ABUNDANT FAMILIES OF JACOBI ELLIPTIC FUNCTION-LIKE SOLUTIONS OF THE (2+1)-DIMENSIONAL BREAKING SOLITON EQUATION AND THE KONOPELCHENKO-DUBROVSKY EQUATION VIA A NEW MAPPING METHOD

EMMANUEL YOMBA,

MAX-PLANCK INSTITUT FÜR MATHEMATIK,
VIVATSGASSE 7,
BONN D-53111,
GERMANY

DEPARTMENT OF PHYSICS,
FACULTY OF SCIENCES,
UNIVERSITY OF NGAOUNDÉRÉ,
BO.POX 454 NGAOUNDÉRÉ, CAMEROON

Abstract. A generalized mapping method is used by the aid of symbolic computation system for constructing exact solutions of some (2+1) dimensional nonlinear partial differential equations (NLPDEs). The key idea of this method is to take advantage of the elliptic equation of first kind involving three parameters which has more new solutions. Some new families of, Jacobi elliptic function-like, combined Jacobi elliptic function-like, rational-like, triangular-function-like, formal soliton-like, combined formal soliton-like and kink-bell soliton-like solutions are obtained for these (2+1) dimensional NLPDEs.

1. Introduction

The investigation of the exact solutions of nonlinear partial differential equations (NLPDEs) play a fundamental role in the description of complex phenomena in various fields of Sciences. The exact solutions if available, of these NLPDEs can provide not only much physical information and more insight into the physical aspect of the problems, but also can help in the stability analysis of solutions and in the verification of numerical analysis. With the availability of computer symbolic systems like Maple or Mathematica which allow us to perfom some complicated and tedious algebraic calculation on a computer as well as help us to find new exact solutions of NLPDEs [1-5], much work on the subject of obtainaing analytical solutions has been done. And in the past decades, there has been significant progression in the development of these methods such as tanh-function method [6,1,2], sech-function method [7,8] and Jacobi elliptic function method [8-10]. Recently, a Jacobi function expansion method was applied to construct periodic wave solutions for some nonlinear equations. The key idea of this method is similar to the tanh method by replacing the tanh function with some Jacobi elliptic functions such as cn ξ, sn ξ and dn ξ [8,11]. To obtain these Jacobi periodic wave solutions, many similar repetition calculations have to be made. Very recently, a unified method called the mapping method has been developed [12,13]. By the means of this method, the solitary wave, the periodic wave and the kink wave solutions can, if they exist, be obtained simultaneously to the equation in concern. Thus, many of tedious and repetitive calculations may be avoided. In this letter, we would like to make a further extension of the mapping method. the rest of this letter is organized as follows: in section 2, we present our extended mapping method and the same time point out the differences between our method and that of Peng [12,13]. In section 3, we demonstrate effectiveness of our method by applying it to the (2+1) dimensional breaking equation and to the (2+1) dimensional Konopelchenko-Dubrovsky equation. We conclude letter in last section.

2. The further extended mapping method

For a given NLPDE, say in three independent variables X= (x,y,t) and dependent variable u, we seek its solutions in the form

$$u(x,y,t) = \sum_{i=0}^{n} A_i(X)\phi^i(\xi(X)), \qquad \textbf{(1)}$$

while ϕ satisfies the first kind elliptic equation
$(\frac{d\phi}{d\xi})^2 = r + a\phi^2(\xi) + b/(2)\phi^4(\xi), \quad \xi = p(y,t)x + q(y,t),$ (**2**)
where r, a and b, are constants. To determine u explicitly, we take the following five steps.

Step 1. Determine n which is fixed by balancing the highest order derivative terms and the highest nonlinear terms in the given NLPDE.

Step 2. Substituting (1) and (2) into the given NLPDE and collecting coefficients of $x^j\phi^k\sqrt{r + a\phi^2(\xi) + b/(2)\phi^4(\xi)}$, $x^l\phi^s$ (j=0,1,...;k=0,1,...; l=0,1,...; s=1,2,...), then setting each coefficient to zero to derive a set of partial differential equations of $A_i(i = 0, 1..., n)$ and ξ

Step 3. Solving the system of partial differential equations obtained in step 2 for A_i and ξ.

Step 4. As (2) possesses the general solution

$$\phi(\xi) = \begin{cases} sn\,\xi;\ cd\,\xi = cn\,\xi/dn\,\xi & : \quad r = 1, \ \ a = -(m^2+1), \ \ b = 2m^2; \\ cn\,\xi & : \quad r = 1 - m^2, \ \ a = 2m^2 - 1, \ \ b = -2m^2; \\ dn\,\xi & : \quad r = m^2 - 1, \ \ a = 2 - m^2, \ \ b = -2; \\ ns\,\xi = 1/sn\,\xi;\ dc\,\xi = dn\,\xi/cn\,\xi & : \quad r = m^2, \ \ a = -(m^2+1), \ \ b = 2; \\ nc\,\xi = 1/cn\,\xi & : \quad r = -m^2, \ \ a = 2m^2 - 1, \ \ b = 2(1 - m^2); \\ nd\,\xi = 1/dn\,\xi & : \quad r = -1, \ \ a = 2 - m^2, \ \ b = 2(m^2 - 1); \\ cs\,\xi = cn\,\xi/sn\,\xi & : \quad r = 1 - m^2, \ \ a = 2 - m^2, \ \ b = 2; \\ sc\,\xi = sn\,\xi/cn\,\xi & : \quad r = 1, \ \ a = 2 - m^2, \ \ b = 2(1 - m^2); \\ sd\,\xi = sn\,\xi/dn\,\xi & : \quad r = 1, \ \ a = 2m^2 - 1, \ \ b = 2m^2(m^2 - 1); \\ ds\,\xi = dn\,\xi/sn\,\xi & : \quad r = m^2(m^2 - 1), \ \ a = 2m^2 - 1, \ \ b = 2; \\ ns\,\xi \pm cs\,\xi & : \quad r = 1/4, \ a = (1 - 2m^2)/2, \ \ b = 1/2; \\ nc\,\xi \pm sc\,\xi & : \quad r = (1 - m^2)/4, \ a = (1 + m^2)/2, \ \ b = (1 - m^2)/2; \\ ns\,\xi + ds\,\xi & : \quad r = m^4/4, \ a = (m^2 - 2)/2, \ \ b = 1/2; \\ sn\,\xi \pm i\ cn\,\xi & : \quad r = m^2/4, \ \ a = (m^2 - 2)/2, \ \ b = m^2/2, \ \ i^2 = -1; \\ C/\xi & : \quad r = 0, \ \ a = 0 \ \ b = 2. \end{cases} \qquad (\mathbf{3})$$

