MATHEMATISCHE ARBEITSTAGUNG 1982

UNIVERSITÄT BONN

Sonderforschungsbereich 40
Theoretische Mathematik
Beringstraße 4
D - 5300 B o n n 1

INHALT

Teilnehmerliste

Programme der Mathematischen Arbeitstagung 1982

Kurzfassungen der Vorträge:

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Mathematisches Institut
der Universität Bonn

Programm der Mathematischen Arbeitstagung 1982 (I)

Dienstag, den 15.6.:

17.00 - 18.00 Uhr: M.F. Atiyah: The Yang-Mills equations and the structure of 4-manifolds

Mittwoch, den 16.6.:

10.30 - 11.30 Uhr: D. Quillen: Determinants of $\bar{\partial}$-operators

15.00 - 16.00 Uhr: J. Coates: Heights on elliptic curves

17.00 - 18.00 Uhr: D. Vogan: Representations with cohomology

Donnerstag, den 17.6.:

9.45 - 10.00 Uhr: Festlegung der nächsten Vorträge

10.00 - 11.00 Uhr: R.S. Palais: Hamilton's work on positively curved 3-manifolds

12.00 - 13.00 Uhr: R. Hartshorne: Space curves

17.00 - 18.00 Uhr: M. Berger: Gromov's filling of Riemannian manifolds

Die Vorträge finden alle im "Großen Hörsaal" (Wegelerstr. 10) statt.

Erfrischungspausen mit Tee: Mittwoch Nachmittag von 16.15 - 17.00 Uhr vor dem Großen Hörsaal; Donnerstag Vormittag von 11.15-12.00 Uhr vor dem Großen Hörsaal; Donnerstag Nachmittag ab 15.30 Uhr im Diskussionsraum Beringstraße 1.

Die Post liegt während der Teepausen aus. Alle Teilnehmer mögen sich bitte in die Teilnehmerlisten eintragen. Teilnehmerlisten und andere Informationen liegen vor dem Diskussionsraum Beringstr. 1 aus. Den Tagungsbeitrag bitte an Frau Gerber (SFB-Büro, Beringstr. 4) bezahlen.

!! Alle Tagungsteilnehmer mit ihren Damen oder Herren sind herzlich zum
!! Empfang des Rektors eingeladen. Zeit: Mittwoch, den 16.6., 20.00 Uhr.
!! Ort: Festsaal der Universität, Hauptgebäude; Eingang von der Straße
!! "Am Hof" durch das Tor gegenüber Buchhandlung Röhrscheid.

Mathematisches Institit
der Universität Bonn

Programm der Mathematischen Arbeitstagung 1982 (II)

Freitag, den 18.6.:

10.00 - 11.00 Uhr: J. Bernstein: Beilison-Bernstein construction

12.45 - ca. 21.00 Uhr: Ausflug nach Leutesdorf. Abfahrt pünktlich um 12.45 Uhr mit Motorschiff "Carmen Silva" am Alten Zoll.

Samstag, den 19.6.:

9.45 - 10.00 Uhr: Festlegung der restlichen Vorträge

10.00 - 11.00 Uhr: S.T. Yau: Manifolds with positive scalar curvature

12.00 - 13.00 Uhr: S. Mori: Rational curves in 3-folds and applications

17.00 - 18.00 Uhr: G. Harder: Tate conjecture for Hilbert modular surfaces

Sonntag, den 20.6.:

10.00 - 11.00 Uhr: L. Siebenmann: M. Freedman's work on 4-dimensional manifolds

Sondervorträge am Donnerstag, den 17.6.: um 15.00 Uhr spricht D.V. Anosov über "Poincaré's approach to problem of closed geodesics"; um 16.00 Uhr spricht V.P. Platonov über "New local-global principles for algebraic groups".

Die Vorträge finden alle im "Großen Hörsaal" (Wegelerstr.10) statt.

Erfrischungspausen mit Tee: Samstag und Sonntag vormittags von 11.15-12.00 Uhr vor dem Großen Hörsaal, Samstag Nachmittag ab 15.30 Uhr im Diskussionsraum Beringstraße 1.

Die Post liegt während der Teepausen aus. Tischtennis im Keller des Hauses Beringstraße 4. Den Tagungsbeitrag bitte an Frau Gerber (SFB-Büro, Beringstraße 4) bezahlen. Alle Teilnehmer mögen sich bitte in die Teilnehmerlisten eintragen. Teilnehmerlisten und andere Informationen liegen vor dem Diskussionsraum Beringstraße 1 aus.

Mathematisches Institut
der Universität Bonn

Programm der Mathematischen Arbeitstagung 1982 (III)

Sonntag, den 20.6.:

12.00 - 13.00 Uhr: S.J. Patterson: Limit sets of Kleinian groups

17.00 - 18.00 Uhr: D. Epstein: On Chapter I of Thurston

Montag, den 21.6.:

10.00 - 11.00 Uhr: S.S. Chern: Web geometry

11.00 - 12.00 Uhr: B. Mazur: $\mathbb{Z}_p$-extensions and heights

17.00 - 18.00 Uhr: F. Adams: Carlsson's prove of Segal's Burnside ring conjecture

Die Referenten werden gebeten, ihre Kurzfassungen bis Sonntag Mittag bei Herrn Kohnen abzugeben, da wir den Tagungsbericht allen Teilnehmern noch vor ihrer Abreise aushändigen möchten.

Die Vorträge finden alle im "Großen Hörsaal" (Wegelerstraße 10) statt.

Erfrischungspausen mit Tee: Sonntag Nachmittag ab 15.30 Uhr im Diskussionsraum Beringstraße 1. Montag Vormittag von 11.15-12.00 Uhr vor dem Großen Hörsaal. Montag Nachmittag ist der Tee im Max-Planck-Institut, Gottfried-Claren-Straße 26, bereits ab 15.00 Uhr. (To reach the Max-Planck-Institut, you can take e.g. tram No.2 from Rheinuferbahnhof--near main railway station--which goes every 10 minutes; leave at Konrad-Adenauer-Platz; the Gottfried-Claren-Straße starts opposite the tram stop.)

Die Post liegt während der Teepausen aus. Tischtennis im Keller des Hauses Beringstraße 4. Den Tagungsbeitrag bitte an Frau Gerber (SFB-Büro, Beringstraße 4) bezahlen. Programme und Informationen liegen vor dem Diskussionsraum, Beringstraße 1, aus.

Titel: M. F. ATIYAH

Autor: YANG-MILLS EQUATIONS AND THE STRUCTURE OF 4-MANIFOLDS

Adresse: MATHEMATICAL INSTITUTE, OXFORD, ENGLAND

§1 Introduction

This lecture is a report on very recent results of S.K. Donaldson (Oxford). Complete proofs have yet to be written up but there seems little doubt that the main result is true. Donaldson's idea is to use the Yang-Mills equations as a tool to solve basic problems in the geometry of 4-manifolds.

Let X be a compact simply-connected 4-manifold, then $H^2(X,Z)$ is free abelian and has a quadratic form given by the cup-product (and a choice of orientation). Poincaré duality implies that this form is unimodular. For topological manifolds one has the recent result:

THEOREM (M. Freedman). All unimodular quadratic forms occur.

In contrast, for smooth manifolds, we have:

THEOREM (S.K. Donaldson). The only positive definite form which occurs is the standard form Σx_i^2

Together with geometric arguments of Freedman one can deduce :

COROLLARY. <u>There is a non-standard differentiable structure on $\mathbb{R}^4$.</u>

§2 Idea of Proof.

Let $\pm\alpha_i$ $(i = 1, \dots, r)$ be all the elements of $H^2(X, \mathbb{Z})$ with $\alpha^2 = 1$. Then $r \leq n$ (rank of H^2) and equality holds iff the quadratic form is standard. Suppose we can find a 5-dimensional space M which is an oriented manifold with boundary X, except that it has $s \leq r$ singular points, each of which looks like the vertex of a cone on $P_2(\mathbb{C})$. Then chopping off these singularities gives an oriented cobordism $X \sim sP_2(\mathbb{C})$. Comparing signatures then gives $n = s$ and so $s = r = n$ and the quadratic form is standard.

Essentially M will be constructed as the moduli space of instantons over X. For this we fix a Riemannian metric on X and look at an $SO(2)$-bundle P over X with $c_2 = -1$.

M is the space of (isomorphism classes of) connections on P which minimize the L^2-norm of the curvature. The singular points arise from abelian solutions, which correspond to line-bundles with $c_1 = \pm \alpha_i$ so that $-c_1^2 = c_2 = -1$: the curvature is the harmonic representative. M is not compact but it has X as natural boundary. A sequence of points in M converging to $x \in X$ represent curvatures which get concentrated around x. When $X = S^4$, $M = B^5$ the unit ball (or hyperbolic 5-space)

In fact M may have additional singularities but these can be eliminated if we displace M slightly, so that it is a generic approximation to the moduli space.

The detailed proofs depend on the existence theorem of C. Taubes (where the positivity of the quadratic form is needed) and regularity theorems of K. Uhlenbeck.

Titel: Determinants of $\bar\partial$-operators

Autor: Daniel Quillen

Adresse: Max-Planck-Institut für Math. Bonn
and MIT

Let M be a Riemann surface and let E be a fixed C^∞ vector bundle over it. A holomorphic structure on E can be identified with a $\bar\partial$-operator $D: E \to E\otimes T^{0,1}$, that is, a differential operator locally of the form $Df = (\partial_{\bar z} f + \alpha f)\, d\bar z$. We shall be concerned with the problem of defining the determinant $\det(D)$ of the operator D in a reasonable way, for example, so as to depend analytically on D.

In order to understand the nature of the problem, consider the example where E is the trivial line bundle over an elliptic curve $\mathbb{C}/\Gamma$ and $Df = (\partial_{\bar z} - \omega) f \cdot d\bar z$, where ω is a complex number. The eigenvalues of $\partial_{\bar z} - \omega$ are $\mu - \omega$, where μ runs over the dual lattice Γ^*, so $\det(D)$ should be a way to give meaning to the infinite product of the $\mu - \omega$. One possibility is the Weierstrass function

$$\sigma(\omega) = \omega \prod_\mu{}' \left(1 - \frac{\omega}{\mu}\right) e^{\frac{\omega}{\mu} + \frac{\omega^2}{2\mu^2}},$$

however equally reasonable choices are $\sigma(\omega + \mu)$ with $\mu \in \Gamma^*$. All these functions differ by exponential factors of the form $e^{a\omega + b}$, and this illustrates the fact that the determinants to be constructed depend on certain choices, different choices leading to such an exponential factor. A completely canonical determinant can't be constructed, because the gauge transformation of multiplying by $e^{\mu\bar z - \bar\mu z}$ transforms $(\partial_{\bar z} - \omega)\, d\bar z$ into $(\partial_{\bar z} - \omega - \mu)\, d\bar z$, and so a canonical determinant would have to be a doubly-periodic entire function of ω.

Returning now to the general situation (M, E), let $\mathcal{A}$ denote the space of holomorphic structures on E, i.e. $\bar\partial$-operators $D: E \to E \otimes T^{0,1}$. Since all these operators have the same symbol one has

$$\Gamma(\mathrm{Hom}(E, E\otimes T^{0,1})) \xrightarrow{\sim} \mathcal{A}, \quad B \mapsto D_0 + B$$

for any $D_0 \in \mathcal{A}$, and hence $\mathcal{A}$ is an infinite-dimensional affine space. The determinant $\det(D)$ we seek is to be an analytic function on $\mathcal{A}$ which is non-zero at D if and only if the operator D is invertible. However according to the Riemann-Roch theorem, the operator D can be invertible only if $\deg(E) = (\mathrm{rank}\, E)(g-1)$, where $g =$ the genus of M. Therefore we assume this to be true. When it is not, the operator D is analogous to a non-square matrix, and so instead of $\det(D)$ one constructs the determinant of a "square submatrix" of D.

