Final Report
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ABSTRACT OF RESEARCH AT THE MPI

My research is in Algebraic Topology, focusing on configuration spaces, diffeomorphism groups, embedding spaces, and moduli spaces of manifolds. During my time at the MPI, I explored these topics through three different projects. The first, which was my main focus, involved using manifold calculus to study knots and links, in particular to understand their concordance invariants. The second project concerned generalized configuration spaces, which model factorization homology. More specifically I studied properties such as homological stability. Lastly, I explored moduli spaces of surfaces from an algebraic perspective, studying how they form structures such as modular operads.

BRIEF DESCRIPTION OF MY RESEARCH AT THE MPI

My research at the MPI can be divided into three major projects:

CONCORDANCE INVARIANTS OF KNOTS AND LINKS VIA MANIFOLD CALCULUS

In collaboration with Peter Teichner and Hyeonhee Jin, I investigated concordance invariants of knots and links through the lens of manifold calculus. A crucial element of this project was the selection of an appropriate functor to which manifold calculus could be applied. The aim was to identify a functor that maps to topological spaces, with path components in bijection with concordance classes of embeddings, while also making other invariants (such as homotopy groups) as computable as possible. One of the main ideas for this project, was to use a functor defined as the classifying space of a concordance category, which was inspired by previous work on the classifying space of cobordism categories. While the choice was convenient and satisfied our original requirements, it was essential to demonstrate that applying manifold calculus to this functor yielded meaningful invariants in the context of knots and links. In this regard, we obtained two significant results:

Theorem 1 (B.-Jin-Teichner)**:** As expected, the concordance tower for long knots stabilizes on π_0, detecting only the Arf invariant.

Theorem 2 (B.-Jin-Teichner)**:** The string link Milnor invariants are detected in the concordance tower for string links.

The results demonstrate that this concordance tower is sufficiently robust to detect some of the known knot and string link invariants. It is our hope that it will produce a universal concordance finite type invariant. Going forward, we intend to continue studying the invariants that can be derived from this concordance tower, with a particular focus on its potential to prove the existence of the conjectured Higher-Order Arf Invariants.

EQUIVARIANT HOMOLOGICAL STABILITY FOR GENERALIZED CONFIGURATION SPACES

This project, which I initiated during my PhD, explores the $\mathbb{D}$-equivariant homology of generalized configuration spaces. These are spaces where particles have labels on a monoid and can collide, in which case the labels are added. This is interesting for several reasons, for instance for being a model for factorization homology. t the MPI, I extended my previous work to show:

Theorem 3 (B.)**:** For oriented surfaces, generalized configuration spaces exhibit $\mathbb{D}$-equivariant homological stability, and we can give an explicit description of the space it stabilizes to.

MODULAR OPERAD OF MODULI SPACES OF SURFACES

It has long been known that moduli spaces of oriented surfaces assemble into interesting algebraic structures. For example, the genus-zero surfaces with at most one boundary component form an operad isomorphic to the Framed Little 2-Discs Operad, which governs spaces with commutative multiplication up to a non-canonical homotopy. Interestingly, the operad of oriented surfaces of all geni with at least one boundary component detects spaces with a multiplication which is commutative up-to canonical homotopy. Together with Marcy Robertson, I investigated how moduli spaces of surfaces with at least one boundary component assemble into more interesting algebraic structures and its relation to the Grothendieck-Teichmuller program of studying the Absolute Galois Group of $\mathbb{Q}$ via its geometric representations.

Theorem 4 (B.-Robertson)**:** The moduli spaces of oriented surfaces with at least one boundary component assemble into a modular operad whose profinite completion has an action of the Absolute Galois Group of $\mathbb{Q}$.

The proof of this result involves explicitly describing the data needed to define a map out of this modular operad of surfaces. One of the main ideas involved relates the relations that certain objects would have to satisfy with loops on a simplicial complex built out of pants decompositions of surfaces.

PAPERS PLANNED, WRITTEN, OR PUBLISHED DURING MY VISIT

During my stay at the MPI, I completed most of the writing for my paper *Decoupling generalised configuration spaces on surfaces*, which is currently under final review at the Transactions of the American Mathematical Society. In collaboration with Marcy Robertson, I also made significant progress on our joint paper *A modular operad of oriented surfaces and the Absolute Galois Group of* $\mathbb{Q}$, which is still in preparation. Lastly, together with Hyeonhee Jin and Peter Teichner, we are preparing a paper on our main results related to the concordance tower.

LECTURES AND COURSES

- Master's Course for the University of Bonn: Advanced Topics in Topology - Homology and Homotopy of Configuration Spaces
- Mini-course *Scanning – from Configuration Spaces to Cobordism Categories* at the Conference Stability in Topology, Arithmetic, and Representation Theory 2023.

OTHER MATHEMATICIANS I HAVE WORKED WITH WHILE AT THE MPI

Peter Teichner, Hyeonhee Jin, Andrea Bianchi, Daniel Bermudez, Yash Deshmukh, Marcy Robertson.