sqrt{\tau}I_{n_3}, ...)$ are in $\mathfrak{t}_2$. Clearly $s_{I_{m_2}}$ and s_{β_2} are not commutative.

It follows that (1) is equivalent to (2).

Now let α be an element in $M_n(E)$. Then we have

$$(\lambda I_{2n} - s_\alpha)(\lambda I_{2n} + s_\alpha) = \begin{pmatrix} \lambda I_n & -\alpha \\ -\bar{\alpha} & \lambda I_n \end{pmatrix} \begin{pmatrix} \lambda I_n & \alpha \\ \bar{\alpha} & \lambda I_n \end{pmatrix} = \begin{pmatrix} \lambda^2 I_n - \alpha\bar{\alpha} & 0 \\ 0 & \lambda^2 I_n - \bar{\alpha}\alpha \end{pmatrix}.$$

Since we have

$$\epsilon(\lambda I_{2n} + s_\alpha)\epsilon^{-1} = \lambda I_{2n} + \theta(s_\alpha) = \lambda I_{2n} - s_\alpha,$$

then

$$\det(\lambda I_{2n} - s_\alpha) = \det(\lambda^2 I_n - \alpha\bar{\alpha}) = (\lambda^2 - \lambda_1)...(\lambda^2 - \lambda_n)$$

where λ_1,..., λ_n are the eigenvalues of $\alpha\bar{\alpha}$ in an algebraic closed closure of F. Thus s_α has $2n$ different eigenvalues if and only $\lambda_i \neq \lambda_j$, if $i \neq j$ and $\lambda_i \neq 0$ for all i. It follows that (2) is equivalent to (3). This ends the proof of the proposition.

We also proved the following corollary in the proof of the lemma.

Corollary 6.2. *Each Cartan subspace of $\mathfrak{s}$ is conjugate to a Cartan subspace of the form*

$$\mathfrak{t}_\alpha = \{s_{\gamma\alpha} = \begin{pmatrix} 0 & \gamma\alpha \\ \gamma\bar{\alpha} & 0 \end{pmatrix} | \gamma \in M_n(F)_{\alpha\bar{\alpha}}\}$$

where $\alpha\bar{\alpha}$ is a regular element in $G_n(F)$ and $M_n(F)_{\alpha\bar{\alpha}}$ is the centralizer of $\alpha\bar{\alpha}$ in $M_n(F)$.

Define a function $D_{\mathfrak{g}}(x)$ on $\mathfrak{g}$ by

$$\det(t - \mathrm{ad}x) = D_{\mathfrak{g}}(x)t^{2n} + \text{terms with higher degrees}, \quad x \in \mathfrak{g}.$$

Then $D_{\mathfrak{g}}(x)$ is a nonzero polynomial on $\mathfrak{g}$. It is well known that $x \in \mathfrak{g}$ is regular if and only if $D_{\mathfrak{g}}(x) \neq 0$. So for $s \in \mathfrak{s}$, we have that $s \in s'$ if and only if $D_{\mathfrak{g}}(s) \neq 0$.

We now consider the H-invariant orbital integrals on $\mathfrak{s}$. Recall that we denote by $\mathfrak{h}$ the Lie algebra of H in $\mathfrak{g}$. Let $\mathfrak{t}_{\mathfrak{s}}$ be a Cartan subspace of $\mathfrak{s}$ and let $\mathfrak{t}'_{\mathfrak{s}} = \mathfrak{t}_{\mathfrak{s}} \cap \mathfrak{s}'$ be the set of regular elements in $\mathfrak{t}_{\mathfrak{s}}$. Suppose $\mathfrak{t}_{\mathfrak{h}}$ is the centralizer of $\mathfrak{t}_{\mathfrak{s}}$ in $\mathfrak{h}$ and T_H is the centralizer of $\mathfrak{t}_{\mathfrak{s}}$ in H. Then, by corollary 6.2, T_H is a torus in H which is conjugate to a Cartan subgroup in $G_n(F)$, and $\mathfrak{t}_{\mathfrak{h}}$ is the Lie algebra of T_H in $\mathfrak{h}$. The orbital integral of $f \in C_c^\infty(\mathfrak{s})$ at a regular element s in $\mathfrak{t}_{\mathfrak{s}}$ is defined by the formula

$$\Phi(s; f; \mathfrak{s}/\mathfrak{t}_{\mathfrak{s}}) = \int_{H/T_H} f(hsh^{-1})dh.$$