Where C is a constant. The Jacobi elliptic functions $sn\,\xi = sn\,(\xi|m)$, $cn\,\xi = cn\,(\xi|m)$, and $dn\,\xi = dn\,(\xi|m)$, where $m(0 < m < 1)$ is the modulus of elliptic function, are double periodic and possess properties of triangular functions, namely

$$sn^2\xi + cn^2\xi = 1, \ \ dn^2\,\xi + m^2 sn^2\,\xi = 1, \ \ \frac{d}{d\xi}sn\,\xi = cn\,\xi dn\,\xi,$$

$$\frac{d}{d\xi}cn\ \xi = -sn\ \xi dn\ \xi, \quad \frac{d}{d\xi}dn\ \xi = -m^2 cn\ \xi sn\ \xi. \tag{4}$$

In addition, we see that other solutions are obtained from solutions (3) in case of degeneracies:

When m→ 0, the Jacobi elliptic functions degenerate to the trigonometric functions solutions of the given NLPDE i.e.

$sn\ \xi \rightarrow \sin\ \xi,\ \ cn\ \xi \rightarrow \cos\ \xi,\ \ dn\ \xi \rightarrow 1.$ **(5)**

When m→ 1, the Jacobi elliptic functions degenerate to the solitonic solutions of the given NLPDE i.e.

$sn\ \xi \rightarrow \tanh\ \xi,\ \ cn\ \xi \rightarrow sech\ \xi,\ \ dn\ \xi \rightarrow sech\ \xi.$ **(6)**

Detailed explanations about Jacobi elliptic functions can be found in Refs.[14-16]. The other 9 Jacobi functions are all expressible in terms of $sn\ \xi,\ cn\ \xi,\ dn\ \xi$.

Step 5. Substituting A_i, ξ and (3) or (5) and (6) into (1) to obtain Jacobi elliptic function-like, combined Jacobi elliptic function-like, rational-like solutions or triangular function-like, and formal soliton-like, combined formal soliton-like, kink bell soliton-like, solutions respectively.

Remark 1.

From the following application, it is easily seen that our algorithm is more powerful than the method presented recently by Peng [12,13]. When A_i in (1) are constant, the method reduces to be the method previously presented by the above author. It is important to note that many solutions of Eq.(2) have been omitted in [13].

3. Application

3.1. (2+1)-dimensional breaking soliton equation

. The (2+1)-dimensional breaking soliton equation of the form [17]

$$u_t + Au_{xxx} + Bu_{xxy} + 6Au_x u + 4Buu_y + 4Bu_x\partial_x^{-1}u_y = 0, \tag{7}$$

describes the (2+1)-dimensional interaction of a Riemann wave propagating along the y-axis with a long wave along the x-axis, in the particular case of A=0, Eq.(7) has been studied [18] and its Painlevé property has been proved [19]. Under the transformation $u_y = v_x$, Eq.(7) is converted into a set of couple NLPDEs

$$\begin{cases} u_t + Au_{xxx} + Bu_{xxy} + 6Au_x u + 4B(uv)_x = 0, \\ u_y = v_x, \end{cases} \tag{8}$$

. Balancing u_{xxx} or u_{xxy} with uu_x 0r $(uv)_x$, ansatz (1) becomes

$$u = f + h\phi(\xi) + g\phi^2(\xi),$$
$$v = F + H\phi(\xi) + G\phi^2(\xi), \qquad \mathbf{(9)}$$
$$\xi = px + q,$$

where f, h, g, F, H, G, p and q are functions of y and t to be determined. Substituting (9) and (2) into (8) and collecting coefficients of power of $x^j\phi^k\sqrt{r+a\phi^2(\xi)+b/(2)\phi^4(\xi)}$, ϕ^s (j=0,1 ;k=0,...3; s=1,...,4), (notice that f, h, g, F, H, G, p and q are independent of x) with the aid of Mathematica, then setting each coefficient to zero, we have

$$f_t + 2Brp^2g_y + 4Brgpp_y = 0, \qquad \mathbf{(10.1)}$$
$$h_t + ABp^2h_y + 2aBhpp_y = 0, \qquad \mathbf{(10.2)}$$
$$g_t + 4aBp^2g_y + 8aBgpp_y = 0, \qquad \mathbf{(10.3)}$$
$$bBp(ph_y + 2p_yh) = 0, \qquad \mathbf{(10.4)}$$
$$3bBp(pg_y + 2p_yg) = 0, \qquad \mathbf{(10.5)}$$
$$hp_t + aBhp^2p_y = 0, \qquad \mathbf{(10.6)}$$
$$2g(p_t + 4aBp^2p_y) = 0, \qquad \mathbf{(10.7)}$$
$$3bBhp^2p_y = 0, \qquad \mathbf{(10.8)}$$
$$12bBgpp_y = 0, \qquad \mathbf{(10.9)}$$
$$6Afhp + 4BFhp + 4BfHp + aAhp^3 + hq_t + aBhp^2q_y = 0, \qquad \mathbf{(10.10)}$$
$$2(6Afgp+4BFgp+4BfGp+3Ah^2p+4BhHp+4aAgp^3+gq_t+4aBgp^2q_y) = 0, \qquad \mathbf{(10.11)}$$
$$3p(6Agh + 4BhG + 4BgH + Abp^2h + bBhpq_y) = 0, \qquad \mathbf{(10.12)}$$
$$2pg(6Ag + 8BG + 6Abp^2 + 6bBpq_y) = 0, \qquad \mathbf{(10.13)}$$
$$f_y = 0, \quad g_y = 0, \quad h_y = 0, \quad hp_y = 0, \quad 2gp_y = 0, \qquad \mathbf{(10.14)}$$
$$Hp - hq_y = 0, \qquad \mathbf{(10.15)}$$
$$2(Gp - gq_y) = 0. \qquad \mathbf{(10.16)}$$

