MATHEMATISCHE ARBEITSTAGUNG 198o

UNIVERSITÄT BONN

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Mathematisches Institut
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Programm der Mathematischen Arbeitstagung 1980 (I)

Freitag, den 13.6.:

17.00 - 18.00 Uhr: M.F. Atiyah: Vector Bundles on Riemann Surfaces

Samstag, den 14.6.:

10.00 - 11.00 Uhr: A. Katok: Counting closed geodesics on surfaces

12.00 - 13.00 Uhr: R. Bott: Equivariant Morse theory

17.00 - 18.00 Uhr: K. Ribet: Mazur and Wiles (cyclotomic fields)

Sonntag, den 15.6.:

10.00 - 11.00 Uhr: A. Borel: L^2 cohomology of arithmetic groups

12.00 - 13.00 Uhr: I. Bakelman: Topological methods in the theory of Monge-Ampère equations

16.45 - 17.00 Uhr: Festlegung der nächsten Vorträge

17.00 - 18.00 Uhr: H. King: Topology of real algebraic varieties

Die Vorträge finden alle im "Großen Hörsaal" (Wegelerstr. 10) statt. Erfrischungspausen mit Tee: Samstag und Sonntag 11.15-12.00 Uhr vor dem Großen Hörsaal, nachmittags ab 15.30 Uhr im Diskussionsraum Beringstraße 1. Die Post liegt während der Teepausen aus. Tischtennis im Keller des Hauses Beringstraße 4. Den Tagungsbeitrag bitte an Frau Gerber (SFB-Büro Beringstr. 4) bezahlen. Alle Tagungsteilnehmer mögen sich bitte in die Teilnehmerliste eintragen. Teilnehmerlisten und andere Informationen liegen vor dem Diskussionsraum Beringstraße 1 aus.

Alle Tagungsteilnehmer mit ihren Damen oder Herren sind herzlich zum Empfang des Rektors eingeladen. Zeit: Freitag, den 13.6., 20.00 Uhr. Ort: Festsaal der Universität (Hauptgebäude), Eingang von der Straße "Am Hof" durch das Tor gegenüber Buchhandlung Röhrscheid.

Mathematisches Institut
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Programm der Mathematischen Arbeitstagung 1980 (II)

Montag, den 16.6.:

10.15 - 11.15 Uhr: J. Milnor: Groups of polynomial growth (Gromov's work)

12.45 - ca. 20.00 Uhr: Ausflug nach Bad Breisig/Bad Hönningen. Abfahrt pünktlich um 12.45 Uhr mit Motorschiff "Carmen Silva" am Alten Zoll.

Dienstag, den 17.6.:

10.00 - 11.00 Uhr: Y.-t. Siu: Andreotti-Fraenkel conjecture

12.00 - 13.00 Uhr: M. Artin: Mori's work

16.45 - 17.00 Uhr: Festlegung der restlichen Vorträge

17.00 - 18.00 Uhr: B. Gross: L-series of elliptic curves

Die Vorträge finden alle im "Großen Hörsaal" (Wegelerstraße 10) statt.

Erfrischungspausen mit Tee: Dienstag vormittags 11.15 - 12.00 Uhr vor dem Großen Hörsaal, nachmittags ab 15.30 Uhr im Diskussionsraum Beringstraße 1.

Die Post liegt während der Teepausen aus.

Tischtennis im Keller des Hauses Beringstraße 4.

Den Tagungsbeitrag bitte an Frau Gerber (SFB-Büro, Beringstraße 4) bezahlen.

Alle Tagungsteilnehmer mögen sich bitte in die Teilnehmerliste eintragen. Teilnehmerlisten und andere Informationen liegen vor dem Diskussionsraum Beringstraße 1 aus.

Mathematisches Institut
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Programm der Mathematischen Arbeitstagung 1980 (III)

Mittwoch, den 18.6.:

10.15 - 11.15 Uhr:	F. Takens: Turbulence and strange attractors
12.00 - 13.00 Uhr:	W. Ziller: Periodic motions in Hamiltonian systems
17.00 - 18.00 Uhr:	P. Slodowy: Simple groups over $\mathbb{C}((t))$ and simple-elliptic singularities

Donnerstag, den 19.6.:

10.15 - 11.15 Uhr:	S. Kudla: Geodesic cycles and the Weil representation
12.00 - 13.00 Uhr:	D. Epstein: A theorem of Thurston with applications to group actions and foliations
16.45 - 17.45 Uhr:	F. Adams: Recent work on homotopy theory

! Die Referenten werden nochmals gebeten, ihre Kurzfassungen bis Mittwoch,
! 16.30 Uhr bei Herrn Schwermer abzugeben, da wir den Tagungsbericht allen
! Teilnehmern noch vor ihrer Abreise aushändigen möchten.

Die Vorträge finden alle im "Großen Hörsaal" (Wegelerstraße 10) statt.

Erfrischungspausen mit Tee: Mittwoch und Donnerstag vormittags von 11.15-12.00 Uhr vor dem Großen Hörsaal, nachmittags ab 15.30 Uhr im Diskussionsraum Beringstraße 1.

Die Post liegt während der Teepausen aus.

Tischtennis im Keller des Hauses Beringstraße 4.

Titel: Vector Bundles over Compact Riemann Surfaces

Autor: M. F. ATIYAH

Adresse: OXFORD

This is a progress report on joint work with R. Bott, and concerns the study of the moduli space of holomorphic vector bundles over a compact Riemann surface X of genus g.

If E is such a holomorphic bundle with rank n and Chern class k we put $\mu(E) = k/n$. Then E is said to be semi-stable if for all $F \subset E \Rightarrow \mu(F) \leq \mu(E)$ (and stable if $<$ holds). We denote by $M(n,k)$ the moduli space of stable bundles of rank n and Chern class k. If $(n,k)=1$ $M(n,k)$ is a compact non-singular algebraic variety. The problem we study is to compute its cohomology.

We fix a C^∞ bundle E for given n,k and let $\mathcal{C}$ denote the space of all complex structures on E. A point of $\mathcal{C}$ can be identified with the corresponding $\bar{\partial}$-operator, showing that $\mathcal{C}$ is an affine space.

If $\mathcal{G}$ denotes the C^∞ automorphism group of E then $\mathcal{G}$ acts on $\mathcal{C}$ and the orbits represent isomorphism classes of holomorphic bundles.

According to G. Harder and M.S. Narasimhan [1] every holomorphic E has a canonical filtration $E = E_1 \supset E_2 \supset \dots \supset E_{r+1} = 0$ with the quotients $F_i = E_i / E_{i+1}$ semi-stable and

$$\mu(F_1) < \mu(F_2) < \dots < \mu(F_r)$$

This induces a stratification of $\mathcal{C}$ according to the type $\lambda = (n_1, k_1; \dots; n_r, k_r)$. For the semi-stable type $\lambda = (n, k)$ we write $\mathcal{C}^s$.
The strata $\mathcal{C}_\lambda$ have the properties:

(1) $\mathcal{C}_\lambda$ is a locally-closed submanifold of finite codimension $d(\lambda)$

(2) There is a partial ordering on the types so that $\overline{b_\lambda} \subset \bigcup_{\mu \geq \lambda} b_\mu$, and

$$\mu > \lambda \implies d(\mu) > d(\lambda)$$

(3) $H_{\mathcal{G}}(b_\lambda) \cong \bigotimes_{i=1}^{r} H_{\mathcal{G}_i}(b_i^s)$ where $\mathcal{G}_i = \mathcal{G}(n_i, k_i)$, $b_i^s = b^s(n_i, k_i)$ and $H_{\mathcal{G}}$ denotes equivariant cohomology

(4) $H^q(B_{\mathcal{G}}) \cong \bigoplus_{\lambda} H_{\mathcal{G}}^{q-d(\lambda)}(b_\lambda)$.

These formulae refer to rational cohomology and enable us inductively to calculate the equivariant cohomology of the semi-stable part b^s. When $(n,k) = 1$ semi-stable is the same as stable, $\mathcal{G}$ acts with constant isotropy group C^* and so we can easily get down to $M(n,k)$.

This approach is a variant on the approach of using the Morse theory of

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the Yang-Mills functional. The results are basically the same as those obtained by the methods of [1] except that we also prove the absence of torsion and in principle get information about the cohomology generators.

References

[1] G. Harder and M. S. Narasimhan, On the cohomology groups of moduli spaces of vector bundles on curves, Math. Ann. 212 (1975), 215-248.

Titel: COUNTING CLOSED GEODESICS ON SURFACES

Autor: ANATOLE KATOK

Adresse: Department of Mathematics, University of Maryland
College Park MD 20742, USA.

Let M be a compact orientable surface of genus $g > 1$, σ a Riemannian metric of class $C^{2+\varepsilon}$ $(\varepsilon > 0)$ on M, V_σ the total volume of M, μ_σ the volume element generated by σ, $P_\sigma(T)$ the number of closed geodesics of length $\leq T$.

The number

$$p_\sigma = \lim_{T \to \infty} \frac{\log P_\sigma(T)}{T}$$

measures the speed of exponential growth of the number of closed geodesics. It follows from classical results of Hilbert and G.D. Birkhoff that $p_\sigma > 0$. For a metric of negative curvature $k(x)$: $-K_1^2 \leq k(x) \leq -K_2^2$

Sinai [1] showed that

$$K_2 \leq p_\sigma \leq K_1$$

In particular, if the curvature is constant and equal to $-K^2$ then $p_\sigma = K = 2\left(\frac{\pi(g-1)}{V_\sigma}\right)^{1/2}$

Margulis [2] has proved (for detailed proof see Bowen [3]) in the case of negative curvature the existence of constants h_σ and C_σ such that

$$\lim_{T \to \infty} \frac{P_\sigma(T) \cdot \exp(-h_\sigma T) \cdot T}{C_\sigma} = 1$$

In particular in this case $p_\sigma = h_\sigma$.

The constant h_σ coincides with the topological entropy of the geodesic flow on the unit tangent bundle to M generated by σ [4]. Roughly speaking, the topological entropy measures the speed of exponential growth of the maximal number of geodesic segments of length $\leq T$ which can be distinguished with finite but arbitrarily fine precision. Furthermore, let N be the universal covering of M, $y \in N$, $V(y,R)$ the ball of radius R on N with the center at y. Then

$$h_\sigma \geq \lim_{R\to\infty} \frac{\log(\text{volume}(V(y,R)))}{R}$$

(A. Manning [4]) and this inequality becomes equality for metrics of negative curvature (Margulis [2]).

The following construction involving closed geodesics is very similar in spirit to the definition of topological entropy.

Let $\gamma_{x_0} : [0,t] \to M$, $\gamma(0) = x_0$ be a closed geodesic of length t parametrized by its length. This representation is unique up to the choice of the initial point x_0. Let us define the distance $d(\Gamma, \Gamma')$ between two closed geodesics Γ, Γ' of length t and t' respectively by

$$d(\Gamma,\Gamma') = |t-t'| + \inf_{\substack{x\in\Gamma \\ x'\in\Gamma'}} \sup_{0\leq s\leq \min(t,t')} d_\sigma(\gamma_x(s), \gamma'_{x'}(s))$$

where d_σ is the distance on M generated by σ.