Since the H orbit of a semisimple element in $\mathfrak{s}$ is closed, the above integral is convergent. Recall that $\mathfrak{h}$, $\mathfrak{s}$ are the 1, -1 eigenspaces of θ in $\mathfrak{g}$. It is easily seen that $\mathfrak{t}_{\mathfrak{g}} = \mathfrak{t}_{\mathfrak{h}} + \mathfrak{t}_{\mathfrak{s}}$ is the centralizer of $\mathfrak{t}_{\mathfrak{s}}$ in $\mathfrak{g}$. Since θ-regular elements in $\mathfrak{s}$ are in $\mathfrak{g}'$, we find that $\mathfrak{t}_{\mathfrak{g}}$ is actually a Cartan algebra in $\mathfrak{g}$. Now we have $\mathfrak{t}_{\mathfrak{g}} \cap \mathfrak{s} = \mathfrak{t}_{\mathfrak{s}}$ and $\mathfrak{t}_{\mathfrak{g}} \cap \mathfrak{h} = \mathfrak{t}_{\mathfrak{h}}$. So we may regard $\mathfrak{s}/\mathfrak{t}_{\mathfrak{s}}$ and $\mathfrak{h}/\mathfrak{t}_{\mathfrak{h}}$ as subspaces of $\mathfrak{g}/\mathfrak{t}_{\mathfrak{g}}$. We identify $\mathfrak{g}/\mathfrak{t}_{\mathfrak{g}}$ with the orthogonal complement $\mathfrak{t}_g^+$ of $\mathfrak{t}_{\mathfrak{g}}$ in $\mathfrak{g}$ under the bilinear form B. Note that $\mathrm{ad}(s)$ for $s \in \mathfrak{s}$ maps $\mathfrak{s}$ into $\mathfrak{h}$ and $\mathfrak{h}$ into $\mathfrak{s}$. So for $s \in \mathfrak{t}'_{\mathfrak{s}}$, we have

$$D_{\mathfrak{g}}(s) = \det(-\mathrm{ad}s|_{\mathfrak{g}/\mathfrak{t}_{\mathfrak{g}}}) = \det(-\mathrm{ad}s|_{\mathfrak{h}/\mathfrak{t}_{\mathfrak{h}} + \mathfrak{s}/\mathfrak{t}_{\mathfrak{s}}}) = \det(-\mathrm{ad}s|_{\mathfrak{t}_{\mathfrak{h}}^+ + \mathfrak{t}_{\mathfrak{s}}^+}),$$

where $\mathfrak{t}_{\mathfrak{h}}^+$ is the orthogonal complement of $\mathfrak{t}_{\mathfrak{h}}$ in $\mathfrak{h}$ and $\mathfrak{t}_{\mathfrak{s}}^+$ is the orthogonal complement of $\mathfrak{t}_{\mathfrak{s}}$ in $\mathfrak{s}$. Let T_H^* be the normalizer of $\mathfrak{t}_{\mathfrak{s}}$ in H and let $W_H(\mathfrak{t}_{\mathfrak{s}}) = T_H^*/T_H$ be the Weyl group. Then $W_H(\mathfrak{t}_{\mathfrak{s}})$ is a finite group. We denote by $|W_H(\mathfrak{t}_{\mathfrak{s}}))|$ the number of the elements in $W_H(\mathfrak{t}_{\mathfrak{s}})$. Then we have (p 106, [RR]):

Weyl Integration Formula. *For $f \in C_c^\infty(\mathfrak{s})$, then*

$$\int_{\mathfrak{s}} f(s)ds = \sum_{\{\mathfrak{t}_{\mathfrak{s}}\}} |W_H(\mathfrak{t}_{\mathfrak{s}})|^{-1} \int_{\mathfrak{t}'_{\mathfrak{s}}} |D_{\mathfrak{g}}(s)|^{1/2} \Phi(s; f; \mathfrak{s}/\mathfrak{t}_{\mathfrak{s}})ds$$

where the sum is over the H-conjugacy classes of the Cartan subspaces of $\mathfrak{s}$ and the Haar measure involved are suitably normalized.

