Memorial Article for Yuri Manin

Edited by Fedor Bogomolov and Yuri Tschinkel

Yuri Manin was born in Simferopol, Crimea, on February 16, 1937, and died in Bonn on January 7, 2023. His father perished on the front fighting against Germany. His mother struggled to survive and raise him through the war and the hardships of the postwar years. Even though Manin lived far away from the main centers of academia, he developed a serious interest in number theory rather early. He was admitted to Moscow State University (MGU) in 1953, and received his PhD in 1961, at which time he was already one of the leading experts in what is now known as *arithmetic geometry*. Even before receiving his PhD, he became a member of the Steklov Institute of Mathematics of the Russian Academy of Sciences, and a few years later, a professor at Moscow State University. After the collapse of the Soviet Union, Manin left Russia for faculty positions at MIT and Northwestern, but soon after he settled in Bonn, joining the Max Planck Institute for Mathematics as one of the directors.

Manin supervised over 50 PhD students, in Russia, the USA, and Germany, many of whom are now professors in leading universities all over the world. Manin's scientific interests spanned an enormous range of fields in mathematics and mathematical physics. His early interests in literature and art shaped his truly unique vision of science, culture, and society. With his deep intuitive understanding of interconnections between different fields he had striking insights even in areas not directly related to his research. One of the most spectacular examples was his idea of quantum computing, which he developed around 1981. He closely collaborated with more than 100 mathematicians. His brilliant lectures were special events for students and faculty alike. He published more than 20 carefully crafted books on very different subjects in mathematics, physics, and philosophy of science.

Through many dramatic changes of circumstances he kept his inner balance and independence of thought. He was lucky to have had unconditional support from his wise and loving wife Xenia Glebovna.

Figure 1. Yuri Manin and his wife Xenia Glebovna, September 2011.

Perhaps the best description of his own view of mathematics was in his quote of Georg Cantor's words, spoken at the ICM in Berlin:
The essence of mathematics lies in its Freedom.

We are grateful to the *Notices* for encouraging us to gather personal memories of some of his students and colleagues.

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Alexander Beilinson

It rarely happens, but if one is lucky enough to see a mathematical problem from the right perspective, the solution suddenly becomes clear: the known pieces by themselves join into an unexpected whole, taking on form and meaning without any effort on your part. Manin's seminars developed in a similar way. Yuri Ivanovich Manin had a wonderfully light personality—he did not wish to be a leader—and new themes and subjects, as if in gratitude, came to life by themselves, in front of our eyes.

Many shades of blue above him.
Green below him, and the world
Is a giant bird before him,
Warbling, trilling, full of songs.[1]

The time of my youth in Russia was benevolent to those who accepted Pushkin's poem "From Pindemonti"[2] as part of their souls. Manin's seminars, like the books of Yuri Koval and the animated movies of Yuri Norstein, were parts of the happiness of that dandelion-light world, of its deeper Liberty.

If one were to formulate how Yuri Ivanovich viewed things, one might first notice his ability to connect facts to arrive at conclusions that often contradicted everyone else's. Or, perhaps, the absence of desire to belong to any association, and to have sway over any other person. Or the rejection of any kind of malice and greed in human relations. And, certainly, the clear kindness.

The 90s, years of dark poverty in Russia, left almost untouched those of us who flew away, like fluff on the wind, into the opening new world. We met far more rarely than in Moscow, where we would see each other or talk on the phone almost every day. Our attitude toward what was around us was changing, and, during the NATO assault on Serbia, I heard from Yuri Ivanovich a grim foreboding of what has happened since and continues to unfold now.

In Manins' Bad Godesberg apartment a glass wall opened onto the Rhine. It seems to me that when you watch a great river from day to day, you become its relative: it starts flowing through you as time flows, always remaining itself, as memory does. And this is happiness.

I am so grateful to Yuri Ivanovich for his gift of joy, full of sun; it became a part of myself.

When I was very young, my most beloved book was *Winnie the Pooh*; Yuri Ivanovich loved it too: "So they went off together. But wherever they go, and whatever happens to them on the way, in that enchanted place on the top of the Forest, a little boy and his Bear will always be playing."

Alexander Beilinson

Vladimir Berkovich

One day in the Fall of 1969, my second year at Moscow University, I was talking with Misha Mandel, a fourth-year computational mathematics student about a problem I had. I had until the end of that school year to find an academic advisor, as every student must have one beginning with their third year. I wanted to have an advisor who is an outstanding mathematician with broad and deep knowledge and taste in mathematics. I hoped that everything else would follow. Misha quietly listened to me and then said there was a young mathematician in mech-math, Yuri Ivanovich Manin, about whom he had heard many good things. I had never heard about Manin and soon found that he would give a course on advanced commutative algebra during the second semester. I attended the first lecture with my friend from high school, Anas Nasybullin. The auditorium was packed, we found seats in the last row, and within a couple of minutes, a short man with a strong voice entered in a suit and a white tie; he did not seem young to us (he was already 33!). His lecture with precise definitions, formulations, and arguments was perfect, and I felt a mathematical aura emanating from him. After the lecture, both Anas and I, said to ourselves "we have found our academic advisor." Of course, there was a remaining nuance that he must agree to have us as his students, but his guidance had already started without him being aware of it.

Somebody gave me a rotaprint of Yuri Ivanovich's lectures on algebraic geometry. I was already slightly familiar with basic notions of classical algebraic geometry and could not say I felt comfortable with it. I also heard about Grothendieck's new approach but never had the chance to learn it, and I thought it might be something

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[1]*Bagritsky's poem "A bird-catcher"*

[2]*Nabokov's translation:* https://ireaddeadpeople.wordpress.com/2014/11/06/alexander-pushkin-to-stroll-in-ones-own-wake

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complicated. From the first pages of that rotaprint, Yuri Ivanovich wonderfully explained the transition from classical algebraic varieties to Grothendieck's schemes. He presented the latter in such a natural and simple way, only he could do it. At the same time, I found Yuri Ivanovich's paper on etale cohomology, Algebraic topology of algebraic varieties, in Uspehi. Again, his explanations were so natural and straightforward. All this strengthened my confidence in choosing him as my academic advisor, and it remained only to overcome that little nuance.

In the Spring of 1970, near the end of the semester, Anas and I told Yura Zarhin and Pasha Kurchanov, our friends from high school, about the choice of Yuri Ivanovich Manin as an advisor. I do not know if this influenced their decision or if they came to it independently, but one day at the end of May, all four of us came up to him, and Pasha said, "Yuri Ivanovich, take us as your students." Yuri Ivanovich laughed and then said that he could not decide this immediately, but he had just finished a course on commutative algebra, offered to give us an exam on it, and after that, he would see. In this way, we became Manin's students. Slightly later, our friend Kolya Chebotarev approached Yuri Ivanovich on the same subject. As Kolya told us, Yuri Ivanovich asked if he had a relation to the famous Russian mathematician Nikolai Chebotarev. Yes, Kolya was his grandson; this was his entrance ticket for joining our group of Manin's students.

A period of intensive learning and immersion in the beautiful world of mathematics started for all of us: class field theory, Grothendieck's *EGA* and *SGA*, works of Tate, Serre, and other mathematicians. The direction of our study was naturally generated by Yuri Ivanovich's lectures, which were, as always, perfect and introduced us to many new things. Especially beneficial for me were Yuri Ivanovich's requests to give a talk in his seminar about a particular paper or so, which forced me to concentrate on the subject and, as a result, do something new. The close presence of brilliant older and younger students of Yuri Ivanovich was highly motivating.

The last three years of study flew by like a dream, and by the end, Anas, Pasha, and I were accepted for graduate studies. (Yura, the brightest of us, could not hope for this because he had poor marks in Marxist–Leninist wisdom.) Besides other things, a graduate student had to pass an exam on a specific big subject, which was usually chosen to be in the student's research area. But Yuri Ivanovich told us that since we would continue to study our fields in any case, each of us had to choose something distant from our research areas. He suggested complex analysis, functional analysis, and mathematical logic. I grabbed the latter since I knew nothing about it, and besides standard textbooks, I had a chance to read his book on mathematical logic before it was published.

Three more years flew quickly by, and we entered life with all its sorrows, surprises, and joys. I attended Yuri Ivanovich's seminar at the university sporadically because I worked as a computer programmer. For several years, I was seriously thinking about devoting myself to this occupation, but mathematics acquired under Yuri Ivanovich's mentorship was boiling in me and, finally, burst out and took me to freedom in Israel. Eventually, Yuri Ivanovich and most of his former students emigrated from the Soviet Union. We did not communicate much since then, but the precious memories of those years and gratitude to Yuri Ivanovich are always with me.