Let for $\delta > 0$ $P_\sigma^\delta(T)$ be the maximal number of closed geodesics of length $\leq T$ with pairwise distances $\geq \delta$. The number

$$P_\sigma^\delta = \varliminf_{T\to\infty} \frac{\log P_\sigma^\delta(T)}{T}$$

is always finite. Let, furthermore

$$P_\sigma^* = \lim_{\delta\to 0} P_\sigma^\delta .$$

Obviously $P_\sigma^* \geq P_\sigma$. If σ is a metric of negative curvature then it follows from the uniqueness of closed geodesic in every free homotopy class on M that for some constant $\delta_\sigma > 0$

$$P_\sigma^{\delta_\sigma} = P_\sigma^* = P_\sigma .$$

MAIN THEOREM:

(i) $\quad P_\sigma^* \geq 2\left(\frac{\pi(g-1)}{v_\sigma}\right)^{1/2}$

(ii) $\quad P_\sigma^* = 2\left(\frac{\pi(g-1)}{v_\sigma}\right)^{1/2}$ iff σ is a metric of constant negative curvature.

This theorem follows from Theorems A and B below:

THEOREM A: $\quad P_\sigma^* = h_\sigma$

This result is true not only for geodesic flows but in a more general setting for any $C^{1+\varepsilon}$ $(\varepsilon>0)$ flow without fixed points on any 3-dimensional compact manifold

This last result is a continuous-time version of Theorem 4.3 from [5] (proofs are completely parallel). Ergodic theory plays the key role in the arguments.

Theorem B[6]. Let σ_1, σ_2 be two conformally equivalent $C^{2+\varepsilon}$ Riemannian metrics on an n-dimensional compact manifold W (i.e $\sigma_2 = \rho\sigma_1$ where ρ is a scalar function). Let us assume that $v_{\sigma_1} = v_{\sigma_2} = v$ and σ_1 is a metric without focal points. Then

$$h_{\sigma_2} \geq \rho_0^{-1} h^{\lambda}_{\sigma_1}$$

where $\rho_0 = \frac{1}{v}\int \rho^{1/2} d\mu_{\sigma_1}$ and h^{λ}_{σ} is the entropy of the geodesic flow generated by σ with respect to Liouville (smooth) invariant measure.

To deduce the Main Theorem from Theorems A and B we apply the classical result of Koebe and find for a given metric σ the conformally equivalent metric σ_0 of constant negative curvature and the same volume. Then

$$P^{*}_{\sigma} = h_{\sigma} \geq \rho_0^{-1} h^{\lambda}_{\sigma_0} = \rho_0^{-1} h^{\lambda}_{\sigma_0} = \rho_0^{-1}\cdot 2\left(\frac{\pi(g-1)}{v_\sigma}\right)^{1/2}$$

Obviously $\rho_0 \leq 1$ and $\rho_0 = 1$ iff $\rho \equiv 1$, i.e if σ itself is a metric of constant negative curvature

Another corollary of theorem B is the following minimax property of metrics of constant negative curvature

<u>Corollary</u>: For any metric σ on M without focal points

$$h^{\lambda}_{\sigma} \leq \left(\frac{2\pi(g-1)}{v_{\sigma}}\right)^{1/2} \leq h_{\sigma}$$

and if one of these two inequalities becomes equality then σ is a metric of constant negative curvature and the second inequality also becomes equality.

References

1. Ja. G. Sinai, The asymptotic behaviour of the number of closed geodesics on a compact manifold of negative curvature, Izv. Akad. Nauk SSSR Ser. Mat. <u>30</u> (1966), 1275-1295 = A.M.S. Transl. (2), 73, (1968), 229-250

2. G. A Margulis. Applications of ergodic theory to the investigation of manifolds of negative curvature
Func. Anal. and its Applications vol. 3, N4 (1969); Certain measures associated with U-flows on compact manifolds, Func. anal. and its appl. <u>4</u>, N1 (1970).

3. R. Bowen Periodic orbits for hyperbolic flows, Amer. J. Math <u>94</u> (1972) 1-30. (?).

4. A. Manning, Topological entropy for geodesic flows, Ann. of Math <u>110</u> (1979), 567-573

5. A. Katok, Lyapunov exponents, entropy and periodic points for diffeomorphisms, Publ. Math. IHES, vol. 51, (1980)

6. A. Katok, Topological entropy versus metric entropy on surfaces of negative curvature, handwritten manuscript.

Titel: Equivariant Morse Theory

Autor: Raoul Bott

Adresse: C/o M. ATIYAH, HOTEL SCHWAN.

If f is a nondegenerate smooth function on the compact manifold M, its Morse Series is given by the formula:

$$(1) \quad \mathcal{M}_t(f) = \sum_p t^{\lambda_p} \qquad p \in Cr(f)$$

where the sum is extended over the critical points of f and for $p \in Cr(f)$ λ_p is the # of negative Eigenvalues of the matrix

$$\left.\frac{\partial^2 f}{\partial x^i \partial x^j}\right|_p .$$

The Morse Inequalities then imply that $\mathcal{M}_t(f)$ dominates the Poincaré Polynomial of M $P_t(M)$ in the following sense:

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there exists a polynomial $Q(t)$ with non-negative coefficients such that:

(2) $\mathcal{M}_t(f) - P_t(M) = (1+t)Q(t)$.

¶ When f has critical sets which are nondegenerate manifolds $\{N\}$ then

(3) $\mathcal{M}_t(f) = \sum t^{\lambda_N} P_t(N)$

will still satisfy the Morse inequalities.

Suppose now that f is ~~..~~ invariant under the action of a ~~smooth the action~~ compact group

G on M. Then if M_G denotes the homotopy quotient of M by G, and we assume that the critical sets of f are non-degenerate manifold N equal is orbits

$$N = G/H_N,$$

one obtains the following equivariant Morse inequalities.

Set

$$(4) \quad \mathcal{M}_t^G(f) = \sum t^{\lambda_N} P_t(BH), \quad H = H_N$$

with BH the classifying space of H. Then

$$(5) \quad \mathcal{M}_t^G(f) - P_t(M_G) = (1+t)\,Q(t)$$

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Applications & examples of this formula are given; in particular it is argued that a better approach to the classical geodesic problem is along these lines.

Titel: Mazur-Wiles (cyclotomic fields)

Autor: Kenneth Ribet
Adresse: Berkeley Math Dept.
Berkeley, Ca. 94720 USA

As was described in the actual talk, Mazur and Wiles are in the process of settling many of the classical problems in the theory of cyclotomic fields, by a method which uses jacobian varieties of modular curves. Their work is not yet available even in preprint form, but a good idea of the techniques involved can be had by consulting Wiles's article in the current volume of Inventiones Math. (which is ?58). The first use of modular curves in the theory of cyclotomic fields was made in an article in Inventiones Math (1976) by Ribet. Finally, a discussion of the "main conjecture" of Iwasawa theory (noy proved for (a wide class of totally real abelian fields) is contained in Coates's article in the Durham Number Theory Conference volume, A. Fröhlich, ed.

In the remainder of this abstract, I will state what was referred to in the talk as the twisted main theorem for $\mathbb{Q}(\mu_{p^n})$. This is the result which is proved directly, and which implies (using arguments from commutative algebra) the main conjecture of Iwasawa theory and its many consequences.

So let p be an odd prime. For $n \geq 0$, let K_n be the field obtained by adjoining to $\mathbb{Q}$ the group of p^{n+1} (st) roots of unity. Let K_∞ be the union of these fields, i.e., the field of p-power roots of unity. Set $G_n = \mathrm{Gal}(K_n/\mathbb{Q}) \approx (\mathbb{Z}/p^{n+1}\mathbb{Z})^*$, and let $\Gamma_n = \mathrm{Gal}(K_\infty/K_n)$, for each n. Let A_n be the p primary part of the ideal class group of K_n for each n, and let A_n^- denote the odd part of this group (the -1 eigenspace under the action of complex conjugation). For each n, the natural map $A_n \longrightarrow A_{n+1}$ induces an injection $A_n^- \hookrightarrow A_{n+1}^-$. Let A_∞^- be the union of the A_n^-. We must consider the twist $A_\infty^-(1)$ of A_∞^-, defined to be the tensor product

$$A_\infty^- \otimes_{\mathbb{Z}_p} \varprojlim (\mu_{p^n}).$$

We consider this as a $\mathrm{Gal}(K_\infty/\mathbb{Q})$-module by having the Galois group act on both factors. Then for each n, we define

$$B_n = A_\infty^-(1)^{\Gamma_n}.$$

This is *a priori* a module for the Galois group G_n, but in fact we see that complex conjugation in G_n acts trivially, so it makes sense to consider B_n as a module for G_n^+, the Galois group (over $\mathbb{Q}$) of the real subfield of K_n. According to a theorem of Coates (Ann of Math, ?1971), B_n is in fact isomorphic to the p-primary part of the group $K_2(R)$, where R is the ring of integers of the real subfield of K_n.

Let us fix n, and set R to be the group ring $\mathbb{Z}_p[G_n^+]$. Then B_n may be viewed as an R-module.

Twisted Main Theorem. The Fitting ideal of the R-module B_n is contained in the **twisted** Stickelberger ideal of R.

We will close this abstract by defining the two ideals in question.

First, the Stickelberger ideal. For $g \in G_n^+$, let ε_g be the characteristic function of g, viewed as a function on $\mathbb{Z}$, periodic mod p^{n+1}. The twisted Stickelberger element is the sum $\sum_{g \in G_n^+} L(-1,\varepsilon_g)g^{-1} \in R \otimes_{\mathbb{Z}} \mathbb{Q}$. The twisted Stickelberger ideal is the set of elements of R which are R-multiples of this sum.

Now, for Fitting ideals. Let R be *any* commutative ring with 1, and let M be a finitely generated torsion R-module. Choose a surjection

$$d: R^q \twoheadrightarrow M.$$

If $a_1, \dots, a_q$ are elements of the kernel of d (each considered as row vectors), we may concatenate them together to make a q by q matrix. The Fitting ideal of M is the ideal of R generated by the determinants of all such matrices.

Titel: L^2-cohomology of arithmetic groups

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1. Let M be a connected (oriented) Riemannian manifold, $A(M)$ the space of smooth exterior differential forms on M. An element $\omega \in A(M)$ is L^2 if $(\omega,\omega) = \int_M |\omega_x|^2 dV_x < \infty$, where dV_x is the Riemannian volume element and $|\omega_x|^2$ the standard norm defined by the metric. Let $A_{(2)}(M)$ or $A_{(2)}$ be the space of $\omega \in A(M)$ for which ω and $d\omega$ are L^2. The L^2-cohomology space of M is then by definition $H^q_{(2)}(M) = (A_{(2)}(M) \cap \ker d) / d(A^{q-1}_{(2)})$

It can also be given a Hilbert-space definition: let $\overline{A_{(2)}}$ be the completion of $A_{(2)}$ with respect to the graph-norm $\langle \omega,\omega\rangle = (\omega,\omega) + (d\omega, d\omega)$. Then d extends to a closed operator $\bar{d} : \overline{A}^q_{(2)} \to \overline{A}^{q+1}_{(2)}$ and it can be shown (cf [5]) that

$$H^q_{(2)}(M) = (\overline{A^q_{(2)}} \cap \ker \bar{d}) / \operatorname{Im} \bar{d} .$$

Let $\mathcal{H}_{(2)}(M)$ be the space of L^2-harmonic forms ($d\omega = \partial\omega = 0$; if M is complete, this is equivalent to $\Delta\omega = 0$, where $\Delta = d\partial + \partial d$, as usual.) We have the natural homomorphisms

$$\begin{tikzcd} H_c(M) \arrow[r] & H_{(2)}(M) \arrow[d, "\alpha"] & \mathcal{H}_{(2)}(M) \arrow[l, "j"'] \arrow[dl, "\beta"] \\ & H(M) & \end{tikzcd}$$

where $H_c(M)$ (resp $H(M)$) refers to cohomology with compact (resp. closed) supports.

