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SCIENTIFIC ACTIVITY REPORT

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This letter is to acknowledge my greatest gratitude for the six months in the academic year February 2023 to August 1st 2023 that I spent in Max Planck Institute of Mathematics, Bonn. At the same time, this letter serves as a Scientific Activity Report of my stay in MPIM Bonn.

From my list of publications below (1) was submitted during my affiliation period with the MPIM Institute, (2), (3) and (4) were published during my time in Bonn while the others are based on my research work carried during my time in Bonn.

1. Publications

Posted:

(1) **Duality and Polyhedrality of cones for Mori Dream Spaces**,
with C. Brambilla, E. Postinghel and L. Santana-Sanchez, in https://arxiv.org/pdf/2305.18536.

Published during the time in Bonn:

(2) **Positivity of divisors on blown up projective spaces, I**, with E. Postinghel, in *Annali della Scuola Normale Superiore di Pisa*, Classe di Scienze, volume XXIV , issue 2, pages 599–618, (2023).

(3) **Cremona Orbits in $\mathbb{P}^4$ and Applications**, with R. Miranda, in *The Art of Doing Algebraic Geometry*, Springer, in honor of Ciro Ciliberto volume, pages 161-185, (2023).

(4) **Weyl cycles on the blow up of $\mathbb{P}^4$ at eight points**, with M. C. Brambilla and E. Postinghel in *The Art of Doing Algebraic Geometry*, Springer, in honor of Ciro Ciliberto volume, pages 1-21, (2023).

In Progress: Algebraic Geometry

(5) **On the F-conjecture in $\overline{\mathcal{M}}_{0,n}$**, with R. Miranda, in progress.

(6) **Cones of curves and Weyl actions**, with C. Brambilla, E. Postinghel and L. Santana-Sanchez.

(7) **The geography of the cones of curves**, with C. Brambilla, E. Postinghel and L. Santana-Sanchez, in progress.

In Progress: Mathematical Physics

(8) Co-Editor: *Lagrangians in Complex Lagrangians, Integrable systems and Quantization*
with L. Schaposnik and M. Mulase.

(9) **Lecture Notes for the Oxford minicourse**, in progress.

(10) **Partial oper theory for Lie groups**, with M. Mulase

(11) **Oper Lagrangians for singular Higgs bundles**, with M. Mulase

(12) **Foliations in for rank 2**, with M. Mulase

(13) **On Frobenius algebras, and TQFTs**, with William Davis.

2. Description of research

2.1. Throughout the time of the visit, I worked on problems related to Algebra, Geometry and Mathematical Physics including: TQFT and CohFT (with William Davis), enumerative geometry (M. Mulase), Hitchin Theory (Mulase, Szabo Szabo), topological recursion on K3 surfaces (J. Sawon) algebra and birational geometry etc. At the same time I invested time in reading new topics in different areas of Mathematics, especially in arithmetic geometry and finding exciting questions and connections with my established results. I will describe below some of the results obtained in birational geometry initiated or developed throughout my stay in Bonn.

2.2. **Algebra and Geometry.** A standard way of studying algebraic curves is to vary the coefficients in the polynomial equations that define the curves and then impose conditions—for example, that the curves pass through a specific collection of points. With too few conditions, the collection of curves remains infinite; with too many, the collection is empty. But with the right balance of conditions, one obtains a finite collection of curves. The problem of "counting curves" in this way—a longstanding problem in algebraic geometry that also arose in string theory—is the main concern of enumerative geometry. Substantial contributions to enumerative geometry were made, by bringing in ideas from physics and deploying a wide range of tools from algebra, combinatorics, and geometry.

If the theory of divisors was vastly investigated in birational geometry, starting by the mathematical school of Kyoto, the theory of curves is not clearly understood as of today. In arbitrary dimension even the basic concepts and examples of curves (or for arbitrary k-cycles) were missing from the literature.