Vladimir Berkovich

Jean-Louis Colliot-Thélène

At the beginning of the 1970s, I started mathematical research and was quickly attracted to work by the Russian school around Shafarevich and Manin. My first encounter had been with (the French translation of) *Theory of Numbers* by Borevich and Shafarevich.

In the period 1963–1972, Manin published many papers on the geometry and arithmetic of (geometrically) rational surfaces, and more generally "varieties close to the rational ones." In the celebrated 1965 Shafarevich seminar on algebraic surfaces, which I read in the German translation, bought in East Berlin, Manin contributed to the section on linear systems with base points, a topic which went back to Max Noether and Beniamino Segre, and which after the famous Manin–Iskovskikh paper on quartic threefolds developed into the study of rigidity.

In that period, it took time for papers written in Russian to be translated. I learned enough basic Russian to be able to decipher papers by Manin, Iskovskikh, Voskresenskiĭ, Bogomolov, and by the Minsk school on linear algebraic groups, which I closely studied with J.-J. Sansuc.

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Ten years later this also helped me to read the papers of the quite distinct algebraic K-theory school in Leningrad (Sankt-Petersburg).

Yuri Manin was among the people who created a bridge between old diophantine problems and the modern algebraic geometry of Serre and Grothendieck. Among the many topics Manin helped develop: birational classification of rational surfaces (which went back to the Italian school, and was then developed into Mori theory and MMP), study of the Cremona group, use of the Brauer group in the study of rationality of varieties and in the study of rational points over number fields.

In 1966, Manin published his big IHÉS paper on Rational Surfaces (in Russian). In 1970, his Nice ICM talk (in French) launched the study of what we now call the Brauer–Manin obstruction. In 1972, Manin published his very attractive book *Cubic forms, algebra, geometry, arithmetic* (in Russian), where among others we can find hints in the direction of descent. One also finds the notion of R-equivalence on rational points, which started its independent life there. Special cubic surfaces, investigated by François Châtelet in 1960, as a possible analogue of descent on elliptic curves, feature in the book. As Manin (and also Peter Swinnerton-Dyer) had predicted, they have turned out to be a testing ground for descent and also for the study of rationality of varieties. In the early 80s, Sansuc and I elaborated an appropriate theory of descent (the full text appeared in the DMJ 1987 volume on the occasion of Manin's 50th birthday) and in works with Daniel Coray and with Swinnerton-Dyer, managed to solve many questions on these surfaces raised in Manin's book. Since then, progress by many people has been achieved—but we still have no systematic algorithm to decide if a given cubic surface over the rationals has a rational point.

Communication with Russia was not simple in the days I am alluding to. In 1982, Jean-Jacques Sansuc and I travelled as "tourists" to Moscow (over two days by train each way), stayed at the old Hotel National where more illustrious people had stayed, a few yards away from Red Square, and met Manin and other Russian mathematicians, for the first time, in the old Steklov Institute. We were smuggled into the main building of MGU, while the guard was not looking. In Manin's seminar, with a packed audience, Sansuc lectured in French on rational points on intersections of two quadrics, with Manin (who spoke perfect French) translating. We were very nicely invited to Manin's home and managed to get hold for him of a copy of the then hardly available book *Master and Margarita*, by Mikhaïl Boulgakov, in a Beriozka shop (opened only to foreigners).

This was for me the first of a series of longer stays in Moscow, where I started a long-lasting collaboration with Alexei Skorobogatov, himself a student of Manin, and also had contacts with Fedor Bogomolov.

At the beginning of the 80s, Manin's manifold interests led him away from "varieties close to rational ones" but by the end of the 80s he had become interested in counting points of bounded height on such varieties, and in works with Batyrev, Franke, and Tschinkel produced conjectures on the behavior of the counting function, with main term depending on the geometry of the underlying variety. Getting the "right" constant in front of the main term was a challenge back in 1990. This was achieved by E. Peyre. The questions Manin then raised, at the interface of analytic number theory and complex algebraic geometry in the MMP style, are still very much open, but they have generated a whole area of research.

Over the years, I met Yuri Ivanovich on various occasions, in Paris and in Bonn. In March 2021, I gave an online talk at his Bonn seminar, where I had the opportunity to recall a 1967 result of his, based on heights, on the lack of finite-to-one parametrizations for rational points of cubic surfaces. My last slide was an image of his handwritten 1982 Moscow dedication of his book on Cubic Forms to Sansuc and me.

Jean-Louis Colliot-Thélène

Vladimir Drinfeld

I was a student of Yuri Ivanovich Manin at Moscow university. Attending his lectures and seminars was a substantial and very happy part of my mathematical life then. This started in 1970/1971, even before I became a student of his. That year he gave a course on the language of schemes, which also included more advanced topics like flat descent. Around that time I also read Manin's brief 1965 survey on étale cohomology, which made me very excited. (By the way, I learned about the existence of Winnie-the-Pooh from the epigraph of the survey.)

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In 1971/1972, Manin and Piatetskii-Shapiro organized a seminar on modular and automorphic forms. Its goal was essentially to learn the representation-theoretic approach to automorphic forms, which was brand new then. This was another formative experience for me.

Rather than describing the later events year by year, let me say that the mathematical subject on which I am now working is very close to Manin's works of the 1960s on formal groups and the Gauss–Manin connection.

Many years ago, I asked Yuri Ivanovich to become my advisor, and it was great luck for me that he agreed. It was especially great luck because of his human qualities. There was a difficult period in my life when Yuri Ivanovich's practical and moral support was crucial for my survival.

Manin was a great lover of literature. As far as I understand, "The 4 quartets" by T. S. Eliot was a favorite poem of his; in particular, he liked the following line:

> For us, there is only the trying. The rest is not our business.

As I am growing old, I cannot help thinking about this line again and again.

Let me now describe some of Manin's contributions to mathematics.

Manin triples: At the beginning of the 1980s, I told Manin about the notion of Poisson–Lie group, which I managed to extract from the works by L. D. Faddeev's school (especially, by E. K. Sklyanin). I also told him about Lie bialgebras, which are infinitesimal analogs of Poisson–Lie groups. A Lie bialgebra is a vector space $\mathfrak{g}$ equipped with a structure of Lie algebra and that of Lie coalgebra so that the two structures are compatible in the following sense: the Lie cobracket $\mathfrak{g} \to \wedge^2\mathfrak{g}$ is a 1-cocycle.

Manin immediately asked me the following question. Consider triples $(\mathfrak{a}, \mathfrak{g}, \mathfrak{g}')$, where $\mathfrak{a}$ is a Lie algebra equipped with a nondegenerate invariant symmetric bilinear form and $\mathfrak{g}, \mathfrak{g}' \subset \mathfrak{a}$ are transversal Lagrangian Lie subalgebras. Then the vector space $\mathfrak{g}$ is dual to $\mathfrak{g}'$, so the Lie bracket on $\mathfrak{g}'$ induces a Lie coalgebra structure on $\mathfrak{g}$ (in addition to the Lie algebra structure). Can it be that $\mathfrak{g}$ is a Lie bialgebra and that one thus obtains an equivalence between the groupoid of finite-dimensional triples as above and the groupoid of finite-dimensional Lie bialgebras?

A simple computation showed that Manin's guess was correct. It played a very important role in the development of the theory of Poisson–Lie groups and quantum groups (in particular, it led to the notion of "quantum double").

Instantons: In 1978, Manin, Atiyah, Hitchin, and I found all G-instantons on the sphere S^4 for all classical compact groups G; this work is known as ADHM. Since I am one of the authors, I cannot praise the work itself, but I can praise algebraic geometry. It is definitely true that ADHM demonstrated the power of 20th century algebraic geometry to theoretical physicists. Indeed, the problem was formulated by physicists who considered it to be very important and the best theoretical physicists tried to solve it. A complete solution was formulated by ADHM in terms of linear algebra, so they could easily understand it. On the other hand, physicists were unable to guess the answer (although they found some nontrivial solutions) because they did not have the relevant tools at their disposal. The reaction of theoretical physicists to ADHM was adequate: they began to learn algebraic geometry very seriously, and by 1990 they knew it, in some sense, better than algebraic geometers.

How did physicists learn algebraic geometry and how did (some) mathematicians learn (some) physics in the period 1975–1990? At least in Moscow, Manin played a crucial role in this process by writing articles and surveys, giving lectures, organizing seminars, and in other ways. As a scientist, Maxim Kontsevich was brought up in this atmosphere.