If M is compact, all these maps are isomorphisms (Hodge, de Rham). In general $\operatorname{Im} \alpha = \operatorname{Im} \beta$ (Kodaira, cf [7]). If M is complete, j is injective (Andreotti-Vesentini, cf [2]).

2. This talk was mainly devoted to the case where $M = \Gamma\backslash X$, with $X = G/K$, G a real (linear) semi-simple group, K a maximal compact subgroup of G and Γ an arithmetically defined subgroup of G. Examples (1) $G = SL_n(\underline{\underline{R}})$, $K = SO_n$, X: space of positive quadratic forms on $\mathbb{R}^n$ of det. 1, $\Gamma = SL_n(\mathbb{Z})$; (2) $G = SO(n,1)^0$, $K = SO(n)$, $X = H^n$ = hyperbolic n-space, $\Gamma = SL_{n+1}(\underline{Z}) \cap SO_n$

3. In joint work with H. Garland (partially announced in [3]), it was shown that the spectrum of Δ on $A_{(2)}$ is bounded below and has no accumulation point, (except $+\infty$). In particular $\mathcal{H}_{(2)}(M)$ is finite dimensional, and consists of automorphic forms. It follows that $H_{(2)}$ finite dimensional $\Longleftrightarrow$ $\mathcal{H}_{(2)} = H_{(2)} \Longleftrightarrow \operatorname{Im} \bar{d}$ is closed.

Theorem 1 $H_{(2)}$ is finite dimensional if $\operatorname{rk} G = \operatorname{rk} K$ or, more generally, if G has no proper parabolic subgroup which is fundamental and cuspidal.

$\operatorname{Rk} G = \operatorname{rk} K$ means that G has a compact Cartan subgroup. A subgroup P of G is parabolic if $G(\mathbb{C})/P(\mathbb{C})$ is a projective variety. A parabolic subgroup P is fundamental if it contains a conjugate of a maximal torus of K, cuspidal if $\Gamma \cap N$ is cocompact in the unipotent radical N of P.

I conjecture that if G has a proper fundamental cuspidal parabolic subgroup P_o, then Γ has a subgroup of finite index Γ' such that $H^q_{(2)}(\Gamma'\backslash X)$ is infinite dimensional for some q, (close to the middle dimension). This has been checked in many cases, in particular if P_o is minimal over $\mathbb{R}$, or if $\Gamma = SL_n(\mathbb{Z})$, or if $X = H^n$, n odd (and then $q = (n+1)/2$).

Remark: Assume X is a bounded symmetric domain. Then $\Gamma\backslash X$ has a compactification $\Gamma\backslash X^*$ which is a normal projective variety (cf [1]). If X is the gen unit ball in $\mathbb{C}^n$, then S. Zucker has shown that $H_{(2)}(\Gamma\backslash X)$ is isomorphic to (Deligne's translation of) the Goresky - MacPherson intersection homology (with middle perversity) of $\Gamma\backslash X^*$ [6]. It is an open question whether this is true more generally.

In the real case, $\Gamma\backslash X$ has various stratified compactifications, the Satake compactification or a manifold with corners. I do not know whether some of them are similarly related to $H_{(2)}$.

3. The proofs of the general results in section 2 involve the spectral decomposition of $L^2(\Gamma\backslash G)$, the space of L^2-functions on $\Gamma\backslash G$, viewed as a unitary G-module via right translations. Recall first that if (π, V) is a unitary G-module, an element $v \in V$ is smooth if $g \mapsto g.v$ is a smooth map from G to V. The smooth vectors form a dense subspace V^∞, stable under G, on which the universal enveloping algebra $U(\mathfrak{g})$ of the Lie algebra $\mathfrak{g}$ of G acts in a natural way. Let now $V = L^2(\Gamma\backslash G)$. Then $V^\infty \subset C^\infty(\Gamma\backslash G)$ by Sobolev's lemma. There is equality if $\Gamma\backslash G$ is compact [4: VII] but not otherwise: for $f \in C^\infty(\Gamma\backslash G)$ to be in $L^2(\Gamma\backslash G)^\infty$, it is necessary that all derivatives Xf, $(X \in U(\mathfrak{g}))$ be L^2.

There are standard isomorphisms

(1) $H^*(\Gamma) = H^*(\mathfrak{g}, \mathfrak{k}; C^\infty(\Gamma\backslash G))$, $H^*(\Gamma\backslash G) = H^*(\mathfrak{g}; C^\infty(\Gamma\backslash G))$,

where $\mathfrak{k}$ is the Lie algebra of K [4: VII]. More precisely, using the projection $G \to X$, one establishes isomorphisms

(2) $A(\Gamma\backslash X) \xrightarrow{\sim} \mathrm{Hom}_{\mathfrak{k}}(\Lambda \mathfrak{g}/\mathfrak{k}, C^\infty(\Gamma\backslash G))$, $A(\Gamma\backslash G) = \mathrm{Hom}(\Lambda\mathfrak{g}, C^\infty(\Gamma\backslash G))$,

which yield (1) by going over to cohomology.

<u>Theorem 2</u> The natural inclusions

$\mathrm{Hom}_{\mathfrak{k}}(\Lambda(\mathfrak{g}, \mathfrak{k}), L^2(\Gamma\backslash G)^\infty) \to A_{(2)}(\Gamma\backslash X)$, $\mathrm{Hom}(\Lambda\mathfrak{g}, L^2(\Gamma\backslash G)^\infty) \to A_{(2)}(\Gamma\backslash G)$

induce isomorphisms in cohomology.

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It follows from the results of [3] quoted earlier that $\mathcal{H}_{(2)}(\Gamma\backslash X) = H^*(\mathfrak{g}, K; V_d)$ and is a finite sum of spaces $H^*(\mathfrak{g}, K; H_i^\infty)$, where H_i runs through a system of constituents of V_d. (See [4] for results on the spaces $H^*(\mathfrak{g}, K; H_i^\infty)$.) By [8], V_{ct} is a Hilbert direct sum of closed invariant subspaces, each of which is a continuous integral of unitarily induced principal series representations. The cohomology with coefficients in one of those can be computed following [4: III], and this leads to Thm 1 and the partial converse statements in § 2.

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Titel: Topological methods in the theory of Monge-Ampere equations.

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We consider the theorems of existence of generalized solutions of the Dirichlet problem for the elliptic Monge-Ampere equations

$$\det \| z_{ij} \| = \varphi(x, z, \triangledown z) \qquad (1)$$

where $\varphi(x, z, p) \in C(\bar{G} \times R \times R^n)$ and G is a bounded convex domain in the Euclidean space E^n.

If the equation (1) is elliptic then each solution $z(x) \in C^2(G)$ must be either strictly convex or strictly concave in G.

We shall consider only the case of convex solutions of the equation (1). Then the function $\varphi(x, z, p)$ must be positive.

The study of the Dirichlet problem for the equation (1) consists of two parts:

A. The function φ has the special form

$$\varphi(x, z, p) = \Psi(x) \cdot R(p) \qquad (2)$$

where $\Psi(x) \geqslant 0$, $\Psi(x) \in L(G)$; $R(p) > 0$ and $\frac{1}{R(p)}$ is locally summable on $\tilde{R}^n$.

B. The function $\varphi(x, z, p)$ satisfies the condition

$$0 \leq \varphi(x, z, p) \leq \Psi(x) \cdot R(p) \qquad (3)$$

in $\bar{G} \times R \times R^n$ where $\Psi(x)$ and $R(p)$ are described above.

We denote by $A(R)$ the number $\int_{R^n} \frac{dp}{R(p)}$.

The main result of the part A.

We suppose that:

1) $$R(p) \leq C_0 (1 + |p|^2)^k \qquad (4)$$

everywhere in R^n and $C_0 = \text{const} > 0$, $k = \text{const} \geqslant 0$;

2) $$0 \leq \Psi(x) \leq a_0 [\, dist(x, \partial G)]^\lambda \qquad (5)$$

for all $x \in G$ belonging to a small neighborhood of ∂G, where $a_0 = \text{const} > 0$, $\lambda = \text{const} \geqslant 0$;

3) Let x_0 be an arbitrary point of ∂G. Then there exists a n-ball U_{x_0} of the radius r_{x_0} such that $x_0 \in \partial U_{x_0}$, $G \subset U_{x_0}$ and

$$r_{x_0} \le r_0 = \mathrm{const} < +\infty .$$

4) $$\int_G \psi(x)\,dx < A(R) ; \qquad (6)$$

the case $A(R) = +\infty$ is not excluded.

5) $$k \le \frac{n+1+\lambda}{2} . \qquad (7)$$

We denote by $W^+(G)$ the set of all convex functions which are defined in G. Each function $z(x) \in W^+(G)$ has the second differential almost everywhere. We define the normal mapping χ_z of Borel subsets of the domain G in Borel subsets of R^n for each convex function $z(x) \in W^+(G)$. If $z(x) \in W^+(G) \cap C^2(G)$ then χ_z is the tangential mapping.

We call the function $z(x) \in W^+(G)$ a generalized solution of the differential equation (1) if $z(x)$ satisfies (1) almost everywhere and the normal mapping χ_z has the absolutely continuous area.

Theorem 1.

The Dirichlet problem

$$\det \| z_{ij} \| = \psi(x) \cdot R(\partial z), \tag{8}$$

$$z \Big|_{\partial G} = h(x) \tag{9}$$

has the unique generalized solution $z(x) \in W^{+}(G)$ if the conditions 1)—5) are fulfilled.

The same result is correct for concave generalized solutions of the Dirichlet problem (8)-(9).

See the proof of this theorem in [1], [2].

Remarks. ①. We have

$$\int_G \psi(x)\,dx = \int_{\chi_z(G)} \frac{dp}{R(p)} \leq \int_{R^n} \frac{dp}{R(p)} = A(R) \tag{10}$$

for all generalized solutions of the equation (8). Therefore the inequality (10) is necessary.

The strict inequality (6) is a sufficient condition of the solvability of the Dirichlet problem (8)-(9).

② If we have the equation

$$\det \| z_{ij} \| = 1 \cdot (1 + |p|^2)^{\frac{n+2}{2}} \tag{11}$$

then all solutions of this equation have Gauss curvature equal to 1. Therefore

$$\frac{1}{2} S_n \geq \int_G K \, d\sigma = \int_G d\sigma = \sigma(z) \tag{12}$$

for each solution $z(x)$ of the equation (11), where S_n is the area of the n-dimensional unit sphere and $\sigma(z)$ is the area of the solution $z(x)$.

Therefore the Dirichlet problem

$$\det \| z_{ij} \| = (1 + |p|^2)^{\frac{n+2}{2}} \tag{13}$$

$$z \Big|_{\partial G} = k x_1 \tag{14}$$

has no solutions in the ball $G: \sum_{i=1}^{n} x_i^2 < \frac{1}{2}$ if the number $k > 0$ is sufficiently big. In this problem the inequality

$$\int_G \Psi(x) \, dx = \int_G dx = \frac{1}{2} \mu_n < A(R) = \mu_n$$

holds, where μ_n is the volume of the n-unit ball in E^n.

