How to calculate the correlation function in twisted SYM $N = 2$, $N_f = 4$ QFT on projective plane

Alexei L. Gorodentsev and Maxim I. Leenson

Steklov Mathematical Institute
Section Algebra
Vavilova 42
Moscow 117333

Russia

Max-Planck-Institut
für Mathematik
Gottfried-Claren-Str. 26
53225 Bonn

Germany

MPI 96-49

INDEPENDENT UNIVERSITY OF MOSCOW
MAX-PLANCK INSTITUTE FÜR MATHEMATIK

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Alexei L. Gorodentsev[1]
Maxim I. Leenson[2]

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§1. Introduction.

1.1. What is this paper about. Let $\mathfrak{M} \stackrel{\text{def}}{=} \mathfrak{M}_{\mathbb{P}_2}(2; -1; k)$ be the moduli space of μ-stable torsion free coherent sheaves F with $\operatorname{rk}(F) = 2$, $c_1(F) = -1$, $c_2(F) = k$ on $\mathbb{P}_2 = \mathbb{P}_2(\mathbb{C})$. $\mathfrak{M}$ is a smooth $4(k-1)$-dimensional projective variety and there exists *the universal* rank 2 torsion free sheaf $\mathcal{F}$ on $\mathbb{P}_2 \times \mathfrak{M}$ defined uniquely up to the twisting by the pull-back of an invertible sheaf on $\mathfrak{M}$ (see, for example, [12, ch.II, §4]).

We fix the standard notations for the projections

$$\begin{array}{ccc} & \mathbb{P}_2 \times \mathfrak{M} & \\ {\scriptstyle p}\swarrow & & \searrow{\scriptstyle \pi} \\ \mathbb{P}_2 & & \mathfrak{M} \end{array} \qquad (1\text{-}1)$$

and put $\mathcal{G} \stackrel{\text{def}}{=} R^1\pi_*\mathcal{F}$. The sheaf $\mathcal{G}$ is locally free and $\operatorname{rk}\mathcal{G} = (k-1)$. The fibers of $\mathcal{G}$ at the closed points $F \in \mathfrak{M}$ are isomorphic to $H^1(\mathbb{P}_2; F)$. We call $\mathcal{G}$ *the universal bundle on* $\mathfrak{M}$.

Let $\mathcal{G}^{\oplus 4} \stackrel{\text{def}}{=} \mathcal{G}\oplus\mathcal{G}\oplus\mathcal{G}\oplus\mathcal{G}$. The sequence of the topological constants $a_k \stackrel{\text{def}}{=} c_{\text{top}}(\mathcal{G}^{\oplus 4}) = c_{4k-4}(\mathcal{G}^{\oplus 4})$, where $k \in \mathbb{N}$, appears as the correlation function in the $N = 2$, $N_f = 4$ supersymmetric Yang-Mills theories with $SO(3)$ gauge group.

Namely, consider $X = \mathbb{CP}_2$ as real smooth 4-manifold equipped with some Riemann metric g and the standart $\text{Spin}^{\mathbb{C}}$-structure $c = 3h$. Also let $F \longrightarrow X$ be a smooth complex 2-dimensional vector bundle with $c_1(F) = -1$, $c_2(F) = k$ equipped with a $SO(3)$-connection a and the corresponding twisted Dirac operator $D_a\colon\ \mathcal{C}^\infty(F \otimes W^+) \longrightarrow \mathcal{C}^\infty(F \otimes W^-)$, where $W^\pm$ are the spinor bundles presented by $\text{Spin}^{\mathbb{C}}$-structure c.

[1]e-mail: gorod@ium.ips.ras.ru
[2]e-mail: leenson@ium.ips.ras.ru

It is easy to check that the correlation functions in the QFT with four flawers, i. e. with four spinor fields

$$\phi_i\colon X \longrightarrow F \otimes W^+\,, \qquad i = 1,2,3,4\,,$$

can be calculated correctly on the classical level, because the β-function of this theory vanishes. Absolute minima of the Yang-Mills Lagrangian are achieved exactly on the (a, ψ_i) that satisfy the equations

$$\left\{\begin{array}{rcl} F_a^+ & = & \sum\limits_{i=1}^{4} (\phi_i \otimes \overline{\phi}_i)_{00} \\ D_a\phi_i & = & 0\,, \quad i = 1,2,3,4 \end{array}\right. , \tag{1-2}$$

where $F_a^+\colon H^0(F) \longrightarrow H^0(\Omega^{2+} \otimes F)$ is the Hodge-selfdual part of the curvature of a. The S^1-action on each of the coupled spinor fields can be killed by the same normalisation of the detrminantal connections as in [16, n° 2.2]. So, for general metric g on X the equations (1-2) have only a finite (up to $SO(3)$ gauge) number of solutions. This number a_k (where $k = c_2(E)$) does not depend on a choice of metric (if the choice is general enough).

To apply the geometry for the calculation of constants a_k, we have to use the Fubini-Study metric g. It is not general: all spinor fields ϕ_i, which satisfy (1-2) must be zero in this case, because of positive scalar curvature of g. Hence, the equations (1-2) take the form $F_a^+ = 0$, i. e. define the usual instantons (antiselfdual connections on F). By Donaldson's theorem, the instantons (considered up to $SO(3)$ gauge) are in 1-1 correspondense with holomorphic μ-stable structures on F, and hence, they are parametrized by the Zarisski open subset of $\mathfrak{M}(2,-1,k)$, which consists of all locally free stable sheaves. So, in order to calculate the actual values of a_k in terms of $\mathfrak{M}$, we have to consider an *obstruction bundle*. Since the obstruction spaces for the existence of non-zero spinor $\phi \in \ker D_a$ coinsides with $\operatorname{coker} D_a = H^1(F)$, the obstruction bundle is exactly the universal bundle $\mathcal{G}$ on $\mathfrak{M}(2,-1,k)$ and the number of solutions of (1-2) for general metric equals $c_{\text{top}}(\mathcal{G}^{\oplus 4})$ (i. e. the number of points where four general obstructions vanish simultaniously).

Constants a_k play an important role in physic and the sum $f(q) = \sum a_k q^k$ is waited by physicists to be equal to the q-decomposition of a modular form, because of the physical S-duality conjecture (see [17, 18]). To check such kind of statements mathematically, we have to find a clear and not too hard way for the calculation of the constants a_k. Such a way is presented in this paper.

1.2. Approach and results. In §2 we give a direct geometrical construction of the moduli space $\mathfrak{M}_{\mathbb{P}_2}(2;-1;k)$. In particular, this reduces the calculation of a_k to the problem that can be solved principally by the Schubert calculus over a ring, which is given by some explicit generators and relations. For this aim we use the approach of G.Ellingsrud and S.A.Strømme from [5]. We extend this approach to the more general *helices* on $\mathbb{P}_2$ and adapt it for the sheaf $\mathcal{G}$.

Namely, consider the *Kronecker moduli space* $\mathfrak{N} = \mathfrak{N}(3; k, k-1)$ defined as the space of the stable orbits (i.e. the geometric factor) of the natural representation of the reductive group

$$\Big(GL_k(\mathbb{C}) \times GL_{k-1}(\mathbb{C})\Big)/\mathbb{C}^{\cdot}\{\mathrm{Id} \times \mathrm{Id}\}$$

in the vector space $\mathbb{C}^{*k} \otimes \mathbb{C}^3 \otimes \mathbb{C}^{(k-1)}$. The variety $\mathfrak{N}$ was studied in [3] and its Chow ring has been calculated via generators and relations in [6]. There exists the *universal Kronecker module* $\mathcal{U}_1^* \otimes \mathbb{C}^3 \otimes \mathcal{U}_2$ over $\mathfrak{N}$, where $\mathcal{U}_1$ and $\mathcal{U}_2$ are the natural universal vector bundles on $\mathfrak{N}$ with $\operatorname{rk}\mathcal{U}_1 = k$ and $\operatorname{rk}\mathcal{U}_2 = (k-1)$ (their Chern classes are the ring generators for $A^*(\mathfrak{N})$, see [6]).

Denote by $\mathfrak{G} = Gr\Big(k-1\,,\ \mathbb{C}^3 \otimes \mathcal{U}_1\Big) \xrightarrow{\text{pr}} \mathfrak{N}$ the relative Grassmann parametrizing all $(k-1)$-subbundles in the bundle $\mathbb{C}^3 \otimes \mathcal{U}_1$ on $\mathfrak{N}$. Let $\mathcal{S}$ be the universal subbundle on $\mathfrak{G}$. The next theorem follows from what we prove in §2.

1.2.1. THEOREM. *The moduli space $\mathfrak{M} = \mathfrak{M}(2; -1; k)$ is isomorphic to the subvariety $\mathfrak{Z} \subset \mathfrak{G}$ defined as the zero scheme of a section of the bundle $\mathbb{C}^3 \otimes \mathcal{S}^* \otimes \mathrm{pr}^*\mathcal{U}_2$ on $\mathfrak{G}$. Under this isomorphism the universal bundle $\mathcal{G}$ on $\mathfrak{M}$ is identified with the restriction of the universal bundle $\mathcal{U}_2$ onto $\mathfrak{Z}$.*

In §3 we study an action of the maximal torus $T \subset PGL_3(\mathbb{C})$ on $\mathfrak{M}$. This action comes from a toric variety structure on $\mathbb{P}_2$ and the fixed points $E \in \mathfrak{M}^T$ of this action are represented by the *toric sheaves* on $\mathbb{P}_2$. Using general technique of A.A.Klyachko ([14, 15]), we enumerate all connected components $Y \subset \mathfrak{M}^T$ in some combinatorial terms.

Namely (see details in §3), each connected component $Y \subset \mathfrak{M}^T$ is given by

- an ordered collection of three non-negative integers d_0, d_1, d_2 such that the sum $d = d_0+d_1+d_2$ is bounded by $0 \le d < c_2(E)$,
- an ordered collection of three positive integers a_0, a_1, a_2, which satisfy three triangle inequalities $a_i < a_j + a_k$ and the equality $(-a_0 + a_1 + a_2)^2 - 4a_1a_2 = 1 - 4(c_2(E) - d)$,
- an ordered collection of three pictures like shown in (3-6) on the page 18 (i-th picture, $i = 0, 1, 2$, is obtained from a pair of Young diagrams (λ_i, μ_i) such that $|\lambda_i| + |\mu_i| = d_i$; the diagrams are shifted with respect to each other by a_j cells in the horizontal direction and by a_k cells in the vertical direction).

Two such data leads to the same Y iff they have the same triples of numbers d_i, a_i and the same (up to a parallel translation) triples of pictures. The figure contained between two polygonal boundaries of the Young diagrams λ_i, μ_i in (3-6) splits into connected components. Let N_i be the number of the components, which are strictly contained inside the intersection of two dotted right angles and let $N = N_0 + N_1 + N_2$. Then the number of Young diagram pair's triples $\{(\lambda_i, \mu_i)\}_{i=0,1,2}$ leading to the same triple of pictures is 2^N and the corresponding component $Y \subset \mathfrak{M}^T$, is isomorphic to

$$\underbrace{\mathbb{P}_1 \times \mathbb{P}_1 \times \cdots \times \mathbb{P}_1}_{N},$$

where the multipliers $\mathbb{P}_1$ are in the natural 1-1 correspondence with the connected pices between the Yong diagram boundaries described above.

Using geometrical construction of $\mathfrak{M}$ presented in §2 we fix some toric structure on the universal bundle $\mathcal{G}$ over $\mathfrak{M}$ and get the corresponding families of toric structures on E runing through any connected component $Y \subset \mathfrak{M}^T$. This let us describe the toric character decomposition of all restrictions $\mathcal{G}|_Y$ (see n° 3.5.2).

In §4 we explain how to apply the Bott residue formula for the calculation of the consitsnts a_k. Unfortunately, we do not get any general closed answer, which express all a_k directly in terms of k and/or some recursive rules, because the combinatorial data required to evaluate the denominator in the Bott formula is too complicated. But we present an explicit description of this data sufficient for the numerical calculation a_k by a computer. The first values of a_k (with respect to the normalizations of $\mathcal{G}$ and $\mathfrak{M}$ given in Th.1.2.1) are the following[3]:

k	1	2	3	4	5	6	7
a_k	0	0	0	13	729	85026	15650066

1.3. About the helices on $\mathbb{P}_2$. All what we need about the helices on $\mathbb{P}_2$ can be read in [9, 11]. We will use the terminology and the notations from these papers. Everybody who is interesting

[3] they are calculated using MAPLE V.3

only in the above numbers can suppose without any loss that the *helix foundations* $\{E_0, E_1, E_2\}$ and $\{{}^{\times}E_2, {}^{\times}E_1, {}^{\times}E_0\}$, which will be used in what follows, are equal to the triples

$$\{\mathcal{O}(-1),\ \Omega_{\mathbb{P}_2}(1),\ \mathcal{O}\} \qquad \text{and} \qquad \{\mathcal{O},\ \mathcal{O}(1),\ \mathcal{O}(2)\}$$

of sheaves on $\mathbb{P}_2$.

The more detailed review of the helix theory and the list of references can be founded in [10].

1.4. About toric varieties and toric bundles. All toric things what we need (with a lot of further references) can be founded in [14, 15]. Our terminology and notations will be very closed to the ones used in these papers. For convenience of readers we recall some basic facts adapted for our framework at the beginning of §3.

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§2. Geometrical description of moduli space.

2.1. The bases used for the representation of sheaves on $\mathbb{P}_2$. Let $\{E_0, E_1, E_2\}$ be a helix foundation on $\mathbb{P}_2$ such that $E_0 = \mathcal{O}(-1)$ and $\mu(E_2) \geq \mu(F) = -1/2$. It follows from [11] that the pair $\{E_1, E_2\}$ in such a foundation is the pair of consequent elements $\{X_\nu, X_{\nu+1}\}$ in the infinite sequence of sheaves $\{X_\nu\}_{1\leq\nu<\infty}$ defined recursively by the relations $X_1 = \Omega_{\mathbb{P}_2}(1)$, $X_2 = \mathcal{O}$, and

$$X_{\nu+2} = \operatorname{coker}\left(X_\nu \xrightarrow{\text{coev}} \operatorname{Hom}(X_\nu, X_{\nu+1})^* \otimes X_{\nu+1}\right).$$

There exists also the natural identification

$$\operatorname{Hom}(X_\nu, X_{\nu+1}) = \begin{cases} \mathbf{V}^* & \text{for even } \nu \\ \mathbf{V} & \text{for odd } \nu \end{cases},$$

where $\mathbf{V}^* = H^0(\mathcal{O}_{\mathbb{P}_2}(1))$. We denote the space $\operatorname{Hom}(E_i, E_j)$ by V_{ij}. As we have just explained, $\dim V_{12} = 3$.

