# RESEARCH SUMMARY

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## 1. Degenerate and irregular topological recursion

Topological recursion is a surprisingly universal mathematical physics tool, that has numerical applications in mathematics, for instance in combinatorics, enumerative geometry, and knot invariants. The theory of topological recursion emerges from the theory of matrix models and is originally due to Chekhov, Eynard, and Orantin. Its main construction is a recursive procedure that produces a system of symmetric meromorphic $n$-differentials $\omega_n^{(g)}$ on $\Sigma^n$, $g \geq 0$, $n \geq 1$, $2g-2+n>0$, where $\Sigma$ is a Riemann surface called the spectral curve (by default, we have no assumptions on $\Sigma$, in particular throughout the text it does not have to be compact unless it is explicitly demanded in some statements). The initial data includes two meromorphic functions, $x$ and $y$, with quite different roles, and a meromorphic bi-differential $B$ on $\Sigma^2$ with the only pole along the diagonal with bi-residue 1. These input data can be either further specified or generalized depending on the problem.

Together with B. Bychkov, P. Dunin-Barkowski, M. Kazarian, and S. Shadrin we use the theory of $x-y$ duality to propose a new definition / construction for the correlation differentials of topological recursion; we call it *generalized topological recursion*. This new definition coincides with the original topological recursion of Chekhov–Eynard–Orantin in the regular case and allows, in particular, to get meaningful answers in a variety of irregular and degenerate situations. We discuss its relation with other versions of topological recursion, including the Chekhov–Norbury and Bouchard–Eynard recursions.

The new definition is also very well compatible with the deformations of the initial data. It behaves nicely in families and provides concrete tools to trace what happens in certain degenerate limits. Furthermore, the new definition provides a greater degree of flexibility, enabling the discovery of novel natural solutions that were previously overlooked by traditional approaches. These solutions possess all the desirable properties typically associated with topological recursion.

The new definition also immediately has quite a few applications. First and foremost, it allows to uniformly extend a variety of results obtained in the previous papers of the authors under the technical assumption of being in general position.

And last but not least: if the spectral curve has genus 0, the differentials constructed by the new definition are always KP integrable. This statement is to appear in a forthcoming paper.

**Dates of stay: 02-30 July**
**MathSciNet classification: 81**

### 1.1. Invited talks.

(1) **Seminar talk**, Integrable Systems Seminar, Zoom, July 17.

### 1.2. List of preprints.

(1) *Degenerate and irregular topological recursion* with B. Bychkov, P. Dunin-Barkowski, M. Kazarian, and S. Shadrin, arXiv:2408.02608.