Next we derive a descent formula for the orbital integrals in the spirit of Harish-Chandra. Let $\mathfrak{P}$ be a parabolic subalgebra of $M_n(E)$. Let $\mathfrak{M}$ be a Levi subalgebra and $\mathfrak{N}$ be the nilpotent radical of $\mathfrak{P}$. We denote by $\mathfrak{P}_{\mathfrak{g}}$ the parabolic subalgebra of $\mathfrak{g}$ of elements of the form

$$\begin{pmatrix} \alpha & \beta \\ \bar{\beta} & \bar{\alpha} \end{pmatrix}, \quad \alpha, \beta \in \mathfrak{P}.$$

Suppose $\mathfrak{M}_{\mathfrak{g}}$ and $\mathfrak{N}_{\mathfrak{g}}$ are the Levi subalgebra and the nilpotent radical of $\mathfrak{P}_{\mathfrak{g}}$ corresponding to $\mathfrak{M}$ and $\mathfrak{N}$ respectively. Let $\mathfrak{P}_{\mathfrak{s}} = \mathfrak{P}_{\mathfrak{g}} \cap \mathfrak{s}$, $\mathfrak{M}_{\mathfrak{s}} = \mathfrak{M}_{\mathfrak{g}} \cap \mathfrak{s}$ and $\mathfrak{N}_{\mathfrak{s}} = \mathfrak{N}_{\mathfrak{g}} \cap \mathfrak{s}$. Then $\mathfrak{P}_{\mathfrak{s}} = \{s_\alpha | \alpha \in \mathfrak{P}\}$. The same thing is true for $\mathfrak{M}_{\mathfrak{s}}$ and $\mathfrak{N}_{\mathfrak{s}}$. We denote by K_H the maximal compact subgroup of H of elements h_g with $g \in G_n(O_E)$. Let

$$\mathfrak{P}_{\mathfrak{h}} = \mathfrak{P}_{\mathfrak{g}} \cap \mathfrak{h}, \quad \mathfrak{M}_{\mathfrak{h}} = \mathfrak{M}_g \cap \mathfrak{h}, \quad \mathfrak{N}_{\mathfrak{h}} = \mathfrak{N}_{\mathfrak{g}} \cap \mathfrak{h}$$

and

$$P_H = \mathfrak{P}_{\mathfrak{h}} \cap H, \quad M_H = \mathfrak{M}_{\mathfrak{h}} \cap H, \quad N_H = I_{2n} + \mathfrak{N}_{\mathfrak{h}}.$$

Then $P_H = M_H N_H$ is a parabolic subgroup of H with M_H as a Levi subgroup and N_H as the unipotent radical. We have the Iwasawa decomposition $H = K_H P_H$. We normalize the Haar measure on H such that

$$\int_H f(h) dh = \int_K \int_N \int_M f(knm) dm dn dk$$

for an integrable function f on H. For $f \in C_c^\infty(\mathfrak{s})$, put

$$\tilde{f}(s) = \int_{K_H} f(ksk^{-1}) dk, \quad s \in \mathfrak{s}$$

and

$$f_{\mathfrak{P}}(m) = \int_{\mathfrak{N}_{\mathfrak{s}}} \tilde{f}(m+s) ds, \quad m \in \mathfrak{M}_{\mathfrak{s}}.$$

Then $f_{\mathfrak{P}} \in C_c^\infty(\mathfrak{M}_{\mathfrak{s}})$.

Now let $\mathfrak{t}_{\mathfrak{s}}$ be a Cartan subspace of $\mathfrak{M}_{\mathfrak{s}}$. Then it is also a Cartan subspace of $\mathfrak{s}$. For $m \in \mathfrak{M}_{\mathfrak{g}}$, put

$$D_{\mathfrak{g}/\mathfrak{M}_{\mathfrak{g}}}(m) = \det(-\mathrm{ad} m|_{\mathfrak{g}/\mathfrak{M}_{\mathfrak{g}}}) = \frac{D_{\mathfrak{g}}(m)}{D_{\mathfrak{M}_{\mathfrak{g}}}(m)}.$$

Let $\mathfrak{N}_F = \mathfrak{N} \cap M_n(F)$ and let $|\quad|_F$ be the absolute value on F. We have:

Lemma 6.3. *If $s = s_\alpha \in \mathfrak{M}_\mathfrak{s}$ with $\alpha\bar{\alpha}$ a regular element in $G_n(F)$, then*

$$|D_{\mathfrak{g}/\mathfrak{M}_\mathfrak{g}}(s)|_F = |det(ad\alpha\bar{\alpha}|_{\mathfrak{N}_F})|_F^4.$$

Proof. Let $\mathfrak{N}_\mathfrak{g}^-$ be the nilpotent radical of the parabolic subalgebra of $\mathfrak{g}$ opposite to $\mathfrak{P}_\mathfrak{g}$. Then we have the decomposition

$$\mathfrak{g} = \mathfrak{N}_\mathfrak{g} + \mathfrak{M}_\mathfrak{g} + \mathfrak{N}_\mathfrak{g}^-.$$

So for $s = s_\alpha \in \mathfrak{M}_\mathfrak{s}$ with $\alpha\bar{\alpha} \in G_n(F)$, we have

$$|D_{\mathfrak{g}/\mathfrak{M}_\mathfrak{g}}(s)|_E = |\det(\mathrm{ad}s|_{\mathfrak{g}/\mathfrak{M}_\mathfrak{g}})|_E = |\det(\mathrm{ad}s|_{\mathfrak{N}_\mathfrak{g}+\mathfrak{N}_\mathfrak{g}^-})|_E$$

$$= |\det(\mathrm{ad}s|_{\mathfrak{N}_\mathfrak{g}})|_E|\det(\mathrm{ad}s|_{\mathfrak{N}_\mathfrak{g}^-})|_E = |\det(\mathrm{ad}s|_{\mathfrak{N}_\mathfrak{g}})|_E^2.$$

Now $\mathfrak{N}_\mathfrak{g} = \mathfrak{N}_\mathfrak{h} + \mathfrak{N}_\mathfrak{s}$ where $\mathrm{ad}s$ maps $\mathfrak{N}_\mathfrak{h}$ into $\mathfrak{N}_\mathfrak{s}$ and maps $\mathfrak{N}_\mathfrak{s}$ into $\mathfrak{N}_\mathfrak{h}$.

We first consider the map $\mathrm{ad}s|_{\mathfrak{N}_\mathfrak{h}} : \mathfrak{N}_\mathfrak{h} \to \mathfrak{N}_\mathfrak{s}$. Here we regard $\mathfrak{N}_\mathfrak{h}$ and $\mathfrak{N}_\mathfrak{s}$ as vector spaces over F. Then $\mathrm{ad}s|_{\mathfrak{N}_\mathfrak{h}}$ is a F-linear map, not E-linear. Note that for $\mathrm{diag}(\beta, \bar{\beta}) \in \mathfrak{N}_\mathfrak{h}$, we have

$$\mathrm{ad}s(\begin{pmatrix} \beta & 0 \\ 0 & \bar{\beta} \end{pmatrix}) = \begin{pmatrix} 0 & \alpha\bar{\beta} - \beta\alpha \\ \bar{\alpha}\beta - \bar{\beta}\bar{\alpha} & 0 \end{pmatrix}.$$

So $\mathrm{ad}s|_{\mathfrak{N}_\mathfrak{h}} : \mathfrak{N}_\mathfrak{h} \to \mathfrak{N}_\mathfrak{s}$ is equivalent to the F-linear map $\beta \mapsto \alpha\bar{\beta} - \beta\alpha$ from $\mathfrak{N}$ to $\mathfrak{N}$. By an elementary computation, we find that its Jacobian is $|\det(\mathrm{ad}\alpha\bar{\alpha}|_{\mathfrak{N}_F})|_F$. Similarly for $s_\beta \in \mathfrak{N}_\mathfrak{s}$ we have

$$\mathrm{ad}s(s_\beta) = \begin{pmatrix} \alpha\bar{\beta} - \beta\bar{\alpha} & 0 \\ 0 & \bar{\alpha}\beta - \bar{\beta}\alpha \end{pmatrix}.$$