Let $\{{}^{\times}E_2, {}^{\times}E_1, {}^{\times}E_0\}$ be the *left dual foundation*. It consists of the sheaves ${}^{\times}E_2 = E_2$, ${}^{\times}E_0 = E_0(3) = \mathcal{O}(2)$, and

$${}^{\times}E_1 = R_{E_2}(E_1) = \operatorname{coker}\left(E_1 \xrightarrow{\text{ev}} \operatorname{Hom}(E_1, E_2)^* \otimes E_2\right)$$

The left dual foundation is uniquely defined by the orthogonality conditions

$$\operatorname{Ext}^{(2-i)}({}^{\times}E_j, E_k) = \begin{cases} \mathbb{C} & \text{for } i = j = k \\ 0 & \text{in the other cases} \end{cases}.$$

If the pair $\{E_1, E_2\}$ coincides with the pair $\{X_\nu, X_{\nu+1}\}$ of consequent elements from the sequence $\{X_\nu\}$ described above, then the dual pair $\{{}^\times E_2, {}^\times E_1\}$ coincides with the pair $\{X_{\nu+1}, X_{\nu+2}\}$ from the same sequence. So, in terms of X_ν, every pair of dual foundations, with is appropriate for us, can be written as

$$\{E_0, X_\nu, X_{\nu+1}\} \qquad \text{and} \qquad \{X_{\nu+1}, X_{\nu+2}, {}^\times E_0\}$$

and there exists the natural coincidence $\mathrm{Hom}(X_\nu, X_{\nu+1}) = \mathrm{Hom}(X_{\nu+1}, X_{\nu+2})^*$.

2.2. Monad on $\mathbb{P}_2$. For $i = 0, 1, 2$ the inequalities $\mu(F) \le \mu({}^\times E_i) \le \mu(F(3))$ between the Mumford slopes imply (via the stability and the Serre duality) that

$$\mathrm{Ext}^0({}^\times E_i, F) = \mathrm{Ext}^2({}^\times E_i, F) = 0 \ .$$

So, only $\mathrm{Ext}^1({}^\times E_i, F)$ can be non-zero. We denote this vector space by U_i and put $u_i = \dim U_i$. The Beilinson spectral sequence (see [9]) shows that F is the cohomology sheaf of the monad

$$0 \to U_0 \otimes E_0 \xrightarrow{\iota_1} U_1 \otimes E_1 \xrightarrow{\iota_2} U_2 \otimes E_2 \to 0 \ . \tag{2-1}$$

Note that $U_0 = H^1(F(-2))$ and $u_0 = k - 1$. In general, it follows from Riemann-Roch that

$$u_i = -\chi({}^\times E_i^*, F) = \mathrm{rk}\,({}^\times E_i) \cdot k + 2c_1({}^\times E_i) - m({}^\times E_i) \ ,$$

where $m({}^\times E_i) \stackrel{\mathrm{def}}{=} (1 + c_1({}^\times E_i)^2)/\mathrm{rk}\,({}^\times E_i)$ is an integer (see [11]), which can be called the *Markov characteristic* of the exceptional vector bundle ${}^\times E_i$ on $\mathbb{P}_2$. In particular, for the simplest pair of the dual foundations $\{\mathcal{O}(-1), \Omega_{\mathbb{P}_2}(1), \mathcal{O}\}$ and $\{\mathcal{O}, \mathcal{O}(1), \mathcal{O}(2)\}$ we have the dimensions $u_1 = k$ and $u_2 = k - 1$.

2.2.1. Some arithmetical conditions on u_i. The classes $\{e_0, e_1, e_2\}$ of the sheaves $\{E_0, E_1, E_2\}$ form an integer semiorthonormal basis of the Mukai lattice $K_0(\mathbb{P}_2)$. In terms of this basis, the class f of F is decomposed as

$$f = -u_0 e_0 + u_1 e_1 - u_2 e_2 \ ,$$

where $u_i = \chi({}^\times e_i, f)$ and the left dual basis $\{{}^\times e_i\}_{i=2,1,0}$ satisfy the relations $\chi({}^\times e_i, e_j) = (-1)^i \delta_{ij}$. If we use this decomposition in order to calculate $\chi(e_0, f)$, then we get

$$\chi(e_0, f) = -u_0 + \chi(e_0, e_1) \cdot u_1 + \chi(e_0, e_2) \cdot u_2 \ .$$

and hence,

$$\chi(e_0, e_1) \cdot u_1 + \chi(e_0, e_2) \cdot u_2 = \chi(e_0, f) - \chi({}^\times e_0, f) = \chi(e_0, f) - \chi(f, e_0) = 3\delta(e_0, f) \ ,$$

where the determinant $\delta(X, Y) \stackrel{\mathrm{def}}{=} \mathrm{rk}\,(X)c_1(Y) - \mathrm{rk}\,(Y)c_1(X)$ coincides by Riemann-Roch with the skew-symmetric part of $\frac{1}{3}\chi(X, Y)$.

On the other side, standard arithmetical properties of exceptional pairs on $\mathbb{P}_2$ (see [11]) imply the identities

$$\chi(e_0, e_1) = 3\delta(e_0, e_1) = 3\mathrm{rk}\,(E_2) \ .$$
$$\chi(e_0, e_2) = 3\delta(e_0, e_2) = 3\mathrm{rk}\,({}^\times E_1) \ ,$$

Hence, we have the identity

$$\mathrm{rk}\,(E_2) \cdot u_1 - \mathrm{rk}\,({}^\times E_1) \cdot u_2 = -\mathrm{rk}\,(E_0) - 2c_1(E_0) \ . \tag{2-2}$$

Since the right side equals 1 for $E_0 = \mathcal{O}(1)$, in this case $u_0 = \dim U_0$ and $u_1 = \dim U_1$ are coprime. The other thing (which will be used below to study the stability) is that

$$\delta(E_0, F) = \delta(E_0, K) = 1 \ ,$$

where $K = \ker(\iota_2)$ is the kernel of the second map from the monad 2-1.

2.3. Monad on $\mathbb{P}_2 \times \mathfrak{M}$. The machinery for the Beilinson decomposition of coherent sheaves on $\mathbb{P}_n$ in terms of dual helix foundations has the straightforward extension (see [13]) to the relative case, where we consider the projectivization of a vector bundle over a base variety (instead of $\mathbb{P}_n$ over a field). In particular, any coherent sheaf $\mathcal{X}$ on $\mathbb{P}_2 \times \mathfrak{M}$ has a natural representation as the limit of the following Beilinson spectral sequence

$$E_1^{\alpha,\beta} = \pi^* \mathcal{E}xt_\pi^\beta(p^*({}^\times E_{2+\alpha}), \mathcal{X}) \otimes p^* E_{2+\alpha} \ ; \quad \alpha = -2, -1, 0 \ , \quad \beta = 0, 1, 2 \quad ,$$

where $\mathcal{E}xt_\pi^\beta(p^*({}^\times E_{2+\alpha}), \mathcal{X}) \stackrel{\text{def}}{=} R^\beta \pi_* \mathcal{H}om(p^*({}^\times E_{2+\alpha}), \mathcal{X})$ and p, π are the projections from (1-1).

So, the universal bundle $\mathcal{F}$ on $\mathbb{P}_2 \times \mathfrak{M}$ is the cohomology sheaf of the monad

$$0 \longrightarrow \pi^* \mathcal{F}_0 \otimes p^* E_0 \longrightarrow \pi^* \mathcal{F}_1 \otimes p^* E_1 \longrightarrow \pi^* \mathcal{F}_2 \otimes p^* E_2 \longrightarrow 0 \ , \tag{2-3}$$

where we denote by $\mathcal{F}_i$ the sheaves $\mathcal{E}xt_\pi^1(p^*({}^\times E_i), \mathcal{F})$ on $\mathfrak{M}$. In the fibers at the points of $\mathfrak{M}$ the monad (2-3) induce various monads (2-1) on $\mathbb{P}_2$.

2.4. Universal complex on $\mathfrak{M}$. The complex of the locally free sheaves (2-3) is *adapted* (in the sense of [8, Ch III, n° 6.2]) to calculate the derived functor $R\pi_* R\mathcal{H}om(p^* E_0, \mathcal{F})$, because

$$R^q \mathcal{H}om(p^* E_0, \ \pi^* \mathcal{F}_i \otimes p^* E_i) = 0 \text{ for } q > 0 \ ;$$

$$R^q \pi_* \mathcal{H}om(p^* E_0, \ \pi^* \mathcal{F}_i \otimes p^* E_i) = \mathcal{F}_i^* \otimes R^q \pi_* \mathcal{H}om(p^* E_0, \ p^* E_i) = \begin{cases} V_{0i} \otimes \mathcal{F}_i^* & \text{for } q = 0 \\ 0 & \text{for } q \neq 0 \end{cases} \ .$$

Hence, applying the functor $R\pi_* R\mathcal{H}om(\, p^* E_0, \ ?\,)$ to the complex (2-3), we get the following complex of locally free sheaves on $\mathfrak{M}$:

$$0 \longrightarrow \mathcal{F}_0 \longrightarrow V_{01} \otimes \mathcal{F}_1 \longrightarrow V_{02} \otimes \mathcal{F}_2 \longrightarrow 0 \ . \tag{2-4}$$

This triple is exact on the left term and has on the middle and on the right the cohomology sheaves $\mathcal{E}xt_\pi^0(p^* E_0, \mathcal{F})$ and $\mathcal{E}xt_\pi^1(p^* E_0, \mathcal{F})$. Both cohomologies can be not locally free[4].

Note that for the simplest triple $\{E_0, E_1, E_2\} = \{\mathcal{O}(-1), \Omega(1), \mathcal{O}\}$ the sheaf $\mathcal{F}_2$ in the right term of (2-4) is exactly the universal sheaf $\mathcal{G}$ on $\mathfrak{M}$.

We are going to construct the closed embedding of $\mathfrak{M}$ into the relative Grassmann $\mathfrak{G}$ over the Kronecker moduli space. Under this embedding the universal complex (2-4) will coincide with the restriction of some natural universal sequence of bundles on the Grassmann $\mathfrak{G}$.

2.5. The map from $\mathfrak{M}$ to $\mathfrak{N}(V_{01}; U_0, U_1)$. The surjection ι_2 from the monad (2-1) gives a *Kronecker module*, i. e. a tensor κ_F in the space

$$U_1^* \otimes V_{12} \otimes U_2 \ . \tag{2-5}$$

[4]Their fibers at the point $F \in \mathfrak{M}$ are $H^0(F(1))$ and $H^1(F(1))$ and these spaces can jump.

In accordance with J.-M.Drezet's results (see [3, Prop.27]) the tensor κ_F is stable with respect to the natural action of the group

$$G = \mathrm{GL}(U_1) \times \mathrm{GL}(U_2)/\mathbb{C}\cdot\{\mathrm{Id} \times \mathrm{Id}\}\ , \tag{2-6}$$

if and only if the corresponding map ι_2 is surjective and its kernel K from the exact triple

$$0 \longrightarrow K \longrightarrow U_1 \otimes E_1 \xrightarrow{\iota_2} U_2 \otimes E_2 \longrightarrow 0$$

is stable sheaf on $\mathbb{P}_2$. In order to check that these conditions hold in our case, we use the other exact triple induced by the monad (2-1):

$$0 \longrightarrow U_0 \otimes E_0 \longrightarrow K \longrightarrow F \longrightarrow 0\ .$$

and note that K belongs to the category spaned by E_1 and E_2, i. e. $\mathrm{Hom}(K, E_0) = 0$. So, by n° 2.2.1 we are in a position to apply the following general lemma.

2.5.1. LEMMA. *Let E be a stable locally free sheaf and F – any sheaf such that*

$$\delta(E, F) = \mathrm{rk}\,(E)c_1(F) - \mathrm{rk}\,(F)c_1(E) = 1\ .$$

If an extension K of the form

$$0 \longrightarrow V \otimes E \longrightarrow K \longrightarrow F \longrightarrow 0$$

satisfy the condition $\mathrm{Hom}(K, E) = 0$, *then F is Mumford-stable if and only if K is.*

PROOF. In terms of an integer 2-dimensional lattice generated by additive functions $(\mathrm{rk}\,, c_1)$, the determinantal conditions $\delta(E, K) = \delta(E, F) = 1$ mean that there are no integer points except for vertices in two parallelograms spaned by the classes of E and K and by the classes of E and F. In particular, any class X, which satisfy the conditions

$$\text{either} \quad \begin{cases} \mu(X) & < & \mu(K) \\ \mathrm{rk}\,(X) & < & \mathrm{rk}\,(K) \end{cases} \qquad \text{or} \quad \begin{cases} \mu(X) & < & \mu(F) \\ \mathrm{rk}\,(X) & < & \mathrm{rk}\,(F) \end{cases}$$

have to satisfy either the inequality $\mu(X) < \mu(E)$ or the system of equalities

$$\begin{cases} \mathrm{rk}\,(X) & = & m \cdot \mathrm{rk}\,(E) \\ c_1(X) & = & m \cdot c_1(E) \end{cases}$$

for some integer m.

Now we consider consequently both implications of the lemma.

Let K be stable. If F is not and admits a torsion free factor $F \longrightarrow Q \longrightarrow 0$. with $\mu(Q) \leq \mu(F)$ and $\mathrm{rk}\,(Q) < \mathrm{rk}\,(F)$, then automatically $\mu(Q) \leq \mu(E) < \mu(K)$. This is impossible, because Q is the factor-sheaf for K too.

Conversely, let F be stable and K be not and let $K \longrightarrow G \longrightarrow 0$ be the minimal stable torsion free Harder-Narasimhan's factor with $\mu(G) \leq \mu(K)$ and $\mathrm{rk}\,(G) < \mathrm{rk}\,(K)$. If $\mathrm{Hom}(E, G) = 0$ then $\mathrm{Hom}(F, G) \neq 0$ and we get the contradiction as above. If thre exists a non-zero map $E \xrightarrow{\psi} G$, then $\mu(G) = \mu(E)$, because $V \otimes E$ is semistable. Since E is stable, locally free, and has the same slope as G, the non-zero map $E \xrightarrow{\psi} G$ is an isomorphism. So, there exists a non-zero map from K to E. This contradicts to the last assumption of the lemma.