From these equations, it follows that

$$p_y = p_t = 0, \quad h_y = h_t = 0, \quad g_y = g_t = 0, \quad f_y = f_t = 0. \qquad \mathbf{(11)}$$

So we obtain that p, h, g and f are constants. And now (10.1)-(10.16) become

$$6Afhp + 4BFhp + 4BfHp + aAhp^3 + hq_t + aBhp^2q_y = 0, \qquad \mathbf{(12.1)}$$
$$2(6Afgp+4BFgp+4BfGp+3Ah^2p+4BhHp+4aAgp^3+gq_t+4aBgp^2q_y) = 0, \qquad \mathbf{(12.2)}$$

$$3p(6Agh+4BhG+4BgH+Abp^2h+bBhpq_y)=0, \qquad (\mathbf{12.3})$$

$$4pg(3Ag+4BG+3Abp^2+3bBpq_y)=0, \qquad (\mathbf{12.4})$$

$$Hp-hq_y=0, \qquad (\mathbf{12.5})$$

$$Gp-gq_y=0. \qquad (\mathbf{12.6})$$

Solving Eqs.(12.1)-(12.6) we can obtain easily the following solutions

$$\begin{gathered}h=0,\\ H=0,\\ g=-\frac{3bp^2(Ap+BK_1)}{3Ap+4BK_1},\\ G=-\frac{3bp(Ap+BK_1)K_1}{3Ap+4BK_1},\\ F=-\frac{1}{4Bp}[2Ap(2ap^2+3f)+K_{2t}+4BK_1(f+ap^2)],\\ q=K_1y+K_2(t),\end{gathered} \qquad (\mathbf{13})$$

where K_2 is an arbitrary function of t, while K_1, p, r, a, b and f are arbitrary constants. Thanks to the arbitrariness of r, a and b, we may choose appropriate values for these parameters in order to obtain the form of solutions given by (3) 0r (5) and (6). Hence, the solutions of Eq.(7) read

$$u=f-\frac{3bp^2(Ap+BK_1)}{3Ap+4BK_1}\phi^2(px+K_1y+K_2(t)), \qquad (\mathbf{14})$$

$$v=-\frac{1}{4Bp}[2Ap(2ap^2+3f)+K_{2t}+4BK_1(f+ap^2)]-\frac{3bp(Ap+BK_1)K_1}{3Ap+4BK_1}\phi^2(px+K_1y+K_2(t)).$$

Therefore, we can consider several cases,

Case 1.

$$u_1=f-\frac{6m^2p^2(Ap+BK_1)}{3Ap+4BK_1}sn^2\xi, \qquad (\mathbf{15.1})$$

$$v_1=-\tfrac{1}{4Bp}[2Ap(-2(m^2+1)p^2+3f)+K_{2t}+4BK_1(f-(m^2+1)p^2)]-\tfrac{6m^2p(Ap+BK_1)K_1}{3Ap+4BK_1}sn^2\xi.$$

Case2

$$u_2=f-\frac{6m^2p^2(Ap+BK_1)}{3Ap+4BK_1}cd^2\xi, \qquad (\mathbf{15.2})$$

$$v_2=-\tfrac{1}{4Bp}[2Ap(-2(m^2+1)p^2+3f)+K_{2t}+4BK_1(f-(m^2+1)p^2)]-\tfrac{6m^2p(Ap+BK_1)K_1}{3Ap+4BK_1}cd^2\xi.$$

Case 3

$$u_3=f+\frac{6m^2p^2(Ap+BK_1)}{3Ap+4BK_1}cn^2\xi, \qquad (\mathbf{15.3})$$

$v_3 = -\frac{1}{4Bp}[2Ap(2(2m^2-1)p^2+3f)+K_{2t}+4BK_1(f+(2m^2-1)p^2)]+\frac{6m^2p(Ap+BK_1)K_1}{3Ap+4BK_1}cn^2\xi.$

Case 4

$$u_4 = f + \frac{6p^2(Ap+BK_1)}{3Ap+4BK_1}dn^2\xi, \qquad (\mathbf{15.4})$$

$v_4 = -\frac{1}{4Bp}[2Ap(2(2-m^2)p^2+3f)+K_{2t}+4BK_1(f+(2-m^2)p^2)]+\frac{6p(Ap+BK_1)K_1}{3Ap+4BK_1}dn^2\xi.$

Case 5

$$u_5 = f - \frac{6p^2(Ap+BK_1)}{3Ap+4BK_1}ns^2\xi, \qquad (\mathbf{15.5})$$

$v_5 = -\frac{1}{4Bp}[2Ap(-2(1+m^2)p^2+3f)+K_{2t}+4BK_1(f-(1+m^2)p^2)]-\frac{6p(Ap+BK_1)K_1}{3Ap+4BK_1}ns^2\xi.$

Case 6

$$u_6 = f - \frac{6p^2(Ap+BK_1)}{3Ap+4BK_1}dc^2\xi, \qquad (\mathbf{15.6})$$

$v_6 = -\frac{1}{4Bp}[2Ap(-2(1+m^2)p^2+3f)+K_{2t}+4BK_1(f-(1+m^2)p^2)]-\frac{6p(Ap+BK_1)K_1}{3Ap+4BK_1}dc^2\xi.$

Case 7

$$u_7 = f - \frac{6(1-m^2)p^2(Ap+BK_1)}{3Ap+4BK_1}nc^2\xi, \qquad (\mathbf{15.7})$$

$v_7 = -\frac{1}{4Bp}[2Ap(2(2m^2-1)p^2+3f)+K_{2t}+4BK_1(f+(2m^2-1)p^2)]-\frac{6(1-m^2)p(Ap+BK_1)K_1}{3Ap+4BK_1}nc^2\xi.$

Case 8

$$u_8 = f - \frac{6(m^2-1)p^2(Ap+BK_1)}{3Ap+4BK_1}nd^2\xi, \qquad (\mathbf{15.8})$$

$v_8 = -\frac{1}{4Bp}[2Ap(2(2-m^2)p^2+3f)+K_{2t}+4BK_1(f+(2-m^2)p^2)]-\frac{6(m^2-1)p(Ap+BK_1)K_1}{3Ap+4BK_1}nd^2\xi.$