So the F-linear map $\mathrm{ad}s|_{\mathfrak{N}_\mathfrak{s}} : \mathfrak{N}_\mathfrak{s} \to \mathfrak{N}_\mathfrak{h}$ is equivalent to the F-linear map $\beta \mapsto \alpha\bar{\beta} - \beta\bar{\alpha}$ from $\mathfrak{N}$ to $\mathfrak{N}$. Its Jacobian is also $|\det(\mathrm{ad}\alpha\bar{\alpha}|_{\mathfrak{N}_F})|_F$. Thus we have

$$|D_{\mathfrak{g}/\mathfrak{M}_\mathfrak{g}}(s)|_E = |[\det(\mathrm{ad}\alpha\bar{\alpha}|_{\mathfrak{N}_F})]^2|_F^2 = |\det(\mathrm{ad}\alpha\bar{\alpha}|_{\mathfrak{N}_F})|_F^4.$$

Proposition 6.4. *For $s \in \mathfrak{t}'_\mathfrak{s}$ and $f \in C_c^\infty(\mathfrak{s})$, we have*

$$\Phi(s; f; \mathfrak{s}/\mathfrak{t}_\mathfrak{s}) = |D_{\mathfrak{g}/\mathfrak{M}_\mathfrak{g}}(s)|_F^{-1/4}\Phi(s; f_\mathfrak{P}; \mathfrak{M}_\mathfrak{s}/\mathfrak{t}_\mathfrak{s}).$$

Proof. Let $s = s_\alpha$ be a regular element in $\mathfrak{s}$. Then $\alpha\bar{\alpha}$ is a regular element in $G_n(E)$. Without loss of generality, we assume that $\alpha\bar{\alpha} \in G_n(F)$. By definition we have

$$\Phi(s; f; \mathfrak{s}/\mathfrak{t}_\mathfrak{s}) = \int_{H/T_H} f(hsh^{-1})dh$$

$$= \int_{K_H}\int_{N_H}\int_{M_H/T_H} f(knmsm^{-1}n^{-1}k^{-1})dmdndk$$

$$= \int_{N_H}\int_{M_H/T_H} \tilde{f}(nmsm^{-1}n^{-1})dmdn.$$

Let $m_s = msm^{-1} \in \mathfrak{M}_{\mathfrak{s}}$. Consider the map $n \mapsto m_s^{-1}nm_sn^{-1}$ from N_H to N_H. It is an isomorphism of algebraic groups over F. If we write $n = h_\beta$, $m = h_\gamma$ and $\gamma_\alpha = \gamma\alpha\bar{\gamma}^{-1} \in M$, we have

$$m_s^{-1}nm_sn^{-1} = \begin{pmatrix} \bar{\gamma}_\alpha^{-1}\bar{\beta}\bar{\gamma}_\alpha\beta^{-1} & 0 \\ 0 & \gamma_\alpha^{-1}\beta\gamma_\alpha\bar{\beta}^{-1} \end{pmatrix}.$$

Then the above map is just the F endomorphism in $\mathfrak{N}$ defined by $\beta \mapsto \bar{\gamma}_\alpha^{-1}\bar{\beta}\bar{\gamma}_\alpha\beta^{-1}$. It is well known that its Jacobian is $|\det(I - \mathrm{Ad}(\alpha\bar{\alpha})^{-1}|_{\mathfrak{N}_F})|_F$ (p. 376, [K]). So the above integral becomes

$$|\det(I - \mathrm{Ad}(\alpha\bar{\alpha})^{-1}|_{\mathfrak{N}_F})|_F^{-1}\int_{M_H/T_H}\int_{N_H} \tilde{f}(msm^{-1}n)dndm$$

$$= |\det(I - \mathrm{Ad}(\alpha\bar{\alpha})^{-1}|_{\mathfrak{N}_F})|_F^{-1}\int_{M_H/T_H}\int_{N_H} \tilde{f}(msm^{-1}(n-I) + msm^{-1})dmdn$$

$$= |\det(I - \mathrm{Ad}(\alpha\bar{\alpha})^{-1}|_{\mathfrak{N}_F})|_F^{-1}\int_{M_H/T_H}\int_{\mathfrak{N}_{\mathfrak{h}}} \tilde{f}(msm^{-1}n + msm^{-1})dndm.$$