□

2.5.2. COROLLARY. *Taking $F \longmapsto \kappa_F$, we define a family of stable G-orbits of the Kronecker modules. Since the family is parametrized by the points of $\mathfrak{M}$, this gives an algebraic morphism $\kappa\colon \mathfrak{M} \longrightarrow \mathfrak{N}$, where $\mathfrak{N} \stackrel{\text{def}}{=} \mathfrak{N}(V_{12}; U_1, U_2)$ is the space of stable orbits (i.e. the geometric factor) of the natural representation of the group G from (2-6) in the vector space (2-5).*

□

Since the dimensions $u_1 = \dim U_1$ and $u_2 = \dim U_2$ are coprime, all semistable tensors in (2-5) are stable (see [3]), the action of G on the set of stable tensors is free (see [6]), and the spaces

$$U_1 \otimes \Big(\det(U_1)^{-\nu_1} \otimes \det(U_2)^{-\nu_2}\Big)^{\dim U_1} \qquad \text{and} \qquad U_2 \otimes \Big(\det(U_1)^{-\nu_1} \otimes \det(U_2)^{-\nu_2}\Big)^{\dim U_2} \tag{2-7}$$

admit the natural structures of G-modules (if ν_1 and ν_2 are such that $\nu_1 u_1 + \nu_2 u_2 = 1$). Hence, $\mathfrak{N}$ is a smooth projective variety and G-modules (2-7) induce some vector bundles on $\mathfrak{N}$. We denote these bundles by $\mathcal{U}_1$ and $\mathcal{U}_2$. Note that by the construction we have the natural line bundle's isomorphism $\det \mathcal{U}_1 = \det \mathcal{U}_2$.

The universal Kronecker module $\mathcal{U}_1^* \otimes V_{12} \otimes \mathcal{U}_2$ has the tautological global section, which can be considered as the homomorphism $\mathcal{U}_1 \longrightarrow V_{12} \otimes \mathcal{U}_2$. This homomorphism induces the map

$$V_{01} \otimes \mathcal{U}_1 \longrightarrow V_{01} \otimes V_{12} \otimes \mathcal{U}_2 \,,$$

which can be composed with the multiplication map

$$V_{01} \otimes V_{12} \otimes \mathcal{U}_2 \xrightarrow{\mu \otimes \mathrm{Id}} V_{02} \otimes \mathcal{U}_2 \,.$$

Denote the resulting map by

$$V_{01} \otimes \mathcal{U}_0 \xrightarrow{\mathrm{i}} V_{02} \otimes \mathcal{U}_2 \tag{2-8}$$

The next corollary follows immediately from the construction.

2.5.3. COROLLARY. *The homomorphism $V_{01} \otimes \mathcal{F}_1 \longrightarrow V_{02} \otimes \mathcal{F}_2$ from (2-4) coincides with the inverse image of (2-8) via the map $\kappa\colon \mathfrak{M} \longrightarrow \mathfrak{N}$ from Crl.2.5.2*

□

The corollary explains why the map $\kappa\colon \mathfrak{M} \longrightarrow \mathfrak{N}$ is not an isomorphism: the kernel of the homomorphism $V_{01} \otimes \mathcal{F}_1 \longrightarrow V_{02} \otimes \mathcal{F}_2$ from (2-4) must contain the subbundle $\mathcal{F}_0 \subset V_{12} \otimes \mathcal{F}_1$. To take this into account we have to lift (2-8) on the Grassmann parametrizing all the subbundles in question.

2.6. The embedding $\mathfrak{M} \hookrightarrow \mathfrak{G}$. Let $\mathfrak{G} \stackrel{\text{def}}{=} Gr\big(u_0\,,\ V_{01} \otimes \mathcal{U}_1\big) \xrightarrow{\mathrm{pr}} \mathfrak{N}$ be the relative Grassmann parametrizing the rank $u_0 = \dim U_0 = \mathrm{rk}\, \mathcal{F}_0$ subbundles in the vector bundle $V_{01} \otimes \mathcal{U}_1$ on $\mathfrak{N}$. Denote by

$$0 \longrightarrow \mathcal{S} \xrightarrow{\mathrm{j}} V_{12}^* \otimes \mathrm{pr}^* \mathcal{U}_1$$

the canonical inclusion of the universal subbundle $\mathcal{S}$ on $\mathfrak{G}$. Let $\mathfrak{Z} \subset \mathfrak{G}$ be the zero scheme of the composition

$$\mathcal{S} \xrightarrow{\mathrm{j}} V_{01} \otimes \mathrm{pr}^* \mathcal{U}_1 \xrightarrow{\mathrm{pr}^*(\mathrm{i})} V_{02} \otimes \mathrm{pr}^* \mathcal{U}_2 \tag{2-9}$$

2.6.1. LEMMA. *The map $\kappa\colon \mathfrak{M} \longrightarrow \mathfrak{N}$ from Crl.2.5.2 lifts to a closed embedding $\mathbf{k}\colon \mathfrak{M} \longrightarrow \mathfrak{G}$, which maps $\mathfrak{M}$ isomorphicaly onto $\mathfrak{Z}$. Under this map the complex (2-4) on $\mathfrak{M}$ is identified with the restriction of the sequence (2-9) onto $\mathfrak{Z}$.*

In particular, for the simplest triple $\{E_0, E_1, E_2\} = \{\mathcal{O}(-1), \Omega(1), \mathcal{O}\}$ *the restriction of the bundle* $\mathrm{pr}^*\mathcal{U}_2$ *onto* $\mathfrak{Z}$ *coincides with the universal bundle* $\mathcal{F}_2 = \mathcal{G}$.

PROOF. It is sufficient to prove that the fiber of the triple (2-9) over any closed point of $\mathfrak{Z}$ induces on $\mathbb{P}_2$ a monad of the form (2-1) with a μ-stable cohomology sheaf.

If we denote the fibers of the bundles $\mathcal{S}$, $\mathrm{pr}^*\mathcal{U}_0$, and $\mathrm{pr}^*\mathcal{U}_1$ by U_0, U_1, and U_2, then for any closed point of $\mathfrak{Z}$ the maps j and pr*(i) give two Kronecker modules

$$\kappa_1 \in U_0^* \otimes V_{01} \otimes U_1 \qquad \text{and} \qquad \kappa_2 \in U_1^* \otimes V_{12} \otimes U_2 \ ,$$

which induce the sequence of sheaf homomrphisms on $\mathbb{P}_2$:

$$0 \to U_0 \otimes E_0 \xrightarrow{\iota_1} U_1 \otimes E_1 \xrightarrow{\iota_2} U_2 \otimes E_2 \to 0 \ . \tag{2-10}$$

Certainly, we have $\iota_2 \cdot \iota_1 = 0$, because this composition considered as a tensor from $V_{02} \otimes \mathrm{Hom}(U_0, U_2)$ coincides with the composition

$$U_0 \xrightarrow{\kappa_1} U_1 \otimes V_{01} \xrightarrow{\kappa_2 \otimes \mathrm{Id}} U_2 \otimes V_{12} \otimes V_{01} \xrightarrow{\mathrm{Id} \otimes \mu} U_2 \ ,$$

which vanishes over $\mathfrak{Z}$ by the construction. Further, the second Kronecker module κ_2 is stable. Hence, by J.-M.Drezet [3, Prop.27] it induces the exact triple

$$0 \to K \to U_1 \otimes E_1 \xrightarrow{\iota_2} U_2 \otimes E_2 \to 0 \tag{2-11}$$

on $\mathbb{P}_2$ and the kernel K of this triple is stable. So, as soon as we prove that ι_1 is an monomorphism we get that the complex (2-10) is a monad and its homology sheaf is stable by Lem.2.5.1.

Since K and E_0 are stable and $\delta(E_0, K) = 1$, the image $I = \mathrm{im}\left(U_0 \otimes E_0 \xrightarrow{\iota_1} K \right)$ is semi-stable and has the same slope as E_0. Hence, all Jordan-Hölder factors of I are equal to E_0 and I is a direct sum of some copies of E_0, because E_0 is exceptional. In other words, $\ker(\iota_1) = W \otimes E_0$ for some subspace $W \subset U_0$. Since the first Kronecker module κ_1 is injective, we can draw the commutative diagram shown on (2-12). The right column of this diagram is the canonical exact triple [5]:

$$0 \to E_1^{\times} \to \mathrm{Hom}(E_0, E_1) \otimes E_0 \xrightarrow{\mathrm{ev}} E_1 \to 0 \ ,$$

tensored by the vector space U_1. The bottom row is induced by the rest part of the diagram and shows that $W = 0$, because $\mathrm{Hom}(E_0, E_1^{\times}) = 0$. So, $\ker(\iota_1) = 0$ and the proof is finished.
□

2.7. How to calculate a_k via Schubert. Extracting $\dim \mathfrak{M}$ from the scalar square of any class $f \in K_0(\mathbb{P}_2)$ coming from $F \in \mathfrak{M}$ (see n° 2.2.1), we get

$$\dim \mathfrak{M} = 1 - u_0^2 - u_1^2 - u_2^2 + h_{01}u_0u_1 + h_{12}u_1u_2 - h_{02}u_0u_2 \ ,$$

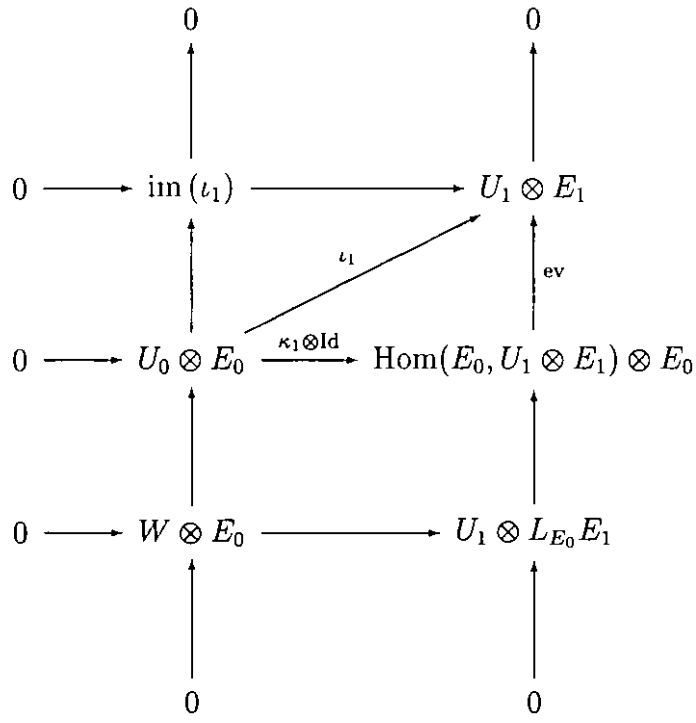
where $h_{ij} = \dim V_{ij}$. Since $\dim \mathfrak{N} = h_{01}u_0u_1 - u_0^2 - u_1^2 + 1 = \dim \mathfrak{M} + u_2(h_{02}u_0 - h_{12}u_1 + u_2)$, we have

$$\dim \mathfrak{G} = u_2(h_{12}u_1 - u_2) + \dim \mathfrak{N} = \dim \mathfrak{M} + h_{02}u_0u_2 \ ,$$

i. e. the codimension of $\mathfrak{M} = \mathfrak{Z}$ in $\mathfrak{G}$

$$d \stackrel{\mathrm{def}}{=} h_{02}u_0u_2 = 3\mathrm{rk}\,({}^{\times}E_1) = 3(k-1)(u_1\mathrm{rk}\,(E_0) - 1) \ .$$

[5] In the terminology of the helix theory, this triple define the left mutation of E_1 by E_0.



(2-12)

is equal to the rank of the bundle $\mathcal{H}om(\mathcal{S},\ V_{02}\otimes \mathrm{pr}^*\mathcal{U}_2)$. Hence, $\mathfrak{Z}$ localizes the top Chern class of this bundle (see [7, Prop.14.1]). Since the rational and numerical equivalences of cycles on $\mathfrak{M}$ coincide to each other (see [4]), we have $A^*(\mathfrak{M}) = \mathfrak{c}_d \cdot A^*(\mathfrak{G})$, where $\mathfrak{c}_d \in A^*(\mathfrak{G})$ is the top Chern class of the bundle $\mathcal{S}^* \otimes V_{02} \otimes \mathrm{pr}^*\mathcal{U}_2$. Hence, the constants a_k can be calculated inside the ring $A^*(\mathfrak{G})$. For the simplest helix foundation we get in this way the formula:

$$a_k = c_{(k-1)^2}(\mathcal{S})^3 \cdot c_{k-1}(\mathcal{U}_2)^4 \ .$$

The ring $A^*(\mathfrak{G})$ is the *Schubert calculus* (see [7, §14.7]) over the ring $A^*(\mathfrak{M})$. An explicit description of the last one via generators and relations can be found in [6, Th.(6.9)].

2.8. Normalization of the universal bundle $\mathcal{G}$. Note that the identification of the bundles $\mathcal{F}_1$, $\mathcal{F}_2$ on $\mathfrak{M}$ with the bundles $\mathcal{U}_1$, $\mathcal{U}_2$ via the embeddind $\mathfrak{M} \hookrightarrow \mathfrak{G}$ gives us a normalization of the universal bundle, i. e. it fixes the $\det\mathcal{G} \in Pic(\mathfrak{M})$ uniquely. In fact, if we multiply the universal sheaf $\mathcal{F}$ on $\mathbb{P}_2 \times \mathfrak{M}$ by $\pi^*\mathcal{L}$, where $\mathcal{L} \in Pic(\mathfrak{M})$, then $\det\mathcal{F}_1$ and $\det\mathcal{F}_2$ are shifted in $Pic(\mathfrak{M})$ by $\mathcal{L}^{\otimes u_1}$ and by $\mathcal{L}^{\otimes u_2}$ respectively. Since u_1 and u_2 are coprime, there is at most one shift, which leads to the identity $\det\mathcal{F}_1 = \det\mathcal{F}_2$. But we have $\det\mathcal{F}_1 = \det\mathcal{U}_1 = \det\mathcal{U}_2 = \det\mathcal{F}_2$ by the construction.

If we take different basic helix foundations $\{E_0, E_1, E_2\} = \{\mathcal{O}(-1), X_\nu, X_{\nu+1}\}$, then we get a sequence of different normalizations of the universal bundle on $\mathfrak{M}$. Numerical experiments show that these normalizations leads to the different values for the constants a_k. In the calculations, which will be present in the following two paragraphs, we will always suppose that the universal bundle $\mathcal{G}$ is normalized via the simplest foundation $\{E_0, E_1, E_2\} = \{\mathcal{O}(-1), \Omega(1), \mathcal{O}\}$.