Case 9

$$u_9 = f - \frac{6p^2(Ap+BK_1)}{3Ap+4BK_1}cs^2\xi, \qquad (\mathbf{15.9})$$

$v_9 = -\frac{1}{4Bp}[2Ap(2(2-m^2)p^2+3f)+K_{2t}+4BK_1(f+(2-m^2)p^2)]-\frac{6p(Ap+BK_1)K_1}{3Ap+4BK_1}cs^2\xi.$

Case 10

$$u_{10} = f - \frac{6(1-m^2)p^2(Ap+BK_1)}{3Ap+4BK_1}sc^2\xi, \qquad (\mathbf{15.10})$$

$v_{10} = -\frac{1}{4Bp}[2Ap(2(2-m^2)p^2+3f)+K_{2t}+4BK_1(f+(2-m^2)p^2)]-\frac{6(1-m^2)p(Ap+BK_1)K_1}{3Ap+4BK_1}sc^2\xi.$

Case 11

$$u_{11} = f - \frac{6m^2(m^2-1)p^2(Ap+BK_1)}{3Ap+4BK_1}sd^2\xi, \qquad (\mathbf{15.11})$$

$v_{11} = -\frac{1}{4Bp}[2Ap(2(2m^2-1)p^2+3f)+K_{2t}+4BK_1(f+(2m^2-1)p^2)]-\frac{6m^2(m^2-1)p(Ap+BK_1)K_1}{3Ap+4BK_1}sd^2\xi.$

Case 12

$$u_{12} = f - \frac{6p^2(Ap+BK_1)}{3Ap+4BK_1}ds^2\xi, \qquad (\mathbf{15.12})$$

$v_{12} = -\frac{1}{4Bp}[2Ap(2(2m^2-1)p^2+3f)+K_{2t}+4BK_1(f+(2m^2-1)p^2)] - \frac{6p(Ap+BK_1)K_1}{3Ap+4BK_1}ds^2\xi.$

Case 13

$$u_{13} = f - \frac{\frac{3}{2}p^2(Ap+BK_1)}{3Ap+4BK_1}[ns\ \xi \pm cs\ \xi]^2, \qquad (\mathbf{15.13})$$

$v_{13} = -\frac{1}{4Bp}[2Ap((1-2m^2)p^2+3f)+K_{2t}+2BK_1(2f+(1-2m^2)p^2)] - \frac{3p(Ap+BK_1)K_1}{2(3Ap+4BK_1)}[ns\ \xi \pm cs\ \xi]^2.$

Case 14

$$u_{14} = f - \frac{3(1-m^2)p^2(Ap+BK_1)}{2(3Ap+4BK_1)}[nc\ \xi \pm sc\ \xi]^2, \qquad (\mathbf{15.14})$$

$v_{14} = -\frac{1}{4Bp}[2Ap((m^2+1)p^2+3f)+K_{2t}+2BK_1(2f+(m^2+1)p^2)] - \frac{3(1-m^2)p(Ap+BK_1)K_1}{2(3Ap+4BK_1)}[nc\ \xi \pm sc\ \xi]^2.$

Case 15

$$u_{15} = f - \frac{3p^2(Ap+BK_1)}{2(3Ap+4BK_1)}[ns\ \xi + ds\ \xi]^2, \qquad (\mathbf{15.15})$$

$v_{15} = -\frac{1}{4Bp}[2Ap((m^2-2)p^2+3f)+K_{2t}+2BK_1(2f+(m^2-2)p^2)] - \frac{3p(Ap+BK_1)K_1}{2(3Ap+4BK_1)}[ns\ \xi + ds\ \xi]^2.$

Case 16

$$u_{16} = f - \frac{3m^2p^2(Ap+BK_1)}{2(3Ap+4BK_1)}[sn\ \xi \pm i\ cn\ \xi]^2, \qquad (\mathbf{15.16})$$

$v_{16} = -\frac{1}{4Bp}[2Ap((m^2-2)p^2+3f)+K_{2t}+2BK_1(2f+(m^2-2)p^2)] - \frac{3m^2p(Ap+BK_1)K_1}{2(3Ap+4BK_1)}[sn\ \xi \pm i\ cn\ \xi]^2.$

Case 17

$$u_{17} = f - \frac{6p^2(Ap+BK_1)}{3Ap+4BK_1}\frac{C^2}{\xi^2}, \qquad (\mathbf{15.17})$$

$v_{17} = -\frac{1}{4Bp}[6Apf+K_{2t}+4BK_1f] - \frac{6p(Ap+BK_1)K_1}{3Ap+4BK_1}\frac{C^2}{\xi^2}.$

Where $\xi = px + K_1y + K_2(t)$.

Remark 2

When m$\to$ 0, these solutions (15.1)-(15.16) degenerate as triangular function-like solutions of Eq.(8), namely

$$u_{18} = f - \frac{6p^2(Ap+BK_1)}{3Ap+4BK_1}\csc^2\xi, \qquad (\mathbf{16.1})$$

$$v_{18} = -\frac{1}{4Bp}[2Ap(-2p^2+3f)+K_{2t}+4BK_1(f-p^2)] - \frac{6p(Ap+BK_1)K_1}{3Ap+4BK_1}\csc^2\xi.$$

$$u_{19} = f - \frac{6p^2(Ap+BK_1)}{3Ap+4BK_1}\sec^2\xi, \qquad (\mathbf{16.2})$$

$$v_{19} = -\frac{1}{4Bp}[2Ap(-2p^2+3f)+K_{2t}+4BK_1(f-p^2)] - \frac{6p(Ap+BK_1)K_1}{3Ap+4BK_1}\sec^2\xi.$$

$$u_{20} = f - \frac{6p^2(Ap+BK_1)}{3Ap+4BK_1}\cot^2\xi, \quad \textbf{(16.3)}$$

$$v_{20} = -\frac{1}{4Bp}[2Ap(4p^2+3f)+K_{2t}+4BK_1(f+2p^2)] - \frac{6p(Ap+BK_1)K_1}{3Ap+4BK_1}\cot^2\xi.$$