But all solutions of the equation (13) can not satisfy the boundary condition (14) if the number $h > 0$ is sufficiently big.

The condition (7) is necessary and sufficient for the satisfaction of the boundary condition (9) in the classic meaning.

The main results of the part B.

We consider the Dirichlet problem

$$\det \| z_{ij} \| = \varphi(x, z, Dz), \tag{15}$$

$$z \Big|_{\partial G} = h(x) \in C(\partial G). \tag{16}$$

where $\varphi(x, z, p) \in C(\bar{G} \times R \times R^n)$ and the inequality

$$0 \leq \varphi(x, z, p) \leq \psi(x) \cdot R(p) \tag{17}$$

holds in $\bar{G} \times R \times R^n$.

Theorem 2. If all the conditions 1) - 5) (see part A) for G, $\psi(x)$ and $R(p)$ are fulfilled then the Dirichlet problem (15)-(16) has at least one generalized solution $z(x) \in W^+(G)$.

<u>The scheme of proof</u>. We denote by $W_h^+(G)$ the set of all convex functions $z(x) \in W^+(G)$ satisfying the boundary condition (17). $W_h^+(G)$ is not empty because the Dirichlet problem (8)-(9) has generalized solutions. The set W_h^+ is a convex set in $C(\bar{G})$. We take an arbitrary function $u(x) \in W_h^+(G)$ and consider the function

$$f_u(x) = \frac{\varphi(x, u, Du)}{R(Du)} .$$

The function $f_u(x)$ is non-negative and $f_u(x) \le \varphi(x)$ everywhere in G and $f_u(x) \in L(G)$. Therefore the Dirichlet problem

$$\frac{\det \| z_{ij} \|}{R(Dz)} = f_u(x), \qquad (18)$$

$$z \big|_{\partial G} = h(x) \qquad (19)$$

has only one generalized solution $z(x) \in W_h^+(G)$.

Thus we constructed the operator

$$A : W_+^h(G) \longrightarrow W_+^h(G)$$

and $z(x) = A(u(x))$. It is clear that the fixed points of the operator A are generalized solutions of the Dirichlet problem (15)–(16). From the condition

$\int_G \varphi(x)\,dx < A(R)$ we prove that the set $A(W_h^+(G))$ is bounded in $C(\bar{G})$. We can take a subsequence $z_{i_m}(x) = A(u_{i_m}(x))$ for each sequence $\{u_i(x)\}$, $u_i(x) \in W_h^+(G)$ which converges in each point $x \in G$ but non-uniformly. From the condition $K \le \frac{n+1+\lambda}{2}$ we prove (very non-trivially) that this subsequence $z_{i_m}(x)$ converges uniformly to any function $z_0(x) \in W_h^+(G)$. Thus the operator $A: W_h^+(G) \to W_h^+(G)$ is compact. From the compactness of the operator A and the uniqueness of the generalized solution of the Dirichlet problem (18)–(19) we prove that the operator A is continuous.

Therefore our theorem is proved by means of the Schauder principle.

If $\varphi(x, z, p) \in C^1(\bar{G} \times R \times R^n)$ and $\varphi_z \geqslant 0$ in $\bar{G} \times R \times R^n$ then the Dirichlet problem (15)–(16) has only one generalized solution. When the condition $\varphi_z \geqslant 0$ is not fulfilled then the Dirichlet problem can have many and even infinite number of generalized solutions. The corresponding examples can be constructed by means of the Krasnoselskii theorems of fixed points (see [2]).

The same topological methods can be used for the study of the Dirichlet problem for more complicated Monge-Ampere equations:

$$\det \| z_{ij} \| = \sum_{k=1}^{n} \Big(\sum_{i_1 < i_2 < \ldots < i_k} b_{i_1 i_2 \ldots i_k}(x, z, Dz)\, \Delta^{(k)}_{i_1 \ldots i_k} \Big) + \\ + \varphi(x, z, Dz)$$

where $\Delta^{(k)}_{i_1 \ldots i_k}$ is a principle minor of the order k of $\det \| z_{ij} \|$. (see [2]).

The Monge-Ampere equations and operators are applied for the obtaining a priori estimates of quasilinear and linear elliptic equations and for the proof of existence theorems of this equations[3],[4]We proved the stability theorems for general elliptic equations also by means of the Monge-Ampere operator(see [5]).

This lecture is based on the results of my works.

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Titel: The Topology of Real Algebraic Varieties

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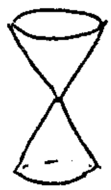
What do the solutions of real polynomial equations look like to a topologist?

More particularly, we say $V \subset R^n$ is a real algebraic set if $V = p^{-1}(0)$ for some polynomial map $p: R^n \to R^k$. The problem I wish to discuss is the characterization of real algebraic sets up to homeomorphism. In other words, what are neccesary and sufficient conditions for a topological space to be homeomorphic to a real algebraic set. (A reference for elementary topological properties of real algebraic sets is [W]).

Let $V \subset R^n$ be a real algebraic set. Then there is a real algebraic set $\Sigma_V \subset V$ so that $V - \Sigma_V$ is a smooth manifold and $\dim V > \dim \Sigma_V$. The points of Σ_V are called singular and those of $V - \Sigma_V$ are called nonsingular or smooth. For example if $V = \{z^2 = x^2 + y^2\}$ then Σ_V is the origin.



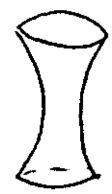
The first step is to understand nonsingular algebraic sets, i.e. those V with Σ_V empty. In the 1930's Seifert showed that any compact parallelizable manifold is diffeomorphic to a nonsingular component of a real algebraic set [S]. The proof was as follows. If M^m is a compact paralellizable manifold it can be imbedded in some R^{m+k} so that it has a neighborhood T and a smooth function $f: T \to R^k$ so 0 is a regular value of f and $f^{-1}(0) = M$. Then extend f to R^{m+k} and approximate by a polynomial p. Then because 0 is a regular value of f, $p^{-1}(0) \cap T$ is a slightly jiggled copy of $f^{-1}(0) = M$.

In the 1950's Nash showed that any compact manifold is diffeomorphic to a nonsingular component of a real algebraic set [N]. The basic idea is similar to Seifert's, except that to classify the normal bundle of M, f must map to the canonical bundle over the Grassmanian. Then approximate f by something algebraic and mess around a bit to finally get M a nonsingular algebraic subset of $R^{m+k} \times R^n$. (The $\times R^n$ is an artifact of the messing around you must do.) Finally in the 70's, Tognoli proved that one can get rid of the extra components, so every compact smooth manifold is diffeomorphic to a nonsingular real algebraic set [T].

So compact nonsingular real algebraic sets are classified up to diffeomorphism.

What about singular algebraic sets? The previous results depended on the fact that locally a submanifold of R^n is the zero set of a smooth function f which has zero as a regular value. So if you perturb f, the zero sets are isotopic. If you have a singular space this is no longer the case. For instance, the cone $z^2=x^2+y^2$ if we approximate by $z^2=x^2+y^2+\varepsilon$ we get something much different.

or



In the 60's Kuiper had the idea of getting around this in certain circumstances by always approximating with something with the same high order Taylor expansion polynomial at a simgular point. [K] This meant that if you knew of polynomials $p_i\colon (R^n,0) \to (R^k,0)$ with an isolated singularity at 0, then any space which had isolated singularities which looked like the singularities of some $p_i^{-1}(0)$'s was homeomorphic to a real algebraic set. This method produced lots of examples but could not hope to classify real algebraic sets, since for instance it depended on knowing examples of polynomials p_i.

The idea of Akbulut and myself was to take a cue from the algebraic resolution of singularities. In a naive form, resolution of singularities says that for every real algebraic set V there is a nonsingular real algebraic set W and a proper polynomial function $f\colon W \to V$ so that $f|_{f^{-1}(V-\Sigma_V)}\colon f^{-1}(V-\Sigma_V) \to V-\Sigma_V$ is a diffeomorphism. One can also specify that $f^{-1}(\Sigma_V)$ is a union of codimension one closed submanifolds of V in general position. For instance, the cone $z^2=x^2+y^2$ is resolved to a cylinder and the map f crushes a circle to a point.

Our idea was to take a topological space X, see if there is some sort of topological resolution $f:Y \to X$. Now Y is a smooth manifold so one can use Seifert-Nash-Tognoli to approximate it by a nonsingular algebraic set W. Them if one has been careful, W can be be blown down algebraicly to a real algebraic set V which is homeomorphic to X.

It seems likely that this approach will allow a complete topological characterization of real algebraic sets. At the moment, these are some of our results.

I) (Characterization of real algebraic sets with isolated singularities) A topological space X is homeomorphic to a real algebraic set V with Σ_V finite iff X is homeomorphic to the interior of a compact smooth manifold with some smooth codimension zero submanifolds crushed to points. [AK1]

II) The interior of any compact PL manifold is homeomorphic to a real algebraic set. [AK2]

III) (Characterization of homotopy types of real algebraic sets) X is homotopy equivalent to an real algebraic set iff X is homotopy equivalent to a finite complex. [AK1]

IV) (Characterization of real algebraic sets of dimension ≤ 2) X is homeomorphic to a compact real alg. set of dimension ≤ 2 iff X is homeomorphic to a polyhedron in which every simplex is properly contained in an even number of closed simplices. (This result was proved independently by Benedetti and ? by the same methods).

V) (All knots are algebraic) Given a knot (or even a link) $K \subset S^3$ there is an algebraic set $V \subset R^4$ so that for all sufficiently small $\varepsilon>0$, $(\varepsilon S^3, \varepsilon S^3 \cap V)$ is diffeomorphic to (S^3,K). (Here

εS^3 is the sphere of radius ε around 0.)

VI) X is homeo. to a real alg. set iff X is locally compact and the one point compactification of X is homeo. to a real alg. set. [AK1]

As an illustration I will give a sketch of a proof of I). First of all, by VI) it is sufficient to prove the result with X and V compact. Next, one can show the comdition on X is equivalent to the condition that X is homeomorphic to a smooth compact boundaryless manifold M with unions of closed codimension one submanifolds in general position crushed to points. (The proof in one direction needs a little trick.) Now by resolution of singularities, if V is a compact algebraic set with Σ_V finite then there is a compact smooth manifold W and a map $f:W \to V$ so f is a diffeomorphism over $V-\Sigma_V$ and $f^{-1}(\Sigma_V)$ = a union of closed codimension one submanifolds in general position. Thus V is homeomorphic to W with these closed codimension one submanifolds crushed to points.

Conversely, take a compact smooth boundaryless manifold M and let $M_i \subset M$ $i=1,\dots,k$ be codimension one closed submanifolds in general position. For simplicity, I will sketch a proof that $M/\cup M_i$ is homeomorphic to a real algebraic set V with Σ_V = a point. Let U_1 be R^n with 0 blown up. Let U_2 be U_1 with a point in the inverse image of 0 blown up. Let $\pi_1:U_1 \to R^n$ and $\pi_2:U_2 \to U_1$ be the blowup maps. Then $(\pi_1 \circ \pi_2)^{-1}(0)$ = two codimension one submanifolds of U_2 in general position. Take a point in their intersection and blow up U_2 at this point to get U_3. Do this k times and we get a U_k and a polynomial map $\pi:U_k \to R^n$ so that $\pi^{-1}(0)$ is a union of k codimension one submanifolds of U_k in general position, call them $S_1,\dots,S_k$. Now imbed M in U_k so that $M \cap S_i = M_i$ and M is in general position with the S_i's. Approximate M by a nonsingular algebraic set $W \subset U_k \times R^m$. Then by general position $\pi(W)$ is homeomorphic to $\pi(M)$ which is homeomorphic to X. Unfortunately there is no guarantee that $\pi(W)$ = an algebraic set but by a very simple trick you can find an algebraic set homeomorphic to $\pi(W)$.