The study of curves in projective space is a well-known problem in algebraic geometry, that goes back centuries. The minimal model program in birational geometry has been formulated via the theory of divisors, and it is an interesting question to understand it via the theory of curves. Mori Dream Spaces are a class of projective varieties that have many nice geometric and algebraic properties. Examples of Mori Dream Spaces include projective toric varieties, smooth Fano varieties etc. A smooth projective variety with finitely generated and free Picard group is a Mori Dream Space precisely when its Cox ring, also called its homogeneous coordinate ring, is finitely generated as an algebra over the base field. This fact provides a strong link between the birational geometry of a Mori Dream Space and the birational geometry of an ambient toric variety.

Papers (1), (6) and (7) were originally planned to form one draft; however they were divided into different projects mainly because they discuss and develop different topics via different techniques.

In paper (1) the authors discuss the polyhedrality of the cones of divisors ample in codimension k on a Mori dream space (answering a question of Choi) and the duality between such cones and the cones of k-moving curves by means of the Mori chamber decomposition of the former. The aim of paper (6) is to unify two different concepts of Weyl k-plane and Weyl k-cycle introduced in my previous work ie (3) and (4) in arbitrary dimension, using the theory of movable curves sweeping them out. Moreover, it discusses the birational geometry for particular spaces with infinte generated Cox rings, using the theory of moving curves.

2.3. **In progress.** I will describe problem (5), in particular the F-Conjecture on moduli spaces on curves. For non-negative integers g and n such that $2g-2+n>0$ denote by $\overline{\mathcal{M}}_{g,n}$ the moduli stack of n-pointed stable curves of genus g. The stack $\overline{\mathcal{M}}_{g,n}$ has a stratification given by topological type, the codimension k strata being the components of the closure in $\overline{\mathcal{M}}_{g,n}$ of the locus of curves with k nodes. The 1-dimensional strata are also called F-curves. The question of describing the ample and the effective cone of $\overline{\mathcal{M}}_g$ goes back to Mumford. The conjecture, verified in numerous cases predicts that for large genus, despite being of general type, $\overline{\mathcal{M}}_g$ behaves from the point of view of Mori theory just like a Fano variety, in the sense that the cone of curves is polyhedral, generated by rational curves.

The F-Conjecture of Fulton states that the Mori cone of curves, $NE_1(\overline{\mathcal{M}}_{0,n})$ is generated by the F-curves. It was proved for $n\leq 7$ by Hu and Keel. This conjecture, was generalized to arbitrary genus by Gibney, Keel and Morrison, further predicting the ample cone of $\overline{\mathcal{M}}_{g,n}$ in the direction of Mumford. Although there is a big volume of work and progress discussing the birational geometry of the moduli space of curves by leading mathematicians in algebraic geometry, these questions are still open.

It is well known that the Mori cone of curves is dual to the cone of Nef divisors so the F-conjecture (a la Fulton) can be attacked both via the theory of curves or via the theory of divisors.

Throughout my postdoctoral year (9 months) spent in Bonn, I tried to prove the F-conjecture of Fulton. The work with E. Postinghel - for example paper (2), was developed to answer the positivity questions of divisors in the sense of Mumford. However, I shortly realized that the methods we developed can not answer the F-conjecture in its full generality, but only cover particular cases. Back then I decided to rather develop the theory of opers and quantum curves.

Changing the point of view it is easy to check via the theory of curves developed together with Miranda, that all the F-curves, in $\overline{\mathcal{M}}_{0,n}$ are movable, for $n\geq 9$, they sweep out subvarieties of $\overline{\mathcal{M}}_{0,n}$ of dimension at least 2.

Possible counterexample to the F-conjecture a la Fulton: Affine plane of order three or Hesse configuration: It is realizable in the complex projective plane as the nine inflection points of an elliptic curve with 12 lines incident to triple of these. We further consider the dual Hesse configuration consisting of 9 lines, 12 points, each line contains 4 points and every point is the intersection of 3 lines. The dual Hesse configuration is known to be rigid (ie it has no moduli) and it gives a curve class in moduli spaces $\overline{\mathcal{M}}_{0,10}$ - in a Kapranov model it is given by a line passing through 12 linear spaces of dimension 5. This curve is expected to be rigid, as its virtual dimension is 0. Since the F-curves for $\overline{\mathcal{M}}_{0,10}$ are movable, a rigid curve is expected to provide a counter-example to the F-conjecture. So far with R. Miranda we implemented a computer program to prove that such a curve will provide a counter-example to the F-conjecture for $n=10$ - in progress.

