Report on scientific activities at MPIM, Bonn
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Subject Classification: 14, 11

1. Research Summary

During my visit to MPIM, Bonn, I continued investigation on my ongoing research project: **Calabi–Yau Varieties and Mirror Symmetry**

Here are the summary of the projects that I worked at MPIM Bonn. The main focuses were on the following three concrete problems:

(a) to investigate the Siegel modularity of certain Calabi–Yau threefolds over $\mathbf{Q}$ of Hodge type $(1,1,1,1)$, that is, the Hodge numbers $h^{p,q}=1$ for all pairs p,q with $p+q=3$. Equivalently, this is a question of the modularity/automorphy of the 4-dimensional Galois representations of arising from Calabi–Yau threefolds over $\mathbf{Q}$ of Hodge type $(1,1,1,1)$.

(b) to investigate the modularity/automorphy question of certain families of double sextic K3 surfaces over $\mathbf{Q}$. This is a joint project with Adrian Clingher (Missouri), Seoyoung Kim (Göttingen) and Andreas Malmendier (Utah State). We focused on those double sextic K3 surfaces of Picard numbers ≥ 16. It turns out that these families of K3 surfaces are of hypergeometric type. We then show that at special fibers, these K3 surfaces have complex multiplication (CM), and hence they are modular. We are trying to determine what kinds of modular forms are associated to these K3 surfaces.

(c) to investigate rigid Calabi–Yau threefolds over $\mathbf{Q}$ and their intermediate Jacobians, from geometric and arithmetic points of view.

Detailed description of research activities Here are more detailed descriptions of the projects. [ ] refers to the number in the list of publications.

(a) Siegel modularity of Calabi–Yau threefolds over Q of Hodge type $(1,1,1,1)$ **[1]**

This is an ongoing project focused on the modularity/automorphy question of the 4-dimensional Galois representations arising from Calabi–Yau threefolds over $\mathbf{Q}$ with the third Betti number $B_3=4$. I have looked into a number of examples of such Calabi–Yau threefolds over $\mathbf{Q}$ having the Hodge numbers $h^{p,q}=1$ for all pairs p,q with $p+q=3$ (so that $B_3=4$). These include 14 one-parameter families of Calabi–Yau threefolds of hypergeometric type, a complete intersection Calabi–Yau threefold of Nygaard and van Geemen, Consani–Scholten quintic Calabi–Yau threefold, and some double octic Calabi–Yau threefolds. I introduced the notion of *real multiplication*, and showed that the Consani–Scholten, and the Cynk–Schütt–van Straten double octic Calabi-Yau threefolds have real multiplication with real quadratic field $\mathbf{Q}(\sqrt{5})$ and $\mathbf{Q}(\sqrt{2})$, respectively. The Consani–Scholten (resp. Cynk–Schütt–van Straten) Calabi–Yau threefold satisfies the Hilbert modularity over $\mathbf{Q}(\sqrt{5})$ (resp. $\mathbf{Q}(\sqrt{2})$ by the recent work of Dieulefait–Pacetti–Schütt (resp. Cynk–Schütt–van Starten). Building on the Hilbert modularity, I establish the Siegel modularity of such Calabi–Yau threefolds over $\mathbf{Q}$, that the L-function of such a Calabi–Yau threefold coincides with the Andrianov L-function of the Siegel modular form. The Siegel modular form has weight 3, genus 2 and level N, and is constructed as a lifting of the Hilbert modular form. These results are to appear in the Proceedings of Calabi–Yau manifolds: Arithemtic and Geoemtry at Pune, India.

(b) Certain families of K3 surfaces and their modularity. [2]

This is a joint work with A. Clingher, S. Kim and A. Malmendier. We start with a double sextic family of

K3 surfaces of 4-parameters defined over **Q** with Picard number 16 considered by Matsumoto et al. Then by the reduction (top-to-bottom) processes, we obtain families of K3 surfaces of $3, 2$- and 1-parameter families with Picard numbers $17, 18$ and 19, respectively. It turns out that all these K3 surfaces are of hypergeometric type in the sense that the Picard–Fuchs differential equations are given by hypergeometric functions or Heun functions. We study two parameter families in detail.

To discuss their modularity, we start with one-parameter family of K3 surfaces of Picard number 19. Atspecial fibers, these K3 surfaces become singular. Thus, these K3 surfaces are modular corresponding to weight 3 modular forms of appropriate levels. We then apply the induction (bottom-to-top) processes, the $2, 3$- and 4-parameter families at special fibers have CM. Therefore, the Galois representations associated to the transcendental lattices are all induced from 1-dimensional representations. To determine what kinds of modular forms would occur is the problem that we are working on. A preprint reporting on this work is on Arxiv.

(c) Intermediate Jacobians of rigid Calabi–Yau threefolds over Q. [3]

This is one of my long-sought after quests. Namely, to understand possible relations between rigid Calabi–Yau threefolds over **Q** and their intermediate Jacobians (Weil and Griffiths). If a rigid Calabi–Yau threefolds is defined over **Q**, it is modular and a modular form of weight 4 is associated to it. The intermediate Jacobians of a rigid Calabi–Yau threefolds are elliptic curves. If they are defined over **Q**, then they are modular and modular forms of weight 2 are associated to them by Wiles. What would be possible relations between these modular forms?

2. Publications

[1] *Siegel modularity of some Calabi–Yau threefolds over* **Q**. To appear in the BP Proceeding of trimester seminar on Calabi–Yau Varieties: Geometry, Arithmetic and Physics. (19 pages as of July 21, 2024).

[2] *Certain families of K3 surfaces and their modularity* (with A. Clingher, S. Kim and A. Malmendier). (34 pages as of July 21, 2024)

[3] *Intermediate Jacobians of a rigid Calabi–Yau threefold over* **Q** (with A. Kiming). (9 pages as of July 21, 2024.) In preparation.

3. Scientific talks and scientific visits

[1] *Certain families of K3 surfaces and their modularity*, Algebraic Geometry Seminar, University of Mainz, Germnay, July 18, 2024.

[2] Newnham College and Kings College, University of Cambridge, England, May 23–29, 2024. Visiting member of High Table at Newnham College, mentoring mathematics students and fellows. Visiting Professor Mark Gross and Minhyong Kim at King's College to discuss mathematics and physics of common interest.

4. Other scientific activities

[1] Serving as external referees for three mathematicians for their possible promotions at their respective institutions in three different countries.

- Professor Daniel Persson, Chalmers University of Technology, Gothenberg, Sweden, to Full Professor.
- Professor Xi Chen, University of Alberta, Edmonton, Canada, to Full Professor.
- Professor Mariaa Vlasenko, The Polish Academy of Sciences (IMPAN), Poland, to a permanent position.

[2] Serving as referees for three research articles for three different journals.

[3] Writing letters of recommendations in support of MPIM membership application of Prof. Shinobu Hosono, and Prof. Sandip Singh.

[4] Attending seminars at MPIM, taking part in some workshops at MPIM, talking with PDFs and graduate students.

[5] Serving as the Managing Editor for the research journal "Communications in Number Theory and Physics", International Press of Boston.

[6] Serving as a reviewer for Mathematical Reviews and Zentralblatt für Mathematik.