§3. Torus action on $\mathfrak{M}$(-1,2,k).

3.1. Toric structure on $\mathbb{P}_2$. Let us fix some homogeneous coordinates $(x_0 : x_1 : x_2)$ on $\mathbb{P}_2$ and consider $\mathbb{P}_2$ as a toric variety with respect to the action of the diagonal subtorus $T \subset \mathrm{PGL}_3(\mathbb{C})$ on this coordinates. We will represent elements of the torus by matrices of the form

$$\begin{pmatrix} 1 & & \\ & t_1^{m_1} & \\ & & t_2^{m_2} \end{pmatrix}$$

and often we will use (m_1, m_2) as the Cartesian coordinates on the torus character's lattice $\Lambda(T)$.

It is convenient to introduce the following three one-parameter subgroups $\tau_i \in \Lambda(T)^*$:

$$\tau_0(t) = \begin{pmatrix} 1 & & \\ & t^{-1} & \\ & & t^{-1} \end{pmatrix}, \quad \tau_1(t) = \begin{pmatrix} 1 & & \\ & t & \\ & & 1 \end{pmatrix}, \quad \tau_2(t) = \begin{pmatrix} 1 & & \\ & 1 & \\ & & t \end{pmatrix},$$

and use sometimes the trigonal coordinates $(m_0.m_1, m_2)$ on $\Lambda(T)$, where

$$m_i(\chi) \stackrel{\text{def}}{=} \langle \tau_i, \chi \rangle .$$

Of course, here (m_1, m_2) are the same as above and $m_0 + m_1 + m_2 = 0$.

Standard affine card $U_i = \{x_i \neq 0\} \subset \mathbb{P}_2$ coincides with $\mathrm{Spec}\mathbb{C}[\sigma_i]$, where $\sigma_i \in \Lambda(T)$ are some angles such that in Cartesian coordinates (m_1, m_2)

$$\begin{array}{ll} \sigma_0 & \text{generated by the characters } (1,0) \text{ and } (0,1) \\ \sigma_1 & \text{generated by the characters } (-1,0) \text{ and } (-1,1) \\ \sigma_2 & \text{generated by the characters } (-1,1) \text{ and } (0,-1) , \end{array}$$

Characters, which are regular on $U_{ij} \stackrel{\text{def}}{=} U_i \cap U_j$, form the following halfplanes $\sigma_{ij} \in \Lambda(T)$:

$$\begin{aligned} \sigma_{12} &= \{(d_1, d_2) \in \Lambda(T) : m_0 = -m_1 - m_2 \geq 0\} \\ \sigma_{02} &= \{(d_1, d_2) \in \Lambda(T) : m_1 \geq 0\} \\ \sigma_{01} &= \{(d_1, d_2) \in \Lambda(T) : m_2 \geq 0\} \quad . \end{aligned}$$

We will always identify the torus T with the open dense orbit $U_{012} \stackrel{\text{def}}{=} U_0 \cap U_1 \cap U_2 = \mathrm{Spec}(\mathbb{C}[\Lambda(T)])$ by puting $t \in T$ into $tp \in U_{012}$, where $p = (1:1:1) \in U_{012}$.

3.2. Torus action on sheaves. The torus T acts on the set of classes of isomorphic torsion free sheaves on $\mathbb{P}_2$ via

$$t : F \longrightarrow t_*(F) . \tag{3-1}$$

So, we have a torus action on $\mathfrak{M} = \mathfrak{M}(2, -1, k)$ and also on the set of sheaves on $\mathfrak{M}$. We are going to apply to this action the Bott residue formula (see [2]). In order to do this, we have to describe the connected components of the fixed point locus $\mathfrak{M}^T \subset \mathfrak{M}$ and to fix a *toric structure* on the universal bundle $\mathcal{G}$.

3.2.1. Toric structures. A sheaf E represent a point of $\mathfrak{M}^T$ iff there exists a a collection of isomorphisms

$$\varphi_t : t_*(E) \longrightarrow E \qquad \text{for every } t \in T .$$

Such a collection φ is called a *toric structure* on a sheaf E if it satisfy the following additional condition: $\forall\, s, t \in T$ the natural diagram

$$\begin{array}{ccc} (t{\cdot}s)_*(E) & \xrightarrow{\varphi_{t\cdot s}} & E \\ {\scriptstyle \mathrm{Id}}\downarrow & & \uparrow{\scriptstyle \varphi_t} \\ t_*(s_*(E)) & \xrightarrow[t_*(\varphi_s)]{} & t_*(E) \end{array}$$

is commutative. A sheaf equipped with a toric structure is called a *toric sheaf*.

The obstruction to the commutativity of the above diagram is represented by a cocycle on T with values in $\mathrm{Hom}(E,E)$ (see [1]). In our case this cocycle vanishes, because E is stable (and $\mathrm{Hom}(E,E)=\mathbb{C}$). So, every sheaf E, which represents a point of $\mathfrak{M}^T$, admits a toric structure.

Certainly, there are many different toric structures on the same sheaf $E \in \mathfrak{M}$. For example, toric structures on the structure sheaf $\mathcal{O}$ are parametrized by the torus characters: the structure $\tilde{\chi}$, which corresponds to the character χ, takes $f \in \mathcal{O}(t^{-1}U)$ to $\tilde{\chi}_t f \in \mathcal{O}(U)$ defined by

$$\tilde{\chi}_t f(x) \stackrel{\mathrm{def}}{=} \chi(t)\cdot f(t^{-1}x)\,.$$

Tensor multiplication of a toric sheaf E by the structure sheaf $\mathcal{O}$ equipped with a toric structure $\tilde{\chi}$ is called a *shift of a toric structure on E by the character χ*. Any two different toric structures on an arbitrary torsion free toric sheaf E can be obtained from each other by the shift by a character.

3.2.2. Uniformal fixation of the toric structures. First of all, on the sheaves $\{\mathcal{O},\mathcal{O}(1),\mathcal{O}(2)\}$ we fix the tautological toric structures coming from the toric variety structure on $\mathbb{P}_2=\mathbb{P}\mathbf{V}$. In these structures T-modules $H^0(\mathcal{O})$, $H^0(\mathcal{O}(1))$, and $H^0(\mathcal{O}(2))$ coincide with the trivial 1-dimensional T-module $\mathbb{C}$ and with the standard representations of T in $\mathbf{V}^*$ and $S^2\mathbf{V}^*$ respectively.

Since the mutations of exceptional bundles are T-equivariant, the previous toric structures induce a toric structure on each exceptional vector bundle E on $\mathbb{P}_2$. In particular, we get the toric structures on the basic sheaves $\{\mathcal{O}(-1),\Omega(1),\mathcal{O}\}$. The corresponding T-modules

$$\begin{aligned} V_{01} &= \mathrm{Hom}(\mathcal{O}(-1),\Omega(1)) \\ V_{12} &= \mathrm{Hom}(\Omega(1),\mathcal{O}) \\ V_{02} &= \mathrm{Hom}(\mathcal{O}(-1),\mathcal{O}) \end{aligned}$$

coincide with standard representations of T in the spaces $\mathbf{V}$, $\mathbf{V}$, and $\Lambda^2(\mathbf{V})$ respectively.

So, we get a natural torus action on all geometrical objects used in §2: T acts on the Kronecker moduli space $\mathfrak{N}(V_{12};U_1,U_2)$ (because it acts on V_{12}), there is a natural toric structure on the bundles $\mathcal{U}_1$, $\mathcal{U}_2$ (induced by the trivial torus action on G-modules (2-7)), and finally, the torus acts on the Grassmanian $\mathfrak{G}$. The last action preserves the subvariety $\mathfrak{M}=\mathfrak{M}(2,-1,k)\subset\mathfrak{G}$ and the restriction of this action onto $\mathfrak{M}$ coincides with the action (3-1). Hence, we have a toric structure on the universal bundle $\mathcal{G}=\mathcal{U}_2|_{\mathfrak{M}}$ and this structure leads to a specific toric structure on each toric bundle E on $\mathbb{P}_2$.

In order to describe the last one, remember that by (2-7) there is T-equivariant isomorphism $\det \mathrm{pr}^*\mathcal{U}_2 = \det \mathrm{pr}^*\mathcal{U}_1$. Taking its fiber over $E\in\mathfrak{M}^T\subset\mathfrak{G}$, we see that T-modules $\det H^1(E)$ and $\det H^1(E(-1))$ must be isomorphic to each other. In other words, the sum of all characters from $H^1(E)$ is equal to the sum of all characters from $H^1(E(-1))$. It is known (see [14, 15]) that after twisting a toric structure on E by a character χ both sets of characters $H^1(E)$ and

$H^1(E(-1))$ are shifted simultaneously by $-\chi$ inside the lattice $\Lambda(T)$. Since $\dim H^1(E) = k-1$ and $\dim H^1(E(-1)) = k$, there exists a unique shift, which leads to the equality between these sums.

So, to get the correct toric structure on a sheaf $E \in \mathfrak{M}^T$ we have to start from any toric structure on E, then calculate the character decompositions for $H^1(E)$ and $H^1(E(-1))$, and then shift the toric structure in order to have the equality between the sums of these characters. We will make all these calculations in §4.

3.3. Rank 2 torsion free toric sheaves on $\mathbb{P}_2$. The technique developed by A.Klyachko (see [14, 15]) presents a description of all toric sheaves up to the isomorphisms, which preserve a toric structure. This description leads to enumeration of all connected components of $\mathfrak{M}^T$. So, let us remember some combinatorial data associated with a toric structure on a sheaf.

Any sheaf E on $\mathbb{P}_2$ is uniquely defined by a triple of the modules $E(U_i) = \Gamma(U_i; E)$. If E is toric, then T acts on each $E(U_i)$. Under this action $t \in T$ maps a section $\sigma\colon \mathcal{O} \longrightarrow E$ to the composition

$$\mathcal{O} \xrightarrow{i_t^{-1}} t_*(\mathcal{O}) \xrightarrow{t_*(\sigma)} t_*(E) \xrightarrow{\varphi_t} E\,,$$

where i_t is the tautological toric structure on $\mathcal{O}$ fixed above. As soon as we have a torus action on $E(U_i)$ we can decompose $E(U_i)$ into a direct sum of eigenspaces $E(U_i)_\chi$ parametrized by the torus characters. Note that if we multiply an eigensection of weight $\chi \in \Lambda(T)$ by a character ξ, which is regular on U_i, then we get a section of weight $\chi - \xi$. So, $\forall\, \xi \in \sigma_i$ we have the inclusion $E(U_i)_\chi \hookrightarrow E(U_i)_{\chi-\xi}$.

3.3.1. Triple of bifiltrations associated with a toric structure. Denote by V the fiber of E over the point $p = (1\colon 1\colon 1) \in U_{012}$. Since any homogeneous section is uniquely determinated by its value at the point p, each $E(U_i)_\chi$ can be considered as a subspace in V. Denote this subspace by $V^i(\chi)$. As we have just seen, these subspaces form an inductive system over the cone $-\sigma_i$, which is opposite to $\sigma_i \subset \Lambda(T)$, i. e.

$$V^i(\chi) \subset V^i(\chi - \xi) \qquad \forall\, \xi \in \sigma_i\,.$$

If we replace the character χ in the notation $V^i(\chi)$ by its coordinates $m_j \stackrel{\text{def}}{=} \langle \tau_j, \chi\rangle$ and $m_k \stackrel{\text{def}}{=} \langle \tau_k, \chi\rangle$ with respect to the sides of the angle σ_i (we always suppose that $j < k$ is the complementary to i pair of indices) and use the notation $V^i(m_j, m_k)$ instead of $V^i(\chi)$, then we can say that for $i = 0, 1, 2$ the subspaces $V^i(m_j, m_k)$ form a triple of bifiltrations of V. These bifiltration have the properties

$$V^i(m_j, m_k + 1) \subset V^i(m_j, m_k) \supset V^i(m_j + 1, m_k) \qquad \forall\, m_j, m_k \in \mathbb{Z} \times \mathbb{Z}\ ,$$

$$V^i(-\infty, -\infty) = V \qquad \text{and} \qquad V^i(\infty, \infty) = 0\,,$$

and a toric structure on E is uniquely defined by such a triple of bifiltrations.

3.3.2. Triple of filtrations associated with a toric structure. Consider now the modules of sections $E(U_{ij}) = \Gamma(U_i \cap U_j; E)$. As above, we can associate with each of these three modules a collection of subspaces $V^{ij}(\chi) \subset V$ parametrized by torus characters $\chi \in \Lambda(T)$. These subspaces correspond to homogeneous sections of E over U_{ij} and satisfy the property

$$V^{ij}(\chi - \xi) \supset V^{ij}(\chi) \qquad \forall\, \xi \in \sigma_{ij} = \{\xi \in \Lambda(T)\colon \langle \tau_k, \xi\rangle \geq 0\}\,.$$

Writting $V^{ij}(m)$ instead of $V^{ij}(\chi)$, where $m = m_k = \langle \tau_k, \chi\rangle$ (and k is complementary to (i, j) index), we get a decreasing filtration

$$V = V^{ij}(-\infty) \supset \cdots \supset V^{ij}(m) \supset V^{ij}(m-1) \supset \cdots \supset V^{ij}(\infty) = 0$$

of V. Certainly, each filtration $V^{ij}(*)$ coincides with the limits of the bifiltrations $V^i(*,*)$ and $V^j(*,*)$:

$$V^i(m,-\infty) = V^j(-\infty,m) = V^{ij}(m) \quad \forall m\in\mathbb{Z}\ ,\ \forall i<j\ ,\ (i,j)\subset\{0,1,2\}\ . \tag{3-2}$$

Now we are in a position to formulate the following result of A.Klyachko extracted from [15].

3.3.3. THEOREM. *The category of toric torsion free rank 2 sheaves with the morphisms, which preserve a toric structure, is equivalent to the category of 2-dimensional vector spaces V equipped with a triple of bifiltrations $V^i(*,*)$, $i=0,1,2$, such that any two of them have the same limits (3-2) (the morphisms in this category must preserve all bifiltrations).*

□

3.4. Toric bundles. Starting from any triple of filtrations $V^{ij}(*)$, one can construct a triple of bilfitrations $\overline{V^i}(*,*)$ defined as

$$\overline{V^i}(m_j,m_k) = V^{ij}(m_1)\cap V^{ik}(m_2)\ . \tag{3-3}$$

Evidently, this triple satisfy the conditions of Th.3.3.3. Hence, such a triple always define a toric sheaf. It is easy to see that this sheaf is locally free. So, we have the following result (see [14]).