Paper (7) investigates a notion of "anticanonical curve class" introduced in previous work D-Miranda and how it determines the birational geometry of different Mori Dream Spaces. The notion of the "anticanonical curve class" is a curve central in the cone of movable curves covering an (open dense subset) of the ambient space. So far, we are able to introduce this notion of the "anticanonical curve class for Mori Dream Spaces (ie generalizations of toric varieties).

2.4. **Geometry and Physics.** It would be realistic to add to the list of in-progress work, projects described in the area of Enumerative Geometry inspired by mathematical discussions with Professors Zagier, Pandharipande and D. Yang in Bonn 2023. These projects are centered around my work on Catalan numbers, and it aims at developing an appropriate mathematical formalism for the definition of an invariant that we introduced in my first joint work with Mulase. In particular topics of interest

- An ELSV formula for Catalan numbers and intersection numbers on moduli spaces $\overline{\mathcal{M}_{g,n}}$
- Connection between the Harer-Zagier recursion formula and Catalan recursion and its implications from the topological recursion formalism.
- Identification of the Cohomological Field theory class corresponding to Catalan numbers.

The original ELSV formula connecting Hurwitz numbers and intersection numbers can also be approached via the topological recursion formalism and polynomiality of the generating functions for Hurwitz numbers. One direction of this equivalence is the Bouchard Marino conjecture, solved by Mulase, Eynard Safnuk while the other direction is due to the work of Shadrin's school [5]. Alternatively, the above projects can be addressed by the work of Okounkov-Pandharipande [9]. In particular, the main questions described in more detail in the last section of this report.

The aim of the paper (9) is to investigate different important enumerative invariants in algebraic geometry. In particular, we focus on DVV type recursions relations to investigate the theory of Catalan numbers, (simple) Hurwitz numbers and Weil Petersson volumes.

We compare the properties of three important examples to relate enumerative properties to the geometry of moduli spaces of curves. For these examples we prove the connection between the spectral curve of the invariants associated to each of these theories and the $(0, 1)$ invariants of the recursion formula given by each of these problems. We investigate the ELSV type recursion relation, and their quantum curves.

The aim of paper (13) in progress with William Davis, is to investigate infinite dimensional vector spaces that can be endowed with a Frobenius like structure. We extend the set of axioms of Atyiah Segal, to define 2D TQFT over Frobenius like structures. We define an equivalent combinatorial set of axioms based on the formalism of ribbon graphs and edge contraction operations. We classify all possible 2D TQFT s and we prove that the two sets of axioms defined are equivalent. For finite dimensional vector spaces ie Frobenius algebras, we recover a classification result of 2D TQFT via ribbon graphs due to D- Mulase.

Some of the work in progress in the area of Hitchin theory (10), (11) and (12) may involve two graduate students from UC Davis and UNC.

The theory of conformal limits via methods in analysis were originally developed for G-opers, G is a Lie group [3]. The technique we used in [3] was inspired by Hitchin's original approach to Non-Abelian Hodge [8] by solving a system of PDEs for the Hermitian metric. In the particular case of connected Lie group being the group $G = SL(n, \mathbb{C})$ these methods were extended for a family lagrangians with stable Hodge bundles [1] using the work of Simpson. These methods have limitations from the analysis techniques including:

- They require to solve a non-linear PDE for the hermitian metric (this is in general a difficult technique.)

- The analysis techniques of the Non-ableian Hodge correspondence do not extend to semi-stable Higgs bundles or to wild Higgs bundles.

The motivation of D-Mulase work on "conformal limits" is to develop an independent *algebraic* approach to Non-Abelian Hodge. With the aim of creating an alternative path to the analytisis techniques, of Hitchin, Simpson we investigate if the conformal limit mechanism can be defined *globally* by approaching the conformal limits theory theory in different steps.