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Titel: Gromov's characterization of groups of polynomial growth

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Let Γ be a finitely generated group. Choose some finite set of generators of the form $S^1 = \{1, x_1, \ldots, x_n, x_1^{-1}, \ldots, x_n^{-1}\}$, and let $|S^r|$ denote the cardinality of the set $S^r \subset \Gamma$ consisting of all r-fold products of generators. Then Γ is said to have <u>polynomial growth</u> if $|S^r|$ is less than some polynomial function of r; or <u>exponential growth</u> if

$$|S^r| \geqslant c^r$$

for some constant $c > 1$. These two properties are of course mutually exclusive. They do not depend on the choice of generating set S^1. It is not known whether every finitely generated group must have either polynomial growth or exponential growth.

Gromov's Theorem. A finitely generated group has polynomial growth if and only if it contains a nilpotent subgroup of finite index.

For solvable groups, this statement had been proved by Wolf and myself in 1968. For subgroups of $GL(n,\mathbb{R})$ (or subgroups of any connected Lie group) it had been proved by Tits in 1972. In both of these cases, Γ must have either polynomial growth or exponential growth. In the nilpotent case, Bass has given the precise estimate, for $r>0$:

$$0 < c_1 r^d \leqslant |S^r| \leqslant c_2 r^d,$$

where the degree d of polynomial growth can be computed from the lower central series $\Gamma = \Gamma_1 \supset \Gamma_2 \supset \ldots$, $\Gamma_{k+1} = [\Gamma, \Gamma_k]$, by the formula

$$d = \sum_k k \cdot \operatorname{rank}(\mathbb{Q} \otimes \Gamma_k / \Gamma_{k+1}).$$

The following four examples may help to illustrate these results.

(1.) The group $\mathbb{Z} \oplus \mathbb{Z}$ with the obvious set S of generators has polynomial growth function

$$|S^r| = 2r^2 + 2r + 1$$

of degree $d = 2$.

(2.) The central extension

$$1 \to \mathbb{Z} \to \Gamma \to \mathbb{Z} \oplus \mathbb{Z} \to 1$$

with generators x, y and relations

$$[x,y] = z, \quad [x,z] = [y,z] = 1$$

has polynomial growth of degree $d=4$. In fact, if $x^i y^j z^k$ belongs to the set S^r then $|i| \le r$, $|j| \le r$, but $|k| \le r^2/4$.

(3.) The extension $1 \to \mathbb{Z} \oplus \mathbb{Z} \to \Gamma \to \mathbb{Z} \to 1$ with generators x, y, t and relations

$$txt^{-1} = x^2 y, \quad tyt^{-1} = xy, \quad [x,y] = 1$$

has exponential growth, since the associated matrix $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$ describing the action of t on $\mathbb{Z} \oplus \mathbb{Z}$ has an eigenvalue $\lambda = \frac{3+\sqrt{5}}{2} > 1$. In fact it is easy to check that the 2^n words $tx^{\varepsilon_1} t x^{\varepsilon_2} \ldots t x^{\varepsilon_n}$, with ε_i equal to 0 or 1, all represent distinct group elements. Hence $|S^{2n}| \geqslant 2^n$.

(4.) The free group $\mathbb{Z} * \mathbb{Z}$, with the obvious generators, has

$$|S^r| = 2 \cdot 3^r - 1 \geqslant 3^r,$$

so the growth is exponential.

Gromov's Theorem is proved by induction on the degree of polynomial growth, starting with the trivial case of degree zero. Using the following two statements, it suffices to construct a non-trivial homomorphism from Γ to $\mathbb{Z}$, in order to carry out the induction.

<u>Lemma 1</u>. If Γ has polynomial growth of degree $\leq d$, and if

$$1 \to N \to \Gamma \to \mathbb{Z} \to 1,$$

then N is finitely generated with

polynomial growth of degree $\leq d-1$.

Lemma 2. If $1 \to N \to \Gamma \to \Delta \to 1$, and if both N and Δ contain polycyclic subgroups of finite index, then so does Γ.

The proofs are reasonably elementary.

Now suppose that Γ can be mapped onto an infinite subgroup of a connected Lie group. Using Tits' theorem, we see that some subgroup of finite index maps onto $\mathbb{Z}$, provided that Γ has polynomial growth; so again it is possible to carry out an inductive argument.

Alternatively, suppose that Γ maps onto finite subgroups of arbitrarily high order in some fixed connected Lie group G. A classical result, due essentially to Jordan, says that there is a fixed integer $n = n(G)$ so that each of these finite groups contains an abelian subgroup of index $\leq n$. Let Γ^* be the intersection of all subgroups of index $\leq n$ in Γ. Then $\Gamma^*/[\Gamma^*, \Gamma^*]$ is infinite, hence maps homomorphically onto $\mathbb{Z}$; so again we can carry out an inductive argument.

Gromov's amazing idea is to construct the required Lie group G, with his bare hands, out of the finitely generated group Γ. He does this by a strange limiting argument as follows.

Let δ_0 be the left invariant "word length metric"

$$\delta_0(x,y) = \min\{r \mid x^{-1}y \in S^r\}$$

on the discrete group Γ, and let $\delta_k = \delta_0/2^k$. If Γ has polynomial growth of degree $\leq d$, then Gromov shows that some subsequence of the sequence of metric spaces (Γ, δ_k) "converges" to a locally compact, locally connected, homogeneous metric space Y of Hausdorff dimension $\leq d$. Roughly speaking, for each r, the balls of radius r in (Γ, δ_k) and in Y can be isometrically embedded in a common metric space so that each point of one is close to some point of the other.

Using the work of Montgomery and Zippin, he shows that the group consisting of all isometries of this limit space Y is a Lie group G, with finitely many components. He then completes the argument by constructing appropriate homomorphisms from subgroups of finite index in Γ to G.

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Titel: The Andreotti-Frankel Conjecture

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Let M be a compact Kähler manifold with Kähler metric $\sum h_{\alpha\bar\beta}\, dz^\alpha d\overline{z^\beta}$. The curvature tensor of M is

$$R_{\alpha\bar\beta\gamma\bar\delta} = \partial_\alpha \partial_{\bar\beta} h_{\gamma\bar\delta} - \sum h^{\lambda\bar\mu}\, \partial_\alpha h_{\gamma\bar\mu}\, \partial_{\bar\beta} h_{\lambda\bar\delta}.$$

M is said to have positive holomorphic bisectional curvature if

$$\sum R_{\alpha\bar\beta\gamma\bar\delta}\, \xi^\alpha \overline{\xi^\beta}\, \eta^\gamma \overline{\eta^\delta} < 0$$

for nonzero (ξ^α) and (η^α). The Andreotti-Frankel conjecture says that a compact Kähler manifold of positive holomorphic bisectional curvature must be biholomorphic to the complex projective space. The dimension 2 case was proved by Andreotti-Frankel [2]. The dimension 3 case was proved by Mabuchi [5]. The general case was recently proved by Mori [6] using the methods of algebraic geometry of positive characteristic and also by Siu-Yau [8] using the methods of harmonic maps and differential geometry. In this talk I will discuss the differential-geometric proof of Siu-

Yau. According to the result of Kobayashi-Ochiai[4] the complex projective space of dimension n is characterized by the fact that its first Chern class equals $\lambda c_1(F)$ for some $\lambda \geq n+1$ and some holomorphic positive line bundle F over it. Since the result of Bishop-Goldberg[1] says that the second Betti number of a compact Kähler manifold M of positive holomorphic bisectional curvature is 1, to prove the conjecture it suffices to show that $c_1(M)$ is λ times the generator of the cohomology group $H^2(M, \mathbb{Z})$ for some $\lambda \geq 1 + \dim M$. This can be done by proving that a generator of the free part of $H_2(M, \mathbb{Z})$ can be represented by a rational curve, because the tangent bundle of M restricted to the rational curve splits into a direct sum of holomorphic line bundles over the rational curve according to a result of Grothendieck [3]. The existence of the rational curve is obtained in the following way. According to a result of Sacks-Uhlenbeck [7], the infimum of the energies of maps from S^2 to M representing the generator of $\pi_2(M)$ can be achieved by a sum of stable harmonic maps f_i from S^2 to M $(1 \leq i \leq m)$. The key step in our proof is to show that each f_i is either holomorphic or antiholomorphic. This is done by using the second variation formula of the energy function. In this second variation formula a 2-parameter variation has to be used to imitate the situation of holomorphic deformation. After this key step we use holomorphic deformations of rational curves in M to show that $m=1$. We do it by showing that in case $m>1$ we can holomorphically deform the images of some holomorphic f_i and some antiholomorphic f_j so that they are tangential to each other at some point. By removing a disc centered at the point of contact from each and joining the two disc boundaries by a suitable surface, we obtain a map from S^2 to M with energy smaller than the minimum energy. Thus $m=1$ and the image of f_1 is a rational curve representing a generator of the free part of $H_2(M, \mathbb{Z})$.

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Titel: Rational curves on projective varieties (work of Mori)

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Let X be a smooth, projective variety of dimension n. Mori has shown that if the canonical class on X is not numerically effective, i.e., if there is a curve C in X such that $(K\cdot C)<0$, then X also contains rational curves with the same property.

Consider the real vector space N of 1-cycles on X, modulo numerical equivalence, and the convex cone $\mathcal{E}$ in N of effective 1-cycles $z=\sum r_i C_i$, $r_i\geq 0$. This cone may not be closed; let $\overline{\mathcal{E}}$ denote its closure. Also, let $\overline{\mathcal{E}}'=\{z\in\overline{\mathcal{E}} \mid (K\cdot z)\geq 0\}$. An <u>extremal curve</u> C is one such that $C=z_1+z_2$ and $z_i\in\mathcal{E}$ implies $z_i=r_iC$ (in N).

<u>Theorem 1:</u> $\overline{\mathcal{E}}=\overline{\mathcal{E}}'+\sum\mathbb{R}^+C$, where the sum runs over extremal rational curves (which may be singular) such that

$$0>(K\cdot C)\geq-(n+1).$$

Suppose X is a surface ($n=2$). The extremal rational curves turn out to be smooth, and so by the genus formula the three values $(K\cdot C) = -1, -2, -3$ allowed in theorem 1 correspond to $(C^2) = -1, 0, 1$. One obtains the following classical result:

Theorem 2: Let X be a surface such that $(K\cdot C) < 0$ for some C. Then one of the following holds:

(i) X contains an exceptional curve
(ii) X is ruled
(iii) $X \approx \mathbb{P}^2$.

Theorem 3: (Hartshorne conjecture)
If the tangent bundle T_X is ample, then $X \approx \mathbb{P}^n$.
This is proved by showing that an extremal curve has the properties of a line in $\mathbb{P}^n$.

