MPIM Final Report

Jonas McCandless

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I had the privilege of being a Postdoctoral Fellow at the Max Planck Institute for Mathematics in Bonn from the 1st of October 2022 to the 31st of July 2024, where I did research under the MathSciNet classification number 55 broadly construed. The MPIM provided a wonderful environment for working intensely on my research projects and ample opportunities for collaboration with leading experts in various fields of mathematics.

During my stay at the MPIM, I completed the major project [2] with A. Krause and T. Nikolaus. In this paper, we identify a shadow of the cyclotomic structure on topological Hochschild homology with coefficients, which allows us to form topological restriction homology TR with coefficients. More precisely, if R is $\mathbb{E}_1$-ring and M is an R-bimodule, then we exhibit a so-called polygonic structure on topological Hochschild homology with coefficients $\mathrm{THH}(R, M)$ by studying THH defined on cyclic graphs. Inspired by the work of Kaledin, we further developed a variant of equivariant homotopy theory for infinite discrete groups (e.g., $G = \mathbb{Z}$) which we call quasifinitely genuine G-spectra. This theory has the important feature that we can form certain infinite sums of transfer maps, and we exhibited this structure on topological restriction homology TR and prove that THH can be recovered by forming the geometric fixedpoints of these infinite sums of transfer maps. This resolves a conjecture of Antieau–Nikolaus.

Furthermore, I completed the project [1] with L. Keenan, establishing a vanishing result for telescopically localized TR. More precisely, we prove that if $n \geq 1$ and R is a connective $\mathbb{E}_1$-ring such that $L_n^{p,f} R = 0$, then $L_{T(k)} \mathrm{TR}(R) = 0$ for every $1 \leq k \leq n$. This builds on the foundational work of Land–Mathew–Meier–Tamme [3] and provides a higher height analog of the result obtained by Mathew in [4]. The key input for this project is the close relationship between TR and the spectrum of curves on algebraic K-theory, which I studied in [5]. We also use the surprising fact that algebraic K-theory commutes with infinite products at the categorical level in many situations.

During my stay, I benefitted tremendously from discussions with (but not limited to) K. Aoki, L. Basualdo Bonatto, T. Barthel, N. Bottman, L. Côte, A. Efimov, K. Hilman, D. Kirstein, C. Kremer, S. Linskens, I. Mor, Y. Shi, I. Sundberg, V. Subramanian, Y. Uday Deshmukh, M. Volpe, N. Wilkins, and N. Zhang. In particular, I enjoyed the opportunity to attend the seminars of the symplectic topology group, as many problems in this field can be addressed using the methods of homotopy theory. Closer to my own research were the seminars of the arithmetic geometry group, which I also followed and benefited from.

In the summer semester of 2024, K. Hilman and I taught an introductory course on algebraic K-theory, aiming to cover the foundational theorems in algebraic K-theory in the modern

formalism of ∞-categories. The course was well-received by the participants, primarily master's and PhD students at the MPIM and the University of Bonn. We produced an extensive set of lecture notes, which we hope will be useful to future practitioners of algebraic K-theory. These can be found on my website. I also supervised a bachelor's thesis on algebraic K-theory and Quillen's plus construction at the University of Bonn.

Together with T. Barthel, K. Hilman, and D. Kirstein, we are organizing the workshop 'Dualisable Categories and Continuous K-theory' at the MPIM in September 2024. The goal of this conference is to update the community on the recent developments around the spectacular work of A. Efimov on the extension of algebraic K-theory to dualisable categories. The workshop will feature two main lecture series by A. Efimov and A. Mathew, supplemented by research talks from M. Ramzi, P. Scholze, and M. Yakerson.

In conclusion, my time at the MPIM has been very productive and beneficial for my career. I formed many valuable contacts at the MPIM, which I can rely on in my future endeavors. I am also glad to have contributed to the mathematics community in Bonn by teaching the course on algebraic K-theory, supervising a bachelor's thesis, and organizing the workshop on dualisable categories.

References

[1] L. Keenan and J. McCandless. A chromatic vanishing result for TR, *arXiv:2310.04388*, accepted for publication in Proceedings of the American Mathematical Society, 2023.

[2] A. Krause, J. McCandless, and T. Nikolaus. Polygonic spectra and TR with coefficients, *arXiv:2302.07686*, 2023

[3] M. Land, A. Mathew, L. Meier, and G. Tamme. Purity in chromatically localized algebraic K-theory, *Journal of the American Mathematical Society*, 2024

[4] A. Mathew. On $K(1)$-local TR, *Compositio Mathematica*, 2021

[5] J. McCandless. Curves on K-theory and TR, *Journal of the European Mathematical Society*, 2023