I am now going to present two constructions of the determinant which turn out to be equivalent. The former construction is more concrete but has the disadvantage that the determinant is constructed only locally over the open set of $\mathcal{A}$ which D is invertible. It starts from the formula

$$(*) \qquad \frac{d}{dt} \log\det(D+tB)\Big|_{t=0} = \mathrm{Tr}(D^{-1}B)$$

which, assuming the trace made sense, gives the differential of the function $\log\det(D)$, and hence the function $\det(D)$ by integrating over curves. The problem is that the operator $D^{-1}B$ is not of trace class and must be "regularized". The Schwarz kernel of the operator D^{-1} has the form $G(z,z')dz'$ where

$$G(z,z') = \frac{i}{2\pi}\,\frac{F(z,z')}{z-z'}$$

and F is smooth $= I$ when $z = z'$.

Choose a Riemannian metric $|\partial/\partial z|^2$ on M and a lifting of $D: E \to E \otimes T^{0,1}$ to a connection $\nabla: E \to E \otimes T^*$. Let $F_b(z,z'): E_{z'} \to E_z$ be smooth, $= I$ when $z = z'$, and flat to the first order on the diagonal: $\nabla F_b(z,z') = 0$ when $z = z'$. Then we define the finite part of G along the diagonal by

$$\mathrm{F.P.}\ G(z',z') = \lim_{z \to z'} \frac{1}{2\pi} \left\{ \frac{F(z,z')}{z-z'} - \frac{F_b(z,z')}{z-z'} + \frac{1}{2} \partial_z \log \left| \partial/\partial z \right|^2 \right\}.$$

This depends only on the choice of metric + connection, but not on F_b or on the local coordinate z. Then we define

$$\mathrm{Tr}^{(reg)}(D^{-1}B) = \int_M \mathrm{tr}(\mathrm{F.P.}\ G(z,z)\, dz\, \beta\, d\bar{z}) \quad \text{if } B = \beta\, d\bar{z}.$$

In order to lift D to a connection ∇ in an analytic way one chooses a fixed ∂-operator $\partial_0: E \to E \otimes T^{1,0}$ and then as D varies one takes ∇ to be the unique connection inducing ∂_0 and D. Then using the regularized trace and $(*)$ the function $\det(D)$ is determined.

The second method of constructing $\det(D)$ uses some general ideas about elliptic operators $D: E \to F$. If one has a family of such operators then the index of this family is a virtual vector bundle over the parameter space, and hence by taking highest exterior powers, it gives a canonical line bundle $\mathcal{L}$ over the parameter space. Thus for the family of $\bar{\partial}$-operators $D: E \to E \otimes T^{0,1}$ parameterized by $\mathcal{A}$ we get a canonical line bundle $\mathcal{L}$ over $\mathcal{A}$. This line bundle is holomorphic over $\mathcal{A}$ and its fibre at D can be identified with

$\lambda(\mathrm{Ker}\, D)^* \otimes \lambda(\mathrm{Coke}\, D)$, where $\lambda(V) = \Lambda^{\dim V}(V)$. In the case of holomorphic structures on a bundle of degree $= (\mathrm{rank}) \times (g-1)$, the line bundle $\mathcal{L}$ is the one associated to the divisor in $\mathcal{A}$ consisting of the D which are not invertible. Hence the problem of constructing the function $\det(D)$ is completely equivalent to trivializing the holomorphic line bundle $\mathcal{L}$.

The concept of analytic torsion due to Ray and Singer allows one to define a metric on the canonical line bundle associated to a family of elliptic operators $D: E \to F$. One chooses metrics on E, F and the underlying manifold, so that sections of E, F have L^2-inner products, and the Laplaceans D^*D, DD^* are defined. Let $\Gamma_{\leq a}(E)$, and $\Gamma_{\leq a}(F)$ be the sum of the eigenspaces of these Laplaceans for all eigenvalues $\leq a$. One has a canonical isomorphism

$$\mathcal{L}_D = \lambda(\Gamma_{\leq a} E)^* \otimes \lambda(\Gamma_{\leq a} F)$$

for any $a \geq 0$. One defines a metric on $\mathcal{L}_D$ by taking the obvious metric coming from the L^2-inner products on sections of E and F and multiplying by the analytic torsion of the Laplacean D^*D, but only using the eigenvalues $> a$:

$$e^{-\zeta'_{D^*D,>a}(0)} \qquad \text{where } \zeta_{D^*D,>a}(s) = \sum_{\lambda > a} \lambda^{-s}$$

and the ζ function is analytically continued to $s=0$ in order to make sense of its derivative there.

In the case of the family of $\bar\partial$-operators $D: E \to E \otimes T^{0,1}$ once metrics are chosen in E and M the space $\Gamma(\mathrm{Hom}(E, E\otimes T^{0,1}))$ has a natural inner product $\|B\|^2 = \int_M tr(B \wedge * B)$, and hence $\mathcal{A}$ can be regarded as a Kähler manifold. The holomorphic line bundle $\mathcal{L}$ over $\mathcal{A}$ with its analytic torsion metric is then determined up to isomorphism by its curvature which is given by the following.

Theorem: The curvature form of $\mathcal{L}$ with the analytic torsion metric is a constant multiple of the Kähler form on $\mathcal{A}$.

This result is valid without any restrictions on the degree of E.

If one fixes an origin D_0 in $\mathcal{A}$, then one obtains a metric on the trivial line bundle over $\mathcal{A}$ with the same curvature as $\mathcal{L}$, by defining the norm-squared of 1 at $D_0 + B$ to be $\exp(-c\|B\|^2)$. Thus one obtains a trivialization of $\mathcal{L}$, and hence a determinant function $\det(D)$ in the case $\deg E = \text{rank} \times (g-1)$. Changing the origin D_0 changes the determinant by the exponential of a linear function on $\mathcal{A}$.

The ideas in the proof of this theorem can be illustrated as follows. Let's consider an invertible D_0. The line bundle $\mathcal{L}$ has a canonical section s non-vanishing near D_0 and $|s|^2$ is the analytic torsion of the Laplacean D^*D. The curvature is $\bar\partial\partial \log|s|^2$ which is 2-form on $\mathcal{A}$. The form $\partial \log|s|^2$ at D is computed to be the linear function

$$B \longmapsto Tr\big((D^*D)^{-s} D^{-1} B\big)\Big|_{s=0}.$$

which is the regularization of $Tr(D^{-1}B)$ obtained by taking the finite part the kernel of D^{-1} using the unique connection ∇ on E which lifts D and which preserves the given metric on E. The connection ∇ does not depend analytically on $D = D_0 + B_1$, but rather has an anti-holomorphic part proportional to B_1^*. This contributes to the finite part an anti-holomorphic term proportional to B_1^*, the rest being holomorphic in B_1. Thus when one computes $\bar{\partial}\partial \log |s|^2$ one obtains only a term proportional to

$$\int \mathrm{tr}\,(B_1^* B) \quad .$$

which is just the Kähler form on $\mathcal{A}$.

Titel: Heights on elliptic curves

Autor: John Coates

Adresse: Université Paris-Sud, Orsay, France.

Let E be an elliptic curve defined over a number field K. Classically, the notion of the (naive) height on E plays an essential role in Mordell and Weil's proof of their celebrated theorem that the group $E(K)$ of K-rational points on E is finitely generated. About 20 years ago, Néron and Tate proved the existence of a canonical height function on E. This is a canonical bilinear form

$$\langle\, ,\, \rangle_\infty : E(K) \times E(K) \longrightarrow \mathbb{R},$$

which has a number of simple characterisations, and which can be easily shown to have a non-zero determinant (on $E(K)$ modulo its torsion subgroup). The determinant of $\langle\, ,\, \rangle_\infty$ appears in Birch and Swinnerton-Dyer's conjectural formula for the leading coefficient of the expansion of the Hasse-Weil L-series of E over K about the point $s = 1$ in the complex plane (although no theoretical result in this direction has been proven so far).

The little progress towards the Birch and Swinnerton-Dyer conjecture which has been made so far has mainly been achieved by <u>p-adic methods</u> (using ideas which have their origin in Iwasawa's theory of $\mathbb{Z}_p$-extensions of number fields). These ideas suggest that, if p is any prime number and L/K is any Galois extension of K whose Galois group Γ is isomorphic to $\mathbb{Z}_p$, then there should exist a natural analogue of $\langle\,,\,\rangle_\infty$ attached to L/K and p (we write ρ for the choice of p and L/K). However, it is not at all obvious how to define such an analogue. We say that p is <u>ordinary</u> for E if E has good ordinary reduction at each prime of K above p. After earlier work of Bernardi, Schneider, and Néron, Mazur and Tate have just proven, for any ordinary p and any ρ, the existence of a canonical bilinear form

$$\langle\,,\,\rangle_\rho : E(K) \times E(K) \longrightarrow \mathbb{Q}_p,$$

which is the desired analogue of $\langle\,,\,\rangle_\infty$. For certain choices of ρ, the form $\langle\,,\,\rangle_\rho$ is degenerate, and it is a very interesting open question to decide for which ρ this can occur (e.g. is this related to the finite generation of the group $E(L)$ modulo torsion?).

The deepest question about $\langle\,,\,\rangle_\rho$ is its probable connexion with a p-adic analogue of the Birch-Swinnerton-Dyer conjecture. Everything indicates that $\langle\,,\,\rangle_\rho$ should occur in the study of the Selmer group of E over L as a module for the group Γ. If E admits complex multiplication, and for a certain choice of ρ, this has just been proven by B. Perrin-Riou. Partial results in this direction for E any elliptic curve and L/K the cyclotomic $\mathbb{Z}_p$-extension have also been recently proven by Schneider. Nearly all this work is still unpublished.

John Coates

Titel: Representations with cohomology

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Let $G_{\mathbb{C}}$ be a reductive algebraic group defined over $\mathbb{R}$, and $G_{\mathbb{R}}$ its group of real points. If (π, V_π) is a continuous Banach space representation of $G_{\mathbb{R}}$, there are cohomology groups

$$H^i_{ct}(G_{\mathbb{R}}, V_\pi)$$

(see [BW]); these arise in the theory of automorphic forms, cohomology of discrete subgroups of $G_{\mathbb{R}}$ etc. In these applications, it is of interest to describe those unitary representations of $G_{\mathbb{R}}$ with non-zero cohomology. To do this, let θ be a Cartan involution of $G_{\mathbb{C}}$ (with respect to $G_{\mathbb{R}}$), and $K_{\mathbb{C}}$ its fixed point group. Then $K_{\mathbb{R}}$ is a maximal compact subgroup of $G_{\mathbb{R}}$. Let $\mathcal{B}$ be the (complete complex algebraic) variety of Borel subgroups of $G_{\mathbb{C}}$, and let $\mathcal{P}$ be a conjugacy class of parabolic subgroups of $G_{\mathbb{C}}$. Each Borel subgroup is contained in a unique element of $\mathcal{P}$; so there is a proper map

$$\mathcal{B} \xrightarrow{\pi} \mathcal{P}$$

which exhibits $\mathcal{B}$ as a fiber bundle over $\mathcal{P}$. Assume now that $\theta\mathcal{P} = \mathcal{P}$. Fix a closed $K_{\mathbb{C}}$ orbit Z_0 of $K_{\mathbb{C}}$ on $\mathcal{P}$, and set

$$Z = \pi^{-1} Z_0 .$$

Then Z is a closed, smooth, $K_{\mathbb{C}}$-invariant subvariety of $\mathcal{B}$. Put

$$W_Z = H^{\operatorname{codim} Z}_{[Z]}(\mathcal{C}_{\mathcal{B}}),$$

the local cohomology of the structure sheaf of $\mathcal{B}$ along Z. The action of $G_{\mathbb{C}}$ on $\mathcal{B}$ induces an action of $\mathfrak{g} = \operatorname{Lie}(G_{\mathbb{C}})$ on W_Z.