Somewhat unexpectedly, the ADHM work led to a revival of interest in homological algebra in Moscow (and probably elsewhere). A crucial step was the description of the derived category of coherent sheaves on the projective space obtained in 1978 by Beilinson and Bernstein–Gelfand–Gelfand. Their motivation was to find a conceptual explanation of the technique of Horrocks and Barth used by ADHM. These works led to a spread of the culture of derived categories (and then differential graded categories) in Moscow. Manin actively participated in this cultural change; in particular, he wrote a textbook on homological algebra jointly with S. Gelfand.

Vladimir Drinfeld

Alexander Goncharov

In 1976, I was fortunate to be admitted to Mechmat MGU[3] bypassing the notorious entrance exams,[4] and on the first Monday of September, shortly after arriving from Ukraine to Moscow, I went to Gelfand's seminar. It was there that I first saw Manin, who gave two talks in February 1977 on soliton equations.

At that time, Moscow was an exciting place where curious students could meet great mathematicians, find all kinds of math and clandestine literature, and discover the math they liked the most.

I came to Manin's seminar in September of 1977. It was the only time when the seminar was designed for beginners. Its main goal was Griffiths's paper "Variations on a theorem of Abel." Among the other topics which Manin suggested at the first meeting was the monodromy of the dilogarithm function. The dilogarithm is the simplest integral of algebraic geometric origin beyond the Abelian integrals. It satisfies Abel's relation—an analog of addition laws for Abelian integrals—which characterizes it uniquely. Manin mentioned that the dilogarithm showed up recently in several unrelated areas, e.g., in Gabrielov–Gelfand–Losik's work on the combinatorial formula for the first Pontryagin class. My fascination with polylogarithms goes back to that time. A year later, Sasha Beilinson gave a series of talks on Manin's seminar on Bloch's 1978 Irvine lectures, where the dilogarithm was one of the key characters. These talks led to Beilinson's conjectures on regulators and crystallization of the very idea of mixed motives.

Manin looked at math through the glasses magnifying the underlying Algebraic Structure. Gelfand presented himself as an analyst. Yet anything he did in math always led, sometimes entirely unexpectedly, to representation theory. For example, his works with Dikii on the KdV equation led to W-algebras—higher analogs of the Virasoro algebra. His works with Ponomarev on classification of linear algebra problems led to quiver varieties, which transformed representation theory, etc. It feels as if they sensed different aspects of the dilogarithm. As we see today, polylogarithms and the relations they satisfy describe the structure of the motivic Galois group, while the quantum deformation of the dilogarithm plays pivotal role in the emerging cluster representation theory of quantum groups.

Yuri Ivanovich influenced the education of generations of mathematicians in a multitude of ways. In Moscow, he conducted two 1.5-hour seminars each week on different subjects, one right after the other. Yuri Ivanovich possessed an amazing intellectual ability to learn any kind of math he found exciting. His attitude toward math was highly social. After mastering a subject, he would give a carefully crafted course, with meticulously prepared notes for each lecture. He never lectured on the same advanced topic twice. Manin's courses attracted a huge audience. And then Manin used his notes to write a book or an expository paper. On top of this, Manin participated in the selection of foreign books to be translated into Russian. He would often either translate a book, or write a beautiful foreword. Since only a small fraction of foreign books were translated, we learned math by reading the books he selected.

After we moved to the West, my relationship with Yuri Ivanovich become much more personal. His kindness, dignity, and decency stand out in my memories.

Alexander Goncharov

Michael Harris

It was common knowledge when I was a Princeton undergraduate that Moscow was not merely an extraordinary mathematical center but that it also possessed a magical aura, where the most unexpected developments were routine. Mathematicians' fascination with Russia was only enhanced by the country's exotic inaccessibility at this stage of the Cold War, and no figure exercised a greater hold on the imaginations of my teachers in number theory and arithmetic geometry than Yuri Ivanovich Manin. For my junior project, Spencer Bloch had me write a report on his 1961 paper on the Hasse–Witt matrix. Nick Katz, my senior advisor, gave me Manin's paper on the Dieudonné modules of formal groups to read. For my senior thesis he suggested I try to answer a question of Manin—in a

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[3]*Abbreviation for the Faculty of Mechanics and Mathematics of the Moscow State University*

[4]*I do not know any Jewish applicant who survived the entrance exams that year. More than 400 students were admitted.*

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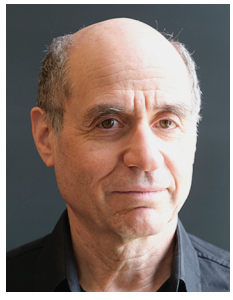
letter he had written to Nick, I think—on an analogue for elliptic curves of the Wieferich criterion for Fermat's last theorem—a question that is still unsolved, as far as I know. My years as a graduate student were marked by a fruitful long-distance exchange between Manin and my advisor Barry Mazur in developing and applying the theory of modular symbols and the construction of p-adic L-functions of modular forms. At the same time, Manin served as a conduit for mathematical news from the Soviet capital.

So it was only natural that, one year past my PhD, I stopped by the Steklov Institute on my way to the Helsinki ICM, to pay a courtesy call on the stranger who had already done so much to shape my mathematical taste. I may have written to warn Manin of my coming but, before she allowed me to wait for the Professor in the library, the woman at the desk by the front door made it clear to me that I had no business being there.[5] Much later I learned that my unplanned visit had subjected Manin to a lengthy bureaucratic nightmare, and that it could have been worse. He must have realized this immediately, but he gave no sign of concern, instead inviting me to join him in an extended conversation about mathematics in Cambridge and Paris, opera in Moscow, and the upcoming ICM. Practically no one from the Moscow school would be allowed to travel to Helsinki, but Manin urged me to spread the word at the Congress that a contingent of colleagues from Moscow would be gathering and hoping to meet visitors in Leningrad, which at the time was accessible for a day trip without a visa. When our conversation ended I drove him home in the Citroën 2 CV, rented in Brussels, that I parked across from the Steklov Institute—after I changed a flat tire, an operation that Manin found incomprehensible.

Eleven years later, Manin hosted my stay for an entire academic year at the Steklov Institute through the National Academy of Sciences exchange program. The optimism of the first years of perestroika had definitely faded and Manin spent most of the year traveling. I mainly experienced his influence second-hand, by attending his 2–3-hour-long seminar at Moscow State University, where Russian colleagues of my generation constantly interrupted each other; someone entering in the middle of a session would have been at a loss to determine who was the speaker and who the audience, a Bakhtinian polyphony that was inspirational but also deeply disorienting. At that point, Manin was mainly thinking about what I thought of as mathematical physics, and I was not. So when we did meet our conversations were mainly about literature and culture in general, as well as politics, about which—unlike most of the Russians I met that year—he had few illusions. He was a big fan of *The Bonfire of the Vanities*, which was still new at the time, and wanted to know more about Tom Wolfe. I cautiously recommended *The Electric Kool-Aid Acid Test* but I don't know whether or not he ever read it.

I have to confess that our conversations that year, and in subsequent years, always left me vaguely uneasy. Manin had a habit of ending his sentences, whatever the subject, with an expectant look. This may just have been Russian body language, but I read it as an invitation to continue the conversation with as much insight and authority as he had just done; the result, I'm afraid, was generally an awkward silence. This only happened infrequently after I moved to Paris in the 1990s, but I did begin a fruitful indirect dialogue with Manin, or rather with his way of thinking, notably through his book *Mathematics as Metaphor*. Faced with predictions of a future dominated by digital technologies and their corporate masters, I take heart from this quotation from an interview[6] published in 2015:

Think! Otherwise no Google will help you.



Michael Harris

Nicholas M. Katz

Manin's 1958 paper "Algebraic curves over fields with differentiation," written while he was still an undergraduate, was transformative, as was his use of those ideas in his 1963 paper on the function field version of the Mordell conjecture. It had been known since Legendre (1811) that the periods of a family of elliptic curves satisfy a differential equation. Once one has a contour integral representation of the periods, one obtains the differential equation by "differentiating under the integral sign." Although this is already present in Fuchs (1869), it is not until Manin that this process of differentiation is explicitly applied, in

[5] *I waited long enough to read an article by Andrianov and Kalinin in the latest issue of* Mat. Sbornik*; this got me started on the projects that would occupy my attention for the next five years.*

[6] *Yuri Manin: "My Life Is Not a Conveyor Belt," in* The Human Face of Computing*, Advances in Computer Science and Engineering, C. S. Calude, ed., Singapore: World Scientific, 2015, 277–286.*

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the case of curves, to the space of differentials of the second kind mod exact, a space which in characteristic zero gives a purely algebraic way to view the H^1 of a smooth, projective, geometrically connected curve. In a July, 1965, footnote to his paper on de Rham cohomology, Grothendieck says that Manin's idea "strongly suggests" the existence of (what came to be called) the Gauss–Manin connection in complete generality. One could argue that the theory of D-modules has its beginnings here.