Mori's most recent work has been an extension of theorem 2 to 3-folds. Consider an extremal rational curve C on X. Then because of the geometry of convex sets, one can find a D in the dual space N^* of divisors, such that D has integer coefficients, $(D\cdot Z)\geq 0$ if $Z\in\overline{\mathcal{E}}$, and $\{(D\cdot Z)=0\}\cap\overline{\mathcal{E}} = \mathbb{R}^+C$. Kleiman's criterion is that a divisor D' is ample iff $(D'\cdot Z)>0$ for all $Z\in\overline{\mathcal{E}}$. Therefore D is nearly ample. However, the map to projective space defined by the linear system $|ND|$ will contract C because $(D\cdot C)=0$. By a detailed analysis of this map, Mori proves

<u>Theorem 4</u>: Let X be a 3-fold such that $(K\cdot C)<0$ for some C. Then one of the following cases occurs:

(i) X contains an exceptional surface of one of the types listed below.

(ii) X is a conic bundle over a smooth surface, with standard degeneracies.

(iii) X is fibred in Fano surfaces over a curve.

<u>(iv) X</u> is a Fano 3-fold.

<u>List</u> of exceptional surfaces occurring in (i):

(a) $E=\mathbb{P}^2$, normal bundle $\mathcal{O}(-1)$ or $\mathcal{O}(-2)$.

(b) E is a ruled surface, and if C is a ruling then $(E\cdot C)=-1$

(c) $E = \mathbb{P}^1 \times \mathbb{P}^1$, with normal bundle $\mathcal{O}_{\mathbb{P}^1}(-1) \otimes \mathcal{O}_{\mathbb{P}^1}(-1)$.

(d) E is a quadric cone in $\mathbb{P}^3$, with normal bundle $\mathcal{O}_{\mathbb{P}^3}(-1)|_E$.

Mori's methods are completely projective, so that the exceptional surfaces found in theorem 4 are projectively contractible. Note that in case (b) for example, there is no theorem asserting that such an E can always be contracted. Nevertheless, $(C\cdot K) = -1$ in this case. So, given any such E, Mori's theorem asserts that X contains some exceptional surface.

Finally, note that theorem 4 has a weakness, as compared to theorem 2, because contraction of E may lead to a singular 3-fold for which the result no longer applies.

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Titel: L-series of elliptic curves

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This talk was a report on some recent results of D. Rohrlich on the L-series of elliptic curves with complex multiplication [3].

Let K be an imaginary quadratic field and let E be an elliptic curve defined over the Hilbert class field H of K with complex multiplication by $\mathcal{O}_K$. Assume that the L-series of E factors as a product over the h ideal class characters φ of K:

$$L(E,s) = \prod_{\varphi} L(\chi\varphi, s)^2 ,$$

where χ is a Hecke character of K. Then E is a $\mathbb{Q}$-curve in the sense of [2], and one conjectures that the rank of the Mordell-Weil group $E(H)$ is equal to $2h \operatorname{ord}_{s=1} L(\chi,s)$.

Using estimates from analytic number theory, Rohrlich is able to show that $L(\chi,1) \neq 0$ in most cases where the conductor of χ divides the different of K. All of the results obtained are consistent with the above conjecture and the rank calculations made in [2]. In fact, when combined with a theorem of N. Arthaud [1], Rohrlich's results yield a successful calculation of $E(H)$ in several new cases.

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* This paper generalizes results of Coates and Wiles (Inv. Math 39 (1977)) to the case $h > 1$.

Titel: Turbulence and strange attractors

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As proposed in [3], the dynamics of a fluid or gas near the onset of turbulence might be modelled as a dynamical system or vectorfield X on a smooth manifold M, both depending on a parameter. The experimental data should then correspond to $y(X_t(p))$, $y: M \to \mathbb{R}$ is a smooth function and X_t is the time t map of X and $p \in M$ is the initial position. The experiments described in [1.] give such data (for discrete values of time $t = 0, \alpha, 2\alpha, 3\alpha, \ldots$). Our theorem below describes algoritms to calculate dimension and entropy of the attractor to which the initial position tends in terms of the experimental data, i.e., in terms of $\{y(X_{i\cdot\alpha}(p))\}_{i=0}^{\infty}$.

If these algoritms, when applied to the actual experimental data in [1.] would not canverge to a finite value this would be a good reason to abandon the descriptions in terms of smooth dynamics in finite dimen. sions given in [2.] and [3.].

If the entropy would come out as a positive number and the dimension as a non-integer, this would be strong evidence in favor of the occuring of strange attractors in the sense of [3].

If the entropy would be zero and the dimension would be in integer this would be evidence for (multi periodic) quasi periodic motion as in [2.].

Theorem. Let M be a smooth compact manifold, X a smooth vectorfield and y a smooth function on M; $p \in M$ and $\alpha \in \mathbb{R}_+$. We assume that

$p \in L^+_X(p)$ (which is automatically satisfied if our initial condition were already in an attractor). Then, for generic such (X, y, p, α) we have the following formulas for the dimension*) of $L^+_X(p)$ (the positive limit set of p) and for the entropy of $X_\alpha | L^+_X(p)$:

$$\text{dimension } L^+_X(p) = \lim_{n\to\infty}\left(\liminf_{\varepsilon\to 0}\left(\frac{\ln|\mathcal{Y}_{n,\varepsilon}|}{n-\ln\varepsilon}\right)\right)$$

$$\text{entropy } X_\alpha|L^+_X(p) = \lim_{\varepsilon\to 0}\left(\limsup_{n\to\infty}\left(\frac{\ln|\mathcal{Y}_{n,\varepsilon}|}{n-\ln\varepsilon}\right)\right).$$

$\mathcal{Y}_{n,\varepsilon} \subset \mathbb{N}$ denote the following subsets :

$0 \in \mathcal{Y}_{n,\varepsilon}$

$k \in \mathcal{Y}_{n,\varepsilon}$ if and only if for all $l < k$, $l \in \mathcal{Y}_{n,\varepsilon}$, $\max(|a_k - a_l|, |a_{k+1} - a_{l+1}|, \ldots, |a_{k+n} - a_{l+n}|) > \varepsilon$,

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*) The definition of dimension, used here, is almost the same as Haussdorf's definition.

where $a_i = y(X_{i,\alpha}(p))$.

(A preprint, containing a proof of this theorem, is available from the Math. Dept. Groningen).

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Titel: Periodic motions in hamiltonian systems.

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One can apply geometric methods in order to find periodic motions in hamiltonian systems of certain type. One needs to require that the hamiltonian $H: T^*M \rightarrow R$ is convex on each cofiber($H_{pp} > 0$). The Jacobi metric is then a Finsler metric and one tries to find closed geodesics or brake orbits of this Finsler metric. We call such hamiltonians convex and say that H is even if $H(q,p)=H(q,-p)$. The following theorems fit in this framework:

<u>Theorem</u> (Birkhoff 1917). Any convex hamiltonian on S^n has one periodic orbit.

<u>Theorem</u> (Fet-Lusternik 1951). Any convex hamiltonian on a compact manifold M has one periodic orbit.

If the hamiltonian H is convex and even we define the potential function $V(q)=H(q,o)$ as in the classical case. The motion is then restricted to the set $M_E=V^{-1}(-\infty,E]$. If M_E has no boundary the above theorems apply and if M_E has boundary we have:

<u>Theorem</u> (Seifert 1947). If M_E is homeomorphic to a disc there exists a brake orbit of energy E.

<u>Theorem</u> (Gluck-Ziller 1980). If M_E is compact there exists a brake orbit of energy E.

If M_E contains a nondegenerate critical point one can find a homoclinic or heteroclinic orbit by these methods.

In special situations one can show that there exists several periodic orbits:

<u>Theorem</u> (Klingenberg 1968). If the sectional curvature satisfies $\frac{1}{4} < K \leq 1$ then the metric has dim M closed geodesics.

In joint work with Ballmann and Thorbergsson I was able to show that this theorem has a generalization to convex hamiltonians,even without any evenness assumption.

<u>Theorem</u> (Ekeland-Lascry 1979). If $H: R^{2n} \longrightarrow R$ has a convex and compact energy hypersurface $H^{-1}(E)$ such that

$$B(1) \subset H^{-1}(E) \subset B(\sqrt{2})$$

then there exist n periodic orbits of energy E.

One hopes that both theorems are true for arbitrary convex hamiltonians.

In certain cases one can also make statements about stability proprties of the periodic orbits obtained by these methods. Let c be a closed geodesic on S^n obtained by Birkhoff's method.

<u>Theorem</u> (Ballmann-Thorbergsson-Ziller 1980).

(1) If $\frac{1}{4} < K \leq 1$ then c is nonhyperbolic, i.e. one eigenvalue of the linearized Poincaré map satifies $|z|=1$.

(2) If $\frac{9}{16} < K \leq 1$ then c is elliptic, i.e. all eigenvalues satisfy $|z| = 1$ and the linearized Poincaré map splits into 2-dimensional rotations or blocks $\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$.

Using the Birkhoff-Lewis fixedpoint theorem and the theorem of Kolmogorov-Arnold Moser this implies that a metric with $\frac{1}{4} < K \leq 1$ has generically infinitely many closed geodesics and that a metric with $\frac{9}{16} < K \leq 1$ has generically a positive measure of invariant tori in the tangent bundle on which the geodesic flow is quasi-periodic, in particular the geodesic flow is not ergodic.

Titel: Simple groups over $\mathbb{C}((t))$ and simply-elliptic singularities

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The main result of Brieskorn's talk at the Arbeitstagung 1970 may be roughly stated as follows:

The semiuniversal deformation $\varphi : X \longrightarrow V$ of a simple singularity (also called Kleinian singularity or rational double point) of type A_r, D_r, E_6, E_7, E_8 can be embedded into the characteristic morphism $\chi : G \longrightarrow \mathbb{C}^r \cong T/W$ of the corresponding simple Lie group G

$$\begin{array}{ccc} X & \hookrightarrow & G \\ \varphi \downarrow & & \downarrow \chi \\ V & \hookrightarrow & T/W \end{array}$$

The morphism χ maps an element $g \in G$ to $(\chi_1(g), \dots, \chi_r(g))$ where χ_i is the trace of the i-th fundamental representation of G.

Moving up in the "hierarchy" of singularities one finds the simply-elliptic ones which are called $\widetilde{E}_8, \widetilde{E}_7, \widetilde{E}_6$ (parabolic singularities of Arnol'd) and $\widetilde{D}_5$ (intersection of two quadrics in $\mathbb{C}^4$) and $\widetilde{A}_4$ (cone in $\mathbb{C}^5$) whose deformation theory was studied by Looijenga, Pinkham and Knörrer.

Now let G be a simple, simply connected algebraic group over $\mathbb{C}$ with maximal Torus T and $W = N_G(T)/T$ the Weylgroup. We have $\dim T = \operatorname{rank} G =: r$. We let $G(K)$ denote the group of points of G over the formal power series field $K = \mathbb{C}((t))$. If G has Dynkin diagram Δ, then to $G(K)$ there is attached the extended Dynkin diagram $\tilde{\Delta}$. Let $\lambda \in \Lambda \cong \mathbb{C}^*$ be the automorphism ${}^{\lambda}p(t) := p(\lambda t)$, $p(t) \in K$, of K. Then λ induces an automorphism of $G(K)$ and we can form the semidirect product $G(K) \rtimes \Lambda$. There is a central extension

$$1 \longrightarrow \mathbb{C}^* \longrightarrow \mathcal{G} \xrightarrow{\varepsilon} G(K) \rtimes \Lambda \longrightarrow 1$$

defined by the inverse of the tame symbol of K.