3.4.1. THEOREM. *The category of all rank 2 vector bundles with the morphisms, which preserve the toric structure, is equivalent to the category of 2-dimensional vector spaces equipped with a triple of filtrations $V^{ij}(*)$, $i<j$, $(i,j)\subset\{0,1,2\}$ (the morphisms in this category must preserve all the filtrations).*

□

Since $\dim V = 2$, each of the filtration $V^{jk}(*)$ can be given by the following data:

- 1-dimensional subspace $L^i\subset V$;
- a number $s_i\in\mathbb{Z}$ such that $V^{jk}(s_i)=V$ and $V^{jk}(s_i+1)=L^i$;
- a number $a_i\in\mathbb{Z}$, $a_i\geq 0$ such that $V^{jk}(s_i+a_i)=L^i$ and $V^{jk}(s_i+a_i+1)=0$ (in other words a_i is the number of 1-dimensional terms of the filtration).

In terms of this data the action of isomorphisms, which preserve a toric structure, coincides with the action of $PGL_2(V)$ on a triple of lines L^i. There is also the very nice stability criterion (see [14, 15]):

3.4.2. THEOREM. *Locally free μ-stable rank 2 toric sheaf E is μ-stable if and only if the corresponding 1-dimensional subspaces L^i are pairwise different and a_i are the side lengths of a triangle (i. e. they are positive integers satisfying the triangle inequalities).*

□

Since $PGL_3(V)$ acts transitively on the triples of pairwise different 1-dimensional subspaces L^i, a stable toric bundle does not depend (up to isomorphism preserving a toric structure) on the choose of the subspaces L^i in V. So, a stable toric bundle E is defined in fact only by six numbers a_i, s_i. For example, the Chern classes of E can be recovered from these numbers by the formulas (see [14])

$$\begin{aligned} c_1(E) &= 2(s_0+s_1+s_2)+a_0+a_1+a_2 \\ c_1(E)^2-4c_2(E) &= (-a_0+a_1+a_2)^2-4a_1a_2 \end{aligned}$$

Moreover, a toric structure on E depends on (and only on!) the numbers s_i. Namely, if a toric structure is twisted by a character χ, then the numbers a_i remain to be the same and the numbers s_i are shifted by the rule

$$s_i \longmapsto s_i - \langle \tau_i, \chi \rangle$$

(so, we see that the sum $s_0 + s_1 + s_2$ is not changed too).

We get

3.4.3. COROLLARY. *There is a 1-1 correspondence between the stable toric bundles (considered up to toric isomorphisms) with* $(\mathrm{rk}\,, c_1, c_2) = (2, -1, k)$ *and the collections*

$$(s_1, s_2, a_0, a_1, a_2) \subset \mathbb{Z} \times \mathbb{Z} \times \mathbb{N} \times \mathbb{N} \times \mathbb{N}$$

such that a_0, a_1, a_2 *satisfy the triangle inequalities and the equality*

$$(-a_0 + a_1 + a_2)^2 - 4a_1a_2 = 1 - 4k\ .$$

If a toric structure on a bundle is changed by twisting by a character $\chi \in \Lambda(T)$, *then the triple* (a_0, a_1, a_2) *remains to be the same and a pair* (s_1, s_2) *is shifted by the vector with coordinates* $(-\langle \tau_1, \chi \rangle, -\langle \tau_2, \chi \rangle)$.

□

3.5. Non-reflexive toric sheaves. From the canonical exact triple

$$0 \longrightarrow E \longrightarrow E^{**} \longrightarrow C_E \longrightarrow 0$$

it follows that if E is a stable toric sheaf with $(\mathrm{rk}\,, c_1, c_2) = (2, -1, k)$, then E^{**} is a stable toric bundle with $(\mathrm{rk}\,, c_1, c_2) = (2, -1, k - d)$, where $d = \dim H^0(C_E)$. The cokernel $C_E = E/E^{**}$ splits into direct sum of three torsion toric sheaves C_i with $\mathrm{Supp}(C_i) = p_i$, where $p_0 = (1\colon 0\colon 0)$, $p_1 = (0\colon 1\colon 0)$, and $p_2 = (0\colon 0\colon 1)$ are three fixed points for the torus action on $\mathbb{P}_2$.

Hence, the combinatorial description of an arbitrary toric sheaf consists of a non-negative integer d, which gives "a jump" $c_2(E) - c_2(E^{**})$, five integers $(s_1, s_2, a_0, a_1, a_2)$, which give a bundle E^{**} with $c_2(E^{**}) = k - d$, and a combinatorial description of three $\mathcal{O}_{\mathbb{P}_2, p_i}$-modules C_i.

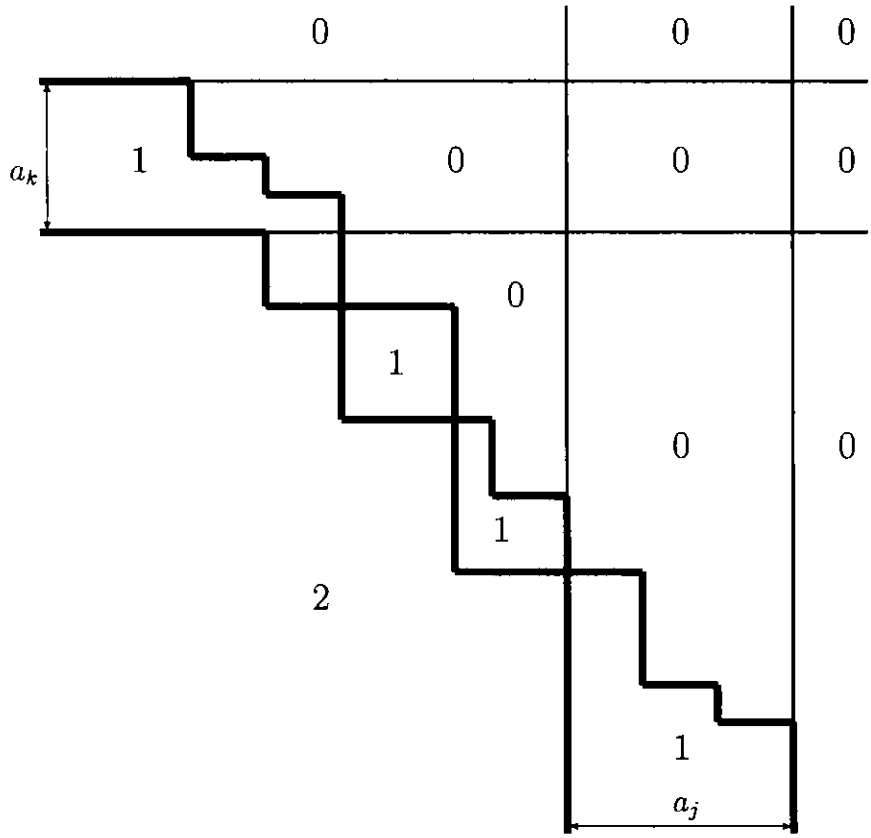
3.5.1. Combinatorial data and moduli of C_i. Suppose E to be given by a triple of bifiltrations $V^i(*, *)$. Then construct the filtrations $V^{ij}(*)$ defined by (3-2). Then make from them the new bifiltrations $\overline{V^i}(*, *)$ defined by (3-3). It is not difficult to see that the vector bundle, which corresponds to this final bifiltration's triple, coincides with E^{**}. Moreover, the canonical inclusion $E \hookrightarrow E^{**}$ corresponds to the natural embedding

$$V^i(*, *) \subset \overline{V^i}(*, *) = V^i(-\infty, *) \cap V^i(*, -\infty)\ ,$$

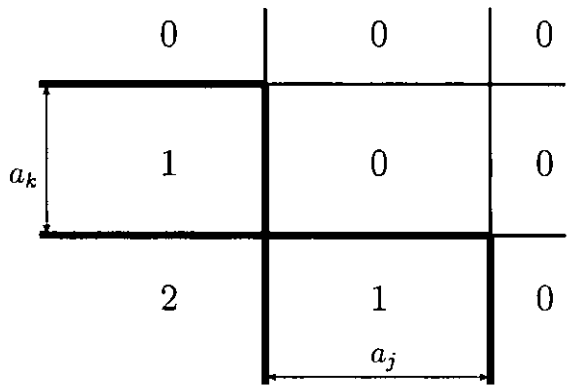
Hence, each C_i can be decomposed via torus action by the formula

$$C_i = \bigoplus_{m_j, m_k} \overline{V^i}(m_j, m_k)/V^i(m_j, m_k)\ .$$

This decomposition can be represented by the following picture. Consider an infinite table with cells indexed by the pairs (m_j, m_k) and put in each cell the corresponding space $\overline{V^i}(m_j, m_k)$. Such a table is divided into several natural zones, which contain the spaces of the same dimension. This looks like



(3-5)



(3-4)

where the numbers indicate the dimensions of the corresponding spaces. Note that the widths of 1-dimensional zones coincide with the numbers a_j, a_k, which define the bundle E^{**} via Crl.3.4.3. The numbers s_i define the placement of the picture: upper cells of the horizontal 1-dimensional strip have $m_k = s_k + a_k$ and the right cells of the vertical 1-dimensional zone have $m_j = s_j + a_j$.

Similar table for the bifiltration $V^i(*,*)$ is obtained from the previous one by changing some dimensions by smaler like in (3-5). An important property of the 1-dimensional zone of this table is that any two cells, which have a common side, have to contain the same 1-dimensional spaces. So, 1-dimensional zone splits into *maximal connected components*: two cells belong to the same component iff they can be connected inside this component by a sequence of cells such that any two consequent elements of this sequence have a common side. All cells of each connected component contain the same 1-dimensional subspace.

Note, that in any case there are exactly two not bounded components. They contain 1-dimensional spaces, which come from the previous picture (i.e. from E^{**} – corresponding cells form the bottom end of the vertical strip and the left end of the horizontal strip). These two 1-dimensional spaces must be different, because of stability, and we can consider them to be fixed by some toric isomorphism.

Other connected components are bounded (there are 2 such components in (3-5)). There are no restrictions on the 1-dimensional spaces placed in each of these components. Hence, each torsion sheaf C_i defines and is uniquely defined by a table like (3-5) and a point from

$$\underbrace{\mathbb{P}_1 \times \mathbb{P}_1 \times \cdots \times \mathbb{P}_1}_{N_i},$$

where N_i is the number of bounded connected components in 1-dimensional zone and each $\mathbb{P}_1 = \mathbb{P}(V)$ parametrize the choice of 1-dimensional subspace in V placed in the corresponding connected component.

So, each connected component $Y \subset \mathfrak{M}^T$ is uniquely defined by a triple of numbers a_i, which give E^{**} (the same for all $E \in Y$), a triple of numbers $d_i = \dim H^0(C_i)$, and a triple of pictures like (3-5). Such a component is isomorphic to the direct product of $N = N_0 + N_1 + N_2$ projective lines, which parametrize the choice of subspaces placed in bounded 1-dimensional zones of the tables (3-5).

3.5.2. Character decomposition and eigensubbundles in $H^0(C_i)$. The picture, which gives the character decomposition for $H^0(C_i)$ follows immediately from (3-5), (3-4): the character χ with the coordinates $m_j = \langle \tau_j, \chi \rangle$, $m_k = \langle \tau_k, \chi \rangle$ is present in $H^0(C_i)$ if and only if a subspace placed in the (m_j, m_k)-cell of (3-5) is smaler then the one placed in the same cell of (3-4). The difference between the dimensions of these subspaces equals the multiplicity of χ.

When E is runnig through a connected component $Y \subset \mathfrak{M}^T$ the eigenspace corresponding to χ form a vector bundle over $Y = \mathbb{P}_1 \times \mathbb{P}_1 \times \cdots \times \mathbb{P}_1$. This bundle is nontrivial iff χ comes from a bounded 1-dimensional zone of (3-5). In this case it equals the pull-back of the universal factor line bundle $\mathcal{O}_{\mathbb{P}_1}(1)$ over the $\mathbb{P}_1$-multiplier, which corresponds to the bounded 1-dimensional zone of (3-5) what χ comes from.

3.5.3. Description of $H^0(C_i)$ by a Young diagram's pair. Combinatorially, it is convenient to represent the character table for $H^0(C_i)$ as a sum of two Young diagrams λ_i, μ_i filled by 1-dimensional spaces like in (3-6) (λ_i, μ_i are any satisfying the condition $|\lambda_i| + |\mu_i| = d_i = \dim H^0(C_i)$). Such a decomposition is not unique: there are exactly 2^{N_i} Young diagram's pairs, which lead to the same resulting picture.