<u>Theorem.</u> <u>Suppose (π, V_π) is an irreducible unitary representation of $G_{\mathbb{R}}$, and $H^{\cdot}_{ct}(G_{\mathbb{R}}, V_\pi) \neq 0$. Then the $\mathfrak{g}$ module of $K_{\mathbb{R}}$-finite vectors in V_π is isomorphic to some W_Z as above.</u>

This proves a conjecture of Zuckerman, extending work of Parthasarathy, Enright, Speh, and Kumaresan. Proofs and related results will appear in [VZ].

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Titel: Hamilton's Work on Positively Curved 3-manifolds

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References: Three manifolds with positive Ricci Curvature.
(To appear in Journal of Diff. Geom.)
The Inverse Function Theorem of Nash + Moser.
(To appear in the Bull. of AMS)

MAIN THEOREM: A compact smooth 3 manifold which admits a metric of positive Ricci curvature also admits a metric of constant positive curvature.

— Such a manifold is diffeomorphic therefore to S^3/Γ where Γ is a finite subgroup of $O(4)$ acting freely on S^3. Such Γ have been classified (Wolf, "Spaces of Constant Curvature", 1967)

— In fact Hamilton defines a flow on the space $\mathcal{M}^+$ of Ricci positive metrics on X^3. If $g \in \mathcal{M}^+$, the orbit $g(t)$ of g under this flow is defined by $g(0) = g$ and

$$(*) \quad \frac{\partial g}{\partial t} = \frac{2}{n} r g - 2\,\mathrm{Ric}(g) \qquad r = \int R(g)\,d\mu \,/\, \mathrm{vol}(g)$$

(here $n = 3$, $\mathrm{Ric}(g)$ is the Ricci tensor of g, and $R(g)$ the scalar curvature of g). He shows that this flow is defined for $0 \le t < \infty$ no matter what the initial condition $g(0)$, and that as $t \to \infty$ $g(t)$ converges in the C^∞ topology to a smooth metric $g(\infty)$ of constant positive curvature. Technically it is easier to deal with the simpler evolution equation

$$(**) \quad \frac{\partial g}{\partial t} = -2\,\mathrm{Ric}(g)$$

given a solution $g(t)$ of $(**)$, if $\psi(t)$ is defined so that $\tilde{g}(t) = \psi(t) g(t)$ has volume one, and if $\tilde{t} = \int \psi(t)\,dt$, then $\tilde{g}(\tilde{t})$ satisfies $(*)$.

We shall work with (**) and only comment briefly further on the above rescaling which goes back to (*).

Hamilton proves first that (**) has unique solutions for short time, and hence for a given initial condition $g(0)$ that the solution exists on a maximal interval $[0,T)$. This would be easy if $Ric(g)$ were strongly elliptic, but it is not, precisely because the equation has the otherwise desireable property of being invariant under diffeomorphisms of X. (Here "strongly elliptic" means each linearization should have positive definite symbol). [To overcome this difficulty he uses a version of the Nash-Moser inverse function theorem to prove a general short time existence theorem for "evolution equations with an integrability condition".]

It is easy to derive from (**) evolution equations for the Riemann tensor, Ric, and R

$$\frac{\partial\, Riem}{\partial t} = \Delta\, Riem + 0\text{-order in } Riem$$

$$\frac{\partial\, Ric}{\partial t} = \Delta\, Ric + 0\text{-order in } Riem \quad (Ric \text{ if } n=3)$$

$$\frac{\partial R}{\partial t} = \Delta R + 2|Ric|^2.$$

also:

$$\frac{\partial}{\partial t}(\nabla^k Riem) = \Delta(\nabla^k Riem) + \sum_{i+j=k} \nabla^i Riem * \nabla^j Riem.$$

(here ∇ = covariant derivative, $\Delta = \operatorname{tr} \nabla\nabla$, and $*$ means a linear combination of contractions of tensor products)

Recall the "maximum principle" for the heat equation: if $\frac{\partial u}{\partial t} = \Delta u + v$ on $X \times [0,T)$ and $v(x,t) \geq 0$ then if $u(x,0) > 0$ then $u(x,t) > 0$. For example this shows that if $R(g(0)) > 0$ then $R(g(t)) > 0$. So far everything works in general dimensions $n > 2$. Now specialize to $n = 3$. Generalizing the maximum principle slightly Hamilton shows from the above equation for the evolution of Ric that if $Ric > 0$ initially then $Ric(g(t)) > 0$ $0 \leq t < T$.

Let $\lambda_1, \lambda_2, \lambda_3$ be the eigenvalues of $Ric(g(t))$, so $R = \sum_i \lambda_i$, $S = tr(Ric^2) = \sum_i \lambda_i^2$ so $S - \frac{1}{3}R^2 = \sum_{i<j} (\lambda_i - \lambda_j)^2$. Clearly this quantity is very important since its vanishing is equivalent to all the λ_i being equal, i.e. to X being Einstein, which in 3 dimensions implies X has constant sectional curvature. Hamilton proves that

$$S - \frac{1}{3}R^2 \leq const.\ R^{2-\epsilon} \qquad t \to T$$

Note that this implies that if $R_{min} \to \infty$ as $t \to T$ then $(S - \frac{1}{3}R^2)/R^2 \to 0$. Since both numerator and denominator scale the same when the metric g is multiplied by a constant, when we rescale to pass from the solution of $(**)$ to that of $(*)$ we will still have this ratio $\to 0$; but after rescaling the denominator will tend to a positive constant so the numerator will approach zero, which is how it is proved that the asymptotic limit metric $g(\infty)$ has constant curvature.

The next two estimates take some of the hardest work and the cleverest tricks:

$$T \leq \frac{3}{R_{min}(g(0))}$$

if $\eta > 0$ $\qquad |\nabla R|^2 \leq \eta R^3 + C(\eta) \qquad t \to T$

A more or less standard argument starting from the evolution equation for $\nabla^k Riem$, using interpolation inequalities, allows Hamilton to bound the L^∞ norm of $\nabla^k Riem$ as $t \to T$ (provided $\|Riem\|_\infty$ is bounded).

Choosing $p > n$ this implies by the Sobolev estimate that (if $\|Riem\|_\infty$ is bounded) that Riem is C^k bounded as $t \to T$ and so is g. Since this is true for all k, by Montel's theorem we would have $g(t) \longrightarrow g(T)$ as $t \to T$ and hence, by the local existence theorem, we could extend our solution beyond T, a contradiction. It follows that $\|Riem\|_\infty$ cannot be bounded as $t \to T$. In dimension 3 this is the same as $\|Ric\|_\infty \to \infty$ as $t \to T$ which clearly implies $R_{max} = \|R\|_\infty \to \infty$ as $t \to T$. On the other hand the estimate above for $|\nabla R|$ easily ~~implies~~ that $R_{min} < (1-\eta) R_{max}$ as $t \to T$ and hence $R_{max}/R_{min} \to 1$ as $t \to T$. (In particular $R_{min} \to \infty$ so we can prove $(S - \frac{1}{3}R^2)/R^2 \longrightarrow 0$ as above).

Now it is time to rescale, to pass to the solution of the equation (*). Hamilton shows that the rescaled $\tilde{T}$ is always ∞ and that $\tilde{R}_{max}$ is bounded. It follows that $\|Riem\|_\infty$ must be bounded, and now the argument above, instead of leading to a contradiction, proves $g(t) \longrightarrow g(\infty)$. And, as indicated above, for this metric $S^2 - \frac{1}{3}R^2 \equiv 0$ so it has constant curvature.

Titel: Space Curves

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We consider the problem of classification of algebraic curves in projective three-space. Two fundamental papers are those of Halphen (1) and Noether (5) in 1882. The main problems are to find for which degree d and genus g there exist irreducible nonsingular curves, and then to describe the irreducible components and dimensions of the Hilbert scheme $H_{d,g}$ which parametrizes them.

The techniques used by Halphen and Noether include a) description of curves on surfaces of degree 1, 2, 3; b) embeddings of abstract curves, when $d \geqslant g + 3$; c) construction of new curves by the residual intersection of two surfaces containing a given curve.

To these techniques have been added in the last fifteen years the techniques of stable vector bundles and reflexive sheaves (see for example (2, §5)).

When d is large with respect to g or vice versa, one obtains good results. But there is a middle range where things get complicated.

As an example, consider curves of degree 13 and genus 18. Here there is one component of the Hilbert scheme corresponding to curves on a quadric surface, another, non-reduced at its generic point, consisting of curves on singular cubic surfaces (see (4, App. B)), and a third whose general curve is not contained even in any quartic surface.

An important recent result is the theorem of Gruson and Peskine (4) which says that for any $d > 0$ and any $0 \leqslant g \leqslant (1/6)d(d-3) + 1$, there exists an irreducible nonsingular curve of degree d and genus g in $\mathbb{P}^3$. This result, stated but incorrectly proved by Halphen, completes the determination of the pairs (d,g) which can occur as the degree and genus of a curve in $\mathbb{P}^3$. (see (2,§2)).

Full details of the proof of the theorem of Gruson and Peskine occur in (3) as well as in their own paper (4).

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Titel: VOLUME ET LONGUEURS DES COURBES NON HOMOTOPES À ZÉRO ("FILLING RIEMANNIAN MANIFOLDS") D'APRÈS M. GROMOV

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Soit (X,g) une variété riemannienne compacte de dimension n ; on peut lui attacher son volume $\mathrm{vol}(g)$ puis, si elle est non-simplement connexe, sa systole $\mathrm{sys}(g)$ qui est, par définition, la borne inférieure de la longueur de ses courbes fermées non homotopes à zéro.

Le problème fondamental est de savoir s'il existe une constante $\sigma(X) > 0$ telle que

INÉGALITÉ ISOSYSTOLIQUE : quelque soit la métrique riemannienne sur X on a $\mathrm{vol}(g) \geq \sigma(X)\, \mathrm{sys}^n(g)$.

C'est évidemment faux pour une variété X de la forme $X = S^1 \times Y$ où S^1 est le cercle et Y compacte quelconque simplement connexe de dimension $n-1$. Par contre on peut espérer une inégalité isosystolique si l'homologie de dimension 1 de X engendre sa classe fondamentale.

Dans cette direction le premier résultat connu était.

Théorème (Loewner 1949) : pour toute métrique riemannienne g sur le tore T^2 on a toujours $\mathrm{vol}(g) \geq \frac{\sqrt{3}}{2}$ et l'égalité est atteinte si et seulement si (T^2, g) est un tore plat équilatéral, c'est à dire semblable à $\mathbb{R}^2/\Lambda$ où $\Lambda = \mathbb{Z}\cdot(0,1) + \mathbb{Z}\cdot\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$.

La démonstration utilise de façon essentielle la dimension deux (représentation conforme). De même que des extensions de Pu (1952) à $\mathbb{R}P^2$ (le plan projectif réel) et aux surfaces X de genre ≥ 2 (Accola et Blatter indépendamment en 1960) puis Hebda en 1981. Mais la constante $\sigma(X)$ n'est pas bonne $\left(\frac{1}{2}\right)$ dans leurs résultats (voir plus bas).

En dimension $n \geq 3$ Gromov vient d'obtenir le premier résultat dans ce domaine :

Théorème : pour tout n il existe $c(n) > 0$ telle que : si X est compacte de dimension n et admet une application $f : X \to K(\pi, 1)$ (dans un $K(\pi,1)$ quelconque) telle que l'image $f_*[X]$ de sa classe fondamentale est non nulle dans $H_*(K(\pi,1))$, alors $\mathrm{vol}(g) > c(n)\,\mathrm{sys}^n(g)$ pour toute métrique riemannienne g sur X.

La démonstration consiste à remplir par une sous-variété minimale Y (de dimension $n+1$) – i.e. $X = \partial Y$ – l'image (encore notée X) de X, dans l'espace normé (par la norme "sup") $L^\infty(X)$ des fonctions continues sur X, par l'application

$$x \longmapsto d(x, \cdot).$$

Cette application est ISOMÉTRIQUE au sens fort, pas au sens riemannien usuel, i.e. $d_X(x,y) = \|x-y\|_{L^\infty(X)}$. Ce fait est essentiel.