Manin's 1963 paper on commutative formal groups in characteristic $p > 0$ was also hugely influential. It made available to a wide audience the theory of Dieudonné modules, their classification up to isogeny, and their relations to Newton polygons for abelian varieties. It clearly suggested their relation to what would become the crystalline H^1 in the case of abelian varieties.

In 1978, I delivered Manin's plenary address at the Helsinki ICM, since he hadn't been allowed to attend. I didn't have the pleasure of meeting Manin in person until 1988, when he gave a lecture at U. Penn., but by that time I had already been deeply influenced by his work for 25 years.

Nicholas M. Katz

Ralph Kaufmann

My first encounter with Yuri Ivanovich Manin was through his writings. In particular, his book on quantum groups was a revelation for me and exemplified just the right balance between motivation and precise abstract structures. In this form of beauty, mathematical knowledge transcends and can become a metaphor. It was clear to me, when Manin came to Bonn to be a director of the Max Planck Institute, that he would be an ideal PhD advisor for me. At that time, there was a seminar that was run by Yuri Ivanovich, Werner Nahm, and Don Zagier on the interactions of mathematics and physics in which I delivered a talk on the Virasoro algebra. After this formal encounter, I mustered the courage to ask him about becoming his student. He informed me that, since he had not yet known me for an extended period of time, I would have to come back in two weeks for a kind of oral entrance exam. Daunting as that was, I passed and began working with him—not, as I had initially thought, on noncommutative geometry, but rather on quantum cohomology.

My first results were on the product structure of quantum cohomology and were included by Yuri Ivanovich in the appendix of a paper he wrote with Maxim Kontsevich. As Yuri Ivanovich put it, pointing to the title page: "you do not appear here, but here." One of the proudest memories of this time was when Yuri Ivanovich wrote me a note saying that my results for higher Weil–Petersson volumes were "beautiful mathematics." This turned into a joint work of Yuri Manin, Don Zagier, and myself, which was the start of a program of study of such volumes and generalizations such as those by Mirzakhani. The years in Beuel, where the Max Planck Institute was located at the time, were full of interactions which shaped me mathematically and philosophically.

I am often asked how it was to have Yuri Ivanovich as an advisor. My response is that for me he was the ideal advisor, the frequency of discussions and mutual understanding being completely in resonance. There was one period, where I did avoid my advisor, and this was when I took some time to write a master thesis in philosophy. I was asked what I was doing, and after three months of dodging the question, when I was finished with the thesis, I came clean. His reaction captures his outlook on the world and his role as an advisor. He first asked what it was about. My answer, "On Frege," evoked an appreciative expression and, as I remember, some words like "aha very interesting" that were followed by a more stern look and the directive to get back to mathematics voiced in the instruction "no more Frege."

Through the years, I was often back in Bonn at MPI which then moved across the Rhine. Entering into Yuri Ivanovich's office was like entering into the hallowed halls of mathematics and culture. His signature shelving of mathematical papers juxtaposed with a collection of books from a wide range of fields reflecting his broad interests. I vividly remember the discussions about mathematics, logic, and the world in his office, with light flooding through the windows overlooking Beethoven and the Münsterplatz.

On a personal level, Yuri Ivanovich and Xenia Glebovna opened their home to my wife Birgit and myself as well as to our sons Julian and Adrian. We still have a book entitled *Leibniz für Kinder* at home which they brought as a gift. These common times and stimulating conversations are many of my happy memories. The first time I met Yuri

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Ivanovich, the image of Hesse's Siddhartha becoming wise while contemplating the river directly came to my mind. Seeing him and Xenia Glebovna in their apartment on the Rhine is an almost prophetic fulfillment of this initial association.

In closing, I will give an excerpt of a speech he delivered when he was inducted into the Order Pour le Mérite for Sciences and Arts (Orden Pour le Mérite für Wissenschaften und Künste)—which he had entrusted me to translate into German for him—that characterizes Yuri Ivanovich perfectly.

> "All my intellectual life was molded by a noble tradition which I somewhat carelessly called the Enlightenment Project. The base of this tradition is the belief that human reason has the highest value, and that the dissemination of science and education will help produce better human beings than we are, who will be living in a better society than we live." (Manin 2007)

Ralph Kaufmann

Matilde Marcolli

It was a long voyage and a beautiful one. Our collaboration started almost immediately after I joined the MPI faculty in the summer of 2000 and lasted to the very end, with our last two joint papers posted on the arXiv less than a month before Yuri died. From the very beginning it was Yuri who started to refer to this as "the last voyage of Ulysses," from Dante's *Divine Comedy*, which he liked to read in the original Italian. If you download from the arXiv the source file of our very first joint paper "Continued fractions, modular symbols, and non-commutative geometry" (math/0102006), you will find it right there at the beginning, hidden by a % in the tex file, *"de remi facemmo ali…,"* the last voyage of Ulysses. We read that canto of the *Divine Comedy* together, and I memorized it to be able to recite it for him, which he asked me to do a few times over the years.

We ended up with 25 joint papers, written over a span of 23 years, though several of them were in fact concentrated in these last few years. Curiously, but perhaps not entirely surprisingly, we collaborated a lot more intensely after I left the Max Planck Institute and relocated to California, than we did during the years when we were both working in the same institute and seeing each other daily. The way I like to think of all these papers is as the outer windows to an inner space, to a very personal place, where a long dialog was unfolding through these two decades, a continuing conversation that cut across the boundaries of different fields and disciplines, across our distance in space and time, and the passing of the years. It was a very special and stable place, filled with its own very special affective as well as intellectual intensity.

We had been in the habit of spending New Year's Eve together at his home in Bonn, every year since I first arrived in Bonn. We continued with our regular New Year events after I moved to California. Every year I returned to Bonn in December, right at the end of our fall term, usually in time to give the last talk of the year in Yuri's "Algebra, Geometry, and Physics" seminar, and I would stay until early January, when I made my return for the start of the next term at Caltech. I always tried to make sure to have something new for the talk I would be giving for him upon arriving in Bonn, something that would be different, surprising, and entertaining. Year after year, I brought back cosmology, linguistics, information theory, and various unexpected motivic incarnations. During those winter breaks, Yuri and I would finish up our current ongoing project (many of our papers are posted to the arXiv on the first of January) and we would start discussing the next thing to think about. We met everyday to work together, including on Christmas day, first mostly at the MPI and in the later years mostly at his home. Sometimes a new project would start in relation to whatever little mathematical trophy I was bringing back from my previous year in California. That's how we ended up writing our own linguistics paper in 2016, for example, just after I had taught my first linguistics class at Caltech in 2015 and had written my first linguistics papers out of that experiment. Other times a new idea came up as a way of returning to previous conversations that had remained dormant for some years. To me it always felt like going home, to a unique place that was always reliably there…until that one time when suddenly it was forever gone.

In Homer's *Odyssey*, the voyages of Ulysses draw a chart of encounters with the multiform liminal creatures of Greek mythology, composite figures that cross the designated boundaries of the realms of nature, the human,

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the animal, and the divine. Our mathematical cartography is usually similarly split into supposedly impenetrable boundaries, and yet there too a pantheon of hybrid chimeras can be envisioned, tantalizing, elusive, luring like the siren's song: noncommutative boundaries of arithmetic varieties, fields with one element, Arakelov holography, phase transitions and noncomputability in spaces of codes, categorified dynamics of neuronal networks, modular and elliptic curves in cosmology, Grassmannian semantics, and other such magical creatures. In Dante's last voyage, Ulysses sails right through the pillars of Hercules, the established and impassable frontier of the system of knowledge of the ancient world, embarked on a heroic, but not solitary, intellectual quest. The voyage ends tragically in a final storm, with the rising mountain of the netherworld looming large on the distant horizon.