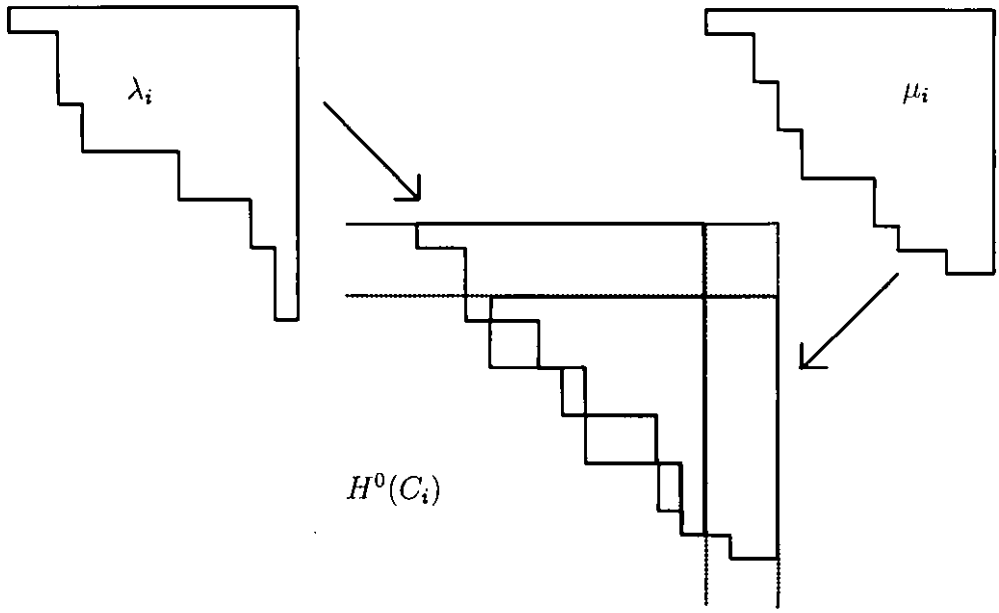
Let us collect all above information in

3.5.4. THEOREM. *Every connected component $Y \subset \mathfrak{M}(2, -1, k)^T$ has a form*

$$\underbrace{\mathbb{P}_1 \times \mathbb{P}_1 \times \cdots \times \mathbb{P}_1}_{N},$$

and there exists a surjective map from the set of all combinatorial data consisting of:

- *an ordered triple of non-negative integers d_0, d_1, d_2 such that the sum $d = d_0 + d_1 + d_2$ is bounded by $0 \leq d \leq (k-1)$,*
- *an ordered triple of positive integers a_0, a_1, a_2, which satisfy three triangle inequalities and the equality $(-a_0 + a_1 + a_2)^2 - 4a_1a_2 = 1 - 4(c_2(E) - d)$,*



(3-6)

– *an ordered triple of Yong diagram's pairs* (λ_i, μ_i) *such that* $|\lambda_i| + |\mu_i| = d_i$

onto the set of all connected components $Y \subset \mathfrak{M}^T$. *For any* Y *there are exactly* $2^{\dim Y}$ *combinatorial data collections, which are mapped into* Y. *They all have the same numbers* a_i, d_i *and the same triples of pictures obtained from the Young diagrams in the way shown in (3-6).* $\mathbb{P}_1$*-multipliers of* Y *naturally correspond to the bounded 1-dimensional zones in three diagrams (3-5).*

§4. Some calculations via Bott formula.

4.1. Bott formula in our framework. Suppose that a torus T acts on a smooth algebraic variety X, $\mathcal{E}$ is a toric bundle on X, and $\mathrm{rk}\,(\mathcal{E}) = \dim(X)$. Then the top Chern class of $\mathcal{E}$ can be evaluated only looking on the restriction $\mathcal{E}|_{X^T}$ of $\mathcal{E}$ onto the fixed points loci X^T.

Namely, let $X^T = \coprod Y$ be the decomposition of X^T into connected components and $\gamma\colon \mathbb{C}^* \longrightarrow T$ be a general one-parametric subgroup (such that $X^\gamma = X^T$). For any connected component Y consider the decompositions

$$\mathcal{E}\,|_Y = \bigoplus_{\chi\in\Lambda(T)} \mathcal{E}^\chi \qquad \text{and} \qquad \mathcal{N}_{X/Y} = \bigoplus_{\chi\in\Lambda(T)} \mathcal{N}^\chi_{X/Y}$$

with respect to the torus action. Let

$$c(\mathcal{E}^\chi) = \prod_i (1 + e^i_\chi) \qquad \text{and} \qquad c(\mathcal{N}^\chi_{X/Y}) = \prod_j (1 + n^j_\chi)$$

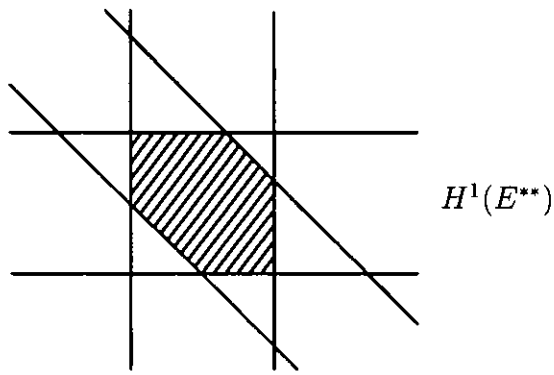
be the formal factorizations of total Chern classes of eigenbundles $\mathcal{E}^\chi$, $\mathcal{N}^\chi_{X/Y}$. It follows from the general Bott residue formula (see [2]) that the top Chern class $c_{\mathrm{top}}(\mathcal{E})$ can be evaluated by the following rule

$$c_{\mathrm{top}}(\mathcal{E}) = \sum_Y \int_Y \frac{\prod_{i,\chi}(\langle\gamma,\chi\rangle + e^i_\chi)}{\prod_{j,\chi}(\langle\gamma,\chi\rangle + n^j_\chi)} \quad ,$$

where the integrand expression is considered as an element of the Chow ring $A^*(Y)$.

We are going to apply this formula to calculate $c_{\text{top}}(\mathcal{G}^{\oplus 4})$, where $\mathcal{G}$ is the universal bundle over $\mathfrak{M} = \mathfrak{M}(2, -1, k)$. As we have seen in the previous paragraph, each connected fixed locus $Y \in \mathfrak{M}^T$ is represented by a family of all torsion free toric sheaves E on $\mathbb{P}^2$, which have a given combinatorial type and are equipped with some special toric structures induced by the canonical toric structure on $\mathcal{G}$ (see n° 3.2.2 above). So, we have to calculate the character's decomposition of $H^1(E, \mathbb{P}_2)$ and describe the corresponding eigensubbundles of $\mathcal{G}|_Y$ in terms of $A^*(Y)$. Then we need to make the same for $\mathcal{N}_{\mathfrak{M}(2,-1,k)/Y}$, i. e. we have to calculate the character's decomposition of $\text{Ext}^1(E, E)$, factorize it by the zero character component, and describe the corresponding eigensubbundles of $\mathcal{E}xt^1_\pi(\mathcal{G}, \mathcal{G})|_Y$ in terms of $A^*(Y)$.

4.2. Character's decomposition for $\mathcal{G}|_Y$. Let E runs through the connected component $Y \subset \mathfrak{M}^T$ given in combinatorial terms of Th.3.5.4. Since $H^1(E) = H^1(E^{**}) \oplus H^0(C_E)$, the diagram of characters, which are present in $H^1(E)$, is the sum of seven pices: the diagram for $H^1(E^{**})$ and six Young diagrams coming from $H^0(C_i)$, $i = 0, 1, 2$ as it was explained in n° 3.5.2 and n° 3.5.3.



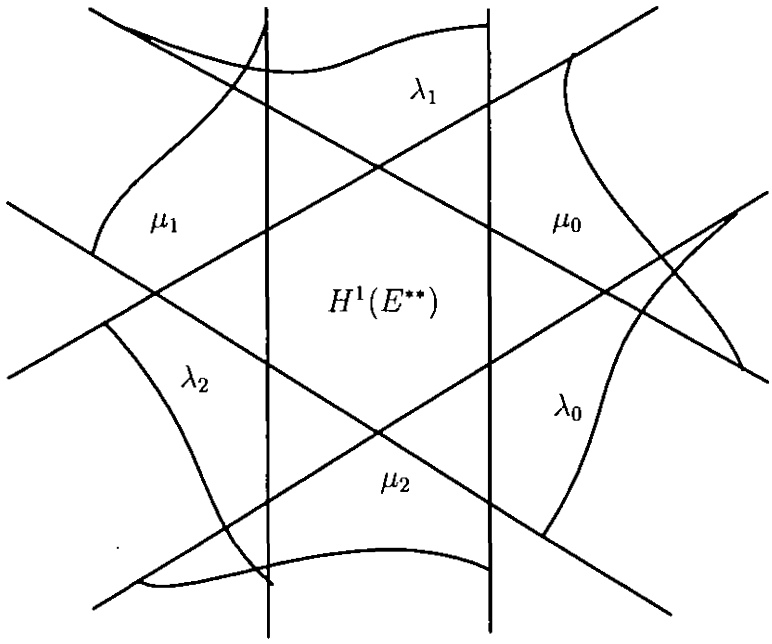
(4-1)

The first pice is calculated directly by looking on the χ-component of the Čhech complex associated with the standard affine covering $\mathbb{P}_2 = \bigcup_{i=0}^{2} U_i$ (see [14]). It is easy to see that in terms of filtrations $V^{jk}(*)$ described in n° 3.3.2, the character χ appears in $H^1(E^{**})$ iff all three spaces $V^{jk}(\chi)$ $((jk) = (01), (02), (12))$ are 1-dimensional. The set of such characters is represented in $\Lambda(T)$ by intersection of three strips like in (4-1). The weights of the slanted, horizontal, and vertical strips are equal to a_0, a_1, a_2 correspondingly. Certainly, all the characters have the multiplicity 1.

The same picture for $E^{**}(-1)$ is obtained by the one step shift of the slanting strip in the right-upper direction like

$H^1(E^{**})$ $\qquad$ $H^1(E^{**}(-1))$

Three Young diagram's pairs are placed into three pairs of outward angles of the above hexagons in such a way that in more symmetric trigonal coordinates on $\Lambda(T)$ we get the picture shown on (4-2).



(4-2)

Recall that this picture must be placed into $\Lambda(T)$ in such a way that the sum of all characters from $H^1(E)$ equals to the sum of all characters from $H^1(E(-1))$.

Eigensubbundle corresponding to the character χ is either trivial or the pull-back of $\mathcal{O}_{\mathbb{P}_1}(1)$. The last case was explained in n° 3.5.2.

4.3. Character's decomposition for $\mathcal{N}_{\mathfrak{M}/Y}$. We compute the character decomposition of $\mathcal{T}_{\mathfrak{M},E} = Ext^1(E,E)$ using the standard spectral sequence associated with the monad (2-1). In this sequence $E_2^{pq} = E_\infty^{pq}$ and E_1^{pq} coincides with the following T-equvariant complex of T-modules

$$0 \to \begin{matrix}\operatorname{End}(U_0)\\ \oplus\\ \operatorname{End}(U_1)\\ \oplus\\ \operatorname{End}(U_2)\end{matrix} \to \mathbf{V}\otimes \begin{matrix}\operatorname{Hom}(H^1(U_0,U_1)\\ \oplus\\ \operatorname{Hom}(H^1(U_1,U_2)\end{matrix} \to \Lambda^2(\mathbf{V})\otimes\operatorname{Hom}(U_0,U_2)\to 0 \qquad (4\text{-}3)$$

where $U_i = H^1(E(i-2))$. This complex has two cohomology spaces: $\mathbb{C}$ in the left term and $\operatorname{Ext}^1(E,E)$ in the middle term. Hence, it gives the character table for $\mathcal{T}_{\mathfrak{M},E}$. Unfortunately, we can not represent the answer by a nice picture like in the previous section. But the calculation has the strightforward algorithmization in terms of set-theoretical and Minkowski sums of character tables. Since we have a full description of all T-modules from (4-3), we can calculate the character table for $\operatorname{Ext}^1(E,E)$ by computer. The character decomposition of $\mathcal{N}_{\mathfrak{M}/Y}$ can be extracted from this table immediately by omitting the zero character. Moreover, using the formal decomposition of Chern polynomials for eigensubbundles in $H^1(E(i))$ presented in n° 3.5.2, we can extract from (4-3) not only the character table but also the formal expansions for the denominators in the Bott formula. We can make this last step also only numerically.

Exact numerical results obtained by this algorithm are the following:

k	1	2	3	4	5	6	7
a_k	0	0	0	13	729	85026	15650066

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	Abbes, Ahmed (Gastf.)	Breite Str. 78 53111 Bonn	MPI 402-240 / Zi.36	
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	Daskalopoulos, G. Prof. (Gastf.)			
	Dierkes, U., Hochschuldozent, Dr. (Univ.Bonn) (Wiss.Ausschuß MPI)	Wilhelm-Levison-Str. 15 53115 Bonn	Be 6, 73-3143 / Zi.5	0228 / 220113
	Dostoglou, Stamatis, Dr. (Gastf.)			
	Emmanouil, Ioannes (Gastf.)			
*	Faltings, Gerd, Prof. Direktor am MPI; Wiss.Ausschuß MPI	Finkenbergstr. 59 53227 Bonn	MPI 402-228 / Zi.28	0228 / 467417
*	Fang, Fuquan, Prof. (Gastf.)	Graurheindorfer Str. 61 53111 Bonn	MPI 402-266 / Zi.38	0228 / 695673
	Feldkamp, Immo (EDV)	Reichensteinstr. 49A 53844 Troisdorf	MPI 402-246 / Zi.15	02241 / 41490
	Fock, Vladimir, Prof. (Gastf.)	Azaleenplatz 6 53840 Troisdorf	MPI 402-247 / Zi.12	
	Fomenko, Anatoly, Prof. (Gastf.)	Konviktstr. 9 53113 Bonn		
	Furusawa, Masaaki, Prof. (Gastf.)			
	Gannon, Terry, Prof. (Gastf.)	Niebuhrstr. 55 53113 Bonn	MPI 402-245 / Zi.16	0228 / 241805

	Name	Adresse	Tel. u. Zi.-Nr. dienstlich	Tel. privat
	Daskalopoulos, G. Prof. (Gastf.)			
	Dierkes, U., Hochschuldozent, Dr. (Univ.Bonn) (Wiss.Ausschuß MPI)	Wilhelm-Levison-Str. 15 53115 Bonn	Be 6, 73-3143 / Zi.5	0228 / 220113
	Dostoglou, Stamatis, Dr. (Gastf.)			
	Emmanouil, Ioannes (Gastf.)			
*	Faltings, Gerd, Prof. Direktor am MPI; Wiss.Ausschuß MPI	Finkenbergstr. 59 53227 Bonn	MPI 402-228 / Zi.28	0228 / 467417
*	Fang, Fuquan, Prof. (Gastf.)	Graurheindorfer Str. 61 53111 Bonn	MPI 402-266 / Zi.38	0228 / 695673
	Feldkamp, Immo (EDV)	Reichensteinstr. 49A 53844 Troisdorf	MPI 402-246 / Zi.15	02241 / 41490
	Fock, Vladimir, Prof. (Gastf.)	Azaleenplatz 6 53840 Troisdorf	MPI 402-247 / Zi.12	
	Fomenko, Anatoly, Prof. (Gastf.)	Konviktstr. 9 53113 Bonn		
	Furusawa, Masaaki, Prof. (Gastf.)			
	Gannon, Terry, Prof. (Gastf.)	Niebuhrstr. 55 53113 Bonn	MPI 402-245 / Zi.16	0228 / 241805