Gromov conclut en utilisant une inégalité isopérimétrique nouvelle pour tout espace normé ; il faut bien sûr trouver une bonne définition des volumes k-dimensionnels dans un espace normé quelconque. Maintenant la définition de la systole, et le fait que X est moralement un $K(\pi,1)$, entraîne l'absurdité suivante : Y se rétracte sur X dès que Y est contenue dans le voisinage métrique de rayon $6.\mathrm{sys}(g)$ de X.

Problèmes naturels :

Problème 1 : calculer, pour une X systolable (i.e. $\sigma(X)>0$) sa constante isosystolique

$$\boxed{\Sigma(X) = \inf\left\{\frac{\mathrm{vol}(g)}{\mathrm{sys}^n(g)} : g \text{ métrique riem. sur } X\right\}}$$

Problème 2 : étudier s'il existe, sur X, des g (dites alors EXTRÉMALES) telles que

$$\boxed{\frac{\mathrm{vol}(g)}{\mathrm{sys}^n(g)} = \Sigma(X)}\,.$$

Etudier de telles g : régularité, nature, etc...

Problème 3 : comparer ces "meilleures" métriques avec les meilleures obtenues parfois par d'autres procédés plus classiques, qui consistent à rendre critique ou minimale des fonctionnelles naturelles (variétés d'Einstein, variétés à volume minimal de Gromov, variétés iso-emboliques, etc...)

Certaines choses :

1) une s atégie est de résoudre le Problème 2 et alors, ayant g, de calculer alors $\Sigma(X)$. Même en admettant ceci (et avec g C^∞) pour les tores T^n, on ne sait pas encore montrer que g est plate, ce qui résoudrait le problème 1 : $\Sigma(T^n)$ serait alors la constante d'Hermite de la Géométrie des Nombres.

2) même pour la bouteille de Klein K^2, on ne connaît pas $\Sigma(K^2)$.

3) Gromov sait montrer que le problème "a de l'existence" (pour des g avec singularités)

lorsque $\boxed{n=2}$

4) Pour le cas des surfaces $X(\gamma)$ orientables de genre γ, lorsque g est à courbure constante ($\equiv -1$!!!) il est classique que $vol(g) \geq k . \frac{\gamma}{\log(\gamma)} . sys^2(g)$. Gromov sait démontrer le :

Théorème : soit X telle que son revêtement universel soit contractible. Si s désigne le nombre minimal de générateurs de $\pi_1(X)$, on a toujours (pour g quelconque on X) :

$$vol(g) \geq k . \sqrt{s} . sys^n(g).$$

Bibliographie :

pour tous renseignements, voir

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Titel: Geometric realization of representations of reductive Lie algebras.

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Let $\mathfrak{g}$ be a reductive complexe Lie algebra. I will describe a construction which gives a convenient geometrical realization of $\mathfrak{g}$-modules. This construction is due to A. Beilinson and myself [1].

First of all, $\mathfrak{g}$-modules can be separated by their infinitezimal characters. Let $U(\mathfrak{g})$ be the universal enveloping algebra of $\mathfrak{g}$, $Z(\mathfrak{g})$ its center. We fix a homomorphism $\theta : Z(\mathfrak{g}) \to \mathbb{C}$ and denote by $\mathcal{M}_\theta(\mathfrak{g})$ the category of $\mathfrak{g}$-modules M with infinitezimal character θ ($M \in \mathcal{M}_\theta(\mathfrak{g})$ if for any $z \in Z(\mathfrak{g})$ $z|_M = \theta(z) \cdot 1_M$). We consider only the character θ_0 which corresponds to the trivial representation of $\mathfrak{g}$. Other characters can be treated similarly (see [1]).

Let G be a complexe reductive Lie group with Lie algebra $\mathfrak{g}$ and B its Borel subgroup. Put $X = G/P$. X is a projective algebraic variety, which is called flag variety of G (Example: $G = GL(n, \mathbb{C})$, X the space of flags in $\mathbb{C}^n$).

Let $\mathcal{O}_X$ be the structure sheaf of X, $\mathcal{D}_X$ be the sheaf of differential operators on X. By definition $\mathcal{D}_X$-module is a quasicoherent sheaf of $\mathcal{D}_X$-modules. The category of (left) $\mathcal{D}_X$-modules I denote by $\mathcal{M}(\mathcal{D}_X)$.

The natural action of G on X gives a morphism $\mathfrak{g} \to$ Vector fields on $X \subset \Gamma(X, \mathcal{D}_X)$. Hence for any $\mathcal{D}_X$-module $\mathcal{F}$ the space $\Gamma(\mathcal{F}) = \Gamma(X, \mathcal{F})$ has a natural structure of $\mathfrak{g}$-module. One can check that $\Gamma(\mathcal{F}) \in \mathcal{M}_{\theta_0}(\mathfrak{g})$.

Theorem (see [1]). The functor $\Gamma: \mathcal{M}(\mathcal{D}_X) \to \mathcal{M}_{\theta_0}(\mathfrak{g})$ $(\mathcal{F} \mapsto \Gamma(\mathcal{F}))$ is an equivalence of categories.

This is a Localization principle, which gives a geometrical realisation of $\mathfrak{g}$-modules. This realization allows to use powerful methodes of algebraic geometry and the theory of $\mathcal{D}$-modules in studying of representations of $\mathfrak{g}$.

The proof of the theorem is relatively easy and standard. The main ingredient of the proof is

Statement. Let $\mathcal{F}$ be any $\mathcal{D}_X$-module. Then $\mathcal{F}$ is acyclic (i.e. $H^i(X,\mathcal{F})=0$ for $i>0$) and $\mathcal{F}$ is generated by its global sections.

This statement is also true for partial flag varieties G/P, where P is a parabolic subgroup of G, for instance for $Gras_{n,k}$ and $\mathbb{P}^n$. Though the proof is not difficult the statement is far from being obvious even for $X=\mathbb{P}^1$. For instance it implies that the sheaf $\mathcal{D}_X \underset{\mathcal{O}_X}{\otimes} \mathcal{O}(-i)$ is acyclic for any i.

Note that analogous statement for right $\mathcal{D}_X$-modules is not true. Indeed $\mathcal{D}_X = \mathcal{O}_X \oplus L$ as left $\mathcal{O}_X$-module, where $L = \{d \in \mathcal{D}_X \mid d(1)=0\}$. Hence right $\mathcal{D}_X$-module $\mathcal{O}(-i) \otimes \mathcal{D}_X = \mathcal{O}(-i) \oplus (\text{something})$ is not acyclic for $i \geqslant 1$.

Now let me demonstrate how to use Localization principle. Consider the problem of classification of irreducible Banach representations of real reductive connected group $G_{\mathbb{R}}$. Let $\mathfrak{g}_{\mathbb{R}}$ be Lie algebra of $G_{\mathbb{R}}$ and $K_{\mathbb{R}}$ be a maximal compact subgroup of $G_{\mathbb{R}}$. Let $\mathfrak{g}$ and K be complexifications of $\mathfrak{g}_{\mathbb{R}}$ and $K_{\mathbb{R}}$ (Exampl $G_{\mathbb{R}} = SL(n,\mathbb{R})$, $\mathfrak{g} = sl(n,\mathbb{C})$, $K = SO(n,\mathbb{C})$).

It is known that irreducible representations of $G_{\mathbb{R}}$ corresponde to irreducible $(\mathfrak{g}, K)$-modules. $(\mathfrak{g}, K)$-module is a vector space V endowed with an action π of $\mathfrak{g}$ and an algebraic action ρ of K which satisfy the compatibility condition $\pi(\xi) = d\rho(\xi)$ for $\xi \in \mathrm{Lie}\, K$. We will consider only $(\mathfrak{g}, K)$-modules with infinitezimal character θ_0. By localization principle they correspond to $(\mathcal{D}_X, K)$-modules, where $(\mathcal{D}_X, K)$-module $\mathcal{F}$ is $\mathcal{D}_X$-module endowed with an algebraic action ρ of K on $(X, \mathcal{F})$ satisfying compatibility condition $\alpha(\xi) = d\rho(\xi)$ for $\xi \in \mathrm{Lie}\, K$, where $\alpha : \mathrm{Lie}\, K \to$ Vector fields on X is the natural morphism.

Note that for any action of an algebraic group K on a nonsingular algebraic variety Y we can consider $(\mathcal{D}_Y, K)$-modules (because the definition does not mention any Lie algebra exept $\mathrm{Lie}\, K$). In a case of transitive action all $(\mathcal{D}_X, K)$-modules $\mathcal{F}$ can be easily described. Namely fix a point $y \in Y$ and consider the stabilizer K_y of y and the fiber $\mathcal{F}_y$ of $\mathcal{F}$ at y. Then the action ρ of K_y on $\mathcal{F}_y$ is trivial on the connected component K_y^0 of K_y and $\mathcal{F}$ is completely determined by the representation of the finite group K_y / K_y^0 in the vector space $\mathcal{F}_y$.

In the case of flag variety X the group K has a finite number of orbits on X.

Using the theory of $\mathcal{D}$-modules it is easy to show that irreducible $(\mathcal{D}_X, K)$-modules are parametrized by pairs $(Y, \mathcal{F})$ where Y is a K-orbit on X and $\mathcal{F}$ an irreducible $(\mathcal{D}_Y, K)$-module.

Using geometrical realization of $(\mathfrak{g}, K)$-modules one can formulate problems, arrising in the representation theory in the language of $\mathcal{D}$-modules. Using recent results on $\mathcal{D}$-modules (due mostly to M. Kashiwara) one can reformulate them as a purely topological questions about topological structure of stratification of X on K-orbits. Then one can go to the etale topology over fields of positive characteristic and using powerful Deligne's theorem answer these question. This long and indirect way allows to obtaine very strong results in representation theory (see [2,3]).

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1

Titel: Manifolds with positive scalar curvature

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Besides its own interest, there were two reasons to study manifolds with positive scalar curvature.

(i) It was proved by Schoen - Yau [1] that if M^4 is a four dimensional spacetime which represents a (non-singular) isolated physical system, then M^4 is topologically the same as $N^3 \times R$ where N^3 is a ~~no~~ three dimensional manifold which admits a complete metric with positive scalar curvature. Hence classifying ~~such~~ such manifolds will give the classification of the topology of space-times that occur in physics.

(ii) It was proved by Lawson-Yau[2] that if M is a manifold that admits an effective non-abelian compact connected Lie group action, then M admits a complete metric with positive scalar curvature. Hence any topological obstruction for M to admit metrics with positive scalar curvature will give an obstruction to such Lie group actions.

Hence the basic question is:

<u>Which manifolds admit complete metrics with positive scalar curvature?</u>

There are two methods to deal with such a question. The first one is very much similar to the method of Bonnet-Synge who used the extremal property of geodesics to prove that diameter of a complete manifold with Ricci curvature $\geq c$ is not greater than $\frac{\pi}{\sqrt{c}}$. (This property was used to establish the fundamental groups of these manifolds are finite) The second method is the method of Bochner who computed and applied the maximum principle to the Laplacian of harmonic one forms. Under the assumption of the positivity of the Ricci curvature, Bochner proved the harmonic one forms do not exist and hence the first Betti number is zero.

Since the positivity of scaler curvature is much weaker than the positivity of the Ricci curvature, both methods have to be strengthened. In the first case, geodesics should be replaced by minimal hypersurfaces. In the second case, the harmonic one forms should be replaced by harmonic spinors. Spinors can be defined only when the second Stiefel Whitney class is zero. In this case, the principle bundle can be lifted to another principle bundle with structure group spin(n). This gives rise to a vector bundle via the spin representation. Sections of such a bundle are called spinors.

In the following, I will describe how these two methods were used to answer our basic question on page 2.

In 1963, Lichnerowicz [3] observed that the Bochner method will give the non-existence of harmonic spinors for compact manifolds M with positive scalar curvature. The Atiyah-Singer index theorem then shows that the $\hat{A}$-genus of M is zero if M is spin. ($\hat{A}$-genus is a Pontryagin number and is part of the KO-characteristic number.) Later Hitchin observed in his thesis[4] that Lichnerowicz's result actually shows that all the KO-characteristic number of M is zero.

