Report

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During my stay at MPI I have mainly worked on length functions induced by isometric group actions. The philosophy is that under appropriate geometric assumptions, the length functions completely determine the group actions up to equivariant (almost) isometry. From this, we can use length functions to construct invariants for deformation spaces of group actions.

1 Written papers

1. Length functions for actions on CAT(0) **cube complexes. [3]** Joint work with *Stephen Cantrell* (University of Warwick).

We studied pairs of length functions associated to "compatible enough" geometric actions of a given group on CAT(0) cube complexes. We showed that they can be simultaneously encoded as potentials in a finite state automaton, which has consequences for the regularity of Manhattan curves and rigidity. The compatibility condition is satisfied if the group is hyperbolic, but it is also satisfied by some virtually special groups with contracting elements (for example, it holds for right-angled Artin groups associated to n-gons for $n \geq 5$). This is the first result in this direction that holds for groups that are not necessarily hyperbolic or relatively hyperbolic.

2. Approximating hyperbolic lattices by cubulations. [1] Joint work with *Nic Brody* (University of California, Santa Cruz).

We approximated the length functions of some cocompact lattices in real hyperbolic spaces by length functions induced by actions on CAT(0) cube complexes. The main result solves a conjecture of David Futer and Dani Wise and has applications to random quotients of such lattices, as well as the existence of quasiconvex subgroups with large exponential growth rates.

2 Accepted papers

Manhattan geodesics and the boundary of the space of metric structures on hyperbolic groups. [2] Joint work with *Stephen Cantrell*.

This work was submitted before starting my stay at MPI, but a critical gap was found and fixed during my stay.

3 Planned papers/ongoing projects

1. The space of co-geodesic currents on hyperbolic groups. Joint with *Didac Martinez-Granado* (University of Luxemburg).

We extend the notion of geodesic currents on surface and free groups to arbitrary hyperbolic groups, by defining appropriate invariant measures on "asymptotic hyperplanes" in the Gromov boundary. There is a natural pairing between co-geodesic currents and usual geodesic currents, generalizing Bonahon's intersection number for surfaces. This construction also generalizes geometric actions on CAT(0) cube complexes, and we show a duality between non-trivial co-geodesic currents and isometric actions on spaces with measured walls.

2. Density of Green metrics and applications. Joint with *Stephen Cantrell* and *Didac Martinez-Granado*.

For an arbitrary hyperbolic group, we show that under a natural (symmetrized) Thurston pseudo-metric the length functions induced by Green metrics for admissible random walks are dense among all the geometric actions of the group on geodesic spaces. This implies continuity of the Bowen-Margulis current on the whole space of geometric actions when the group is CAT(-1), and potentially for all hyperbolic groups. For word metrics, we also expect this result to have applications to the existence of Patterson-Sullivan measures with continuous Radon-Nikodym derivatives.

3. Approximation of maximal quotients of Birkhoff averages. Joint with *Jairo Bochi* (PennState University) and *Federico Rodriguez Hertz* (PennState University).

We show that the maximal quotient of Birkhoff averages of positive Hölder potentials on an Anosov dynamical system can be approximated by the Birkhoff averages along periodic orbits of length L, with error of order $o(L^\kappa)$ for any $\kappa > 0$. We use this to obtain uniform approximations of the maximal quotients of length functions associated to negatively curved Riemannian metrics on closed manifolds.

4. Extension of Leighton's theorem for lattices on CAT(0) **cube complexes.** Joint with *Macarena Arenas* (University of Cambridge)

We explore an extension of Leigthon's theorem in the context of cubical groups: whether two virtually compact special non-positively cube complexes with isomorphic universal covers have isomorphic finite covers. We also wonder

whether having (quantified) quasi-isometric universal covers implies the existence of a (quantified) quasi-isometry between these covers that is equivariant for some finite index subgroups of the fundamental groups.

5. The joint translation spectrum and Manhattan manifolds. Joint with *Stephen Cantrell* and *Cagri Sert* (University of Warwick).

We define and study a version of the joint spectrum (introduced for subsets of matrices) for group actions of hyperbolic groups on hyperbolic metric spaces. We identify the joint translation spectrum with the image of the gradient function of a corresponding Manhattan manifold: a higher dimensional version of the well-known and studied Manhattan curve. As a consequence we deduce many properties about the joint translation spectrum. For example we show that it is given by the closure of the set of all possible drift vectors associated to finite, symmetric, admissible random walks on the acting group.

6. Stable isoperimetric rations on hyperbolic groups. Joint with *Stephen Cantrell* and *Cameron Gates Rudd* (MPIM).

For hyperbolic groups, we want to approximate the first eigenvalue of the combinatorial Laplacian of a presentation complex in terms of stable commutator lengths and length functions, with comparison errors controlled by geometric data (cardinality of the generating set, hyperbolicity constants, etc.). This would be an analog of the comparison between the first eigenvalue of the Hodge Laplacian and length functions+stable commutator lengths for closed hyperbolic manifolds.

7. Virtually abelian subgroups of higher-rank lattices. Joint with *Grigori Avramidi* (MPIM) and *Jacques Audibert* (MPIM).

We want to understand the behavior of virtually abelian groups of torsion-free higher-rank lattices. We expect restrictions coming from the Weyl group and arithmetic data. In particular, we expect that the Promislow subgroup is never a subgroup of such lattices. In the long term, we also expect to say something about (non) diffuseness in higher-rank lattices, at least up to finite-index subgroups.

References

[1] N. Brody, E. Reyes. Approximating hyperbolic lattices by cubulations. https://arxiv.org/abs/2404.01511, arXiv preprint, 2024.

[2] S. Cantrell, E. Reyes. Manhattan geodesics and the boundary of the space of metric structures on hyperbolic groups. To appear in *Commentarii Mathematici Helvetici*. https://arxiv.org/abs/2210.07136.

[3] S. Cantrell, E. Reyes. Rigidity phenomena and the statistical properties of group actions on CAT(0) cube complexes. https://arxiv.org/abs/2310.10595, arXiv preprint, 2023.