The Mathematics of Post-Quantum Cryptography

Max Planck Institute for Mathematics, Bonn December 4, 2024

Welcome

- Speakers: please upload your slides in case you haven't done so. (Ask one of the organizers in case you lost the link.)
- All participants: please sign the participant list!
- Coffee/tea: there will be coffee, tea, fruit and water in the tea room on the 4th floor,
- Additionally, cakes and cookies for the 4pm breaks.
- Please note: no food or drinks (except water) are allowed in the lecture hall!

Conference Dinner

- Only registered participants (registered for dinner) can participate.
- Please inform Pieter if you registered for dinner but are not able to come.
- \bullet We meet at the reception at $18{:}30$ to walk to the restaurant.
- Restaurant: Nees (in front of Poppelsdorfer Schloss), Meckenheimer Allee 169.
- Dinner starts at 19:30.
- For all dinner questions: ask Pieter!

- Public-Key cryptography: usually relies on computational hardness assumptions.
- That means essentially: we assume, a certain problem cannot be solved efficiently (in polynomial time).

Example 1 (RSA problem)

Given a composite positive integer *n*, exponent $e \in \mathbb{Z}$ and

$$c = m^e \pmod{n}$$

(for secret m), find m.

• This is easy if factoring integers is easy.

Example 2 (Discrete logarithm problem)

Given elements a, b of a group G, with $b = a^k$ find k.

Introduction

• For **Quantum computers**, there are algorithms (Shor, 1994), that solve these problems (factoring, DLP) in polynomial time.



2023 OPINION-BASED ESTIMATES OF THE LIKELIHOOD OF A DIGITAL QUANTUM COMPUTER ABLE TO BREAK RSA-2048 IN 24 HOURS, AS FUNCTION OF TIME

Range between average of an optimistic (top value) or pessimistic (bottom value) interpretation of the likelihood intervals indicated by the respondents "The 25-year timeframe was not not explicitly considered in the questionnaire.



BSI publications



Federal Office for Information Security

Status of quantum computer development Entwicklungsstand Quantencomputer



Post-Quantum Cryptography

- **Post-Quantum cryptography** instead relies on the assumed quantum and computational hardness of certain mathematical problems:
- Lattice problems: Shortest Vector Problem, Closest Vector problem
- Decoding problems: For a (linear, binary code), solve the decoding problem
- Problems related to isogenies between elliptic curves
- Systems of multivariate polynomial equations
- Group actions
- Equivalence/Isomorphism problems
-?

A look at our program: today

- 13:20: Wouter Castryck: Interpolating **isogenies** between elliptic curves: destructive and constructive applications
- 14:25: Peter Stevenhagen: Lattices in Number Theory
- 15:15: Leo Ducas: Principles of Lattice Cryptography, and cryptanalysis by lattice reduction
- 16:40: Monika Trimoska: Algebraic cryptanalysis applied to equivalence problems
- 17:30: problem session

A look at our program: tomorrow

- 9:00: Hugues Randriam: The syzygy distinguisher (codes)
- 9:45: Sabrina Kunzweiler: Isogeny-based group actions in cryptography
- 11:00: Aurel Page: Hardness of isogeny problems and equidistribution
- 14:05: Severin Barmeier: Utility and usability of projective resolutions (somewhat related to Hugues talk)
- 15:00: Wessel van Woerden: Dense and smooth lattices in any genus