In 1974 in the Stanford conference on differential geometry, Physicists proposed to geometers an old problem in general relativity which is called the positive mass conjecture. A special case of this question is whether non-trivial metric with non-negative scalar curvature on R^3, which is euclidean outside a compact set, exists or not. This question motivates the study of manifolds with positive scalar curvature strongly. Since the KO-characteristic numbers are trivial for three dimensional manifolds are trivial, it was thought that spinors will not be as useful as in higher dimensional manifolds.

Then in 1977, Schoen and Yau [5] applied the theory of minimal surfaces to settle the positive mass conjecture and study the structure of three dimensional manifolds with positive scalar curvature.

They proved that if π_1 of a complete three dimensional manifold with positive scalar curvature contains the fundamental group of a compact surface with genus ≥ 1, then M admits no metric with positive scalar curvature. If one is willing to assume some standard conjectures in the theory of three manifolds, one can give a good picture of compact three dimensional manifold with positive scalar curvature. In any case, soon after this work, we find that it is better to go to the non-compact cover of these manifolds.

Given a compact manifold M with scalar curvature $\geq c \geq 0$, one can take a cover $\hat{M}$ of M with $\pi_1(\hat{M}) = \mathbb{Z}$ in case $\exists\ \mathbb{Z} \subset \pi_1(M)$. By using the circle that represents the generator of $\pi_1(\hat{M})$, one can construct another circle that links with it and construct a minimal surface bounded by this circle. Letting this circle go to infinity, one constructs a complete minimal surface Σ. By looking at the second variation of this surface, one can construct as in Fischer-Colbrie–Schoen[ ] a positive function φ which satisfies $\Delta\varphi + (c-K)\varphi = 0$. Then we try to "symmetrize" this equation. We take a metric on $\Sigma \times S^1$ by $ds^2 + \varphi^2 dt^2$. It turns out that this metric still has scalar curvature $\geq c \geq 0$. By minimizing area of equivariant surfaces in $ds^2 + \varphi^2 dt^2$, we can reduce the previous equation to an ordinary differential equation and conclude that the diameter of $\Sigma \le \frac{2\pi}{\sqrt{c}}$. Hence if $c>0$, Σ must be compact and one can show that Σ is either S^2 or RP^2.

This argument shows that if a compact 3 manifold M has a metric with $R>0$ then $M = M_1 \# M_2 \# \cdots \# M_k$ where M_i is either $S^2 \times S^1$ or has finite fundamental group. It turns out that if the Poincaré - Smith Conjecture is true, then compact manifold with finite fundamental group has metric positive scalar curvature and their connected sums with $S^2 \times S^1$ also admit such metrics.

The above argument can be generalized to higher dimensional manifolds. Given a n-dimensional M^n, one construct a minimal hypersurface M^{n-1} in its

Titel:

Autor:

Adresse:

universal cover. Then we perform the symmetrization process to $M^{n-1} \times S^1$ and construct M^{n-2}. This procedure can be continued. It can be used to prove that if M represents a non-trivial homology class in a compact manifold with non-positive curvature, then M admits no metric with positive scalar curvature. This arguments were basically done in 1979 [7] [8]. However, at that time, we did not know how to avoid the singularities of the minimal submanifolds and we have to restrict ourselves to $\dim M \leq 7$.

At this point, Gromov-Lawson [9] found that Lichnerowicz' method can be generalized to deal with spin manifolds with non-positive curvature whose

Titel:

Autor:

Adresse:

fundamental group is residually finite.

Independently Schoen-Yau[7] and Gromov-Lawson[9] found that in the category of manifolds with positive scalar curvature, one can perform surgery with codimension ≥ 3. Hence this enables one to reduce our question to a simpler by spin cobordism theory. This works particularly well for dim ≥ 5 and manifold simply connected [9].

Recently Gromov-Lawson were able to drop the assumption on the fundamental group being residually finite & proved the three dimensional manifold result by generalizing Lichnerowicz argument to non-compact manifold. At the same time we were able to drop our assumption on dim ≤ 7 because we found that singularities on minimal hypersurface can be avoided.

The final conclusions seem to lead to the following suggestion that if a compact manifold M admits a metric with $R > 0$, then

$$\exists\, f : \begin{tikzcd} M \arrow[rr] \arrow[dr, "h"'] & & K(\pi_1(M), 1) \\ & N^n \arrow[ur, "k"'] & \end{tikzcd}$$

where $k_*[N] \in H^n(K(\pi_1(M),1))$, f_* is injective & generically, if $f^{-1}(x)$ is spin, then $f^{-1}(x)$ is spin cobordant to zero.

Hopefully a condition similar to this one will be a necessary and sufficient condition for a manifold to admit a metric with positive scalar curvature.

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Titel: Rational curves on 3-folds and applications

Autor: Shigefumi Mori

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In this talk, we consider Fano 3-folds as an application because it is the class of 3-folds to which our theory of extremal rays applies best (Mori - Mukai [5]).

Def. An n-dim. n.s. proj. var. X over $k=\bar{k}$ is a Fano n-fold if $-K_X$ is ample. Let $\rho(X)$ be the rank of Pic X.

ex. If X is a hypersurface of $\mathbb{P}^{n+1}$ of degree $d \leq n+1$, then X is a Fano n-fold with $\rho = 1$.

First of all, the following general result is known.

Thm. (Janos Kollar) For an arbitrary char k, every Fano n-fold is uni-ruled, i.e. there exist an (n-1)-fold Y and a dominating rational map $\mathbb{P}^1 \times Y \dashrightarrow X$.

The proof is obtained by modifying the proof of Theorems 4, 5 in [3].

From now on, we will work over $\mathbb{C}$.

Fano 3-folds with $\rho = 1$ are classified by Iskovskih [1], [2] based on Shokurov's works [6], [7].

Applying the theory of extremal rays [4], one can classify Fano 3-folds with $\rho \geq 2$. For example, one has

<u>Thm</u> (i) There are exactly 87 deformation types,

(ii) If $\rho(X) \geq 6$, then $X \simeq \mathbb{P}^1 \times$ (del Pezzo surface).

We say that a Fano 3-fold is <u>primitive</u> if X is not a blow-up of any Fano 3-fold along an irreducible non-singular curve.

This definition turns out to be reasonable by

<u>Thm</u> If X is a Fano 3-fold with $\rho \geq 2$, then $\rho(X) = 1$, or X has a structure of a conic bundle over $\mathbb{P}^2$ or $\mathbb{P}^1 \times \mathbb{P}^1$.

There are 30 classes of primitive Fano 3-folds: 17 with $\rho = 1$, 9 with $\rho = 2$, 4 with $\rho = 3$. The following proposition is basic in seeing if one gets a Fano 3-fold by blowing up a 3-fold.

<u>Prop.</u> Let $f : X \to Y$ be the blow-up of a n.s. proj. 3-fold Y along a n.s. irred. curve $C \subset Y$. If X is a Fano 3-fold and if $Z \subset Y$ is an irreducible curve such that $(Z \cdot -K_Y) = 1$, then $C = Z$ or $C \cap Z = \phi$.

Using this, one can show the following roughly because Y contains a curve C such that $(C \cdot -K_Y) = 1$ [7].

<u>Cor.</u> If Y is a Fano 3-fold with $\rho = 1$ and if some curve-blow-up of Y is again a Fano 3-fold, then $-K_Y \sim rL$ for some $r \geq 2$ and a divisor L.

There are exactly 7 classes of Fano 3-folds Y with $\rho = 1$ satisfying this necessary condition.

We conclude this note by an example of Fano 3-fold with $\rho = 5$ (largest non-trivial case).

ex. Let $Q \subset \mathbb{P}^4$ be a quadric hypersurface. Let C be a n.s. conic on Q and p_1, p_2, p_3 distinct points on C. Let $\psi : Y \to Q$ be the blow-up of Q at 3 points p_1, p_2, p_3 and $C' \subset Y$ the proper transform of C. Let $\varphi : X \to Y$ be the blow-up along C'. Then X is a Fano 3-fold with $\rho = 5$, while Y is not a Fano 3-fold.

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Titel: On the Tate conjecture for Hilbert modular surfaces

Autor: G. Harder

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This is a report on joint work of Langlands Rapoport and the speaker. These results will be published in a paper which is in course of preparation [HLR]. Many of the ideas are already contained in a paper by Oda [Od] which is going to appear.

If $F/\mathbb{Q}$ is a real quadratic extension we consider the Shimura varieties

$$S_{K_f}(\mathbb{C}) = GL_2(F)\backslash GL_2(\mathbb{A})/K_\infty K_f$$

where $K_\infty = Z_\infty^0(SO(2) \times SO(2)) \subset GL_2(\mathbb{R}) \times GL_2(\mathbb{R}) = G_\infty$ and K_f is open and compact in $GL_2(\mathbb{A}_f)$. Then $S_{K_f}(\mathbb{C})$ is the set of complex points of the canonical model $S_{K_f}/\mathbb{Q}$. We consider the "cuspidal" cohomology groups

$$\varinjlim_{K_f} H^2_{cusp}(\bar{S}_{K_f}, \mathbb{Q}_\ell) = H^2(\mathbb{Q}_\ell)$$

and the map from

$$NS = \varinjlim_{K_f} NS(\bar{S}_{K_f}) \longrightarrow H^2(\mathbb{Q}_\ell)$$

where $NS(\bar{S}_{K_f})$ is the Neron - Severi group of algebraic cycles. We want to investigate to what extend we can verify the Tate conjecture in this case.

We may decompose the cohomology under the action of the adele group

$$H^2_{cusp}(\bar{\mathbb{Q}}_\ell) = \bigoplus_{\pi_f} H^2(\bar{\mathbb{Q}}_\ell)(\pi_f)$$

Here the π_f are ~~[illegible]~~ finite components of certain automorphic representations of the adele group and each component occurs with multiplicity 4. The action of the Galois group $Gal(\bar{\mathbb{Q}}/\mathbb{Q})$ on $H^2_{cusp}(\bar{\mathbb{Q}}_\ell)$ commutes with the action of $Gl_2(\mathbb{A}_f)$ and therefore as a $Gal(\bar{\mathbb{Q}}/\mathbb{Q}) \times Gl_2(\mathbb{A}_f)$-module we have

$$H^2(\bar{\mathbb{Q}}_\ell)(\pi_f) = V_{\pi_f} \otimes W_{\sigma_{\pi_f}}$$

where V_{π_f} is a copy of π_f and where $W_{\sigma_{\pi_f}}$ is a 4 dimensional Galois module.

The representation σ_{π_f} of the Galois group can be computed in terms of the representation π_f of $GL_2(\mathbb{A}_f)$ (See [La], [PS], and [Ca]) and this allows us to express the L-function of $S_{K_f}/\mathbb{Q}$ as a product of certain L-functions $L(\pi_f, \rho, s)$ associated to π and a representation ρ of the dual group.

We may ask for the validity of the Tate conjecture on each individual piece π_f and then we ~~get~~ have to verify.

multiplicity of $NS(\pi_f)$ = multiplicity of the Tate character in $W_{\sigma_{\pi_f}}$ if we restrict to small subgroups $Gal(\bar{\mathbb{Q}}/E)$ in $Gal(\bar{\mathbb{Q}}/\mathbb{Q})$.

Then we find three cases

I. There is no character $\eta: I_F/I_\infty \cdot F^\times \to S^1$ such that $\pi_f \otimes \eta_f$ is self conjugate.*) ~~Then~~

*) With respect to the involution induced by $Gal(F/\mathbb{Q})$.

In this case the Tate character does not occur in $W_{\sigma_{\pi_f}}$ if we restrict to arbitrary open subgroups of the Galois group. Hence

$$NS(\pi_f) = 0$$

in this case and moreover $L(\pi_f, \rho, s)$ is holomorphic and non zero at $s=1$. In this case the Tate conjecture is true.