$$u_{21} = f - \frac{6p^2(Ap+BK_1)}{3Ap+4BK_1}\tan^2\xi, \quad \textbf{(16.4)}$$

$$v_{21} = -\frac{1}{4Bp}[2Ap(4p^2+3f)+K_{2t}+4BK_1(f+2p^2)] - \frac{6p(Ap+BK_1)K_1}{3Ap+4BK_1}\tan^2\xi.$$

$$u_{22} = f - \frac{3p^2(Ap+BK_1)}{2(3Ap+4BK_1)}[\sec\ \xi \pm \tan\ \xi]^2, \quad \textbf{(16.5)}$$

$$v_{22} = -\tfrac{1}{4Bp}[2Ap((p^2+3f)+K_{2t}+2BK_1(2f+p^2)] - \tfrac{3p(Ap+BK_1)K_1}{2(3Ap+4BK_1)}[\sec\ \xi \pm \tan\ \xi]^2.$$

$$u_{23} = f - \frac{3p^2(Ap+BK_1)}{2(3Ap+4BK_1)}[\csc\ \xi \pm \cot\ \xi]^2, \quad \textbf{(16.6)}$$

$$v_{23} = -\tfrac{1}{4Bp}[2Ap((p^2+3f)+K_{2t}+2BK_1(2f+p^2)] - \tfrac{3p(Ap+BK_1)K_1}{2(3Ap+4BK_1)}[\csc\ \xi \pm \cot\ \xi]^2.$$

Where $\xi = px + K_1y + K_2(t)$.

Remark 3

When m→ 1, these solutions (15.1)-(15.16) degenerate as soliton-like solutions of Eq.(8), namely

$$u_{24} = f - \frac{6p^2(Ap+BK_1)}{3Ap+4BK_1}\tanh^2\xi, \quad \textbf{(17.1)}$$

$$v_{24} = -\frac{1}{4Bp}[2Ap(-4p^2+3f)+K_{2t}+4BK_1(f-2p^2)] - \frac{6p(Ap+BK_1)K_1}{3Ap+4BK_1}\tanh^2\xi.$$

$$u_{25} = f - \frac{6p^2(Ap+BK_1)}{3Ap+4BK_1}\coth^2\xi, \quad \textbf{(17.2)}$$

$$v_{25} = -\frac{1}{4Bp}[2Ap(-4p^2+3f)+K_{2t}+4BK_1(f-2p^2)] - \frac{6p(Ap+BK_1)K_1}{3Ap+4BK_1}\coth^2\xi.$$

$$u_{26} = f + \frac{6p^2(Ap+BK_1)}{3Ap+4BK_1}\ sech^2\xi, \quad \textbf{(17.3)}$$

$$v_{26} = -\frac{1}{4Bp}[2Ap(2p^2+3f)+K_{2t}+4BK_1(f+p^2)] + \frac{6p(Ap+BK_1)K_1}{3Ap+4BK_1}\ sech^2\xi.$$

$$u_{27} = f - \frac{6p^2(Ap+BK_1)}{3Ap+4BK_1}\ csch^2\xi, \quad \textbf{(17.4)}$$

$$v_{27} = -\frac{1}{4Bp}[2Ap(2p^2+3f)+K_{2t}+4BK_1(f+p^2)] - \frac{6p(Ap+BK_1)K_1}{3Ap+4BK_1}\ csch^2\xi.$$

$$u_{28} = f - \frac{3p^2(Ap+BK_1)}{2(3Ap+4BK_1)}[\coth\ \xi \pm csch\ \xi]^2, \quad \textbf{(17.5)}$$

$v_{28}=-\frac{1}{4Bp}[2Ap((-p^2+3f)+K_{2t}+2BK_1(2f-p^2)]-\frac{3p(Ap+BK_1)K_1}{2(3Ap+4BK_1)}[\coth\ \xi\pm csch\ \xi]^2.$

$$u_{29}=f-\frac{3p^2(Ap+BK_1)}{2(3Ap+4BK_1)}[\tanh\ \xi\pm i\ sech\ \xi]^2, \qquad (\mathbf{17.6})$$

$v_{29}=-\frac{1}{4Bp}[2Ap(-p^2+3f)+K_{2t}+2BK_1(2f-p^2)]-\frac{3p(Ap+BK_1)K_1}{2(3Ap+4BK_1)}[\tanh\ \xi\pm i\ sech\ \xi]^2.$
Where $\xi=px+K_1y+K_2(t)$.

3.2. **The (2+1)-dimensional Konopelchenko-Dubrovsky (KD) equation**
. Consider the (2+1)-dimensional Konopelchenko-Dubrovsky equation

$$\begin{cases} U_t-U_{xxx}-6\beta UU_x+\frac{3}{2}\alpha^2U^2U_x-3W_y+3\alpha U_xW=0, \\ W_x=U_y, \end{cases} \qquad (\mathbf{18})$$

where $\alpha,\ \beta$ are real parameters [20]. Our proposed method gives the equation a series of exact solutions as follows. By expand the solution of (18) in the form

$$U(x,y,t)=\sum_{i=0}^{n}A_i(X)\phi^i(\xi(X)), \qquad (\mathbf{19})$$

$$W(x,y,t)=\sum_{i=0}^{l}B_i(X)\phi^i(\xi(X),$$

while ϕ satisfies the first kind elliptic equation (2). Balancing the highest derivative with the highest nonlinear terms in (18) gives n = l = 1 and thus we have:

$$\begin{aligned} U(x,y,t)&=f(y,t)+h(y,t)\phi\ (\xi), \\ W(x,y,t)&=F(y,t)+H(y,t)\phi(\xi), \\ \xi&=p(y,t)x+q(y,t) \end{aligned} \qquad (\mathbf{20})$$

where ϕ satisfies (2). Inserting (20) and (2) into (18) by using the symbolic computation yield a serie of powers of $x^j\phi^k\sqrt{r+a\phi^2(\xi)+b/(2)\phi^4(\xi)},\ \ \phi^s$ (j=0,1 ;k=0,1,2; s=0,1,2), then setting each coefficient to zero,to derive a set of partial differential equations as follows

$$f_t-3F_y=0, \qquad (\mathbf{21.1})$$

$$h_t-3H_y=0, \qquad (\mathbf{21.2})$$

$$hp_t-3Hp_y=0, \qquad (\mathbf{21.3})$$

$$12\beta fhp-3\alpha^2f^2hp-6\alpha Fhp+2ahp^3-2hq_t+6Hq_y=0, \qquad (\mathbf{21.4})$$