Now consider the map $n \mapsto m_sn$ from $\mathfrak{N}_{\mathfrak{h}}$ to $\mathfrak{N}_{\mathfrak{s}}$. Then its Jacobian is $|\det(L_\alpha|_{\mathfrak{N}})|_E = |\det(L_{\alpha\bar{\alpha}}|_{\mathfrak{N}_F})|_F$ where L_α is the map on $\mathfrak{N}$ defined by $L_\alpha(\beta) = \alpha\beta$ for $\beta \in \mathfrak{N}$ and $L_{\alpha\bar{\alpha}}$ is defined similarly. Then $L_{\alpha\bar{\alpha}}|_{\mathfrak{N}_F} \circ (1 - \mathrm{Ad}(\alpha\bar{\alpha})^{-1}|_{\mathfrak{N}_F}) = \mathrm{ad}\alpha\bar{\alpha}|_{\mathfrak{N}_F}$. So the integral becomes

$$|\det(I - \mathrm{Ad}(\alpha\bar{\alpha})^{-1}|_{\mathfrak{N}_F})|_F^{-1}|\det(L_{\alpha\bar{\alpha}}|_{\mathfrak{N}_F})|_F^{-1}\int_{M_H/T_H}\int_{\mathfrak{N}_{\mathfrak{s}}} \tilde{f}(msm^{-1} + n)dndm$$

$$= |\det(\mathrm{ad}\alpha\bar{\alpha}|_{\mathfrak{N}_F})|_F^{-1}\int_{M_H/T_H} f_{\mathfrak{P}}(msm^{-1})dm$$

$$= |D_{\mathfrak{g}/\mathfrak{M}_{\mathfrak{g}}}(s)|^{-1/4}\Phi(s; f_{\mathfrak{P}}; \mathfrak{M}_{\mathfrak{s}}/s).$$

7. The nilpotent orbits (II)

We continue with our study of the nilpotent orbital integrals on $\mathfrak{s}$. Recall that for any nilpotent orbit in $\mathfrak{s}$, there is an element s_{α_0} in it such that α_0 is a nilpotent matrix in $M_n(F)$. We denote by $\mathfrak{P}$ the parabolic subalgebra of $M_n(E)$ stabilizing the flag $\{\ker\alpha_0^{n_i}\}$ in E^n. Let Λ be the distribution defined by the orbital integral on the H orbit of s_{α_0} in $\mathfrak{s}$. Our purpose is to prove that the Fourier transform $\hat{\Lambda}$ of Λ is represented by a locally integrable function on $\mathfrak{s}$, i.e. there is a locally integrable function Θ on $\mathfrak{s}$ such that

$$\hat{\Lambda}(f) = \int_{\mathfrak{s}} f(s)\Theta(s)ds, \quad f \in C_c^\infty(\mathfrak{s}).$$

For this we will follow the arguments in [vD] as well as in the proof of Lemma 4.7.6 in [S].

We denote by $C_{\mathfrak{s}}$ (resp. $C_{\mathfrak{M}_{\mathfrak{s}}}$) the set of the conjugacy classes of Cartan subspaces in $\mathfrak{s}$ (resp. $\mathfrak{M}_{\mathfrak{s}}$) under H (resp. M_H). Both $C_{\mathfrak{s}}$ and $C_{\mathfrak{M}_{\mathfrak{s}}}$ are finite sets. Given a Cartan subspace $\mathfrak{t}$ of $\mathfrak{M}_{\mathfrak{s}}$, we write $C_{\mathfrak{t}}$ as a subset of $C_{\mathfrak{M}_{\mathfrak{s}}}$ consisting of all the M_H conjugacy classes of Cartan subspaces of $\mathfrak{M}_{\mathfrak{s}}$ which are H conjugate to $\mathfrak{t}$. Let $Y_{\mathfrak{t}} = \{y \in H | y^{-1}\mathfrak{t}y \subset \mathfrak{M}_{\mathfrak{s}}\}$. Then it is easily seen that the map $y \mapsto y^{-1}\mathfrak{t}y$ defines a one to one correspondence between $T_H^*\backslash Y_{\mathfrak{t}}/M_H$ and $C_{\mathfrak{t}}$. Here T_H^* is the normalizer of $\mathfrak{t}$ in H. Let y_1, ..., $y_{l_{\mathfrak{t}}}$ be a set of representatives for $T_H^*\backslash Y_{\mathfrak{t}}/M_H$. Set $\mathfrak{t}_i = y_i^{-1}\mathfrak{t}y_i$. Then $\mathfrak{t}_1$, ..., $\mathfrak{t}_{l_{\mathfrak{t}}}$ is a set of representatives of $C_{\mathfrak{t}}$. For $s \in \mathfrak{t}'$, define