II. π_f is self conjugate, then $\pi = \pi_0 \times_{\mathbb{Q}} F$ is a lifting of a form over $\mathbb{Q}$. We assume that π_0 has no complex multiplication,

then

$$\mathrm{mult}\, NS(\pi_f) = \mathrm{mult}\, \mathrm{Tate}|_{Gal(\bar{\mathbb{Q}}/E)} (W_{\sigma_{\pi_f}}) = 1,$$

the cycles are given by the construction of Hirzebruch-Zagier, i.e. by the diagonal embedding $\Gamma\backslash H \hookrightarrow SL_2(\mathcal{O}) \backslash H \times H$. ($\Gamma = SL_2(\mathbb{Z})$).

The L-function $L(\pi_f, \rho, s)$ has a first order pole at $s=1$ and this pole stays a first order pole after solvable extensions.

III. $\pi = \pi_0 \times_{\mathbb{Q}} F$ and π_0 has complex multiplication. In this case we have

$$\text{mult. Tate}\,\Big|_{Gal(\bar{\mathbb{Q}}/E)} (W_{\sigma_{\pi_f}}) = 2$$

and in general the NS-part exhausted by the ~~B~~ cycles of Hirzebruch Zagier is of multiplicity one ~~in general~~. There are some cycles missing. One can prove their existence by using an argument of Oda in [Od] if one has certain non vanishing of certain L-values. Again $L(\pi_f, \rho, s)$ has a first order pole which becomes a second order pole after ~~an~~ suitable extension (over which the second cycle becomes rational).

Hence the Tate conjecture is true in the cases I, II as far as the comparison of the $NS(\pi_f)$ group and the multiplicity of the Tate character is concerned. The order of the pole of the L-function is correct over solvable extension.

In case III the situation is not yet quite so clear, we have to check the non vanishing of some members in a family of values of L-functions.

Titel:

Autor: G. Harder

Adresse:

If we compare the methods in [HLR] to Oda's method one might say that we are dealing with the l-adic realisation of the motif which is superior if one has to exclude the existence of cycles and Oda uses the Hodge realisation, which in this case is better if one wants to construct cycles.

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Titel: The topological Poincaré conjecture in dimension 4 as proved by M. H. Freedman
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Freedman's results are even stronger. He classifies all closed simply connected topological 4-manifolds with the property (inevitable?) that the complement of a point admits a C^∞ smooth structure. Up to homeomorphism they are classified by their unimodular intersection form on the 2-dimensional homology and their Kirby-Siebenmann smoothing obstruction in $\mathbb{Z}_2$; these invariants are arbitrary subject to the single condition that, for even forms, ($x \cdot x$ even for all x) the smoothing obstruction is (signature/8) mod 2. Freedman works with A. Casson's theory of flexible 2-handles (1973) and his main result is that every (open) flexible Casson 2-handle is homeomorphic to the standard open 2-handle $B^2 \times \mathbb{R}^2$. His complete proof has just appeared in preprint form at U. of Calif. San Diego.*
An important auxiliary result is that smooth proper h-cobordism implies homeomorphism for simply connected smooth 4-manifolds that (if noncompact) are simply connected at each end.

*see also Sém Bourbaki, Fevrier 1982.

In conclusion I will indicate a proof of a result that has puzzled many participants at this Tagung. Fact: If there exists no closed smooth M^4 with a definite intersection form of signature 16 (Donaldson, Oxford), and we grant Freedman's above results, then there exists an exotic differentiable structure on $\mathbb{R}^4$, i.e. there exists a smooth M^4 homeomorphic to $\mathbb{R}^4$ but not diffeomorphic to $\mathbb{R}^4$. Proof In the Kummer surface K^4, the flexible handle theory of Casson (see Mandelbaum Bull. AMS 1979±ε review article, or notes of Guillou, Orsay) produces three disjoint open sets M_1, M_2, M_3; such that each M_i embeds smoothly as an open subset $f_i M_i \subset S^2 \times S^2$ which has the proper homotopy type of $S^2 \times S^2 - (\text{point})$; in fact M_i is by construction the interior of a flexible handlebody with one 0-handle and two flexible 2-handles of Casson. Hence, by Freedman, M_i must admit a homeomorphism $h_i: M_i \to S^2 \times S^2 - (\text{point})$. Consider $N_i = S^2 \times S^2 - f_i h_i^{-1}(S^2 \times S^2 - \text{open ball})$; N_i is explicitly proper h-cobordant to any open ball in N_i; hence it is homeomorphic to $\mathbb{R}^4$.

If all three N_i are diffeomorphic to R^4, then smooth 3-spheres near infinity, transported by the f_i clearly split K^4 smoothly as a connected sum $K_0^4 \# S^2 \times S^2 \# S^2 \times S^2 \# S^2 \times S^2$ where K_0 has, like K^4, signature 16 and $H_2 K_0$ has rank $22 - 3 \cdot 2 = 16$ a contradiction.
(I believe Casson knew this argument in 1973-4.)

Titel: Limit Sets of Kleinian Groups

Autor: S. J. Patterson

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Consider $D^{n+1} = \{\underline{x} \in \mathbb{R}^{n+1} : \|x\|^2 < 1\}$ endowed with the metric

$$ds^2 = (1-\|x\|^2)^{-2} \cdot \|d\underline{x}\|^2$$

as a model for $(n+1)$-dimensional hyperbolic geometry. Let $Con(n)$ be the group of isometries of D^{n+1} onto itself; this is $SO(n+1,n)$. Let $S^n = \{\underline{x} \in \mathbb{R}^{n+1} : \|x\|^2 = 1\}$, then the action of $Con(n)$ extends continuously to S^n.

Let

$$j(g,\underline{x}) = (1-\|g\underline{x}\|^2)/(1-\|\underline{x}\|^2) \qquad \begin{matrix} g \in Con(n) \\ \underline{x} \in \overline{D^{n+1}} \end{matrix}$$

so that

$$j(g_1g_2,x) = j(g_1,g_2\underline{x})\, j(g_2,\underline{x})$$

This is the conformal distorsion, and its $(n+1)^{th}$ power is the Jacobian. One has

$$\|g x_1 - g x_2\|^2 = j(g,x_1) \cdot j(g,x_2)\, \|x_1 - x_2\|^2 \qquad (x_1,x_2 \in \overline{D^{n+1}})$$

and we define

$$L(x_1,x_2) = 1 + \|x_1-x_2\|^2/(1-\|x_1\|^2)(1-\|x_2\|^2).$$

Thus one has

$$L(gx_1,gx_2) = L(x_1,x_2) \qquad (g \in Con(n))$$

and this is related to the hyperbolic distance between x_1 and x_2 by.

$$L(x_1,x_2) = \cosh^2(d_{hyp}(x_1,x_2)).$$

Now let G be a discrete subgroup of $Con(n)$. We define the <u>limit set</u> $L(G)$ of G to be the smallest non-trivial closed subset of S^n which is invariant under G (and to be $\emptyset$ in the exceptional case that G is finite).

A Kleinian group is for the purposes here a discrete group $G \subset \mathrm{Con}(n)$, with $L(G) \neq \emptyset, S^n$. We shall also need a numerical invariant $\delta(G)$, the exponent of convergence of G, which is defined as

$$\delta(G) = \operatorname{Inf}\{s \in \mathbb{R} : \sum_{g \in G} L(x_1, gx_2)^{-s} < \infty\}$$

for some $x_1, x_2 \in D^{n+1}$. It does not depend on the choice of x_1, x_2 (by the triangle inequality). One has by a simple packing argument that

$$\delta(G) \leq n$$

whereas, if G is not finite (when $\delta(G) = -\infty$) one has $0 \leq \delta(G)$. One has moreover that if G is "non-elementary" (its Zariski closure is not of dimension 0,1) then (Beardon)

$$0 < \delta(G) \leq n.$$

The study of the limit set is motivated by the Ahlfors-Bers simultanous uniformization theorem, by Thurston's work on hyperbolic geometry, and by studies in the ergodic theory of the geodesic flow on $G \backslash D^{n+1}$ and the study of the Laplace operator on $G \backslash D^{n+1}$. In recent times pictures of these have become available and are very beautiful, cf. [2]. They give a very natural class of fractals.

Let, for $P, P' \in S^n, P \neq P'$, $[P, P']$ be the geodesic arc joining P and P'. Then let

$$FN(G) = \overline{\bigcup_{\substack{P, P' \in L(G) \\ P \neq P'}} [P, P']}$$

This is the Fenchel-Nielssen region for G. If there exists a compact subset K of D^{n+1} so that

$$GK \supset FN(G)$$

then one says that G is <u>convex</u> <u>cocompact</u>. This is the simplest class of groups. One has

<u>Theorem</u> ([3], [9]) If G is a convex cocompact Kleinian group then

$$\dim_H(L(G)) = \delta(G)$$

and $L(G)$ has finite, positive measure with respect to the $\delta(G)$-dimensional Hausdorff measure.

(Here $\dim_H(S)$ = Hausdorff dimension of S).

The proof brings out a strong relationship with the spectral theory of the Laplace operator (cf. refs above and [10]) One has a measure (for <u>all</u> G) μ supported on $L(G)$ such that

$$d\mu(gx) = j(g,x)^{\delta(G)}\, d\mu(x).$$

Since this is what Hausdorff measure would satisfy the construction of this measure is a crucial step. Note that it yields the <u>invariant</u> measure

$$d\mu(x_1)\, d\mu(x_2) \,/\, \|x_1 - x_2\|^{2\delta(G)}$$

on $S^n \times S^n - (\mathrm{Diag})$; this is supported on $L(G) \times L(G)$ and carries the interesting part of the geodesic flow, (cf [9]).

If G is not assumed to be <u>convex</u> <u>cocompact</u> then one has that

$$\dim_H(L(G)) \geq \delta(G)$$

and it is not easy to determine conditions for equality. It is, for example, the case that equality holds for all finitely generated Fuchsian groups ($n=1$).

One can show

Example ([4],[5]). Let $n, \varepsilon > 0$ be given. Then there exists a discrete subgroup G of $\mathrm{Con}(n)$ such that

(i) $\delta(G) < \varepsilon$

(ii) $L(G) = S^n$.

The proof is based on ideas from [1]. This is about as bad as one can expect.

Another interesting case is if we take a discrete group G and a normal subgroup G_1 (non-finite). As $L(G_1)$ is G-invariant

$$L(G) = L(G_1).$$

One can ask whether $\delta(G) = \delta(G_1)$ holds. There is a negative result:

Example ([4]) If $n=1$ there exists a finitely generated Fuchsian group G, and a normal subgroup G_1 of G with $\delta(G_1) < \delta(G) = 1$.

In this example G/G_1 is a surface group.

With these examples in mind it came as a surprise when M. Rees proved the following theorem:

Theorem ([6]). Suppose $n=1$, and that G is finitely generated. Then if G_1 is a normal subgroup of G with G/G_1 abelian. Then

$$\delta(G_1) = \delta(G).$$

This is a remarkable theorem, and she can prove a sharper result than that quoted here. Presumably the condition that $n=1$ is redundant.

The proof first uses the reinterpretation of the ergodic theory described above in terms of symbolic dynamics as explained by C. Series in [7], [8]. Then in terms of the symbolic dynamics she carried out various delicate estimates, using novel methods, to establish the required inequalites.

On studying this one is led by the example and the proof of Rees to believe that the following holds:

Let G be a finitely generated discrete subgroup of $Con(n)$ and G_1 a normal subgroup. Then

$\delta(G_1) = \delta(G)$ if G/G_1 is of polynomial type

$\delta(G_1) < \delta(G)$ if G/G_1 is of exponential type

There is as yet no further evidence, so that I shall leave this as a problem.

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Titel: On Chapter I of Thurston

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Let M be a compact 3-manifold, possibly with boundary, which is homotopy equivalent to some complete hyperbolic 3-manifold N, possibly with infinite volume. Let $\pi = \pi_1 M = \pi_1 N$.