We consider the theory of lagrangian foliations for the Dolbeault and the De Rham over a curve of genus at least 2 moduli spaces over the group

(1) $SL(n, \mathbb{C})$ (arbitrary rank Higgs bundle and connections over the special linear group).
(2) Higgs bundles over any connected and simply connected Lie group G.
(3) parabolic and wild Higgs bundles.
(4) Higgs bundles over projective complex algebraic varieties.

For defining one lagrangian inside the Dolbeault moduli space, Hitchin used the work of Kostant to realize particular Higgs bundles with split vector bundles as globally defined Higgs fields using the Lie algebra of the special linear group $sl(2, \mathbb{C})$ representations.

Kostant studied representations of a connected and simply connected Lie group with the goal of connecting the representation theory of its Lie algebra with the topology of the Lie group. His theorem implies that any such Lie group contains nside a unique (up to conjugation) copy of $sl(2, \mathbb{C})$ - he names this a principal TDS (three dimensional subalgebra).

The D-Mulase algebraic approach to the conformal limits theory avoids the analysis part of Non Abelian Hodge and uses exclusively the work of Kostant for principal TDS and the construction of the Hitchin section to find an algebraic holomorphic correspondence to the lagrangian of opers.

A natural question arises

Question 2.1. *Can one use Kostant's sub-principal TDS to extend the concept of Hitchin section to arbitrary lagrangians on the Dolbeault moduli space of holomorphic Higgs bundles?*

The general perspective gained during the time of the visit and multiple discussions in the area of Higgs bundles suggest different generalization of D- Mulase work of [4] originally constructed for the group $G = SL(n, \mathbb{C})$ - are possible to cover a larger range of cases.

Namely in our work on opers [4] we presented an algebraic approach to the theory of conformal limits originally introduced as a generalization of a theorem of Gunning of rank 2 vector bundles, degree zero on a Riemann surface C of genus at least 2. It uses the theory of extension classes of $K_C^{\frac{1}{2}}$ and $K_C^{-\frac{1}{2}}$ for the group $G = SL(n.\mathbb{C})$. We generalized this construction to arbitrary rank, and created a $\bar{h} \in \mathbb{C}$ family degree 0 vector bundle via a clever manipulation of matrix theory. Furthermore, this mechanism can be extended to the construction of a λ-connection by adding a constant multiple of Higgs bundle on the Hitchin section to the external differentiation d where λ is a complex number. The result of [4] is that this algebraic construction construction coincides with the analysis construction to Gaiotto's conformal limit for the $SL(n, \mathbb{C})$ opers. We are able to generalize the algebraic result to a general Lie group using the work of Kostant

Theorem 2.2. *[Construction of G-opers] Let G be a complex connected and simply connected Lie group. The algebraic correspondence between the Hitchin section and opers lagrangian [4] is independent of the group G.*

Moreover, it further agrees with Gaiotto's conformal limit (analysis) of [3] ia a Gauge transformation.

In the particular case when the Lie group $G = SL(n, \mathbb{C})$ our result of [3] was further generalized by the conformal limits in [1] or certain Higgs bundles whose limit point, asa Hodge bundle, remain stable under the $\mathbb{C}^*$ action. This result uses Simpson's constructions of the Hodge bundles, that is carefully defined for $SL(n, C)$. If G is a general Lie group, Simpson indicates the steps for obtaining the limiting Hodge bundles however his description is not complete.

Sketch of the proof of Theorem 2.2. The passage from $SL(n, \mathbb{C})$ to a general Lie group G is done by the work of Kostant. Take G a Lie group in the hypothesis of Theorem 2.2, and α its associated Lie algebra.

Kostant theorem ensures that every such Lie algebra α contains a unique principal TDS (three dimensional subalgebra) ie a Lie algebra $\beta \cong sl(2, \mathbb{C})$ up to conjugation so that $\beta \subset \alpha$ is viewed as an adjoint representation of β so that α factors into k irreducible components of a given type determined by the Poincare polynomial of the Lie group G or rank k.

