MPI Final Report

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Abstract

During my stay at the MPI, I made substantial progress in the following two projects. [1] I have been engaged in a collaboration project with Naoki Imai and Alex Youcis, which aims at applications of recent development of p-adic Hodge theory to the theory of integral models of Shimura varieties. We have constructed the so-called prismatic realization for Shimura varieties of abelian type at primes of hyperspecial level, and applied it to obtain a prismatic characterization of the integral canonical model of the Shimura variety, and to establish a Serre–Tate type theorem for Shimura varieties of abelian type. I have also worked on another project with Eamon Quinlan-Gallego and Daichi Takeuchi, where we seek for a relationship between the Bernstein–Sato theory in positive characteristic and the monodromy eigenvalues on nearby cycles. We have established such a relationship for homogeneous isolated singularities.

Brief description of my research at the MPI

In the first mentioned project, construction of the prismatic realization for Shimura varieties is motivated by the philosophy that any Shimura variety (of abelian type) should be the moduli space of abelian varieties/motives with certain structures, as in the case of Siegel modular varieties (or more generally those of PEL type). The prismatic realization on a Shimura variety is supposed to be the prismatic object (prismatic F-gauge) obtained as the prismatic realization of the universal abelian motive associated to the hypothetical moduli interpretation. From this point of view, one could say that our result gives an approximation of the universal abelian motive, as the prismatic theory is "close" to being universal among the p-adic cohomology theories.

According to the same philosophy, the integral canonical model should be defined by the same but integrally written (hypothetical) moduli description. However, in current understanding of mathematics, it seems out of reach to work out this philosophy, and people have tried to use other available structures to talk about the integral canonical model, which has led to some characterizations. However our characterization is better than any of them in the following two points: it does not rely on a somewhat ad-hoc extension property that all the previous ones relied on; it characterizes the integral models of individual finite level, while previously the characterization was only known at the infinite level (or equivalently for the tower of integral models).

As the original Serre–Tate theorem that provides an equivalence between (p-adic) deformations of an abelian variety and those of the associated p-divisible group, our Serre–Tate type theorem gives an equivalence between deformations of a point of a Shimura variety of abelian type (which according to the above philosophy is the same thing as an abelian variety/motive with some structures) and

[1] The first two digits of the MathSciNet classification number: 14.

those of the corresponding "p-divisible group with structures." Previously, it was difficult even just to make sense of "p-divisible groups with structures," and possible only in some restricted cases by some ad-hoc means. This is an example of objects that make sense systematically in the framework of prismatic F-gauge.

Concerning relationships between the Bernstein–Sato theory and nearby cycle monodromy, in characteristic zero there is an old result known as the theorem of Kashiwara and Malgrange. It states that, for a hypersurface singularity, the monodromy eigenvalues of the nearby cycle are recovered precisely as the exponential of ($2\pi i$ times) the roots of Bernstein–Sato polynomial. For instance, it can be thought of as a differential calculus type description of the monodromy eigenvalues, or also as a cohomological interpretation of the roots of the Bernstein–Sato polynomial (up to integer shift), which are deeply related to many important invariants in the study of singularities. Given that there is a (partially) successful theory of Bernstein–Sato type also in positive characteristic (where the set of Bernstein–Sato roots are defined but not the multiplicity of each root), it is natural to ask if there is a similar relation to the monodromy eigenvalues of nearby cycles. We have observed a subtle phenomenon that involves the Frobenius structure on the nearby cycle complex, and formulated a conjecture: we expect that the monodromy eigenvalues on the *unit root* nearby cycle complex are precisely the exponential (in a suitable p-adic sense) of the Bernstein–Sato roots. As evidence, we have verified this conjecture in the case of homogeneous isolated singularities.

List of papers: During my visit, I wrote and submitted to the arXiv the following two papers.

- With Naoki Imai and Alex Youcis: A Tannakian framework for prismatic F-crystals, arXiv preprint arXiv:2406.08259, (2024).
- With Naoki Imai and Alex Youcis: The prismatic realization functor for Shimura varieties of abelian type, arXiv preprint arXiv:2310.08472, (2024).

Lectures and courses: None (expect that I supervised a master student, Max von Consbruch, with thesis title 'Explicit aspects of non-abelian Lubin–Tate theory').

Other mathematicians (guests) I have worked with: Alex Youcis (National University of Singapore), Daichi Takeuchi (RIKEN).