We consider the set F of representions $\pi \to \mathrm{Isom}^+ H^3 \approx PSL(2,\mathbb{C})$ with discrete, faithful image. F is a subspace of

$$\mathrm{Isom}^+ H^3 \times \ . \ \ - \ . \ . \times \mathrm{Isom}^+ H^3$$

where the number of factors is equal to the number of generators of K.

<u>Theorem</u> (Chuckrow) F is closed.

We are interested in $AH(M)$, the quotient of F by conjugation by $\mathrm{Isom}^+ H^3$

<u>Lemma</u> $AH(M)$ is Hausdorff.

<u>Theorem</u> (Mostow) Suppose N has finite volume, then $AH(M)$ = one point.

The boundary of M is said to be <u>multiply incompressible</u> if whenever 2 curves in ∂M are homotopic in M, the homotopy can be pushed into ∂M, keeping the curves fixed.

<u>Theorem</u> (Thurston) If $\partial M \subset M$ is multiply incompressible, then $AH(M)$ is compact.

This result is one of the key pieces in a large jigsaw puzzle whose object is to put hyperbolic structures on many 3-manifolds.

The ideas occuring in the proof are

1. Ideal triangulations
2. Geodesic laminations and pleated surfaces
3. Peaceful division of disputed territory
4. Homological calculations.

Thurston's paper has been submitted to the Annals of Mathematics.

1

Titel: Web Geometry

Autor: S. S. CHERN

Adresse:

I. Def of a d-web: d foliations in "general position"; dim, codim.

II. Historical. Blaschke started the subject in 1929. Hexagonal webs. Theorem of Graf and Sauer. Gronwall's conjecture. Hexagonal d-webs. Exceptional 5-web in the plane. Quadrilateral webs.

III Why study web geometry?

a) Generalization of geometry of projective varieties.

b) Theorem of Remmert and Stein. Rigid polyhedral domains

c) Canonical web in the "enhanced configuration" of Gelfand-McPherson. Orbit structure of Lie group action.

IV Abelian equations

Consider d-web in R^n of codim k, each foliation being given by the submersion

$$R^n \xrightarrow{\pi_i} R^k, \quad 1 \leqq i \leqq d.$$

Let Ω_i be form of degree p in R^k. An equation of the form

$$\sum \pi_i^* \Omega_i = 0$$

is called an abelian equation. This generalizes

Abel's theorem.

Rank = max. no. of linearly indep't abelian equations.

a) If codim $k=1$, rank $\leqq \pi(d,n)$, where $\pi(d,n)$ depends only on d, n, and is called the Castelnuovo number. It is the max genus of an algebraic curve of degree d in P^n, which does not belong to P^{n-1}. We have $\pi(d,2) = \frac{1}{2}(d-1)(d-2)$, $\pi(2n,n) = n+1$.

b) If dim $= n-k = 1$, Damiano proved that rank $\leqq \pi^*(d,n)$. $\pi^*(n+3,n) = \frac{1}{2}(n+1)(n+2)$.

Fundamental Problem. Determine all webs of max rank, and, at this stage, of codim or dim 1.

V. Max rank webs of codim 1.

Lie-Wirtinger Thm. Web geometric proof.

St. Donat's proof of Trelli's Thm.

Bol's exceptional web

Bol's thm for $n=3$, $d \geqq 6$.

Generalization of Chern-Griffiths

VI Max rank webs of dim 1.

Characteristic forms and abelian equations.

Work of Gelfand, McPherson, Damiano.

Autor: CHERN Seite Nr.: 3

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Titel: Z_p-extensions and heights of elliptic curves

Autor: B. Mazur

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Let E be an elliptic curve over a number field K. The theorem of Mordell-Weil asserts that E(K), the group of K-rational points of E, is a finitely generated abelian group.

What is the behavior of the <u>rank</u> of E(K) as K varies? The object of this talk is to indicate how some new tools, notably the p-adic <u>height</u>, may lead one to recast this question in more specific terms, and perhaps provide a key to help us understand it.

Let $\wp$ denote a Z_p-extension of K. That is, $\wp$ is a sequence of field extensions $K \subset K_n \subset K_{n+1} \subset \ldots$ such that K_n/K is cyclic of degree p^n.

Say that $\wp$ is <u>nonsingular</u> for E if the rank of $E(K_n)$ is bounded, independent of n. It is <u>singular</u> if the rank of $E(K_n)$ tends to infinity with n.

Suppose that E has good, ordinary reduction at all primes of K dividing p. Then, modulo the Shafarevitch-Tate conjecture, one can show that in singular Z_p-extensions, the rank of $E(K_n)$ grows systematically, and significantly, in the sense that there exists apositive integer M such that

$$\text{rank } E(K_n) = M \cdot p^n + O(1) .$$

There <u>are</u> singular Z_p-extensions. Indeed, if L/K is the anti-cyclotomic Z_p-extension of the quadratic imaginary field K, then Kurchanov has shown that for any CM elliptic curve E of conductor prime to p, and ordinary at p, there is a finite field extension K'/K such that E is defined over K', and LK'/K' is singular for E.

If $E_{/K}$ is a CM elliptic curve with complex multiplicaton by the quadratic imaginary field K, suppose that p splits in the ring of integers of K $(p = \pi\bar{\pi})$ and that $\pi, \bar{\pi}$ are endomorphisms of E. Define the <u>characteristic</u> Z_p-extensions of E to be the fields L_{π}, $L_{\bar{\pi}}$ generated over K by the coordinates of the π-power, and $\bar{\pi}$-power division points of E, respectively.

<u>Theorem</u> 1 : (Coates, Perrin-Riou, Rubin--Wiles)

Under the hypotheses above, the two characteristic Z_p-ext-ensions of E are nonsingular. Thecyclotomic Z_p-extension is also nonsingular. There are at most a finite number of Z_p-extensions of K which are singular for E.

One should remark that the finiteness assertion in Theorem 1 is ineffective. To locate the singular Z_p-extensions for E, we consider the p-adic height pairing. Specifically, there is a canonical, <u>computable</u>, pairing

$$(\, , \,)_{\rho} \; : \; E(K) \times E(K) \longrightarrow Q_p$$
$$(x \, , \, y) \longmapsto (x,y)_{\rho}$$

for $\rho : \mathrm{Gal}(\bar{K}/K) \longrightarrow Q_p$ any continuous homomorphism, which is trilinear in x,y, and ρ, and symmetric in x andy.

One can show

<u>Theorem</u> 2: If the Shafarevitch-Tate conjecture holds, then (under the above hypotheses on $E_{/K}$):

If the $\wp$-height pairing $(\,,)_\wp$ is nondegenerate, the Z_p-extension of K cut out by $\wp$ is nonsingular.

Note that the determinant of the $\wp$-height pairing as function of $\wp$ yields a homogeneous form of degree r on the space of Z_p-extensions of K , where r is the rank of E(K). It would then follow (modulo the Shafarevitch-Tate conjecture) that the singular Z_p-extensions are fewer than r, and could be calculated-- provided that the form is not identically zero. It would be very interesting to get some detailed numerical calculations.

The remainder of the talk was devoted to a discussion of the p-adic height. Many people have contributed to the theory: Coates, Bernardi, Perrin-Riou, Schneider, Neron.

The talk concluded with a brief description of an algorithmic construction obtained jointly with Tate.

Titel: Carlsson's Proof of Segal's Conjecture

Autor: J. F. Adams

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§1. The original conjecture. Let G be a finite group and BG its classifying space. Atiyah says there is an isomorphism

$$R(G)^{\wedge} \xrightarrow{\alpha^{\wedge}} K(BG).$$

Segal replaced K-theory by cohomotopy and the representation ring $R(G)$ by the Burnside ring $A(G)$. He defined a suitable homomorphism

$$A(G)^{\wedge} \xrightarrow{\alpha^{\wedge}} \pi^0(BG)$$

and conjectured that $\alpha^{\wedge}$ is an isomorphism for all finite groups G. At the Northwestern topology conference, Easter 1982, G. Carlsson announced a proof of this conjecture, modulo results of his predecessors.

§2. Non-equivariant generalisations. Let G and H be two finite groups. Take BG, adjoin a disjoint base-point to get $BG \sqcup P$, and take the suspension spectrum of that to get $\underset{\sim}{B}G = \Sigma^{\infty}(BG \sqcup P)$; similarly for H. Let $\underset{\sim}{I}$ be a spectrum which we choose as a test space. Consider the problem of computing

$$[\underset{\sim}{I} \wedge \underset{\sim}{B} G, \underset{\sim}{B} H]$$

(morphisms in the category of spectra). For example, with $\underset{\sim}{I} = \underset{\sim}{S}^a$, $H = 1$ we get

$$[\underset{\sim}{S}^a \wedge \underset{\sim}{B} G, \underset{\sim}{B} 1] = \pi^{-a}(BG).$$

As in §1, one must define a group $A(\underset{\sim}{I}, G, H)$ and a homomorphism

$$A(\underset{\sim}{I}, G, H) \xrightarrow{\alpha} [\underset{\sim}{I} \wedge \underset{\sim}{B} G, \underset{\sim}{B} H],$$

and then proceed as before. I explain only the nature of the definition.

The groups $A(\underset{\sim}{I}, G, H)$ are defined in terms of the special case $G = 1$; we have

$$[\underset{\sim}{I} \wedge \underset{\sim}{B} 1, \underset{\sim}{B} H] = [\underset{\sim}{I}, \underset{\sim}{B} H]$$

and we are willing to treat these groups as known. There is a list of formal properties of functors such as $[\underset{\sim}{I} \wedge \underset{\sim}{B} G, \underset{\sim}{B} H]$. Here I take T to be fixed, so that I consider functors $M(G, H)$ of two variables, contravariant in G, covariant in H, the value $M(G, H)$ being an abelian group, and so on.

Prop. 1. Suppose given a suitable contravariant functor $N(H)$ of one variable H, for example $N(H) = [\underset{\sim}{I}, \underset{\sim}{B} H]$. Then there is an essentially unique functor $M(G, H)$ of two variables G, H which
(i) has the formal properties listed,
(ii) has $M(1, H) = N(H)$, and
(iii) is universal with respect to (i), (ii).

One calculates $M(G, H)$ from a formula of the following form.

$$M(G, H) = \bigoplus_K N(K).$$

Here K runs over a finite set of groups computable from G and H.

Definition. $A(\underset{\sim}{I}, G, H)$ is the universal functor corresponding to $N(H) = [\underset{\sim}{I}, \underset{\sim}{B} H]$.

The existence of the map

$$A(\underset{\sim}{I}, G, H) \xrightarrow{\alpha} [\underset{\sim}{I} \wedge \underset{\sim}{B}G, \underset{\sim}{B}H]$$

follows from the universal property.

It follows from the formal properties that $A(\underset{\sim}{I}, G, H)$ is a module over the Burnside ring $A(G)$, so one can complete it for the usual I-adic topology and so obtain

$$A(\underset{\sim}{I}, G, H)^\wedge \xrightarrow{\alpha^\wedge} [\underset{\sim}{I} \wedge \underset{\sim}{B}G, \underset{\sim}{B}H].$$

<u>Conjecture</u>: $\alpha^\wedge$ is an isomorphism for all finite spectra I and all finite groups G, H.

<u>Prop 2</u>. This is true when G and H are elementary abelian 2-groups.

It is believed that the proof can be carried over to elementary abelian p-groups [Adams, Gunawardena and Miller; in preparation]. This is the result Carlsson needs to quote.

<u>Note</u>. Among the formal properties listed, the first says on what category the functors N, M are defined. It will

the category of finite groups and homomorphisms.

§3. Equivariant generalisations. One introduces equivariant cohomotopy π^{ρ}_{G}. Let X be a suitable G-space, say a finite G-CW-complex. One has an isomorphism

$$\pi^{\rho}(EG \times_G X) \xleftrightarrow{\cong} \pi^{\rho}_{G}(EG \times X)$$

for $\rho \in \mathbb{Z}$.