There was no grandiose plan guiding our long voyage of exploration, no holy grail hypothesis to chase. It was a peaceful state of meditation, a voyage of curiosity rather than a conquering campaign. It meant a lot to me to be able to finally discover, through our joint work, that mathematics does not need to be a bloody battlefield out there, does not have to be governed by aggression, territoriality, violence, like I have too often experienced it elsewhere. It can also be that peaceful shared inner space and that long shared voyage of discovery beyond *"dov' Ercule segnò li suoi riguardi acciò che l'uom più oltre non si metta."*

Matilde Marcolli

Ivan Penkov

I first met Yuri Ivanovich Manin at the beginning of September 1978, when I started my second year as an undergraduate mathematics major at MGU. My father, a Bulgarian probabilist and statistician who had good connections to some of his Soviet colleagues (despite being firmly anti-Soviet in his political thinking), had attended a lecture of Manin in the spring of 1975 and was deeply impressed by his intellectual power and personal charisma.

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In September 1978 I had just returned to Moscow after two very miserable years of military service in Bulgaria and was looking for an intellectual challenge, when I remembered my father's remarks and quickly decided to give Professor Manin's seminars a try. Certainly, I was not free of fear that this might turn into a humiliating experience as fellow students had told me that only the most brilliant students can keep up with the pace of Professor Manin. Nevertheless, my curiosity was strong and I went to the opening meeting of his seminar on number theory. That fall, the seminar's main topic was Mordell's conjecture (which was proved five years later, in 1983 by Gerd Faltings). Manin gracefully planned out several consecutive talks on the subject and distributed them among his advanced students. The very first introductory talk, basically following material from the book *Algebraic Number Theory* (J.W.S Cassels and A. Fröhlich, eds.), remained unassigned, when I raised my hand and said that I could try to understand and prepare some basic facts from the book. Manin looked surprised and quickly figured out that I was a second-year student who basically knew nothing. Nevertheless, with his confidence-inspiring intellectual generosity which I have experienced ever since, Manin gently allowed me to try and added that Misha Tsfasman, a graduate student, should support me in this endeavor.

This was a magical moment for me, and it basically determined my entire life. Two weeks later, I gave a somewhat lousy talk (without the help of Misha it would have been much worse) and became a steady participant in Professor Manin's seminars.

Yuri Ivanovich and I became closer by 1980 when he visited Bulgaria and I happened to be his local guide. Over many years, we loved to remember our visit to Plovdiv, together with Yuri Ivanovich's wife Xenia Glebovna, where we ended up spending several days more than planned because my Soviet-made car broke down and could not be repaired. As a result, we had the gift of three joyful days together in a fascinating town. Later on, in 1993 I had the great honor of Yuri Ivanovich visiting me at the University of California at Riverside, where I had just received tenure. At that time, my home was in Lake Arrowhead, in the mountains of Southern California. Yuri Ivanovich and Xenia Glebovna were of course invited to spend a day in the mountains, and I was the driver. On the way up, Yuri Ivanovich started to get uncomfortable and asked Xenia Glebovna to pass him some books and folders from the back seat so that he could sit on them. I could not understand what was the matter, until I realized that Yuri Ivanovich was sitting on a heated seat, turned on to the maximum, on a warm California day. We all laughed loudly when we figured out what had happened: in the early morning I had driven my son to school on the

mountain, and he had turned on the passenger seat heater. Driving back, I could not see whether the passenger seat was on or off, while Yuri Ivanovich did not realize that a seat in a car could be heated! We made it to Lake Arrowhead in a good mood, and I still remember this incident.

We, the students in the Mekh-Mat of the late 70s and early 80s were spoiled in the sense that there was a cohort of superb professors whom we could choose as advisors: A. A. Kirillov, V. A. Arnold, S. P. Novikov, etc., and of course Manin. Choosing a scientific advisor can be like falling in love. When this happens, one star in the sky starts shining brighter and more enticingly, as if it were the only star in the sky. From the day of our first meeting in September 1978 until our last conversation on Skype around December 17, 2022, Manin was this special star on my scientific and, more generally, intellectual sky. Manin had the ability to confidently guide his students and collaborators through mathematical landscapes and to gently demonstrate by example how to unravel mathematical truths. Having been influenced by Grothendieck, he looked for the "abstract core" in every mathematical theme. He would then study this, possibly new, mathematical structure with all available tools. Very much like turning on a light!

I observed this in person when Manin introduced supergrassmannians and flag supervarieties. At that time, around 1980, supergeometry was a hot topic and many key structures had not yet emerged. With the confidence of a master, Manin defined complex supergrassmannians as supervarieties representing certain natural functors and proved their existence and smoothness. Then we, his students, started carrying out detailed studies of the structure of supergrassmannians and flag supermanifolds and eventually built a somewhat incomplete Bott–Borel–Weil theory for flag supermanifolds. The problem of computing all cohomology groups of all line bundles of flag supermanifolds is still open today.

One may talk about the "chemistry" in a relationship. This applies in particular to the relationship of a scientific advisor with his or her students. In my case, this perfect chemistry was the feeling of constant support from Manin, which unlocked my self-confidence and eventually turned my mathematical dreams into mathematical reality. My (totally subjective) feeling was that we had similar mathematical dreams, and the process of collaboration in the only three joint papers that we wrote, was the process of sharing our dreams. My advisor's acknowledgment of my modest mathematical thoughts was absolutely crucial for my becoming a mathematician. Another fantastic privilege of being Manin's student was meeting some of the most prominent mathematicians of my generation. It suffices to mention Alexander Beilinson and Vladimir Drinfeld. Seeing them "in action" in the Manin seminars was a life-shaping experience. Early on I knew that meeting Manin in 1978 was the blessing of my life, and this has carried me through life ever since.

Ivan Penkov

Vadim Schechtman

My brightest memories of Yuri Ivanovich Manin belong to the blessed time at the end of the 1980s when the miraculous breath of freedom came to our country. I remember the remarkable beginning of his advisor Igor Rostislavovich Shafarevich's talk at the meeting of the Moscow Mathematical Society in 1987 devoted to the 50th anniversary of Yuri Ivanovich. Shafarevich had mentioned five great mathematicians: Golod, Anosov, Novikov, Arnold, and Manin who all had been born around 1937 and entered Moscow University in 1953. I remember meeting him and his good lady Xenia Glebovna in their small apartment on Vernadsky Ave. I remember our collaboration with Yuri Ivanovich on higher Bruhat orders which originated from the Zamolodchikov's string analogs of the Yang–Baxter equations. I remember sitting with Sasha Beilinson and Yuri Ivanovich writing the foreword to the *K-theory in Moscow* collection (Lecture Notes in Math. 1289); the other contributors to this volume were Boris Feigin, Boris Tsygan, Vladimir Hinich, and Mariusz Wodzicky. We ended our foreword with a joking citation from *Eugene Onegin*. Yuri Ivanovich was a man of a universal culture. Here is an example of what I have learned from him. The first phrase of his article on quantum groups sounded like "The aim here is to resist the charismatic influence of Drinfeld." At the time, I did not know the meaning of the word "charismatic;" so my interpretation was that Drinfeld was mischievously trying to badly influence his teacher Manin. I was a bit mystified, and this gave me an occasion to learn the true meaning of this word. Sitting with Yuri Ivanovich and Xenia Glebovna on the balcony of their apartment in

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the charming city of Bonn, viewing the majestic Rhein was a true blessing. …und ruhig fließt der Rhein.

Vadim Schechtman

Alexei Skorobogatov

I became Manin's student in 1980 after Andrey Levin insisted that I should attend Manin's seminar. At that time, the overall scenery of the Department of Mechanics and Mathematics at the Moscow University was pretty drab: one would not normally bump into any luminaries who were around but not visible. Almost none of them were involved in undergraduate teaching. What one saw was a mass of compliant mathematical functionaries in indistinguishable grey suits, some of them prominent in notorious entrance examinations designed to stop Jewish students from being admitted to the University. The atmosphere was not very inspiring.

The experience of Manin's seminar was overwhelming, it was akin to going abroad. People just looked different, I could not understand a word they said, and it was all immensely attractive. Immediately after Manin accepted Andrey and me, alongside Mikhail Kapranov and Vera Serganova, he told us to read Sasha Beilinson's famous paper "Coherent sheaves on $\mathbb{P}^n$ and problems in linear algebra." I fondly remember the study group that Sasha organized for us where he carefully explained the notion of a derived category. This was all very nice, but I hardly knew what a sheaf was. The mathematics of Manin's seminar was amazing, but it was an expanding universe! It made things worse for me that at the time Manin had a keen interest in physics and was moving away from his earlier work in arithmetic geometry. I decided that mathematics was hard enough and if I wanted to survive I had to leave physics alone. At the time, my discussions with Manin touched on Hitchin's work on twistor spaces, the Calabi conjecture recently proved by Yau, and the beginnings of hyper-Kähler geometry. Although they did not lead me directly to do any research by myself, soon I found a problem that I was able to solve. The result was a two-page note in which I proved that the Kuga–Satake abelian variety of a Kummer surface attached to an abelian surface A is isogenous to a power of A. I remember a useful discussion with Misha Kapranov in a university canteen. Only later, I realized that it was always like this: Manin eagerly discussed mathematics with his students but did not give them problems to solve. Unless one was fortunate enough to do research on a subject of Manin's current interest, one was left to learn from preprints and other students. Crucial for my PhD was my joint work with Mikhail Tsfasman. I started working with Jean-Louis Colliot-Thélène during his visit to Moscow in 1986.

