FINAL REPORT

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The stay at MPIM is between July 26 and September 30, 2024.

The first two digits of the MathSciNet classification number is 57.

We had fruitful discussions with Prof. Finashin between July 26 and 31, 2024. It can be gathered under two main topics.

- On algebraic relations between Dehn twists in the mapping class group of punctured torus.

- On the expression the Dynnikov coordinates of the isotopy class of simple closed curves in M in terms of (p, q)-torus coordinates.

The following studies were carried out between August 1 and September 30, 2024.

- To fill the gap in the work titled "Automorphisms of the mapping class group of the n-punctured real projective plane", the Birman-Hilden theory was studied.
- Studies have been carried out on the application of Dynnikov coordinates to the geometric group theory. In particular, a recipe to determine the orbits of reducible and pseudo-Anosov elements of the pure mapping class group of a sphere M with four marked points with respect to a certain generating set via Dynnikov coordinates (Dynnikov matrices) is given. Using this information and thanks to the ping pong lemma, we will present alternative proofs to classical results:

  (i) The group generated by Dehn twists t_c and t_d is isomorphic to the free group of rank 2. Here, c and d are two distinct isotopy classes of simple closed curves on M (let us take one of the punctures to lie at infinity, we can consider M as the thrice-punctured disc, where the punctures are aligned in the horizontal diameter of the disc) such that each of c and d contains two punctures and that c intersects d precisely at two points and they intersect the x-axis $\mathbb{R}$ at most twice.

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(ii) The group generated by the Dehn twists t_c, t_d and $(t_c t_d)^{-1}$ is isomorphic to the free group of rank 3, where the curves c and d are as above.

We can compute the distance from a vertex in the curve complex X of M to an appropriate vertex in the specific vertex set of X by encoding the vertices of X using Dynnikov coordinates.

An algorithm which is determination orbits of the actions of pseudo-Anosov maps $t_d t_c^{-1}$ and $t_d^{-1} t_c$ is given, utilizing the actions of t_c and t_d. This algorithm offers greater practicality, as it reveals how the iteration changes geometrically, regardless of the starting point. Moreover, the following are observed:

i) If the pairs in the sequence of values generated by the iteration of $t_d t_c^{-1}$, starting from the point $(0,1)$, are combined with the pairs in the sequence of values generated by the iteration of $t_d t_c^{-1}$, starting from the point $(0,-1)$, respectively; we obtain a sequence of isosceles triangles. The areas of these triangles are 1, 1, 9, 49, 289, and so on.

ii) The pairs in the sequence of values generated by the iteration of $t_d t_c^{-1}$, starting from the point $(0,1)$ satisfy the equation $2a^2 - b^2 = 1$, except for the pair $(0,1)$. These are the NSW numbers (named after Newman, Shanks, and Williams) integers m that solve the Diophantine equation $2n^2 = m^2 + 1$. As the starting points change, it is observed that the pairs in the value sequence of values generated by the iteration of $t_d t_c^{-1}$ satisfy the equation $2a^2 - b^2 = Det(A)$, where $A = \begin{bmatrix} 2a & b \\ b & a \end{bmatrix}$.

In summary:

(1) The key gaps in the article titled "Automorphisms of the mapping class group of the n-punctured real projective plane" were addressed and finalized here.
(2) The results obtained using the Dynnikov coordinates mentioned above are written in the paper titled "An application of Dynnikov Coordinates in D_3" here.
(3) "On computing the entropy of braids, J.-O. Moussafir", "Topological Entropy of Braids on the Torus, M.D. Finn and J.-L. Thiffeault", and "A Maximum Entropy Approach, D. D'Alessandro, M. Dahleh,and I. Mezi´c" papers were studied. Use M.D. Finn and J.-L. Thiffeault's method, studies to calculate the topological entropy of mapping classes on the real projective plane started.
(4) The relationships between the powers of pseudo-Anosov mapping classes in the mapping class group of M, using Dynnikov coordinates, have been observed, and further studies on these relationships are planned.

In August, discussions were held with Prof. Zorich, Prof. Degtyarev, and Prof. Itenberg regarding the powers of pseudo-Anosov mapping classes and the topological entropy problem. I also attended workshop and most of the seminars, which sparked new research questions for me. Finally, I would like to thank the institute for providing an excellent working environment.

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