$$12\beta ph^2-6\alpha^2fh^2p-6\alpha hHp=0, \qquad (\mathbf{21.5})$$

$$-3hp(\alpha^2h^2-2bp^2)=0, \qquad (\mathbf{21.6})$$

$$f_y=0, \quad h_y=0, \quad -hp_y=0, \qquad (\mathbf{21.7})$$

$$Hp-hq_y=0. \qquad (\mathbf{21.8})$$

From Eqs.(21.3) and (21.7) we have

$$p_y=p_t=0, \ \ h_y=0, \ \ f_y=0. \qquad (\mathbf{22})$$

From (21.6) and (21.8) we deduce

$$h^2=\frac{2}{\alpha^2}bp^2, \qquad (\mathbf{23.1})$$

$$H=\frac{h}{p}q_y. \qquad (\mathbf{23.2})$$

Using (23.1) and (23.2) into (21.5), we obtain

$$f=\frac{2\beta}{\alpha^2}-\frac{1}{\alpha p}q_y. \qquad (\mathbf{24})$$

Using (23.2) and (24) into (21.4) leads to

$$F=\frac{2\beta}{\alpha}[\frac{2\beta}{\alpha^2}-\frac{1}{\alpha p}q_y]+\frac{1}{\alpha p^2}q_y^2+\frac{1}{3\alpha}ap^2+\frac{\alpha}{2}[\frac{2\beta}{\alpha^2}-\frac{1}{\alpha p}q_y]^2-\frac{1}{3\alpha p}q_t. \qquad (\mathbf{25})$$

We obtain from (22) that p is constant, thus, h is also constant from (23.1), then (21.2) gives

$$H_y=0, \qquad (\mathbf{26})$$

Taking into account (23.2), we have

$$q_{yy}=0. \qquad (\mathbf{27})$$

Taking into account (24), (25) and (26) we verify that (21.1) is satisfied. Relation (27) can be integrated twice as

$$q=K_1(t)y+K_2(t), \qquad (\mathbf{28})$$

Hence, the solution of Eq.(18) reads

$$U=[\frac{2\beta}{\alpha^2}-\frac{1}{\alpha p}K_1(t)]\pm\frac{\sqrt{2b}}{\alpha}p\phi(px+K_1(t)y+K_2(t)),$$

$$W=F\pm\frac{\sqrt{2b}}{\alpha}K_1(t)\phi(px+K_1(t)y+K_2(t)), \qquad (\mathbf{29})$$

$$F=\frac{2\beta}{\alpha}[\frac{2\beta}{\alpha^2}-\frac{1}{\alpha p}K_1(t)]+\frac{1}{\alpha p^2}K_1^2(t)+\frac{1}{3\alpha}ap^2+\frac{\alpha}{2}[\frac{2\beta}{\alpha^2}-\frac{1}{\alpha p}K_1(t)]^2-\frac{1}{3\alpha p}[K_{1t}y+K_{2t}]$$

where $K_1,\ K_2$ are arbitrary functions of t and p, r,a and b are arbitrary constants. We may Choose properly (r,a,b) in ordinary differential equation (2) such that the corresponding solution $\phi(\xi)$ of it, is one of, Jacobi elliptic functions, or combined Jacobi elliptic function, or rational function . The relations between values of (r,a,b) and corresponding, Jacobi elliptic function, combined Jacobi elliptic function, rational solutions $\phi(\xi)$ of Eq.(2) are given in relation (3). Only the function $\phi(\xi)$ with b > 0 from the relation (3) will be chosen. In what follows we discuss the specific expression of ϕ