The [G-opers Constructions] of the Theorem 2.2 The original proof of [4] is independent of the $SL(n, \mathbb{C})$. It uses *exclusively* relations that the generators of the β adjoint representation satisfy. In particular, it uses only the information of a $sl(2, \mathbb{C})$ representation in higher dimensional Lie algebra encoded in the theory of three generators, ie a square diagonal matrix of arbitrary rank H, one lower diagonal matrix X_- and its transpose X^+, where X^+ is a nilpotent matrix and $[X_-, X^+] = H$.

Lagrangians in the de Rham moduli space corresponding to stable Hodge bundles (fixed points under the $\mathbb{C}^*$ action) with longest variation of Hodge structure are called in the literature as "partial opers". The oper lagrangian or the Hitchin section corresponds to one of this lagrangians for the special case when the line bundle is $K_C^{\frac{1}{2}}$. However, the other lagrangians can not be understood as principal TDS by the simple reason that Higgs fileds cannot be globally defined if they are not on the Hitchin section.

Question 2.3 (From G-opers to "partial G-opers" via the work of Kostant)**.** *Can the algebraic approach of Theorem 2.2 via the work of Kostant improve the theory of Collier-Wenthworh-Simpson and make the passage from $SL(n, \mathbb{C})$ to an arbitrary complex Lie group?*

In [1] the authors use the Bialynicki-Birula stratification on the space of λ connections, this agrees with the $\mathbb{C}^*$ actions of Simpson.

Our preliminary computations suggest that such extension can be possible by generalizing the square matrix H from a diagonal shape to a matrix with diagonal blocks (partial opers). The original diagonal shape of the matrix H is determined by the split vector bundle on the Hitchin section.

Remark this particular choice of the matrix H is compatible to the Hodge bundles of obtained via the Bialynicki-Birula stratification - ie this matrix determines the vector bundle class on the lagrangian inside the Hitchin moduli space. One needs to identify the choice of the matrices X_- and X^+ in such a way that X_- is a stable Hodge bundle they satisfy the relations of the $sl(2, \mathbb{C})$ generators and further extend the Kostant result to "sub-principal TDS".

To conclude, it was proved in [4] that a compatible choice of the three matrices involved H, X_-, X^+ satisfying the same relations of $sl(2, \mathbb{C})$ will further extend to λ connections.

It seems that the mechanism of algebraic confromal limits can be extended to wild Higgs bundles. A great passage from the world of Enumerative Geometry of invariants of moduli spaces of curves and the Hitchin moduli space of Higgs bundle is encoded in the theorem below.

Theorem 2.4. *The algebraic construction of conformal limits from a Higgs moduli space with wild singularities holds in the following two examples*

(1) The quantum curve of the Catalan numbers is an oper in the de Rham moduli space of connections over $\mathbb{P}^1$.
(2) The Airy Quantum curve is an oper in the de Rham moduli space of connections over $\mathbb{P}^1$.

3. Activities and Discussions

Instead of honoring travel invitationss, I attended most of the special programs and events in MPIM Bonn Seminar on Algebra, Geometry and Physics:A Homage for Yuri Manin, Motives and Automorphic Forms" in honor of Günter Harder, Arbeitstagung 2023 on Condensed Mathematics etc and invested my time in learning a new theory.

Mini-courses:

(1) **2023** (3 hours) *On Ribbon Graphs and Geometry of Moduli Spaces of curves*, Oxford University **Workshop and school on Complex Lagrangians, Integrable Systems, and Quantization**.

Talks:

(1) *MPIM Bonn Oberseminar.*

(2) *University of Heidelberg,*

(3) *University of Bonn, Reading seminar in Algebraic Geometry,*

3.1. **Interactions with other guests.** My visit to MPIM overlapped with the HCM special trimester on number theory - during this time mathematical activity was influenced by the intensive program in the HCM. Due to this overlap, in my second half of my stay in Bonn, I followed the invitations of Justin Sawon and Evgeny Shinder and attended the reading seminars and courses in the University of Bonn and interacted with the group of postdocs in Algebraic Geometry studying the K3 theory.