	Name	Adresse	Tel. u. Zi.-Nr. dienstlich	Tel. privat
	Getzler, Ezra, Dr. (Gastf.)	Wilhelm-Levison-Str. 18 53115 Bonn	MPI 402-250 / Zi.19	0228 / 225397
	Gilkey, Peter B., Prof. (Gastf.)			
	Gindikin, Simon, Prof. (Gastf.)	Mozartstr. 52 53115 Bonn	MPI 402-256 / Zi.42	
*	Goncharov, Alexander, Prof. (Gastf.)	Sebastianstr. 187 53115 Bonn	MPI 402-264 / Zi.39	0228 / 611830
	Gorodentsev, Alexei, Prof. (Gastf.)	Roisdorfer Weg 15 53121 Bonn	MPI 402-224 / Zi.24	
	Goto, Yasuhiro, Dr. (Gastf.)	Hauptstr. 86 53229 Niederkassel	MPI 402-263 / Zi. 43	
	Govorov, Valentin, Prof. (Gastf.)	Mozartstr. 52 53115 Bonn	MPI 402-269 / Zi.44	
	Grossberg, Michael D., Dr. (Gastf.)	Rheindorfer Str. 143 53225 Bonn	MPI 402-268 / Zi.45	
	Guha, Partha, Dr. (Gastf.)	Schumannstr. 36 53113 Bonn	MPI 402-238 / Zi.23	0228 / 241880
*	Hambleton, Ian, Prof. (Gastf.)	Goldbergweg 15 53177 Bonn	MPI 402-225 / Zi.25	0228 / 327523
	Hanamura, Masaki, Dr. (Gastf.)	Burggartenstr. 4 53115 Bonn	MPI 402-249 / Zi.18	
*	Harder, Günter, Prof. Direktor am MPI; Wiss.Ausschuß MPI	Wegelerstr. 2 53115 Bonn	B 1 73-7786 / Zi.15 MPI 402-227 / Zi.27	0228 / 656152

	Name	Adresse	Tel. u. Zi.-Nr. dienstlich	Tel. privat
	Getzler, Ezra, Dr. (Gastf.)	Wilhelm-Levison-Str. 18 53115 Bonn	MPI 402-250 / Zi.19	0228 / 225397
	Gilkey, Peter B., Prof. (Gastf.)			
	Gindikin, Simon, Prof. (Gastf.)	Mozartstr. 52 53115 Bonn	MPI 402-256 / Zi.42	
*	Goncharov, Alexander, Prof. (Gastf.)	Sebastianstr. 187 53115 Bonn	MPI 402-264 / Zi.39	0228 / 611830
	Gorodentsev, Alexei, Prof. (Gastf.)	Roisdorfer Weg 15 53121 Bonn	MPI 402-224 / Zi.24	
	Goto, Yasuhiro, Dr. (Gastf.)	Hauptstr. 86 53229 Niederkassel	MPI 402-263 / Zi. 43	
	Govorov, Valentin, Prof. (Gastf.)	Mozartstr. 52 53115 Bonn	MPI 402-269 / Zi.44	
	Grossberg, Michael D., Dr. (Gastf.)	Rheindorfer Str. 143 53225 Bonn	MPI 402-268 / Zi.45	
	Guha, Partha, Dr. (Gastf.)	Schumannstr. 36 53113 Bonn	MPI 402-238 / Zi.23	0228 / 241880
*	Hambleton, Ian, Prof. (Gastf.)	Goldbergweg 15 53177 Bonn	MPI 402-225 / Zi.25	0228 / 327523
	Hanamura, Masaki, Dr. (Gastf.)	Burggartenstr. 4 53115 Bonn	MPI 402-249 / Zi.18	
*	Harder, Günter, Prof. Direktor am MPI; Wiss.Ausschuß MPI	Wegelerstr. 2 53115 Bonn	B 1 73-7786 / Zi.15 MPI 402-227 / Zi.27	0228 / 656152

	Name	Adresse	Tel. u. Zi.-Nr. dienstlich	Tel. privat
*	Hirzebruch, Friedrich, Prof. (emer.Wiss.Mitglied) Wiss.Ausschuß MPI	Thüringer Allee 127 53757 St. Augustin	MPI 402-244 / Zi.54	02241 / 332377
*	Hu, Yi, Prof. (Gastf.)			
	Husemöller, Dale, Prof. (Gastf.)	Mozartstr. 52 53115 Bonn	MPI 402-260 / Zi.37	
*	Huybrechts, Daniel, Dr. (Wiss.Ang.)	Bunsenstr. 8 40215 Düsseldorf	MPI 402-256 / Zi.42	0211 / 373401
	Izhboldin, Oleg, Prof. (Gastf.)	Niebuhrstr. 55 53113 Bonn	MPI 402-234 / Zi.34	
	Jarisch, Dagobert (Verw.Leiter MPI)	Fockestr. 1 53125 Bonn	MPI 402-222 / Zi.5 Rheinaustr. 134	0228 / 255375
	Kabanov, Dr. (Gastf.)	Niebuhrstr. 55 53113 Bonn		
*	Kapranov, Mikhail, Prof. (Gastf.)	Clemens-August-Str. 2-4 53115 Bonn	MPI 402-224 / Zi.24	0228 / 632048
	Kaufmann, Ralph, Dipl.-Phys. (WH)	Hatschiergasse 13 53111 Bonn	MPI 402-261 / Zi.40	0228 / 656485
	Kestermann, Rainer (Wiss.Ang.)	Gottfried-Claren-Str. 2 53225 Bonn	MPI 402-239 / Zi.14	0228 / 460453
	Kimura, Takashi, Prof. (Gastf.)			
	Kleinjung, Thorsten, Dipl.-Math. (WH)	Ippendorfer Allee 23 53127 Bonn	MPI 402-266 / Zi.38	0228 / 284954

	Name	Adresse	Tel. u. Zi.-Nr. dienstlich	Tel. privat
*	Hirzebruch, Friedrich, Prof. (emer.Wiss.Mitglied) Wiss.Ausschuß MPI	Thüringer Allee 127 53757 St. Augustin	MPI 402-244 / Zi.54	02241 / 332377
*	Hu, Yi, Prof. (Gastf.)			
	Husemöller, Dale, Prof. (Gastf.)	Mozartstr. 52 53115 Bonn	MPI 402-260 / Zi.37	
*	Huybrechts, Daniel, Dr. (Wiss.Ang.)	Bunsenstr. 8 40215 Düsseldorf	MPI 402-256 / Zi.42	0211 / 373401
	Izhboldin, Oleg, Prof. (Gastf.)	Niebuhrstr. 55 53113 Bonn	MPI 402-234 / Zi.34	
	Jarisch, Dagobert (Verw.Leiter MPI)	Fockestr. 1 53125 Bonn	MPI 402-222 / Zi.5 Rheinaustr. 134	0228 / 255375
	Kabanov, Dr. (Gastf.)	Niebuhrstr. 55 53113 Bonn		
*	Kapranov, Mikhail, Prof. (Gastf.)	Clemens-August-Str. 2-4 53115 Bonn	MPI 402-224 / Zi.24	0228 / 632048
	Kaufmann, Ralph, Dipl.-Phys. (WH)	Hatschiergasse 13 53111 Bonn	MPI 402-261 / Zi.40	0228 / 656485
	Kestermann, Rainer (Wiss.Ang.)	Gottfried-Claren-Str. 2 53225 Bonn	MPI 402-239 / Zi.14	0228 / 460453
	Kimura, Takashi, Prof. (Gastf.)			
	Kleinjung, Thorsten, Dipl.-Math. (WH)	Ippendorfer Allee 23 53127 Bonn	MPI 402-266 / Zi.38	0228 / 284954

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	Koch, Marion, (WH)	Johannesstr. 41 53225 Bonn	MPI 402-216 / Druckerei	0228 / 420448
	Koya, Yoshihiro, Prof. (Gastf.)	Niebuhrstr. 55 53113 Bonn	MPI 402-274 / Zi.20-2	
	Krämer, Evelyn (WH)	Stockenstr. 13 53113 Bonn	MPI 402-239 / Zi.14	
	Lando, Sergei, Prof. (Gastf.)	Hauptstr. 86 53859 Niederkassel	MPI 402-262 / Zi.41	
	Lang, Serge, Prof. (Gastf.)	Hotel Mozart Mozartstr. 1 53115 Bonn		
	Le, Hong Van, Dr. (Heisenberg-Stip.)	Kaiser-Konrad-Str. 93 53225 Bonn	MPI 402-268 / Zi.45	0228 / 469304
	Lee, Ronnie, Prof. (Gastf.)	Rheinaustr. 221 53225 Bonn	MPI 402-247 / Zi.12	
	Leprevost, Franck, Dr. (Gastf.)	Elliger Höhe 13 53177 Bonn	MPI 402-238 / Zi.23	0228 / 321492
	Levin, Andrey, Dr. (Gastf.)	Königswinterer Str. 154 53227 Bonn	MPI 402-278 / Zi. 20-4	0228 / 467665
*	Lieb, Ingo, Prof. (Univ.Bonn) (Wiss.Ausschuß MPI)	Heinrich-Heine-Str. 69 53225 Bonn	W 10 73-2243 / Zi.346	0228 / 469287
	Lieberum, Jens (Gastf.)	Argelanderstr. 128 53115 Bonn	MPI 402-265 / Zi.38	0228 / 219751

	Name	Adresse	Tel. u. Zi.-Nr. dienstlich	Tel. privat
	Koch, Marion, (WH)	Johannesstr. 41 53225 Bonn	MPI 402-216 / Druckerei	0228 / 420448
	Koya, Yoshihiro, Prof. (Gastf.)	Niebuhrstr. 55 53113 Bonn	MPI 402-274 / Zi.20-2	
	Krämer, Evelyn (WH)	Stockenstr. 13 53113 Bonn	MPI 402-239 / Zi.14	
	Lando, Sergei, Prof. (Gastf.)	Hauptstr. 86 53859 Niederkassel	MPI 402-262 / Zi.41	
	Lang, Serge, Prof. (Gastf.)	Hotel Mozart Mozartstr. 1 53115 Bonn		
	Le, Hong Van, Dr. (Heisenberg-Stip.)	Kaiser-Konrad-Str. 93 53225 Bonn	MPI 402-268 / Zi.45	0228 / 469304
	Lee, Ronnie, Prof. (Gastf.)	Rheinaustr. 221 53225 Bonn	MPI 402-247 / Zi.12	
	Leprevost, Franck, Dr. (Gastf.)	Elliger Höhe 13 53177 Bonn	MPI 402-238 / Zi.23	0228 / 321492
	Levin, Andrey, Dr. (Gastf.)	Königswinterer Str. 154 53227 Bonn	MPI 402-278 / Zi. 20-4	0228 / 467665
*	Lieb, Ingo, Prof. (Univ.Bonn) (Wiss.Ausschuß MPI)	Heinrich-Heine-Str. 69 53225 Bonn	W 10 73-2243 / Zi.346	0228 / 469287
	Lieberum, Jens (Gastf.)	Argelanderstr. 128 53115 Bonn	MPI 402-265 / Zi.38	0228 / 219751

	Name	Adresse	Tel. u. Zi.-Nr. dienstlich	Tel. privat
	Lin, Vladimir, Prof. (Gastf.)	Breite Str. 70 53111 Bonn	MPI 402-267 / Zi.46	
	Lott, John, Prof. (Gastf.)	Auf der Schleide 81 53225 Bonn	MPI 402-260 / Zi.37	0228 / 478706
	Lück, Wolfgang, Dr. (Gastf.)		MPI 402-245 / Zi.19	
	Lustig, Martin, Priv.Doz., Dr. (Gastf.)		MPI 402-237 / Zi.22	
	Mäkelä, Marianna (Vorz. Wiss. Mitglieder)	Voigtelstr. 5 50933 Köln	MPI 402-229 / Zi.29	
*	Manin, Yuri, Prof. Direktor am MPI; Wiss.Ausschuß MPI	Ölidenweg 2 53129 Bonn	MPI 402-271 / Zi.20-3	0228 / 237407
*	Manolache, Nicolae, Prof. (Gastf.)	Am Schildchen 14 53844 Troisdorf	MPI 402-231 / Zi.31	0228 / 456018
	Maurmann, Sven, Dipl.-Math. (Wiss.Ang.)	Hausdorffstr.115 53129 Bonn	MPI 402-236 / Zi.14	
	Merkulov, Sergej, Prof. (Gastf.)	Oppelner Str. 132-136 53119 Bonn		
	Mersmann, Gerd, Dipl.-Math. (Gast)	Florentiusgraben 25a 53111 Bonn	MPI 402-261 / Zi.40	0228 / 653472
	Mettlach, Melanie (Vorz. Geschäftsf. Direktor und Prof. Hirzebruch)	Nogenter Platz 4 53721 Siegburg	MPI 402-243 / Zi.53	02241 / 69581

	Name	Adresse	Tel. u. Zi.-Nr. dienstlich	Tel. privat
	Lin, Vladimir, Prof. (Gastf.)	Breite Str. 70 53111 Bonn	MPI 402-267 / Zi.46	
	Lott, John, Prof. (Gastf.)	Auf der Schleide 81 53225 Bonn	MPI 402-260 / Zi.37	0228 / 478706
	Lück, Wolfgang, Dr. (Gastf.)		MPI 402-245 / Zi.19	
	Lustig, Martin, Priv.Doz., Dr. (Gastf.)		MPI 402-237 / Zi.22	
	Mäkelä, Marianna (Vorz. Wiss. Mitglieder)	Voigtelstr. 5 50933 Köln	MPI 402-229 / Zi.29	
*	Manin, Yuri, Prof. Direktor am MPI; Wiss.Ausschuß MPI	Ölidenweg 2 53129 Bonn	MPI 402-271 / Zi.20-3	0228 / 237407
*	Manolache, Nicolae, Prof. (Gastf.)	Am Schildchen 14 53844 Troisdorf	MPI 402-231 / Zi.31	0228 / 456018
	Maurmann, Sven, Dipl.-Math. (Wiss.Ang.)	Hausdorffstr.115 53129 Bonn	MPI 402-236 / Zi.14	
	Merkulov, Sergej, Prof. (Gastf.)	Oppelner Str. 132-136 53119 Bonn		
	Mersmann, Gerd, Dipl.-Math. (Gast)	Florentiusgraben 25a 53111 Bonn	MPI 402-261 / Zi.40	0228 / 653472
	Mettlach, Melanie (Vorz. Geschäftsf. Direktor und Prof. Hirzebruch)	Nogenter Platz 4 53721 Siegburg	MPI 402-243 / Zi.53	02241 / 69581