$$U_1 = [\tfrac{2\beta}{\alpha^2} - \tfrac{1}{\alpha p}K_1(t)] \pm \tfrac{\sqrt{2b}}{\alpha}p\ sn(px + K_1(t)y + K_2(t)),$$
$$W_1 = F \pm \tfrac{\sqrt{2b}}{\alpha}K_1(t)\ sn(px + K_1(t)y + K_2(t)), \qquad \mathbf{(30.1)}$$
$$U_2 = [\tfrac{2\beta}{\alpha^2} - \tfrac{1}{\alpha p}K_1(t)] \pm \tfrac{\sqrt{2b}}{\alpha}p\ cd(px + K_1(t)y + K_2(t)),$$
$$W_2 = F \pm \tfrac{\sqrt{2b}}{\alpha}K_1(t)\ cd(px + K_1(t)y + K_2(t)), \qquad \mathbf{(30.2)}$$
$$U_3 = [\tfrac{2\beta}{\alpha^2} - \tfrac{1}{\alpha p}K_1(t)] \pm \tfrac{\sqrt{2b}}{\alpha}p\ ns(px + K_1(t)y + K_2(t))\ ,$$
$$W_3 = F \pm \tfrac{\sqrt{2b}}{\alpha}K_1(t)\ ns(px + K_1(t)y + K_2(t)), \qquad \mathbf{(30.3)}$$
$$U_4 = [\tfrac{2\beta}{\alpha^2} - \tfrac{1}{\alpha p}K_1(t)] \pm \tfrac{\sqrt{2b}}{\alpha}p\ dc(px + K_1(t)y + K_2(t))\ ,$$
$$W_4 = F \pm \tfrac{\sqrt{2b}}{\alpha}K_1(t)\ dc(px + K_1(t)y + K_2(t)), \qquad \mathbf{(30.4)}$$
$$U_5 = [\tfrac{2\beta}{\alpha^2} - \tfrac{1}{\alpha p}K_1(t)] \pm \tfrac{\sqrt{2b}}{\alpha}p\, nc(px + K_1(t)y + K_2(t)),$$
$$W_5 = F \pm \tfrac{\sqrt{2b}}{\alpha}K_1(t)\ nc(px + K_1(t)y + K_2(t)), \qquad \mathbf{(30.5)}$$
$$U_6 = [\tfrac{2\beta}{\alpha^2} - \tfrac{1}{\alpha p}K_1(t)] \pm \tfrac{\sqrt{2b}}{\alpha}p\ cs(px + K_1(t)y + K_2(t)),$$
$$W_6 = F \pm \tfrac{\sqrt{2b}}{\alpha}K_1(t)\ cs(px + K_1(t)y + K_2(t)), \qquad \mathbf{(30.6)}$$
$$U_7 = [\tfrac{2\beta}{\alpha^2} - \tfrac{1}{\alpha p}K_1(t)] \pm \tfrac{\sqrt{2b}}{\alpha}p\ sc(px + K_1(t)y + K_2(t)),$$
$$W_7 = F \pm \tfrac{\sqrt{2b}}{\alpha}K_1(t)\ sc(px + K_1(t)y + K_2(t)), \qquad \mathbf{(30.7)}$$
$$U_8 = [\tfrac{2\beta}{\alpha^2} - \tfrac{1}{\alpha p}K_1(t)] \pm \tfrac{\sqrt{2b}}{\alpha}p\ ds(px + K_1(t)y + K_2(t)),$$
$$W_8 = F \pm \tfrac{\sqrt{2b}}{\alpha}K_1(t)\ ds(px + K_1(t)y + K_2(t)), \qquad \mathbf{(30.8)}$$
$$U_9 = [\tfrac{2\beta}{\alpha^2} - \tfrac{1}{\alpha p}K_1(t)] \pm \tfrac{\sqrt{2b}}{\alpha}p[ns(px + K_1(t)y + K_2(t)) \pm cs(px + K_1(t)y + K_2(t))],$$
$$W_9 = F \pm \tfrac{\sqrt{2b}}{\alpha}K_1(t)[ns(px + K_1(t)y + K_2(t)) \pm cs(px + K_1(t)y + K_2(t))], \qquad \mathbf{(30.9)}$$
$$U_{10} = [\tfrac{2\beta}{\alpha^2} - \tfrac{1}{\alpha p}K_1(t)] \pm \tfrac{\sqrt{2b}}{\alpha}p[nc(px + K_1(t)y + K_2(t)) \pm sc(px + K_1(t)y + K_2(t))],$$
$$W_{10} = F \pm \tfrac{\sqrt{2b}}{\alpha}K_1(t)[nc(px + K_1(t)y + K_2(t)) \pm sc(px + K_1(t)y + K_2(t))], \qquad \mathbf{(30.10)}$$
$$U_{11} = [\tfrac{2\beta}{\alpha^2} - \tfrac{1}{\alpha p}K_1(t)] \pm \tfrac{\sqrt{2b}}{\alpha}p[ns(px + K_1(t)y + K_2(t)) + ds(px + K_1(t)y + K_2(t))],$$
$$W_{11} = F \pm \tfrac{\sqrt{2b}}{\alpha}K_1(t)[ns(px + K_1(t)y + K_2(t)) + ds(px + K_1(t)y + K_2(t))], \qquad \mathbf{(30.11)}$$
$$U_{12} = [\tfrac{2\beta}{\alpha^2} - \tfrac{1}{\alpha p}K_1(t)] \pm \tfrac{\sqrt{2b}}{\alpha}p[sn(px + K_1(t)y + K_2(t)) \pm i\ cn(px + K_1(t)y + K_2(t))],$$
$$W_{12} = F \pm \tfrac{\sqrt{2b}}{\alpha}K_1(t)[sn(px + K_1(t)y + K_2(t)) \pm i\ cn(px + K_1(t)y + K_2(t))], \mathbf{(30.12)}$$
$$U_{13} = [\tfrac{2\beta}{\alpha^2} - \tfrac{1}{\alpha p}K_1(t)] \pm \tfrac{\sqrt{2b}}{\alpha}p\tfrac{C}{(px+K_1(t)y+K_2(t))}, \quad W_{13} = F \pm \tfrac{\sqrt{2b}}{\alpha}K_1(t)\tfrac{C}{(px+K_1(t)y+K_2(t))}. \mathbf{(30.13)}$$

Where F is determined by the third equation of (29), a and b are given by their corresponding values of the chosen solution in (3).

Remark 4

When $m \to 0$, The solutions (30.1)-(30.12) degenerate as triangular function-like solutions as follows

$U_{14} = [\frac{2\beta}{\alpha^2} - \frac{1}{\alpha p}K_1(t)] \pm \frac{\sqrt{2b}}{\alpha}p\, csc(px + K_1(t)y + K_2(t))$,
$W_{14} = F \pm \frac{\sqrt{2b}}{\alpha}K_1(t)\ csc(px + K_1(t)y + K_2(t)),$ **(31.1)**
$U_{15} = [\frac{2\beta}{\alpha^2} - \frac{1}{\alpha p}K_1(t)] \pm \frac{\sqrt{2b}}{\alpha}p\, sec(px + K_1(t)y + K_2(t)),$
$W_{15} = F \pm \frac{\sqrt{2b}}{\alpha}K_1(t)\ sec(px + K_1(t)y + K_2(t)),$ **(31.2)**
$U_{16} = [\frac{2\beta}{\alpha^2} - \frac{1}{\alpha p}K_1(t)] \pm \frac{\sqrt{2b}}{\alpha}p\ \cot(px + K_1(t)y + K_2(t)),$
$W_{16} = F \pm \frac{\sqrt{2b}}{\alpha}K_1(t)\ \cot(px + K_1(t)y + K_2(t)),$ **(31.3)**
$U_{17} = [\frac{2\beta}{\alpha^2} - \frac{1}{\alpha p}K_1(t)] \pm \frac{\sqrt{2b}}{\alpha}p\ \tan(px + K_1(t)y + K_2(t)),$
$W_{17} = F \pm \frac{\sqrt{2b}}{\alpha}K_1(t)\ \tan(px + K_1(t)y + K_2(t)),$ **(31.4)**
$U_{18} = [\frac{2\beta}{\alpha^2} - \frac{1}{\alpha p}K_1(t)] \pm \frac{\sqrt{2b}}{\alpha}p[csc(px + K_1(t)y + K_2(t)) \pm \cot(px + K_1(t)y + K_2(t))],$
$W_{18} = F \pm \frac{\sqrt{2b}}{\alpha}K_1(t)[csc(px + K_1(t)y + K_2(t)) \pm \cot(px + K_1(t)y + K_2(t))],$ **(31.5)**
$U_{19} = [\frac{2\beta}{\alpha^2} - \frac{1}{\alpha p}K_1(t)] \pm \frac{\sqrt{2b}}{\alpha}p[sec(px + K_1(t)y + K_2(t)) \pm \tan(px + K_1(t)y + K_2(t))],$
$W_{19} = F \pm \frac{\sqrt{2b}}{\alpha}K_1(t)[sec(px + K_1(t)y + K_2(t)) \pm \tan(px + K_1(t)y + K_2(t))],$ **(31.6)**