During my visit I had daily conversations running long computations with Szilard Szabo on the theory of rank 2 parabolic Higgs bundles (about 1 month), Enno Keßler on super Gromov Witten theory in genus zero (about 1 month), Motohico Mulase about connections arising from the theory of ribbon graphs (about 1 month), William Davis (3 months) and Luuk Stehouer about the theory of ribbon graphs and their information in the enumerative geometry, with Don Zagier and Di Yang about the theory of Catalan numbers (in the last days of my stay). After my visit I kept good connections with few of other visitors including Szilard Szabo, Enno Keßler, Sasha Petrov or Ruijie Yang.

Professors Zagier and Pandharipande expressed their interest in visiting UNC. Szilard Szabo visited UNC in October 2023 and was invited to attend a BIRS workshop that I co-organized; we are continuing our weekly discussions online regarding Hitchin theory since then, and planning future joint activities. Enno Keßler expressed his interest to continue our mathematical discussions during my visits in Europe (Germany, IHES etc). We will be inviting Sasha Petrov and Ruijie Yang (now based in the US) for series of lectures in UNC (that accepted non formal invitations).

During my stay at MPIM-Bonn, I had discussions with guest members of the Institute on various topics related to my research area. Some of the invited MPIM guests that I interacted with include

- Motohico Mulase (Professor) - combinatorial constructions of connections over Riemann surfaces of arbitrary genus from the ribbon graph data on the Riemann surface. Discussions with Motohico Mulase include different projects on different topics that are emphasized in the Research Section.

- Justin Sawon (Professor) Together with MPIM visitors Justin Sawon and Motohico Mulase, we ave been trying to extend Eynard-Orantin topological recursion to general Lagrangian fibrations. We understand the quantization of the spectral curves of the GL Hitchin system. Then Baraglia and Huang proved that key geometric information of the Hitchin system, namely the special K¨ahler metric on the base, can be computed from the Eynard-Orantin invariants of the spectral curves. This result was extended by Rayan and his student to include Higgs fields with singularities. While in Bonn we worked on better understanding this computation. Specifically, there are some normalizations of forms, and bases for spaces of forms on the curves involved, that we now understand much better. Next we plan to extend these computations to spectral curves in Beauville-Mukai systems and other Lagrangian fibrations by Prym varieties. The difficulty at the moment is how to find all the ingredients for topological

- Rahul Pandharipande (Professor): Discussions with Rahul Pandharipande were generated by the Hirzebruch lecture of 2023 at University of Bonn and continued during the colloquium dinner and long walks after the dinner.

  My interest in the work of Rahul Pandhapripande goes back to my work on Catalan numbers and Cohomological field theory. We defined the generalized Catalan numbers as an integer numbers counting ribbon graphs. The Catalan numbers satisfy a recursive relation (of the DVV type) that is analogous to a combinatorial formula known in the literature as the Cut and Join Equation for the Hurwitz numbers. We further relate the Catalan numbers to intersection numbers on moduli space of curves via the recursion relation formalism. Our theory on Catalan numbers is incomplete. One of the properties missing is a concrete formula relating Catalan numbers and intersection classes of ψ_i integrals. In Hurwitz theory this is the celebrated ELSV (Ekedahl, Lando, Shapiro, Vainshtein) formula, relating Hurwitz numbers, ie a count of maps of curves over $\mathbb{P}^1$ with simple ramification points and prescribed profile at zero, and intersection theory on moduli spaces of stable curves $\overline{\mathcal{M}_{g,n}}$.

  In a series of influential papers on Gromov-Witten theory around 2000 Okounkov and Pandharipande established a new proof of the Witten-Kontsevich theorem by exploiting the theory of Hurwitz numbers. In their papers, Okounkov and Pandharipande use asymptotics of Hurwitz numbers and their Laplace transform relating the Hurwitz theory to generating series for ψ_i integrals.