Manin was the epitome of cool. He dressed impeccably, and his mathematics was elegant. Mathematics was always at the center: what mattered was a "correct" approach to a problem—a cleaner, more transparent and instructive proof based on an idea, a proof that would allow us to see farther and clearer. It did not matter whose proof it was; mathematics was never an exercise of one's power or a strife for recognition. If I had to come up with one quote from Manin it would be this: "a good proof is a proof that teaches us something." I remember how he called me once to tell me that he had read my paper and thought that it contained the "right" way to prove a certain result on which he had worked earlier. I would even say that mathematics for Manin was not about solving problems: this would happen almost automatically if we achieved deeper understanding and asked the right questions. Manin had a unique ability to see a far-reaching theoretical potential in mathematical facts and observations that could be perceived by others as specific and isolated, and he was happy to share these visions. His research papers are scattered with open questions and suggestions; the same was true of his courses and seminars.

It was always a great pleasure for me and my wife Anna to visit Yuri Ivanovich and his beloved wife Xenia Glebovna in their appartment in Bonn. We talked about poetry, philosophy, and all sorts of things. I will finish by saying that besides being a towering figure in mathematics, Yuri Ivanovich Manin was a very good person, honest, sincere, gentle, and sensitive, with a wonderful sense of humor. All who had the good fortune to know him will miss him a lot.

Alexei Skorobogatov

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Vyacheslav V. Shokurov

My recollections about Manin start with the Evening Mathematical School (EMS) meetings, an activity for schoolchildren organized by E. B. Dynkin. Starting in the fall of 1963, Dynkin invited several young bright stars of the faculty of mechanics and mathematics of Moscow State University to EMS. One of them was Yuri Ivanovich Manin who was only 26 years old, and just recently got his doctoral degree, the habilitation. He lectured not only on mathematics but also on linguistics: how to guess the meaning of a Serbian word. I understood that Manin was not only a mathematician but also had an interest in humanities. Later I found out that he was much broader than I could imagine.

Over time, lectures by professors of Moscow State University became a regular activity of EMS during the usual daytime classes of school No. 2, where both activities were held. During my senior year 1965–1967 in school No. 2, they were delivered by Y. I. Manin and E. B. Vinberg. Both were wonderful lecturers, and the schoolchildren liked them very much. Both were algebraists, but their styles were slightly different and complementary: Manin was more functorial, while Vinberg was more concrete. These lectures laid down a foundation of mathematical education for me and my classmates (B. Dubrovin, I. Cherednik, and S. Dobrokhotov).

Naturally, at MGU I started attending Manin's special courses and seminars. This was a remarkable period of the Moscow mathematical school and perhaps even of all of Soviet mathematics. Manin was one of the prominent figures of the time. Of course, in the late 1960s and early 1970s there were more famous and established mathematicians in Moscow, e.g., Kolmogorov, Pontryagin, Markov, Gelfand, Shafarevich. Manin belonged to the next generation of brilliant mathematicians. He was a very versatile person, and not only in mathematics. His lectures on recent mathematical advances, e.g., on Matiyasevich's solution of the Hilbert 10th problem, were accessible to a wide mathematical audience and at the same time deep and insightful.

In 1968, I became Manin's student, first as an undergraduate and later as a graduate student, until 1975. My PhD thesis was about modular symbols, Kuga's modular varieties and Shimura's integrals. However, later my research shifted to a previous interest of Manin and his former student V.A. Iskovskikh—birational geometry.

In the fall of 1968, Manin gave me some notes about the geometry of canonical embeddings of algebraic curves. It turned into my first publication, concerning a theorem of Noether–Enriques on canonical curves. Actually, I had started to work on this paper only at the end of 1969. Manin's notes were based on a paper by Babbage from 1939. In school, I had studied English and a little bit of French, but not enough even for mathematical literature. Manin proposed that I learn not only English and French, but also German. Another more important obstacle was my lack of knowledge of modern techniques in algebraic geometry: sheaves and their cohomologies. Fortunately, Manin already lectured on this subject in his courses and corresponding literature already appeared in mathematical bookstores of Moscow.

By the summer of 1970, my paper was finished and I came to show Manin the final version. Usually in the summer, Manin leased a cottage near Moscow, and it took some time to get there. Since I was young, I was neither tired nor hungry after the trip. Nonetheless, according to Russian tradition, Manin fed me and only after that we started to discuss the paper. Soon the paper was approved by him with a few minor remarks. One of which was about the acknowledgment. In the original version the author thanks Manin for help in writing the paper but he preferred a more modest role: "for posing the problem." Actually, the paper and Manin's guidance became important in my subsequent research because canonical curves are curve sections of K3 surfaces and Fano 3-folds.

After that, Manin suggested I investigate moduli of curves of general type. These moduli were just rigorously introduced by Deligne and Mumford. However, after Manin turned to modular forms and I finished my PhD thesis, my interest deviated from Manin to Iskovskikh—to birational geometry. Only recently, I got interested in moduli but mainly not of curves and not only of general type.

In the fall of 2017, I visited the Max Planck Institute in Bonn to congratulate Manin on his 80th birthday. I had a few meetings with him and delivered a talk in his seminar about complements and recent advances of Caucher Birkar. I had the feeling that these would be our last personal meetings. Once he told me during this visit that he was thinking about noncommutative modular symbols. I am not aware of his results in this direction but certainly this could be interesting to new generations of mathematicians.

Vyacheslav V. Shokurov

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Michael Tsfasman

I first met Manin as a sophomore, when I started to attend his courses and seminars, understanding as yet almost nothing. My impression was mostly of how he lectured. In his hand he had a sheet of paper on which he had the plan of what to write on the blackboard and where. Then he took the second sheet, almost empty except for a rectangle to write on the erased piece of the board. Usually, he gave two courses and led two seminars. Every year the topic of his courses changed. Later he would tell me that every seven years one should completely change the subject in order not to be bored. For a long time, these subjects were mostly centered in between algebraic geometry and number theory, then crept towards mathematical physics. The sportive side of mathematics was quite foreign for him, his goal almost never was to solve a difficult problem, though sometimes he did it, but rather to understand the subject. I would even dare to say that to understand the object, he took an important piece of mathematical reality and tried to observe and study it from different angles. For Manin, mathematics does not stand alone, it is a part of science, and science is a part of culture. He was a man of culture, rather a scholar than just a scientist. His interests were as wide as we see only in the time of Ancient Greece or the Renaissance. Besides mathematics and physics—to mention just a little part—he was interested in biology, in the origin of human speech, in medieval French, in poetry. His poems and poetry translations are not many, but they are of extremely high quality.

Yuri Ivanovich was an illustrious teacher. I asked him to become my PhD advisor when I was 25, rather late by Moscow standards, and having already a published paper. I had just read his marvelous book *Cubic Forms*, was fascinated by its subject, and wanted to choose it as my subject. He said he would be honored to have me as a student (you can imagine how pleased I was) and warned me that since he had already shifted to another subject, it would be more difficult for me. Then I told him that I was on a kind of a blacklist of the Soviet regime, and it could be difficult for me to enter the doctoral school of Moscow State. He said we should try anyway and helped me to overcome the difficulties. A year and a half later, I was desperate, half of my PhD time had elapsed with no interesting result obtained. I dared to ask him how he would define what is a good thesis. After a short reflection he answered, "A good thesis is what a good PhD student does in three years." I was soothed and soon started getting some new results in arithmetic of rational surfaces. A year later at his seminar he asked a question about curves over a finite field, I managed to answer it, though I had never before worked over a finite field. He told me that the question is related to coding theory and gave me a book on it. In a while we wrote our first paper on algebraic geometry codes, learning the definition of a code on the way. This was by no means my best paper, but definitely my most cited. After that, algebraic geometry over finite fields and global fields became the center of my interest. My previous activity was crowned by a large survey on the arithmetic of rational varieties that I wrote with Manin.