Remark 5

When m$\to$ 1 the solutions (30.1)-(30.12) degenerate as formal soliton-like, combined formal soliton-like and kink bell soliton-like solutions of Eq.(18)

$U_{20} = [\frac{2\beta}{\alpha^2} - \frac{1}{\alpha p}K_1(t)] \pm \frac{\sqrt{2b}}{\alpha}p\ \tanh(px + K_1(t)y + K_2(t)),$
$W_{20} = F \pm \frac{\sqrt{2b}}{\alpha}K_1(t)\ \tanh(px + K_1(t)y + K_2(t)),$ **(32.1)**
$U_{21} = [\frac{2\beta}{\alpha^2} - \frac{1}{\alpha p}K_1(t)] \pm \frac{\sqrt{2b}}{\alpha}p\ \coth(px + K_1(t)y + K_2(t))$,
$W_{21} = F \pm \frac{\sqrt{2b}}{\alpha}K_1(t)\ \coth(px + K_1(t)y + K_2(t)),$ **(32.2)**
$U_{22} = [\frac{2\beta}{\alpha^2} - \frac{1}{\alpha p}K_1(t)] \pm \frac{\sqrt{2b}}{\alpha}p\ csch(px + K_1(t)y + K_2(t)),$
$W_{22} = F \pm \frac{\sqrt{2b}}{\alpha}K_1(t)\ csch(px + K_1(t)y + K_2(t)),$ **(32.3)**
$U_{23} = [\frac{2\beta}{\alpha^2} - \frac{1}{\alpha p}K_1(t)] \pm \frac{\sqrt{2b}}{\alpha}p[\coth(px + K_1(t)y + K_2(t)) \pm csch(px + K_1(t)y + K_2(t))],$
$W_{23} = F \pm \frac{\sqrt{2b}}{\alpha}K_1(t)[\coth(px + K_1(t)y + K_2(t)) \pm csch(px + K_1(t)y + K_2(t))],$ **(32.4)**
$U_{24} = [\frac{2\beta}{\alpha^2} - \frac{1}{\alpha p}K_1(t)] \pm \frac{\sqrt{2b}}{\alpha}p[\tanh(px + K_1(t)y + K_2(t)) \pm i\ sech(px + K_1(t)y + K_2(t))],$
$W_{24} = F \pm \frac{\sqrt{2b}}{\alpha}K_1(t)[\tanh(px + K_1(t)y + K_2(t)) \pm i\ sech(px + K_1(t)y + K_2(t))],$ **(32.5)**

4. CONCLUSION

We have developed a modification of an extended mapping. The validity and reliability of the method is tested by applying it to the (2+1)-dimensional breaking soliton equation and the (2+1)-dimensional Konopelchenko-Dubrovsky equation. We have successfully recovered abundant explicit solutions including Jacobi elliptic function-like, combined Jacobi elliptic function-like, triangular periodic-like, combined triangular periodic-like, soliton-like, combined soliton-like and rational-like solutions. Our method is similar to that of Peng [12,13] in the sens that, it use the elliptic equation of first kind involving three parameters instead of specific functions as in previous method. Thus, this help to avoid many tedious and repetitive calculations. In the constrast to the method of Peng [12,13], we are able to go beyong the constant coefficients for the series by introducing coefficient functions. Thus, our method can used to obtain more types of solutions of the NLPDEs.

Acknowledgement.
This research was carried out during the author's stay at the Max–Planck Institut für Mathematik in Bonn. I thank this institution for the generous hospitality and support.

References

[1] B.R. Duffy, E.J. Parkes, Phys.Lett. A 214 (1996) 271.
[2] E.J. Parkes, B.R.Duffy, Phys. Lett. A 229 (1997) 217.
[3] Z.B. Li, Exact solitary wave solutions of nonlinear evolution equat ions. In:X.S. Gao, D.M. Wang, editors. Mathematics Mechanization and application. Academic Press; 2000.
[4] Z.B. Li, M.L. Wang, J. Phys. A 26 (1993) 6027.
[5] E.J. Parkes, J. Phys. A 27 (1994) L497.
[6] W. Malfliet, Am.J. Phys. 60 (1992) 650.
[7] E.J. Parkes, Z. Zhu, B.R. Duffy, H.C. Huang, Phys. Lett. A 248 (1998) 219.
[8] S.K. Liu, Z. Fu, S.D. Liu, Q. Zhao, Phys. Lett. A 289 (2001) 69.
[9] Z. Fu, S.K. Liu, S.D. Liu, Q. Zhao, Phys. Lett. A 290 (2001) 72.
[10] E.J. Parkes, B.R.Duffy, P.C. Abbott, Phys.Lett. A 295 (2002) 280.
[11] S.K. Liu, Z.T. Fu, S.D. Liu, Q. Zhao, Acta Phys. Sin. 51 (2002) 10.
[12] Y.Z. Peng, Chin.J. Phys.41 (2003) 103.
[13] Y.Z. Peng, Phys. Lett. A 314 (2003) 401.
[14] D.V. Patrick, Elliptic function and elliptic curves. Cambridge: cambidge University Press, 1973.
[15] K. Chamdrasekharan, Elliptic functions. Berlin: Springer-Verlag, 1985.
[16] F. Bowman, Introduction to Elliptic functions with applications, Universities, London, 1959.
[17] B.G. Konopelchenko, Solitons in multidimensions, World Scientific, Singapore, 1993.
[18] O.I. Bogoyavlensky, Izv Akad. Nauk SSSR, Ser. Mat.53 (1989) 907.
[19] R. Radha, M.Lakshmanan, Phys. Lett. A 197 (1995) 7.
[20] B. Konopelchenko, V. Dubrovsky, Phys. Lett. A 102 (1984) 15.

E-mail address: `eyomba@yahoo.com`