The group $\mathcal{G}$ acts on the fundamental representations $\varrho_0, \dots, \varrho_r$ of the affine Kac-Moody-Lie algebra associated to $\mathcal{G}$. These representations are infinite dimensional. However for a part of the group $\mathcal{G}$ convergent characters can be defined. Let $\nu\colon \mathcal{G} \longrightarrow \mathbb{C}^*$ denote the composition $\mathcal{G} \xrightarrow{\varepsilon} G(K) \rtimes \Lambda \xrightarrow{\pi_2} \Lambda$ and let $\mathcal{G}^c$ denote the elements in $\mathcal{G}$ which are conjugate into an Iwahori subgroup of $\mathcal{G}$. Define $\mathcal{G}^c_{>\lambda_0} := \{ g \in \mathcal{G}^c \mid |\nu(g)| > \lambda_0 \}$ for any $\lambda_0 \in \mathbb{R}^*$.

<u>Theorem:</u> There is a $\lambda_0 \geq 1$ such that the map
$\chi : \mathcal{G}^0_{>\lambda_0} \longrightarrow \mathbb{C}^{r+1} \times \Lambda$, $g \longmapsto (\text{trace } \rho_i(g), \nu(g))$
is well defined.

Let $\mathcal{T} \subset \mathcal{G}$ denote the "maximal" $\mathbb{C}$-Torus $\varepsilon^{-1}(T(\mathbb{C}) \times \Lambda)$.

<u>Lemma.</u> $N_{\mathcal{G}}(\mathcal{T})/\mathcal{T}$ is isomorphic to the affine Weyl group $W \ltimes \mathbb{Z}^r =: \tilde{W}$. The group $\tilde{W}$ acts effectively and discontinuously on $\mathcal{T}$.

The restriction of χ to $\mathcal{T} \cap \mathcal{G}^0_{>\lambda_0}$ may be identified with the natural quotient $\mathcal{T}_{>\lambda_0} \longrightarrow \mathcal{T}_{>\lambda_0}/\tilde{W}$. The image of χ is equal to $\mathcal{T}_{>\lambda_0}/\tilde{W} \cong (\mathbb{C}^{r+1} \setminus \{0\}) \times \Lambda_{>\lambda_0}$

<u>Theorem</u>: Let $s \in \mathcal{T}_{>\lambda_0}$. Then the centralizer $Z(s)$ of s is a finite-dimensional reductive group over $\mathbb{C}$. The fiber $\chi^{-1}(\chi(s))$ is isomorphic to the associated bundle $\mathcal{G} \times^{Z(s)} (\mathcal{U}(s))$, where $\mathcal{U}(s)$ is the unipotent variety of $Z(s)$.

<u>Corollary:</u> Each fiber of χ is the union of finitely many conjugacy classes

Using an Iwahori subgroups of $\mathcal{G}$ one can construct a simultaneous resolution for χ similarly as was done by Grothendieck and Springer for the case of groups over $\mathbb{C}$.

Now let $\varphi : X \longrightarrow V$ be a semiuniversal deformation of a simply-elliptic singularity. Let $V_f = \{ v \in V \mid \varphi^{-1}(v)$ is smooth or has at most simple singularities$\}$ and let $X_f = \varphi^{-1}(V_f)$

Using Looijenga's description of V_f one can identify V_f with $\mathcal{T}_{>1} / \tilde{W}$ and a part of V_f (excluding certain j-values for the elliptic singularities) with $\mathcal{T}_{>\lambda_0} / \tilde{W}$ such that the map φ

$$\begin{tikzcd} X_f \arrow[rr, dashed] \arrow[dr, "\varphi"'] & & \mathfrak{g}^0_{>\lambda_0} \arrow[dl, "\chi"] \\ & V_f & \end{tikzcd}$$

can be embedded <u>locally</u> over V_f into χ

Unfortunately we do not obtain a global inclusion $X_f \subset \mathfrak{g}^0_{>\lambda_0}$. We also miss the simply-elliptic deformations. However, we believe that χ can be extended over $\mathbb{C}^{r+1} \times \Lambda_{>\lambda_0}$ and that then even X can be embedded into a partially compactified $\mathfrak{g}^0$.

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Titel: Geodesic cycles and the Weil representation

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The results discussed here are part of on going joint work with J. Millson [3]. These results are rather appropriate for the Arbeitstagung since they have one of their origins in the work of Hirzebruch and Zagier [2], who discovered a relation between intersection numbers of certain curves on Hilbert modular surfaces and the Fourier coefficients of elliptic modular forms. They also showed that these relations were connected with the Doi-Naganuma lifting [1] which produces Hilbert modular forms from elliptic modular forms. It now appears that the phenomena which they discovered are quite general; and that, at least from one point of view, the source of these phenomena is the Weil representation — or in more classical terminology, the theory of Θ-functions.

I will discuss the special case, $SO(n,1)$, where things are most complete.

Let V be a $\mathbb{R}$ vector space, $(\,,\,)$ a non-degenerate inner product on V with signature $(n,1)$. Let $D = \{Z \in V \mid (Z,Z) = -1\}^0$ be one component of the hyperboloid of two sheets in V. Let $G = SO^0(V)$ be the connected component of the special orthogonal group of $V, (\,,\,)$. Let $\Gamma \subset G$ be an arithmetic subgroup such that the quotient $M = \Gamma\backslash D$ is a smooth compact manifold, $\dim_{\mathbb{R}} M = n$.

Let $Fr_k^+(V) = \{X = (X_1, \dots, X_k) \in V^k \mid (\,,\,)|_{\text{span}X} > 0 \text{ , } \dim \text{span} X = k\}$ and let $Fr_k^+(V)_{\mathbb{Q}}$ denote the set of such k-frames which are 'commensurable with Γ'.

For $X \in Fr_k^+(V)_{\mathbb{Q}}$, let $D_X = \{ z \in D \mid (z, X) = 0 \}$, let $G_X = \{ g \in G \mid gX = X \}$ and let $\Gamma_X = \Gamma \cap G_X$. Then we may define a codimension k cycle in M by

$$\begin{tikzcd} D_X \arrow[r] \arrow[d] & D \arrow[d] \\ M_X = \Gamma_X \backslash D_X \arrow[r, "\iota_X"] & \Gamma \backslash D = M \end{tikzcd}$$

$C_X = (\iota_X)_*(1_X) \in H_{n-k}(M, \mathbb{Z})$.

First we construct the Poincaré dual form to C_X.

There is a geodesic fibration $\pi : D \to D_X$ which is G_X-equivariant. Let μ_X be the G_X-invariant volume form on D_X, and let

$$\varphi_{X,s} = \|\varphi_X\|^{\frac{s}{n-k}} \varphi_X$$

where

$$\varphi_X = * \pi^* \mu_X \quad .$$

Let

$$\omega_{X,s} = \kappa(s)^{-1} \sum_{\gamma \in \Gamma_X \backslash \Gamma} \gamma^* \varphi_{X,s} \quad .$$

<u>Theorem I.</u> i) $\omega_{X,s}$ converges absolutely for $Re(s) > \frac{k-1}{2}$, and has a meromorphic continuation to the whole s-plane.

ii) $\Delta \omega_{X,s} = -2s(2s+n-2k+1)(\omega_{X,s} - \omega_{X,s+1})$.

iii) $\omega_{X,s}$ is regular at $s=0$ and $\omega_{X,0}$ is <u>the</u> harmonic Poincaré dual form to C_X.

Next we consider the whole family of cycles parameterized by $X \in Fr_k^+(V)_{\mathbb{Q}}$.

Let k be a totally real field, $|k:\mathbb{Q}| = g$. Let W be an n-dimensional k vector space and $(\,,\,)$ a non-degenerate k-bilinear form on W such that $sig(W_\lambda, (\,,\,)_\lambda) = (n,1)$ if $\lambda = 1$, and $(n+1, 0)$ if $\lambda > 0$, where

k_λ, $\lambda=1,\dots,g$ are the real completions of k and W_λ are the corresponding completions of W. Let L be an $\mathcal{O}_k$-lattice in W, L^* the dual lattice and $\Gamma \subset \Gamma_L$ a congruence subgroup of the unit group of $L,(\,,\,)$. Then we may take $V=W_1$ in the previous construction, imbed W and L into V and identify Γ with a subgroup of $SO^0(V,(\,,\,))$. Then $Fr_k^+(V)_{\mathbb{Q}} = Fr_k^+(W)$ where $Fr_k^+(W)$ is the set of totally positive k-frames in W. Note that if $X \in Fr_k^+(W)$, then $\beta = (X,X) \in M_k(k)$ with ${}^t\beta = \beta \gg 0$.

Let $Fr_k^+(W)_{\mathbb{Z}} = Fr_k^+(W) \cap (L^*)^k$, and for any $\beta \in M_k(k)$ and $h \in (L^*/L)^k$ with ${}^t\beta = \beta \gg 0$, let

$$C_\beta = \sum_{\substack{X \in Fr_k^+(W)_{\mathbb{Z}} \\ (X,X)=\beta,\ X \equiv h (L^k) \\ \mathrm{mod}\,\Gamma}} c_X ,$$

so that

$$C_\beta \in H_{n-k}(M,\mathbb{Z}).$$

Let $\mathfrak{H}_k$ be the Siegel space of genus k. Then for $\tau = (\tau_1,\dots,\tau_g) \in (\mathfrak{H}_k)^g$ and any $C \in H_k(M)$, define

$$I(\tau, C) = \sum_\beta C \cdot C_\beta \; e(\mathrm{tr}_k \mathrm{tr}(\beta\tau)) .$$

<u>Theorem II.</u> i) $I(\tau,C)$ *is a Hilbert-Siegel modular cusp form of weight* $\frac{1}{2}(n+1)$ for a certain congruence subgroup $\tilde{\Gamma} \subset Sp(k,\mathcal{O}_k)$, so that we have a map

$$I : H_k(M) \longrightarrow S_{\frac{1}{2}(n+1)}(\tilde{\Gamma}) .$$

ii) Assume that $4k < n+1$, and let $\mathcal{H}^k(M)$ be the space of harmonic k-forms on M. Then there is a 'lifting' defined by a Θ-kernel

$$L : S_{\frac{1}{2}(n+1)}(\tilde{\Gamma}) \longrightarrow \mathcal{H}_k(M) \otimes \mathbb{C}$$

whose image is precisely the span of the harmonic dual forms to the cycles C_β. Explicitly we have

$$L(\varphi_\beta) = \kappa\, \omega_{\beta,0}$$

where $\omega_{\beta,0}$ is the harmonic dual form to C_β and φ_β is the βth Poincaré series:

$$\varphi_\beta(\tau) = \sum_{\gamma \in \tilde{\Gamma}_\infty \backslash \tilde{\Gamma}} N(\det(c\tau+d))^{-\frac{1}{2}(n+1)}\, e(\mathrm{tr}\,\mathrm{tr}(\beta\gamma\tau))$$

in $S_{\frac{1}{2}(n+1)}$.

iii) L and I are adjoints up to a constant, i.e.:

$$\langle \varphi, I(C) \rangle = 2^{k/2} \langle L(\varphi), C \rangle$$

where $\langle \varphi, I(C) \rangle$ is the Petersson inner product in $S_{\frac{1}{2}(n+1)}(\tilde{\Gamma})$ and

$$\langle L(\varphi), C \rangle = \int_C L(\varphi) .$$

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Titel: A theorem of Thurston with applications to group actions and foliations

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Let H be a topological group with a generating compact neighbourhood of the identity. If H is discrete then this condition is equivalent to H being finitely generated. Let X be a Banach space (for example $\mathbb{R}^n$) and let

$$[F] : H \times X, H \times O \longrightarrow X, O$$

be the germ of a group action keep O fixed. This means that F is defined on a neighbourhood U of $H \times O$ in $H \times X$ and that the associative law holds near $H \times H \times O$ in $H \times H \times X$ and $F(id, x) = x$.