  My question of interest and discussions regarding the work of Rahul was centered around implementing the techniques of the Hurwitz theory of Okounkov-Pandharipande to identify the ELSV formula type formula for Catalan numbers in terms of intersection classes on $\overline{\mathcal{M}_{g,n}}$. In particular, can one obtain a new proof to the Kontsevich model by considering appropriate asymptotics of the Catalan numbers via the Laplace transform? I aim to continue this sequence of ideas by mutual visits to our home institutions.

  (During the official inaugural dinner conversation I facilitated interactions of young scientists eg Enno Keßler and Luuk Stehouer inviting them to ask questions and to join us.)

- Don Zagier (Professor): In the influential paper of J. Harer and D. Zagier (Invent. Matem 1986) computing the Euler characteristic of the moduli space of curves the authors use a recursion formula that is closely related to the recursion of the Catalan numbers of D.-Mulase. However, concrete connections between the two recursions formulas and their consequences are not yet known.

- Szilard Szabo (Professor): Long discussions on Hitchin Theory started in MPIM Bonn and continued until now online on a weekly basis. We are planning many activities in the near future and mutual visits - the first official visit to UNC and Banff happened in October 2023.

  It is a standard result in Hitchin's theory that any connection can be understood as an oper with singularities. We start understanding this fact this fact in the theory of Hitchin systems via the Lagrangian foliations. Our discussions centered around the theory of rank 2 parabolic connections on a Riemann Surface of arbitrary genus with n punctures and a stability condition reflecting the full flag filtaration combinatorial data. If the base curve is $\mathbb{P}^1$, with 4 marked points, the moduli space of rank 2, degree 0 connections can be understood as a blown up of a Hirzebruch surface $\mathbb{F}_2$ at 8 points located on the fibers of the four marked points. It turns out that for this particular example, the Non-Abelian Hodge correspondence, the Hitchin and de Rham moduli spaces are globally understood. The Simpson's conjecture regarding the lagrangian foliation holds - such foliation is a fibration (closed fibers). We would like to identify conformal limits even if the $\mathbb{C}^*$ action and the analysis approach may not exist. We believe the algebraic approach can be implemented in this theory.

- Richard Wentworh (Professor): Together with Szilard Szabo, we discussed the constructions of the Fock Goncharov coordinate system on the Moduli space of de Rham connections. Together with Justin Sawon discussed Teichmuller theory. (Lunch appointments regarding the topic of conformal limits were cancelled due to pandemic toward the end of the visit.)

- Di Yang (Professor): My interest in the work of Yang revolves around one question connecting a particular enumerative problem to the theory of moduli spaces of curves. Invariants in enumerative geometry are described as intersection numbers on moduli space of curves. For example, Hurwitz numbers are correlators of ψ_i classes (1st Chern classes of the tautological bundle) and the Hodge class of the Chern bundle on the moduli space of curves. Similarly the Weil Petersson volume is the correlator between ψ_i classes and the exponential of the κ_1 class. My discussions with Di Yang revolved around the Catalan recursion and the Catalan numbers description as a ψ_i class intersection number and a known cohomological field theory class.

- Enno Keßler (postdoctorand): I helped Enno in long discussions during my last weeks in Bonn defining the formalism of super Gromov-Witten invariants in genus 0. These super GW invariants are defined by the integral over the pullback of homology classes along the evaluation maps divided by the equivariant Euler class of the normal bundle of the embedding of the moduli space of stable spin maps into the moduli space of super stable maps. However to make this theory consistent with the torus localization techniques and to compute the first invariants genus 0 super invariants Enno had to overcome many difficulties.

  Enno acknowledged my help in his recent arxiv preprint (joint with Artan Seshmani and ST Yau) 2311.09074 and we would like to meet during my next visits in Europe.