	Name	Adresse	Tel. u. Zi.-Nr. dienstlich	Tel. privat
	Moree, Pieter, Dr. (Wiss. Ang.)	Hunsrückstr. 4 53119 Bonn	MPI 402-279 / Zi.20-1	0228 / 651923
*	Nahm, Werner, Prof. (Univ.Bonn) (Wiss.Ausschuß MPI)	Robelstr. 28 53123 Bonn	N 12 73-2319 / Zi.217	0228 / 627013
*	Neretin, Yuri, Prof. (Gastf.)	Troschelstr. 6 53115 Bonn	MPI 402-248 / Zi.17	0228 / 220868
	Netsvetaev, Nikita, Prof. (Gastf.)			
*	Nippert, Petra (Verwaltung MPI)	Wallen 3 53545 Linz	MPI 402-223 / Zi.3 Rheinaustr. 134	02644 / 1661
	Oertel, Ulrich, Prof. (Gastf.)	Heidebergenstr. 84 53229 Bonn	MPI 402-237 / Zi. 22	0228 / 486136
	Olshanetzky, Mikhail, Prof. (Gastf.)	Eulenweg 15 53129 Bonn	MPI 402-240 / Zi.36	
*	Pearce-Jahre, Christine (Bibliothek)	Max-Bruch-Str. 2 53121 Bonn	MPI 402-252 / Zi.2	0228 / 628117
	Pedersen, Erik, Prof. (Gastf.)		MPI 402-264 / Zi.39	
	Penkov, Iwan, Prof. (Gastf.)	Mozartstr. 52 53115 Bonn		
	Poletaeva, Elena, Prof. (Gastf.)			
*	Pragacz, Piotr, Dr. (Gastf.)	Herderstr. 57 C 16 53173 Bonn	MPI 402-269 / Zi.44	0228 / 351799

	Name	Adresse	Tel. u. Zi.-Nr. dienstlich	Tel. privat
	Moree, Pieter, Dr. (Wiss. Ang.)	Hunsrückstr. 4 53119 Bonn	MPI 402-279 / Zi.20-1	0228 / 651923
*	Nahm, Werner, Prof. (Univ.Bonn) (Wiss.Ausschuß MPI)	Robelstr. 28 53123 Bonn	N 12 73-2319 / Zi.217	0228 / 627013
*	Neretin, Yuri, Prof. (Gastf.)	Troschelstr. 6 53115 Bonn	MPI 402-248 / Zi.17	0228 / 220868
	Netsvetaev, Nikita, Prof. (Gastf.)			
*	Nippert, Petra (Verwaltung MPI)	Wallen 3 53545 Linz	MPI 402-223 / Zi.3 Rheinaustr. 134	02644 / 1661
	Oertel, Ulrich, Prof. (Gastf.)	Heidebergenstr. 84 53229 Bonn	MPI 402-237 / Zi. 22	0228 / 486136
	Olshanetzky, Mikhail, Prof. (Gastf.)	Eulenweg 15 53129 Bonn	MPI 402-240 / Zi.36	
*	Pearce-Jahre, Christine (Bibliothek)	Max-Bruch-Str. 2 53121 Bonn	MPI 402-252 / Zi.2	0228 / 628117
	Pedersen, Erik, Prof. (Gastf.)		MPI 402-264 / Zi.39	
	Penkov, Iwan, Prof. (Gastf.)	Mozartstr. 52 53115 Bonn		
	Poletaeva, Elena, Prof. (Gastf.)			
*	Pragacz, Piotr, Dr. (Gastf.)	Herderstr. 57 C 16 53173 Bonn	MPI 402-269 / Zi.44	0228 / 351799

	Name	Adresse	Tel. u. Zi.-Nr. dienstlich	Tel. privat
	Rabi, Reuben (Gastf.)			
	Ramero, Lorenzo, Dr. (Gastf.)	Niebuhrstr. 55 53113 Bonn	MPI 402-268 / Zi.45	
	Rapoport, Michael, Prof. (Univ.Wuppertal) (Wiss.Ausschuß MPI)	Triebelsheide 52 42111 Wuppertal	Math.Inst.U.W'tal 0202 / 439-2663	0202 / 721700
	Reznikov, Andre, Dr. (Gastf.)	Konstantinstr. 86 53179 Bonn	MPI 402-247 / Zi.12	
	Romann, Klaus (Verwaltung MPI)	Königswinterer Str. 391 53227 Bonn	MPI 402-255 / Zi.4 Rheinaustr. 134	
	Rosellen, Markus (Gast)	Auf der Bergwiese 13 53639 Königswinter		02244 / 80593
	Rozenberg, Alexander, Prof. (Gastf.)	Ernst-Moritz-Arndt-Str. 18 53225 Bonn	MPI 402-221 / Zi.21	
	Rudakov, Alexei, Prof. (Gastf.)	Eulenweg 15 53129 Bonn		
	Salomonsen, Gorm, Dipl.-Math. (Gastf.)	Hunsrückstr. 4 53119 Bonn	MPI 402-245 / Zi.16	0228 / 651923
*	Sarlette, Manuela (Schreibbüro MPI)	Steinergasse 65 53347 Alfter	MPI 402-270 / Zi.6 Rheinaustr. 134	
	Schächl, Hans (EDV)	Kölnstr. 90 53111 Bonn	MPI 402-245 / Zi.15	0228 / 651613
	Schneider, Ruth (WH)	Hödenbuschweg 2 53547 Kasbach	MPI 402-252 (Bibliothek)	

	Name	Adresse	Tel. u. Zi.-Nr. dienstlich	Tel. privat
	Rabi, Reuben (Gastf.)			
	Ramero, Lorenzo, Dr. (Gastf.)	Niebuhrstr. 55 53113 Bonn	MPI 402-268 / Zi.45	
	Rapoport, Michael, Prof. (Univ.Wuppertal) (Wiss.Ausschuß MPI)	Triebelsheide 52 42111 Wuppertal	Math.Inst.U.W'tal 0202 / 439-2663	0202 / 721700
	Reznikov, Andre, Dr. (Gastf.)	Konstantinstr. 86 53179 Bonn	MPI 402-247 / Zi.12	
	Romann, Klaus (Verwaltung MPI)	Königswinterer Str. 391 53227 Bonn	MPI 402-255 / Zi.4 Rheinaustr. 134	
	Rosellen, Markus (Gast)	Auf der Bergwiese 13 53639 Königswinter		02244 / 80593
	Rozenberg, Alexander, Prof. (Gastf.)	Ernst-Moritz-Arndt-Str. 18 53225 Bonn	MPI 402-221 / Zi.21	
	Rudakov, Alexei, Prof. (Gastf.)	Eulenweg 15 53129 Bonn		
	Salomonsen, Gorm, Dipl.-Math. (Gastf.)	Hunsrückstr. 4 53119 Bonn	MPI 402-245 / Zi.16	0228 / 651923
*	Sarlette, Manuela (Schreibbüro MPI)	Steinergasse 65 53347 Alfter	MPI 402-270 / Zi.6 Rheinaustr. 134	
	Schächl, Hans (EDV)	Kölnstr. 90 53111 Bonn	MPI 402-245 / Zi.15	0228 / 651613
	Schneider, Ruth (WH)	Hödenbuschweg 2 53547 Kasbach	MPI 402-252 (Bibliothek)	

	Name	Adresse	Tel. u. Zi.-Nr. dienstlich	Tel. privat
	Scholz, Elke (WH)	Wilhelmstr. 21 53879 Euskirchen	MPI 402-0 / Zentrale	02251 / 58857
	Shang, Zaijiu, Dr. (Gastf.)	Niebuhrstr. 55 53113 Bonn	MPI 402-279 / Zi.20-1	
	Shi, Jian-Yi, Prof. (Gastf.)	Im Tannenwinkel 6 53639 Königswinter	MPI 402-261 / Zi.40	02244 / 4840
	Smyth, Brian, Prof. (Gastf.)	Holzgasse 52b 53227 Siegburg	MPI 402-238 / Zi.23	
	Speh, Birgit, Prof. (Gastf.)	Herderstr. 59 53173 Bonn	MPI 402-242 / Zi.56	
	Stamnas, Erasmia (WH)	Friedrich-Hegeler-Str. 5 53757 St. Augustin	MPI 402-252 (Bibliothek)	
	Stein, Yosef, Dr. (Gastf.)	Burbankstr. 63 53229 Bonn	MPI 402-267 / Zi.46	
	Stolz, Stefan, Prof. (Gastf.)			
	Suter, Silke (Wiss.Ang.) (Wiss.Ausschuß MPI)	Thomas-Mann-Str. 49 53111 Bonn	MPI 402-232 / Zi.32	0228 / 651811
	Swiatkowski, Jacek, Dr. (Gastf.)	Azaleenplatz 6 53840 Troisdorf	MPI 402-274 / Zi.20-2	
*	Tabachnikov, Serge, Prof. (Gastf.)	Frankenweg 32 53225 Bonn	MPI 402-249 / Zi.18	0228 / 469892
	Tian, Qingchun, Prof. (Gastf.)			

	Name	Adresse	Tel. u. Zi.-Nr. dienstlich	Tel. privat
	Scholz, Elke (WH)	Wilhelmstr. 21 53879 Euskirchen	MPI 402-0 / Zentrale	02251 / 58857
	Shang, Zaijiu, Dr. (Gastf.)	Niebuhrstr. 55 53113 Bonn	MPI 402-279 / Zi.20-1	
	Shi, Jian-Yi, Prof. (Gastf.)	Im Tannenwinkel 6 53639 Königswinter	MPI 402-261 / Zi.40	02244 / 4840
	Smyth, Brian, Prof. (Gastf.)	Holzgasse 52b 53227 Siegburg	MPI 402-238 / Zi.23	
	Speh, Birgit, Prof. (Gastf.)	Herderstr. 59 53173 Bonn	MPI 402-242 / Zi.56	
	Stamnas, Erasmia (WH)	Friedrich-Hegeler-Str. 5 53757 St. Augustin	MPI 402-252 (Bibliothek)	
	Stein, Yosef, Dr. (Gastf.)	Burbankstr. 63 53229 Bonn	MPI 402-267 / Zi.46	
	Stolz, Stefan, Prof. (Gastf.)			
	Suter, Silke (Wiss.Ang.) (Wiss.Ausschuß MPI)	Thomas-Mann-Str. 49 53111 Bonn	MPI 402-232 / Zi.32	0228 / 651811
	Swiatkowski, Jacek, Dr. (Gastf.)	Azaleenplatz 6 53840 Troisdorf	MPI 402-274 / Zi.20-2	
*	Tabachnikov, Serge, Prof. (Gastf.)	Frankenweg 32 53225 Bonn	MPI 402-249 / Zi.18	0228 / 469892
	Tian, Qingchun, Prof. (Gastf.)			

	Name	Adresse	Tel. u. Zi.-Nr. dienstlich	Tel. privat
*	Tokunaga, Hiro-o, Dr. (Gastf.)	Schumannstr. 36 53115 Bonn	MPI 402-263 / Zi.43	0228 / 21583
	Tschinkel, Yuri, Dr. (Gastf.)	Mozartstr. 52 53115 Bonn	MPI 402-262 / Zi.41	0228 / 659071
	Ueda, Masaru, Prof. (Gastf.)	Ferdinandstr. 66 53127 Bonn	MPI 402-279 / Zi.20-1	
*	Umehara, Masaaki, Prof. (Gastf.)	Niebuhrstr. 55 53113 Bonn	MPI 402-234 / Zi.34	0228 / 215444
	Venkataramana, Tyakal, Prof. (Gastf.)	Breite Str. 70 53111 Bonn	MPI 402-240 / Zi.36	
	Voevodsky, Vladimir (Gastf.)			
	Vogel, Wolfgang, Prof. (Gastf.)		MPI 402-256 / Zi.42	
	Weber, Christian, Dipl.-Math. (WH)	Venusbergweg 3 53115 Bonn	MPI 402-265 / Zi.38	0228 / 265313
	Winter, Peter (Hausmeister MPI)	von-Sandt-Str. 32 53225 Bonn	MPI 402-219 / Zi.11 MPI 402-280	0228 / 470230
	Wisniewsky, Jaroslav, Prof. (Gastf.)	Austr. 75 53343 Wachtberg	MPI 402-262 / Z.41	
	Yu, Guoliang, Prof. (Gastf.)	Mozartstr. 52 53115 Bonn		

	Name	Adresse	Tel. u. Zi.-Nr. dienstlich	Tel. privat
*	Tokunaga, Hiro-o, Dr. (Gastf.)	Schumannstr. 36 53115 Bonn	MPI 402-263 / Zi.43	0228 / 21583
	Tschinkel, Yuri, Dr. (Gastf.)	Mozartstr. 52 53115 Bonn	MPI 402-262 / Zi.41	0228 / 659071
	Ueda, Masaru, Prof. (Gastf.)	Ferdinandstr. 66 53127 Bonn	MPI 402-279 / Zi.20-1	
*	Umehara, Masaaki, Prof. (Gastf.)	Niebuhrstr. 55 53113 Bonn	MPI 402-234 / Zi.34	0228 / 215444
	Venkataramana, Tyakal, Prof. (Gastf.)	Breite Str. 70 53111 Bonn	MPI 402-240 / Zi.36	
	Voevodsky, Vladimir (Gastf.)			
	Vogel, Wolfgang, Prof. (Gastf.)		MPI 402-256 / Zi.42	
	Weber, Christian, Dipl.-Math. (WH)	Venusbergweg 3 53115 Bonn	MPI 402-265 / Zi.38	0228 / 265313
	Winter, Peter (Hausmeister MPI)	von-Sandt-Str. 32 53225 Bonn	MPI 402-219 / Zi.11 MPI 402-280	0228 / 470230
	Wisniewsky, Jaroslav, Prof. (Gastf.)	Austr. 75 53343 Wachtberg	MPI 402-262 / Z.41	
	Yu, Guoliang, Prof. (Gastf.)	Mozartstr. 52 53115 Bonn		

	Name	Adresse	Tel. u. Zi.-Nr. dienstlich	Tel. privat
	Zagier, Don, Prof. Geschäftsf. Direktor am MPI; Wiss.Ausschuß MPI	Thomas-Mann-Str. 49 53111 Bonn	MPI 402-233 / Zi.33	0228 / 653045
	Zaidenberg, Mikhail, Prof. (Gastf.)			

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	Zagier, Don, Prof. Geschäftsf. Direktor am MPI; Wiss.Ausschuß MPI	Thomas-Mann-Str. 49 53111 Bonn	MPI 402-233 / Zi.33	0228 / 653045
	Zaidenberg, Mikhail, Prof. (Gastf.)			