Theorem 1. Let F be C^1 i.e. let $\partial F/\partial x$ be continuous. Then at least one of the following conditions holds:

1) There is a neighbourhood of $H \times O$ on which F is trivial — i.e. $F(h, x) = x$
2) The homomorphism $H \longrightarrow GL(X)$ given by $h \longmapsto Dh(0) = \dfrac{\partial F}{\partial x}(h, 0)$ is non-trivial.
3) There is a continuous homomorphism $H \to \mathbb{R}$ which is non-trivial.

This is a reformulation of a result of Thurston, Topology Vol 13 (1974) pp. 347-352, with some minor frills added. The proof of this result which Thurston published is fine for discrete groups, but fails for topological groups. Thurston has a correction to the proof (unpublished), but nevertheless the result has been referred to in print as an open problem.

It is proposed to give a proof of this result together with applications such as the following:

Theorem 2. Let M be a connected manifold and let $\mathcal{H}$ be a finite dimensional subalgebra of the Lie algebra of vector fields on a manifold. Let $U \neq \emptyset$ be an open subset of M such that $\mathcal{H}|U$ is zero. Then $\mathcal{H} = 0$ on M.

Theorem 3 Let M be a connected manifold and let $F: H \times M \longrightarrow M$ be a C^1 action of a topological group H generated by a compact neighbourhood of the identity. Suppose there is an open subset $U \neq \emptyset$ of M on which H acts trivially. Then either the action is trivial throughout M or there is a continuous non-trivial homomorphism $H \longrightarrow \mathbb{R}$

Theorem 4 (Thurston-Reeb Stability Theorem) Let M be a compact connected manifold with a transversely oriented C^1-foliation. Suppose there is one leaf L which is compact and such that $H^1(L) = 0$. Then each leaf is diffeomorphic to L and the foliation is given by the fibres of a smooth fibre bundle $M \longrightarrow S^1$.

Theorems 3 and 4 are due to Thurston. Theorem 2 was undoubtedly known to Thurston. However, to prove it one needs the mild reformulation of the theorem in his paper, given above as Theorem 1.

Theorem 4 is of fundamental significance in the theory of codimension 1 foliations.

Proof of Theorem 1. Let $K=K^{-1}$ be compact and generate H. We suppose that in each neighbourhood U of $H\times 0$ there is a point (h,x) such that $h.x \neq x$. We suppose that $\frac{\partial F}{\partial x}(h,0) = id : X \to X$ for each $h \in H$.

For each $r>0$, $\exists\, x \in X$ with $\|x\|<r$ and $h\in K$ with $hx \neq x$.

Let $G(g,x) = gx - x$. If all terms make sense, then

$$G(g,y) - G(g,x) = \int_0^1 DG(g, ty+(1-t)x)\,dt\,.(y-x)$$

Putting $y = h.x$, we obtain

$$\begin{aligned} G(gh,x) &= gh.x - x \\ &= g.hx - x \\ &= G(g,hx) + G(h,x) \\ &= G(g,x) + G(h,x) + \int_0^1 DG(g, ty+(1-t)x)\,dt\,.G(h,x) \end{aligned}$$

So $G(gh,x) = G(g,x) + G(h,x) + A(g,h,x)\,G(h,x)$ (1)

$A(g,h,0)=0$ for all $g,h\in H$, where A is continuous and A is defined on a neighbourhood of $H\times H\times 0$ in $H\times H\times X$ and has values in $\mathrm{Lin}(X,X)$.

We inductively choose r_n so that

i) $G(g,x)$ is defined for $g\in K^{2n}$ and $\|x\|\leq r_n$;

ii) if $g,h\in K^n$ and $\|x\|\leq r_n$ then $A(g,h,x)$ is defined and the above equation is true and $\|A(g,h,x)\| \leq 1/2^{n+1}$;

iii) $r_n < r_{n-1}/2$;

iv) $0<\sup\{\|G(g,x)\| ; g\in K, \|x\|\leq r_n\} < \infty$ and we call this number M_n.

Let $g_n \in K$ and $x_n\in X$ with $\|x_n\|\leq r_n$ such that $\|G(g_n,x_n)\| > M_n/2$. By induction on m, using i) and ii), we see that

$$m\leq n,\ g\in K^m,\ \|x\|\leq r_n \implies \|G(g,x)\| \leq 2m\,M_n \quad (2).$$

By the Hahn-Banach theorem, there is a linear functional $\lambda_n : X \to \mathbb{R}$ such that $\|\lambda_n\| = 1/M_n$ and

$$1/2 \leq \lambda_n G(g_n,x_n) \leq 1. (3).$$

Let $T_n(g) = \lambda_n G(g, x_n)$ so that $T_n : K^{2^n} \to \mathbb{R}$.
Then if $h \in K^m$, $g \in K^n$ and $m \le n$, we have by (2)

$$|T_n(h)| \le 2m \qquad (4)$$

and by (1) and (2)

$$|T_n(gh) - T_n(g) - T_n(h)| \le m/2^n \qquad (5).$$

By induction on r we define a compact neighbourhood U_r of the identity in H, such that

v) $U_1 = K$, $U_r \subseteq U_{r-1}$ and $U_r^r \subseteq K$

and vi) $|T_n(gh) - T_n(g)| \le 1/r$ for $n \le r$, $h \in U_r$ and $g \in K^n$.

By induction on r, we obtain from (5) with $m = 1$ that

$$h \in U_r \implies |T_n(h^r) - r T_n(h)| \le r/2^n \qquad (6).$$

Now, if $h \in U_r$, then $|T_n(h^r)| \le 1$, and so (6) implies that $|T_n(h)| \le \frac{1}{2^n} + \frac{1}{r}$ (7).

By vi) if $n \le r$ and by (5) and (7) if $n > r$,

$$|T_n(gh) - T_n(g)| \le 2/r \text{ if } g \in K^n \text{ and } h \in U_r \quad (8).$$

The sequence $T_m, T_{m+1}, T_{m+2}, \ldots$ is equicontinuous on K^m by (8) and bounded on K^m by (4). It follows from Ascoli's Theorem that there is a convergent subsequence $T_{i_1}, T_{i_2}, \ldots$ which is in fact defined and convergent on every K^n. The limit $T : H \to \mathbb{R}$ is continuous and $T(id) = 0$. If we fix g, h and m, and let n tend to infinity in (5), we see that T is a homomorphism. Finally T is non-trivial since we may assume that g_n tends to a limit g and then by (3)

$$T(g) \geqslant \underrightarrow{\lim}\, T_n g_n \geqslant 1/2.$$

We may assume $\mathfrak{h}$ is simple by standard Lie algebra results.

Proof of Theorem 2 Integrate $\mathfrak{h}$ to get a germ of an action of the connected Lie group H. Let U be the complement of the support of $\mathfrak{h}$. By Theorem 1 $\bar{U}$ is open. Hence $U = \bar{U} = M$.

The proof of Theorem 3 is the same.

The proof of Theorem 4 is standard and is due to Reeb.

Theorem 1 is false if the assumption of differentiability is dropped. Let H be the universal cover of $PSL(2,\mathbb{R})$. This acts on $\mathbb{R}$, the universal cover of $P^1(\mathbb{R}) = S^1$, and hence on $\mathbb{R} \cup \{\infty\} = S^1$, with a fixed point. There are many perfect finitely presented subgroups of H.

Theorem 1 is false if compactness assumptions on H are removed. ~~Thus~~ It can be deduced from results of Sergeraert and Mather that the group of C^∞ diffeomorphisms of $\mathbb{R}$ keeping 0 fixed, with compact support, and which are C^∞ tangent to the identity, is perfect.

Titel: Systematic phenomena in p-primary stable homotopy, as revealed by BP-theory

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Let X and Y be (say) finite CW-complexes. One may wish to study the (graded) group $[X,Y]^S_*$ of stable homotopy classes of stable maps from X to Y, at a given prime p. One has a spectral sequence

$$\mathrm{Ext}^{s*}_{A_*}(\tilde{H}_*(X;F_p),\tilde{H}_*(Y;F_p)) \underset{s}{\Rightarrow} [X,Y]^S_{*(p)};$$

here A_* is the dual of the algebra A^* of stable operations on mod p cohomology, and $\tilde{H}_*(X;F_p)$, $\tilde{H}_*(Y;F_p)$ are considered as comodules over A_*. One may seek to replace ordinary homology by a suitable generalised homology theory; this was first done by S.P. Novikov, who replaced ordinary cohomology by complex cobordism and so obtained the Novikov spectral sequence. I give a homology version.

$$\mathrm{Ext}^{s*}_{MU_*(\underline{MU})}(\widetilde{MU}^*(X),\widetilde{MU}^*(Y)) \underset{s}{\Rightarrow} [X,Y]^S_*.$$

Here $\underline{MU}$ means the Thom spectrum, which consists of the sequence of spaces $MU(n)$; and if $\underline{E}$ is a spectrum, I write E_* for the corresponding homology theory —

in this case complex bordism. It has to be explained in what sense $MU_*(\underline{MU})$ is a Hopf algebra and in what sense $\widetilde{MU}_*(X)$, $\widetilde{MU}_*(Y)$ are comodules over it. This spectral sequence is good at all primes at once. At a given prime p, the spectrum $\underline{MU}$ splits as a sum of suspensions of the Brown-Peterson spectrum $\underline{BP}$. One then has a spectral sequence

$$\mathrm{Ext}^{s*}_{BP_*(\underline{BP})}(\widetilde{BP}_*(X), \widetilde{BP}_*(Y)) \underset{s}{\Rightarrow} [X, Y]^S_{*(p)}.$$

One has a fair ability to compute these Ext groups; this rests on the method of formal groups and the insights of Jack Morava.

The principal achievement of this method, so far, is to disentangle phenomena in p-primary stable homotopy which show systematic behavior or periodicity of various different kinds. Here I must recall that $\pi_*(BP)$ is a polynomial algebra $\mathbb{Z}_{(p)}[v_1, v_2, \cdots]$ on generators v_i of degree $2(p^i - 1)$ which can be explicitly defined. Some phenomena show systematic behavior related to v_1; these are periodic with period $2(p-1)$ or some multiple of $2(p-1)$; they are well understood. Some show systematic behaviour related to v_2; these are periodic with period $2(p^2-1)$ or some multiple of $2(p^2-1)$; we do not yet have complete control of them, but it looks as if we could. There certainly exist phenomena which have periodic behaviour related to v_3; these are periodic with period $2(p^3-1)$ or some multiple of $2(p^3-1)$; at this point our power to perform the calculations begins to tail off.

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