<u>Conjecture</u>: The projection $EG \times X \to X$ induces an isomorphism

$$\pi^{\rho}_{G}(X)^{\wedge} \longrightarrow \pi^{\rho}_{G}(EG \times X)^{\wedge}$$

for all $\rho \in RO(G)$.

§4. Carlsson's proof. Carlsson proves the conjecture stated in §3 for all finite groups G.

§5. Further prospects. One should be able to prove something when G is a compact Lie group. We have an isomorphism

$$A(U(1))^{\wedge} \xrightarrow{\cong} \varprojlim_{n} \pi^{0}(CP^{n})$$

provided A is tom Dieck's Burnside ring. One should aim at the conjecture stated in §3.

Titel: Poincaré's approach to the problem of closed geodesics (closed geodesics on closed convex surfaces).

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1. In 1905 Poincaré publisched a paper [1] on geodesics on closed convex surfaces (here "convex" means "strictly positive curvature"). Its main part is devoted to an attempt to prove the existence of one nonselfintersecting closed geodesic (n.c.g.) on any such surface. Later a stronger result was obtained in the calculus of variations in the large: There are 3 n.c.g. for any riemannian metric on S^2 (without any assumption on the curvature). However, Poincaré's approach still seems to be of some interest because it uses entirely different arguments.

I am discussing the proof given in §4 of [1]. Details can be found in [2]. In §7 Poincaré gives another proof. See [3], [4] for a precise treatment of this proof.

2. Let M be a manifold with a riemannian metric g. We consider the geodesic flow $\{\varphi_t^g\}$ in SM, the sphere bundle over M. (The points of SM are rays in the tangent spaces starting from the origins of these spaces). A closed geodesic (c.g.) c is called nondegenerate if the corresponding periodic trajectory of $\{\varphi_t^g\}$ is nondegenerate.

Poincaré considers a one-parameter analytic family $g(\mu)$, $0 \leq \mu \leq 1$, of convex riemannian metrics such that $g(1)$ is a given metric and $g(0)$ has an odd number of n.c.g., all of them being nondegenerate (e.g., $g(0)$ can be a metric on an ellipsoid with approximately equal principal axes). The nondegeneracity of a c.g.c implies that for a small μ there exists a c.g. c_μ which depends analytically on μ and $c_0 = c$. There arises a temptation to consider analytic continuation for large μ. Poincaré assumes implicitly that the only singularities which can arise during this process are branch points similar to the branch points of analytic functions. Then it is easy to see that as μ varies, two c.g. can come together, coincide and disappear, or the inverse situation can occur so that two n.c.g. are born. A n.c.g. can coincide only with one other n.c.g. at a time so that the total number of n.c.g. remains odd.

3. In order to justify this assumption one must rule out the phenomenon which is called the "blue sky catastrophe" (in which the length of $c_\mu \to \infty$ when $\mu \to \mu_1$). In our case (positive curvature) this phenomenon is excluded by the existence of an "a priori bound" for the length of a n.c.g. This was pointed out by Birkhoff [5] (a well-known exact estimate was given by Toponogov later). One can construct a metric on S^2 with arbitrarily long n.c.g. (Anosov, Calabi), so this approach does not work if the curvature changes sign.

4. Some additional work still has to be done. Consider, for example, the possibility of a continuum of n.c.g. for some μ. The latter seems to be exceptional. Perhaps Poincaré had in mind a typical 1-parameter family? May be he appealed to the analyticity because this makes it possible to avoid some pathologies?

Nowadays we can deal with this question using purely real techniques (such as Sard's theorem and Abraham's transversality theorem). In such a way one can prove [2] that for a "typical" family $g(\mu)$, the set

$$(*) \quad \{(v, \omega, \mu) \in SS^2 \times (0,\infty) \times [0,1] : \{\varphi_t^{g(\mu)} v\} \text{ is a closed trajectory corresponding to a n.c.g. of length } \omega\}$$

is a compact 2-manifold (with boundary). Factoring by the natural $O(2)$-action, we obtain a 1-dimensional manifold L and a map $p: L \to [0,1]$ such that the points of $p^{-1}(\mu)$ are in 1-1 correspondence with the n.c.g. of $g(\mu)$ and $\partial L = L \cap p^{-1}\{0,1\}$. (Connected component of L corresponds to what Poincaré calls "continuous series of n.c.g."). This proves that for a dense set of convex metrics, the total number of n.c.g. is odd. For arbitrary metrics one uses limit arguments (together with an a priori estimate).

5. In the proof it is shown that for any family $g(\mu)$ the set $(*)$ is isolated in the sense of Fuller [6]. Then one can use Fuller's index instead of transversality arguments.

6. Although 4 is sufficient for the proof of existence of n.c.g., it does not yet justify Poincaré's point of view about ("typical")

bifurcations. In 4 we deal only with a geodesic flow and its action on tangent vectors of SS^2, while bifurcations depend upon second order approximations. Gruntal obtained the following results about bifurcations. For "typical" $g(\mu)$ a Poincaré map for the degenerate n.c.g. c_{μ_1} has (in appropriate coordinates) the form

$$\begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} x + y + \dots \\ y + a(\mu - \mu_1) + b x^2 + \dots \end{pmatrix}$$

with $a, b \neq 0$. This result allows us to describe bifurcations which lead to the appearance or disappearance of n.c.g.

7. There is another type of bifurcation of n.c.g. which happens when one (and hence two) of their multiplicators (eigenvalues of the linear part of the Poincaré map) equals to -1. Typically this leads to a "change of stability" (an elliptic n.c.g. becomes hyperbolic, and vice versa). At this point Poincaré forgot about these bifurcations and made a statement that for any convex metric on S^2 there exists a nonhyperbolic n.c.g. Gruntal has constructed a counterexample to this statement [7].

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19. VII. 1982

Аносов

Titel: V. P. P. New local-global principle for algebraic groups.
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Let G be a simple algebraic group defined over a field K of algebraic numbers, and let $G(K)$ be the group of K-rational points of G.

The problem on the structure of $G(K)$ is old, but untill recently nothing was known about the structure of $G(K)$ for K-anisotropic group G. If G is a K-isotropic group then in almoust all cases the structure of $G(K)$ is known ([1], [2]).

The key question here is: When is the group $G(K)$ projectively simple (i.e the quotient of $G(K)$ by its center is simple)? In my talk at the congress in Vancouver [1] I formulated the following conjecture.

Conjecture The group $G(K)$ is projectively simple if and only if for all non-archimedean valuations v of K the local groups $G(K_v)$ are projectively simple

It is not hard to schow that this is actually equivalent to the following conjecture: if G is a simple and simply-connected algebraic group, and if G is K_v-isotropic for all non-archimedean valuations v, then $G(K)$ is projectively simple.

This conjecture is the local-global principle of new type. All previous versions of the local-global principle were related to the local-global isomorphisms of some objects. Therefore, as a rule, proofs of these versions amounted to the proof of the injectivity of some natural maps of cohomologies.

The basic purpose of my talk is to tell about new results on the structure of $G(K)$ for the case when $G(K) = SL(1, \mathcal{D})$, where $\mathcal{D}$ is a finite dimensional division algebra over field K, $[\mathcal{D}:K] = n^2$, and $SL(1,\mathcal{D}) = \mathcal{D}^{(1)} = \{a \in \mathcal{D} \mid Nrd_{\mathcal{D}/K}(a) = 1\}$. Let V_f be a set of non-archimedean valuations of K,

$$T = \{v \in V_f \mid \mathcal{D}_v = \mathcal{D} \otimes_K K_v \text{ is a division algebra}\};$$

let U_v denote the group of units of K_v and

for $a \in K^*$ $V(a) = \{v \in V_f \mid a \notin U_v\}$.

For groups G of A_n-type there is a natural generalization of conjecture stated above [3]:

Conjecture[1]. For every noncentral normal subgroup $N \subset \mathcal{D}^{(1)}$ there exists open normal subgroup $W \subset \prod_{v \in T} \mathcal{D}_v^{(1)}$ such that $N = \mathcal{D}^{(1)} \cap W$. In particular, the group $\mathcal{D}^{(1)}$ is projectively simple for $T = \emptyset$.

This generalization is a natural one as one can see from the following theorem by Margulis [3]: Let G be a simple simply-connected group. Then any noncentral normal subgroup $H \subset G(K)$ has finite index.

For $[\mathcal{D}:K]=4$ this conjecture was stated by Kneser (1956) in a somewhat different form. In this case the conjecture was proved in [4], [5] (see also [6]).

Here we consider a general situation and develop the multiplicative arithmetic for division algebras of arbitrary index.

Theorem 1. Let $\mathcal{D}$ be a division algebra of index n, and $\forall v \in T$ $v(n)=0$. Then the commutant

$$[\mathcal{D}^{(1)},\mathcal{D}^{(1)}] = \mathcal{D}^{(1)} \cap \prod_{v\in T} [\mathcal{D}_v^{(1)},\mathcal{D}_v^{(1)}];$$

in particular, $\mathcal{D}^{(1)}=[\mathcal{D}^{(1)},\mathcal{D}^{(1)}]$ for $T=\emptyset$.

Let L be a maximal cyclic subfield of the division algebra $\mathcal{D}$ and $L^{(1)}=L\cap\mathcal{D}^{(1)}$. The following statement plays an important rule in the proof of the theorem 1.

Theorem 2. $\mathcal{D}^{(1)}=L^{(1)}[\mathcal{D}^{(1)},\mathcal{D}^{(1)}]$ if and only if for every $v\in T$ the field L_v is maximal unramified subfield of the division algebra $\mathcal{D}_v$.

Theorem 3. Let $\mathcal{D}^{(1)}=L^{(1)}[\mathcal{D}^{(1)},\mathcal{D}^{(1)}]$. Then

$$L^{(1)} \cap \Big(\prod_{v\in T} [\mathcal{D}_v^{(1)},\mathcal{D}_v^{(1)}]\Big) \subset [\mathcal{D}^{(1)},\mathcal{D}^{(1)}].$$

Theorem 1 follows immediately from the theorems 2, 3. The part of the proofs of the theorems 2, 3, which is the moust difficult in technical respect, is contained in the following statement.

Theorem 4. Let $\mathcal{U} = \{x \in \mathcal{D}^* \mid Nrd_{\mathcal{D}/K}(x) \in \mathcal{U}_v, \forall v \in T\}$ and for $y \in \mathcal{D}^*$ the element $x \in (\mathcal{U}_v[\mathcal{D}_v^{(1)}, \mathcal{D}_v^{(1)}]) \cap \mathcal{U}$ $\forall v \in T \cap V(Nrd_{\mathcal{D}/K}(y))$. Then $x^{-1}y^{-1}xy \in [\mathcal{D}^{(1)}, \mathcal{D}^{(1)}]$, in particular, $\mathcal{D}^{(1)} = [\mathcal{D}^{(1)}, \mathcal{D}^{(1)}]$ for $T = \emptyset$.

For the proof of theorem 4 generalization of the Hasse's principle for some non-normal extensions is essential. To be more precise, we need to solve the following problem: given a finite set $S \subset V_f$, find a class of fields for which validity of Hasse's principle depends upon their local behaviour in points of S.

Theorem 5. Let $S = \{v_1, v_2\} \subset V_f$. Then for every $n > 1$ there exists the extension $L = L(S)$ of K of the degree n with the property: if $M \supset K$ and $M \otimes_K K_{v_i} \overset{K_{v_i}}{\simeq} L \otimes_K K_{v_i}$ $(i = 1, 2)$, then Hasse's principle holds for M.

One can reformulate the theorem 5 in another way. Let $G = SL(n, \mathbb{C})$ and let $\mathcal{T}$ be the variety of maximal tori of G. Then for any $S = \{v_1, v_2\} \subset V_f$ there exists an open subset $W \subset \mathcal{T}_S = \prod_{v \in S} \mathcal{T}(K_v)$ such that for any torus $R \subset \mathcal{T}(K) \cap W$ the Shafarevich-Tate group $Ш(R) = 0$

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