$$\Theta(s) = \frac{1}{|D_{\mathfrak{g}}(s)|^{1/4}} \sum_{i=1}^{l_{\mathfrak{t}}} \sum_h |D_{\mathfrak{M}_{\mathfrak{g}}}(hs_ih^{-1})|^{1/4}, \quad h \in W_H(\mathfrak{t}_i)/W_{M_H}(\mathfrak{t}_i)$$

where we set $s_i = y_i^{-1}sy_i \in \mathfrak{t}_i$ for $i = 1, 2, ..., l_{\mathfrak{t}}$. Then for a general element s in $\mathfrak{s}$, we define

$$\Theta(s) = \begin{cases} \Theta(s_0), & \text{if } s = hs_0h^{-1},\ h \in H \text{ and } s_0 \in \mathfrak{M}'_{\mathfrak{s}} \\ 0, & \text{otherwise.} \end{cases}$$

Here Θ is a well defined function on $\mathfrak{s}$ which is locally constant on the regular set $\mathfrak{s}'$ and is supported in $\bigcup_{h\in H} h\mathfrak{M}'_{\mathfrak{s}}h^{-1}$. It also satisfies $\Theta(hsh^{-1}) = \Theta(s)$ for $h \in H$ and $s \in \mathfrak{s}$.

Proposition 7. *The function Θ is locally integrable on $\mathfrak{s}$.*

Proof. To prove Θ is locally integrable, it is enough to show that for any function $f \in C_c^\infty(\mathfrak{s})$, we have

$$\int_{\mathfrak{s}} |f(s)\Theta(s)|ds < +\infty.$$

Or equivalently

$$I_f = \sum_{\{\mathfrak{t}\}\in C_{\mathfrak{s}}} |W_H(\mathfrak{t})|^{-1} \int_{\mathfrak{t}'} |D_{\mathfrak{g}}(s)|^{1/2}\Phi(s;|f|;\mathfrak{s}/\mathfrak{t})\Theta(s)ds < +\infty$$

by the Weyl integration formula. For convenience, we assume $f \geq 0$ from now on. Because $\Theta(s) = 0$ unless there is $h \in H$ such that $hsh^{-1} \in \mathfrak{M}_{\mathfrak{s}}$, any class in $C_{\mathfrak{s}}$ which contributes to the above sum must be H-conjugate to a Cartan subspace of $\mathfrak{M}_{\mathfrak{s}}$. Let $X_{\mathfrak{s}} = \{\mathfrak{t}\}$ with $\mathfrak{t} \subset \mathfrak{M}_{\mathfrak{s}}$ be a set of representatives for such classes in $C_{\mathfrak{s}}$. So we get

$$I_f = \sum_{\mathfrak{t}\in X_{\mathfrak{s}}} |W_H(\mathfrak{t})|^{-1} \int_{\mathfrak{t}'} |D_{\mathfrak{g}}(s)|^{1/2}\Phi(s;f;\mathfrak{s}/\mathfrak{t})\Theta(s)ds.$$

Now for a given Cartan subspace $\mathfrak{t}$ of $\mathfrak{M}_{\mathfrak{s}}$, we have

$$\int_{\mathfrak{t}'} |D_{\mathfrak{g}}(s)|^{1/2}\Phi(s;f;\mathfrak{s}/\mathfrak{t})\Theta(s)ds$$

$$= \sum_{i=1}^{l_{\mathfrak{t}}} \sum_{h\in W_H(\mathfrak{t}_i)/W_{M_H}(\mathfrak{t}_i)} \int_{\mathfrak{t}'} |D_{\mathfrak{g}}(s)|^{1/2}\Phi(s;f;\mathfrak{s}/\mathfrak{t})|D_{\mathfrak{g}}(s)|^{-1/4}|D_{\mathfrak{M}_{\mathfrak{g}}}(hs_ih^{-1})|^{1/4}ds.$$