At some point, Yuri Ivanovich told me that there is something we lack in the professor-student relationship. In the nineteenth century, professors had a so-called "*jour fixe*:" once a week every student could visit his professor on social grounds. And the series of Manin's *jours fixes* started. Every Friday his students, in many cases with their wives, came to his small one-bedroom in the southwest of Moscow. It was forbidden to discuss mathematics. Each time he proposed a subject to start with, it could be some development in science other than mathematics, the possible future influence of computers on social life, the origin of speech, etc. Once I was late and Manin explained to me that they were discussing why it was so easy for us to do mathematics, and so difficult to communicate with people. My reaction was that it was difficult for me to do math and quite easy to communicate. Another impression of these *jours fixes* was the hospitality and active participation of his wife. For my wife and me, these meetings were very important. One of these Fridays fell on Manin's fiftieth birthday. The situation in the country had drastically changed with perestroika, the iron curtain starting to fall, and the air of freedom sweeping out the suffocating air of the Soviet regime. His reaction to these changes was that of pure joy. It reminded him of his youth. One of his reminiscences was that he first came to Moscow at the end of Stalin's rule, and next soon after Stalin's death. The visible difference was that the first time the whole of Moscow was full of solid fences, and the next time they somewhat disappeared.


Michael Tsfasman

New Russia happened to be far away from his dreams, and he became pessimistic. I would have liked him to be wrong.... Once he said to me that the experience of his generation is totally negative and should be erased from the memory of younger generations. Manin spent the last three decades partly in the USA and mostly in Germany. He used to come to Moscow almost every year and I cherish our rare meetings.

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Alexander Voronov

Yuri Ivanovich Manin was a remarkable thinker, mathematician, and advisor. I am glad that we, his students, carry his legacy in the way we do mathematics, teach, and work with our own students.

Manin had scores of students back in the day and at times was even overwhelmed by them. When I approached him for the first time and asked if he would take me as a student, he told me he was not accepting any new students, as he already had too many (in fact, he had 11 just within the class of students one year more senior than I). Since my facial expression apparently showed that my whole world of hopes had just collapsed, he nevertheless gave me a chance. No wonder that the admission interview promised to be tough.

Manin asked me what mathematics I had been learning. Among other things, I told him that I was attending a topics course on the "Winnie-the-Pooh conjecture," given by Kostrikin. Manin's eyes sparked with curiosity. So, he asked me to describe the problem and explain the unusual name. The name is based on a verse from Milne's classic *The House at Pooh Corner*.

> And the cuckoo isn't cooing,
> But he's cucking and he's ooing,
> And a Pooh is simply poohing
> Like a bird.

In a Russian translation:

Возьмём это самое слово «опять».
Зачем мы его произносим,
Когда мы свободно могли бы сказать
«Ошесть», и «осемь», и «овосемь»?

The name is a pun because the words "опять, ошесть, осемь," and "овосемь" are pronounced the same way one would in Russian pronounce A_5, A_6, A_7, and A_8, which refer to the classical Lie algebra series A_n. The original English verse is also a pun, but the reference to the series A_n is totally "lost in translation:"

The conjecture was related to the fact that the existence of an orthogonal decomposition for a complex simple Lie algebra of type A_n had been settled for A_6, A_7, and A_8, but not for A_5 (and still has not). I successfully described the problem and recited the verse to Manin, but the real test came later: he asked me whose Russian translation it was. I started digging into the deep corners of my memory, but could not find anything. So, I came up with a wild guess: Boris Pasternak. I knew that Pasternak had made numerous Russian translations of English poetry (Shakespeare, Byron, Keats, to name a few), and it sounded to me like a reasonable guess. Yuri Ivanovich, in his gentle manner suggested that the translator was "probably" Boris Zakhoder and nevertheless immediately told me that he was admitting me as a student. My initial shame was quickly superseded by the feeling of happiness….

When I was Manin's student, he did not have regularly scheduled meetings with his students. They happened on an ad hoc basis, initiated by Manin. Sometimes, he would just see me in his seminar or call me up and say "Sasha I have an idea of a project that might be interesting for you. Would you like to come over to my place next Wednesday at noon?" That was exciting as I anticipated Manin outlining a beautiful piece of mathematics in front of me.

Manin held the plank of general culture very high. He was well versed in many languages, which was rather unusual for the time of isolation of the Soviet Union from the rest of the world, known as the Iron Curtain, and the dominance of one language on the vast lands of the Soviet empire. Once, when I only started to attend his seminars, still eyeing him as a prospective advisor, I arrived at a seminar a few minutes late, just to hear a tape recorded voice speaking German, while everybody including Manin, sat in total silence. This lasted for some long minutes with very few people understanding a word. At the end of the recording, Manin, who was obviously inspired by it, explained that it was Hilbert speaking on his own twenty-first problem on what had become known as the Riemann–Hilbert correspondence. That seminar was actually a joint venture with Sergei Gelfand, which resulted in them writing their famous book on Homological Algebra, which ends in a beautiful chapter on D-modules.

Another time, at one of the student-advisor meetings at Manin's home, Manin said: "I suggest that you read a modern French course on algebraic surfaces, such as this one,"—and handed me over Beauville's *Astérisque* volume on complex algebraic surfaces. When I exclaimed: "But it is in French!"—Manin looked at me with genuine surprise: "Oh, Sasha! Do not you read French?" It was said in an unassuming way, but I realized that if I wanted to be in this intellectual circle in which reading mathematics in French is the norm, I'd better do it. So, within a month, I was reading Beauville's text with great interest and secret pride.

At some point, Manin told me that he was turning 50 and decided it was time for him (as an old man, as I thought back then) to connect to the younger generation. This is how he and his wife Xenia Glebovna started those Friday gatherings, later known as *jours fixes*, in their apartment with numerous students and their significant others. Xenia Glebovna played a wonderful host role in these. Her warmth, energy, and wit often helped break the ice and

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engage some of us less-sociable mathematicians in interesting nonmath discussions initiated by Yuri Ivanovich.

In retrospect, the six years I spent being Manin's undergraduate and then graduate student, were packed with mathematical events. There was amazing mathematics happening in front of my eyes, as Manin created new mathematics for his weekly lectures in various topics courses, and you would be among the first people on Earth to learn about it.

Manin's 1986 "critical dimensions" paper is especially dear to my heart. The paper pointed at the critical dimension 26 showing up in the Mumford isomorphism $\lambda_2 \cong \lambda_1^{\otimes 13}$ of line bundles over the moduli space of algebraic curves, directly related to the Polyakov string measure. I took it as a challenge to look for a similar statement in supergeometry, which led to proving the super Mumford isomorphism $\lambda_{3/2} = \lambda_{1/2}^{\otimes 5}$ in my PhD thesis.

These are just a few random flashes of memory I recollect from these years, which made an enormous impression on my future life, both professional and cultural. And now, many years later, I feel lucky to be a member of Manin's school of Enlightenment and carry on his wisdom, philosophy, and style, and pass it on to my students.

Alexander Voronov

Don Zagier

My first meeting with Yuri Ivanovich Manin was in Moscow in 1987, where I spent two months (one of about a dozen visits that I made to Russia in the Soviet and post-Soviet periods, but the only one that was more than a few days) that happened to include Yuri's 50th birthday. Of course at that time he was not yet "Yuri" to me, but "Professor Manin" or (since we were then mostly talking in Russian, mine being rather primitive) "Yuri Ivanovich." To my great surprise, since we had only just met, he invited me to one of the two birthday parties that he and his wife Xenia Glebovna gave in their Moscow apartment. One of these was a more private affair, for his students and intimate friends, and included poetry and maybe even some dancing, while the other, the one I attended, was for further colleagues and friends, including new ones. The experience was unforgettable, in particular because his apartment was like a beacon of culture in the midst of all the drabness of Soviet Moscow, overflowing with books in a multitude of languages and on a multitude of subjects (including, to my great delight, even several frivolous novels in English and other languages, not just books on intellectual subjects), and the conversations with him and the others were of a similar level. Altogether it was more like a salon in Paris in some earlier epoch than like anything I had ever experienced in the United States or in Europe.

