Report

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Date of my stay: 01/10/2023-30/09/2024

MathSciNet classification: 22 Topological groups, Lie groups

Abstract: During my stay at the MPI, I worked on several projects. Two of them lead or will lead to a publication. The first one is a collaboration with Michael Zshornack (arXiv:2310.15121). It started before I arrived at the MPI but was mostly written the first months of my stay. Together with Michael, we then worked on a different project on which we gave up after a few months. The last months, I worked with Sami Douba, Gye-Seon Lee and Ludovic Marquis on a new paper and wrote most of it already.

Description of my research: The paper of Michael Zshornack and I is about the representation variety of morphisms from the fundamental group of a closed surface to the special linear group over the reals. This variety has a component of very interesting geometric significance called the "Hitchin component". *With Michael, we showed the the $\mathbb{Q}$-points of the Hitchin component are dense in the Hitchin component for the euclidean topology.* The proof is very short, but surprisingly uses the symplectic structure of the Hitchin component. There is a group of Hamiltonian diffeomorphisms that acts transitively on this component and which have a very nice description in terms of representations. Out of a single $\mathbb{Q}$-point, these diffeomorphisms can be use to produce infinitely many $\mathbb{Q}$-points and eventually a dense subset.

After this, Michael and I started to think about the representation variety of morphisms from the fundamental group of a closed surface to the special linear group over the *p-adics*. We constructed an interesting invariant associated to representations that is the analogue of the "Euler number" over the reals. Our hope was that there are special values of this invariant with geometric significance. However, our conclusions were quite the opposite and we gave up on it.

With Sami Douba, Gye-Seon Lee and Ludovic Marquis we construct interesting new examples of Zariski-dense subgroups of the integer special linear group. Such subgroups have attracted a lot of attention recently and are now called "thin groups". *In the paper, we give a general recipe to construct Zariski-dense reflection subgroups.* To do this, we prove a general result on the Zariski-closure

of reflection groups. In particular, we prove that the integer special linear group always contain a Zariski-dense subgroup isomorphic to the fundamental group of a closed surface. We also produce interesting exemples of higher dimensional manifolds which embeds in the integers special linear group Zariski-densely.

Papers:

- *Rational approximation for Hitchin representations*, joint with Michael Zshornack, preprint, arXiv:2310.15121

- *Zariski-closure of reflection groups*, joint with Sami Douba, Gye-Seon Lee and Ludovic Marquis, in preparation

Lectures and courses: none

Mathematicians I worked with: During my stay, I started a project with Grigori Avramidi and Eduardo Reyes which didn't lead anywhere.