- Luuk Stehouer (former phd student) and William Student (phd student): The discussions centered around a combinatorial approach to the Givental Teleman result regarding the classification of the Cohomological Field Theories for the semi-simple Frobenius algebras. Namely using the theory of ribbon graphs and edge contractions formalism defined from the category of ribbon graphs one can extend the classification result for 2D TQFT to the CohFT. With this combinatorial approach to CohFT I would like to develop a formalism for twisting the Catalan recursion by the CohFT defined on some Frobenius algebra A that would conjecturally relate it to Gromov Witten invariants of an orbifold X where A relates to the orbifold cohomology of X. The main example is for X a point, $A = \mathbb{Q}$, the CohFT is trivial and the Catalan recursion recovers Witten Kontsevich intersection numbers. Similarly, if X is the classifying stack of a finite group G the Gromov-Witten theory of BG was computed by Jarvis-Kimura. The orbifold cohomology is centered around the degree 0 part, and the CohFT becomes a 2D TQFT. The Catalan numbers recursive formalism can be twisted by the 2D TQFT and produce the invariants computed by Jarvis Kimura.

- Wadim Zudilin (Professor) - Discussions regarding combinatorial formulas for generalized hypergeometric functions with rational coefficients and applications to number theory. Golyshev predicted based on argument of motives that generating functions arising as exponential functions depending on hypergeometric functions are algebraic functions. In an influential paper on Hirzebruch Lectures, Zagier proved that Golyshev predictions hold for three such examples by verifying they satisfy certain differential equation. Recent generalizations of this result relating it to a classical problem of Abel give connection to p-adic geometry via a criterium of Gauss eg by Delaygue and Rivoal exposed by talks of Kontsevich. I was interested in the nature of the algebraic equations (for example rationality questions) satisfied by such tuples of numbers that naturally appears in physics.

  The discussions generated by a seminar talk of Prof. Wadim Zudilin continued the following day during lunch and tea time together with professor Don Zagier. I further gave such combinatorial projects regarding generalizations of results in this area to students in UNC.

- Alexander Petrov (Postodctorand MPIP) Discussions with Sasha include general facts on the theory of rigid analytic varieties, p-adic Simpson correspondence, Hodge symmetry over p adics, Gauss-Manin connection.

  We will be inviting Sasha to give series of lectures in UNC in the Fall 2024 and continue our discussions on Hodge symmetry in p-adics geometry together with my graduate student Kexuan Yang.

- Ruijie Yang (former Postoctorand MPIM/ Assistant Professor now). Topics from birational algebraic geometry to theory of differential operators.

  We are planning on inviting Ruijie to give seminars in UNC during the Fall semester 2024.

- Evgeny Shinder (Professor):Different trends in the area of Mathematical Physics and Topological recursion. Relations with Algebraic Geometry. Discussions continued in the reading seminar on K3 surfaces at University of Bonn.

- Joseph Hoisington (Postoctorand MPIM) Joseph introduced us to the theory of stable harmonic maps, the energy in homotopy classes of mappings from complex projective spaces to Riemannian manifolds. Part of the blackboard discussions based on Joseph's reseach area were held together with Enno Keßler.

- Yajnaseni Dutta (former Postdoctorand University Bonn/ Assistant Professor now) Theory of vector bundles and relations to the Higgs bundles theory. Curves on K3 surfaces, relations between the theory of lagrangians in the Hitchin theory and lagrangians inside K3 surfaces.

- Maxim Smirnov (Assistant Professor) Topics in Mathematical physics and Algebraic geometry Quantum Cohomology, Motivic Quantum Cohomology.

- Quick conversation with Professors Bernd Sturmfels and Gaetan Borot during the one day visits of special events (Homage for Yuri Manin) on tropical geometry techniques on the moduli spaces of curves etc.

- Eugenia Rosu (Assistant Professor), Zohreh Ravanpak (Postdoctorand)

The discussions mentioned above have been useful for my research projects and some of them continued outside MPIM Bonn.

The opportunity to engage in numerous timely discussions with the experts in mathematics and physics is very rare occasion, and I am grateful for MPIM to provide such a fantastic place to facilitate the lively interactions. My visit was an exceptional experience for my mathematical research. Thank you for your leadership and vision of the Institute!

Olivia Dumitrescu,

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