My two months in Moscow were one of the high points of my life, not just because of meeting with Manin, but also because of the way mathematics was done there, which I had never seen before and which perhaps has never existed anywhere else, before or since. Almost none of the many people I met there worked in their university or institute office, and indeed many (like Sasha Beilinson) did not even have an affiliation with a mathematical institution. Instead, mathematical discussions took place in people's homes, often in their kitchens, with people dropping in unexpectedly and unannounced, and with an unheard-of level of intensity. It seemed to me then, and I still feel now, that this high level was possible, not despite but precisely because of the difficult external conditions and the fact that one was in no danger of being distracted by frivolities like shopping or going to restaurants (two activities that were more or less nonexistent). For most of the people I met, many of whom became friends for life, a normal academic life with normal academic duties was impossible, as was foreign travel and many of the other things that scientists in the West took for granted. The result was a hugely increased focus on the two things that mattered: friends and mathematics.

I have one other striking memory from this visit. On one occasion I attended a meeting of the Moscow Mathematical Society. There were two lectures as far as I can remember, a survey talk on Diophantine equations that I gave and a talk by Shafarevich on Yuri's mathematical work in honor of his 50th birthday. At the end of that talk somebody stood up and attacked Manin viciously, saying that honoring him in this way was a farce, that it was well known that he was a mediocre mathematician and that his most famous results were either wrong or due in essence to others. I was horrified and wanted to sink through the floor, but Yuri simply ignored the attack as if it had never happened. This was of course the only sensible way to deal with it, but not one that a lesser person could have carried out, and I was hugely impressed to witness it.

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Only a very few years later there came a turn of events that neither I nor anybody else could have predicted: the great thaw of the perestroika and glasnost years and the possibility for Soviet mathematicians to emigrate. I am pretty sure that the idea that Yuri might be tempted to come to Bonn and to the Max Planck Institute for Mathematics, despite the innumerable offers he received from the United States, came originally from me, since I had often talked with my wife Silke about my meeting with him and we hoped that he could be tempted by a city that was nearer physically and culturally to the world he would be leaving behind. In fact, both we and, separately, Friedrich Hirzebruch met with him and Xenia in the United States to "woo" him, and the courtship was eventually successful. Xenia tells the anecdote that at that time he was hesitating about whether or not to sign a contract that MIT had offered him and that Bob MacPherson, whom he knew well from many meetings in Moscow, told him "You know, Yuri, a contract here is not a prison—even if you sign it you are still free!" He eventually did sign it but nevertheless ended up coming to Bonn, to my lasting joy and that of Hirzebruch and many other colleagues. For several years, until the horrendous bureaucracy of the post-9/11 years forced him to stop, he also had an affiliation with Northwestern University in Illinois and spent several months there every year, always, I believe, staying in the same small hotel where they knew him and where he felt at home. But his main home, physically and intellectually, was the MPI in Bonn where he worked for 30 years and to which he contributed more than any other mathematician through his presence, his works, and his seminars.

During those years we became close friends as well as colleagues. We had innumerable mathematical discussions, though in the end we wrote only two joint papers, each with an additional coauthor (Ralph Kaufmann and Paula Cohen). I had hoped that there would be more, in particular in connection with periods of modular forms, which was a subject to which he had made crucial contributions and in which my own work (with John Lewis and others) was also of special interest to him, but this never happened.

Working with Yuri made me aware in a way that I never had been before of the two types of mathematicians, what Yuri's friend Freeman Dyson called the "frogs" and the "eagles": those who look at mathematics from a position in the grass and perceive everything from below, and those who look from a position in the sky and see everything from above. Needless to say, I belonged to the first category and Yuri to the second, but the new insight was that a collaboration between an eagle and a frog was not only possible, but in many ways even more fruitful than one between members of the same species. This became particularly evident in the paper that we wrote about modular forms and pseudodifferential operators (the third collaborator, Paula Cohen, joined only at the end to add a "super" version that Yuri was very keen to have). Unlike other joint papers that I have written in which the two authors sat repeatedly at the same table and worked out pieces of mathematics and pieces of text together, we worked independently and then met at frequent intervals to discuss our progress. What I remember is that on several occasions Yuri told me excitedly "Don, I have great news to report. The result that you showed me last week with a complicated computational proof I can now understand purely conceptually, with no need for computations at all!" On other occasions, I would report with equal enthusiasm "Yuri, I also have great news to report. The result that you showed me last week with a difficult conceptual proof I can now prove by direct computation that anybody can understand, with no need for any thinking at all!" And, rather amazingly, this worked out well: what he thought of as progress and what I thought of as progress were exactly opposite, but somehow everything converged in the end to results that were pleasing to both of us.

Another thing that I remember with fondness from our collaboration was that Yuri once told me that he had a private tradition of proving a "Christmas theorem" as a Christmas present to himself every year if he could find one, which of course he almost always could. I loved this notion and proved my own Christmas theorem that year (I think it was 1993), the proof of a complicated combinatorial identity for our joint paper that I had discovered experimentally some time before. But I am afraid that I have not kept up the tradition since. Perhaps I can try again in the future, as a tribute to him.

As a mathematician, as an intellectual, and as a person, Yuri Manin was one of the few truly great people that I have ever met. Like so many others, I would like to thank him for having enriched my life and shown me the level that a human being can achieve.

Don Zagier

Yuri G. Zarhin

The first time I saw Yuri Ivanovich was at a meeting of the Moscow Mathematical Society in Spring 1969 where John Tate gave a talk in English. Manin translated, and was elegant and charming.

The following semester I started to attend his seminar, and in Fall 1970 I became one of the *Maniniacs*—that is how fellow students called Manin's advisees, because the latter were so impressed by their advisor that they talked about him to everybody. I still remember the feeling of a forthcoming feast on Wednesdays and Thursdays—the days of Manin's seminars and special courses at Mekhmat. Besides official courses and seminars, we usually met every other week. I was told that I should ask questions and I tried to do my best. Usually, my questions resulted in a half-hour lecture by Yuri Ivanovich that I tried to comprehend in the following days and weeks. Once, trying to invent a good question, I asked whether two abelian varieties A and B over a number field K are isogenous if their ranks over every finite field extension of K coincide. (I had noticed that the equality of ranks means that the Galois modules of all algebraic points of A and B modulo torsion are isomorphic.)[7] After this question, Yuri Ivanovich decided I should learn *heights* and I started to read about Weil's heights and distributions, Néron–Tate heights, Néron pairings and their generalizations that were introduced around that time by Manin. In 1972, we published a joint paper that provided explicit upper bounds for the absolute value of the difference between the heights of Weil and Néron–Tate on abelian varieties in terms of the corresponding theta constants.

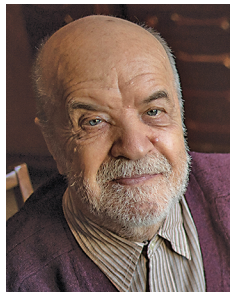
Since I never attended any graduate school, it was a difficult problem to find a place in the USSR where I could defend my PhD thesis (get a kandidat degree) and even to pass qualifiers (kandidat minimum). However, enormous joint efforts of Yuri Ivanovich, Solomon Grigor'evich Mikhlin, Dmitri Konstantinovich Faddeev, and Georgiy Ivanovich Petrashen' helped solve this problem in 1975 and I defended my thesis at the St.Petersburg (then Leningrad) branch of the Steklov Institute.

Later, we did not meet as often as in my student years but we started to talk about things not directly related to mathematics. In the middle of the 1980s, I heard from Yuri Ivanovich that he was again urged by the authorities to repent for signing *the letter of the 99* (in defense of Aleksander Yesenin-Volpin). "Yuri Ivanovich, you published a textbook in logic. Please do it in such a way that suits you." Manin's answer was that during the previous 17 years he had not given any reasons to believe that his point of view on the subject has changed.

I vividly remember the celebration of Yuri Ivanovich's 50th birthday that took place in February 1987 in his Moscow apartment. I was siting next to Andrey Tyurin; our vis-á-vis were Don Zagier and Miles Reid. I. R. Shafarevich and Vjacheslav Vsevolodovich Ivanov tried to recall the last time they had seen each other: it turns out that it was in 1960, at the funeral of Boris Pasternak. Yuri Ivanovich recited his own translations into Russian of verses of Rudyard Kipling. Now Manin's translations and his own verses are published in his book *Mathematics as Metaphor*.

The last time I saw Manin was in May 2022, in his Bonn apartment. Yuri Ivanovich, Xenia Glebovna, my wife, and I were sitting on the balcony with a view of the Rhine river, drinking tea with apple pie, talking about everything but trying not to mention the war. I hoped to see him this year again. But it will not happen anymore. It is hard for me to accept that my teacher is no longer with us.

Yuri G. Zarhin



Fedor Bogomolov

Yuri Tschinkel

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[7]*Despite a certain progress in the case of elliptic curves over the rationals, this half-century old question remains unsettled.*

Credits

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