For any $s \in \mathfrak{t}'$ and $h \in W_H(\mathfrak{t}_i)$, it is routine to verify that

$$\Phi(s;f;\mathfrak{s}/\mathfrak{t}) = \Phi(s_i;f;\mathfrak{s}/\mathfrak{t}_i) = \Phi(hs_ih^{-1};f;\mathfrak{s}/\mathfrak{t}_i).$$

So the above sum of integrals becomes

$$|W_H(\mathfrak{t})| \sum_{i=1}^{l_{\mathfrak{t}}} |W_{M_H}(\mathfrak{t}_i)|^{-1} \int_{\mathfrak{t}_i'} |D_{\mathfrak{g}}(s)|^{1/4}|D_{\mathfrak{M}_{\mathfrak{g}}}(s)|^{1/4}\Phi(s;f;\mathfrak{s}/\mathfrak{t}_i)ds$$

$$= |W_H(\mathfrak{t})| \sum_{i=1}^{l_{\mathfrak{t}}} |W_{M_H}(\mathfrak{t}_i)|^{-1} \int_{\mathfrak{t}_i'} |D_{\mathfrak{M}_{\mathfrak{g}}}(s)|^{1/2}\Phi(s;f_{\mathfrak{P}};\mathfrak{M}_s/\mathfrak{t}_i)ds.$$

Here we used the descent formula for the orbital integrals as proved in the last section. Thus we obtain

$$I_f = \sum_{\mathfrak{t}\in X_{\mathfrak{s}}} \sum_{i=1}^{l_{\mathfrak{t}}} |W_{M_H}(\mathfrak{t}_i)|^{-1} \int_{t_i'} |D_{\mathfrak{M}_{\mathfrak{g}}}(s)|^{1/2}\Phi(s;f_{\mathfrak{P}};\mathfrak{M}_{\mathfrak{s}}/\mathfrak{t}_i)ds$$

$$= \sum_{\{\mathfrak{t}\} \in C_{\mathfrak{M}_{\mathfrak{s}}}} |W_{M_H}(\mathfrak{t})|^{-1} \int_{\mathfrak{t}'} |D_{\mathfrak{M}_{\mathfrak{g}}}(s)|^{1/2} \Phi(s; f_{\mathfrak{P}}; \mathfrak{M}_{\mathfrak{s}}/\mathfrak{t}) ds$$

$$= \int_{\mathfrak{M}_{\mathfrak{s}}} f_{\mathfrak{P}}(m) dm$$

by the Weyl integration formula for $f_{\mathfrak{P}}$ on $\mathfrak{M}_{\mathfrak{s}}$. Since $f_{\mathfrak{P}}$ is in $C_c^\infty(\mathfrak{M}_{\mathfrak{s}})$, we have

$$I_f = \int_{\mathfrak{M}_{\mathfrak{s}}} f_{\mathfrak{P}}(m) dm < \infty.$$

This ends the proof of the proposition.

We actually proved that for $f \in C_c^\infty(\mathfrak{s})$,

$$\int_{\mathfrak{s}} f(s)\Theta(s) ds = \int_{\mathfrak{M}_{\mathfrak{s}}} f_{\mathfrak{P}}(m) dm.$$

On the other hand, by proposition 5.1, we have

$$\hat{\Lambda}(f) = \int_{\mathfrak{P}_{\mathfrak{s}}} (\int_{K_H} f(kpk^{-1}) dk) dp = \int_{\mathfrak{P}_{\mathfrak{s}}} \tilde{f}(p) dp$$

$$= \int_{\mathfrak{M}_{\mathfrak{s}}} \int_{\mathfrak{N}_{\mathfrak{s}}} \tilde{f}(m+n) dn dm = \int_{\mathfrak{M}_{\mathfrak{s}}} f_{\mathfrak{P}}(m) dm.$$

So we obtained the following proposition.

Proposition 8. *For $f \in C_c^\infty(\mathfrak{s})$, we have*

$$\hat{\Lambda}(f) = \int_{\mathfrak{s}} f(s)\Theta(s) ds.$$

Hence $\hat{\Lambda}$ is locally integrable.

Theorem 2.3 follows. Theorem 1.1 is then a consequence of theorem 2.1, theorem 2.3 and proposition 4.

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