Max-Planck-Institut für Mathematik
Bonn

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Max-Planck-Institut für Mathematik
Preprint Series 2012 (9)

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MPIM 12-09

EXACTLY FILLABLE CONTACT STRUCTURES WITHOUT STEIN FILLINGS

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Abstract. We give examples of contact structures which admit exact symplectic fillings, but no Stein fillings, answering a question of Ghiggini.

1. Introduction

It is a fundamental problem is contact topology to determine which contact 3-manifolds admit symplectic fillings. There are many varieties of symplectic fillings. In addition to weak and strong fillings it is natural to consider those contact manifolds that are *exactly fillable*, i.e. those contact manifolds that bound exact symplectic manifolds. One may also require that the filling have a complex structure, in which case one considers *Stein* fillable contact structures, which are in particular exactly fillable. The relationship between these various notions is depicted in following sequence of inclusions:

$$\{\text{Stein fillable}\} \subset \{\text{Exactly fillable}\} \subsetneq \{\text{Strongly fillable}\} \subsetneq \{\text{Weakly fillable}\} \subsetneq \{\text{Tight}\}.$$

Examples of strongly fillable contact structures that are not exactly fillable were found by Ghiggini (cf. [9], p. 1685). Examples of non strongly fillable, weakly fillable contact structures were first discovered by Eliashberg (cf. [6]). Finally there exist tight contact structures that are not weakly fillable by [7]. So all these inclusions are strict except possibly for the first. The main result of this paper is that the first inclusion is also strict.

Theorem 1.1. *There exist exactly fillable contact structures that admit no Stein fillings.*

This answers a question raised by Ghiggini whilst studying the relationship between strong and Stein fillability ([9], p. 1686). The contact structures of Theorem 1.1 are obtained by using the fact that the Brieskorn spheres $\Sigma(2,3,6n+5)$ considered in [9] can be realised as coverings of Seifert fibred manifolds that are compact quotients of $PSL(2,\mathbb{R})$. One then constructs an exactly fillable contact structure on the connected sum $\overline{\Sigma}(2,3,6n+5)\#\Sigma(2,3,6n+5)$ using the $PSL(2,\mathbb{R})$-structure in an explicit way. By a result of Eliashberg a connected sum of contact manifolds is Stein fillable if and only if each of the summands is Stein fillable. However, the contact structure on $\overline{\Sigma}(2,3,6n+5)$ is Ghiggini's non-Stein fillable contact structure, which is in fact not even exactly fillable. This then exhibits the failure of Eliashberg's result for exact fillings. Moreover, since the contact structure on $\overline{\Sigma}(2,3,6n+5)$ is a perturbation of a taut foliation, we further deduce that perturbations of taut foliations on homology spheres are not necessarily Stein fillable.

Date: February 8, 2012.
2000 *Mathematics Subject Classification.* Primary 57R17, 53D10; Secondary 32Q28.

Acknowledgments: The author would like to thank Prof. D. Kotschick for his continued encouragement and support. He would also like to thank K. Cieliebak, T. Vogel and P. Ghiggini for his careful reading and helpful suggestions concerning an earlier version this manuscript. The hospitality of the Max Planck Institute für Mathematik in Bonn, where part of this research was carried out, is gratefully acknowledged.

Conventions: All contact structures will be assumed to be smooth and oriented. Unless otherwise stated all homology groups will be taken with rational coefficients.

2. Fillings of non-prime Manifolds

In his fundamental paper on filling by holomorphic disks [3], Eliashberg states a result about decomposing fillings of non-prime manifolds. The aim of this section is to give a proof of a slightly modified form of this statement, which ensures that the proof outlined *in situ* is valid. For this we will need to recall the notion of a symplectic filling with J-convex boundary.

Definition 2.1. *A triple (X, ω, J) consisting of a symplectic manifold (X, ω) with boundary and an almost complex structure J that is tamed by ω is called a J-*convex *filling of a contact manifold (M, ξ) if the contact structure ξ is given as the set of complex tangencies in $\partial X = M$.*

We also recall the definition of the boundary connected sum.

Definition 2.2. *Let X_1, X_2 be two connected n-dimensional manifolds with boundary. The boundary connected sum $X = X_1 \#_\partial X_2$, is defined as the manifold obtained by attaching a 1-handle $B^{n-1} \times I$ to $X_1 \sqcup X_2$ so that $B^{n-1} \times \{0\} \subset \partial X_1$ and $B^{n-1} \times \{1\} \subset \partial X_2$.*

We may now state the aforementioned result.

Theorem 2.3 ([3], Section 8)**.** *Let (X, ω, J) be a symplectic filling with J-convex boundary $M = M_1 \# M_2$ that decomposes as a non-trivial connected sum. Further, assume that $H_3(X) = 0$. Then X decomposes as a boundary connect sum $X = X_1 \#_\partial X_2$, with $\partial X_1 = M_1$ and $\partial X_2 = M_2$. Moreover X_1, X_2 are J-convex fillings of M_1, M_2 with the induced contact structures.*

The assumption on $H_3(X)$ is not made explicitly in [3] but seems necessary in order that the proof outlined there goes through. Moreover, this condition is automatically satisfied for Stein fillings, yielding the following corollary.

Corollary 2.4. *Let (X, ω, J) be a Stein filling with boundary $M = M_1 \# M_2$ that decomposes as a non-trivial connected sum. Then X decomposes as a boundary connect sum $X = X_1 \#_\partial X_2$, with $\partial X_1 = M_1$ and $\partial X_2 = M_2$. Moreover X_1, X_2 are Stein fillings of M_1, M_2 with the induced contact structures.*

Remark 2.5. We note that the connect sum operation is well-defined on tight contact manifolds by [2]. In this way Corollary 2.4 implies that a connect sum of contact manifolds $(M_1, \xi_1) \# (M_2, \xi_2)$ is Stein fillable if and only if (M_1, ξ_1) and (M_2, ξ_2) are Stein fillable.

Before embarking on the proof we will need to quote several facts about *J-convex* functions on almost complex manifolds. The main source for these results is a book in preparation of Cieliebak and Eliashberg (cf. [1]). We begin with some definitions and basic results.

Definition 2.6. *A function $f : X \to \mathbb{R}$ on an almost complex manifold is called J-convex if the 2-form*

$$\Omega = -dd^{\mathbb{C}} f$$

has the property $\Omega(\xi, J\xi) > 0$ for all non-zero $\xi \in T_pX$, where $d^{\mathbb{C}} f_p(\xi) = df_p(J\xi)$. A hypersurface $H \subset M$ is called J-convex if there exists a J-convex function $f : U \to (-1, 1)$ defined on a neighbourhood U of H so that $f^{-1}(0) = H$.

It is an elementary fact that a compact hypersurface H is J-convex if and only if the codimension 1 set of complex tangencies defines a contact structure on H ([1], Section 2.3). Thus a J-convex filling has J-convex boundary in the sense of Definition 2.6.

We next note the following lemma, which gives J-convex neighbourhoods of certain hypersurfaces.

Lemma 2.7 ([1], Sect. 2.7)**.** *Let H be a properly embedded compact hypersurface in an almost complex manifold (X, J). Assume that there exists a function $\phi : H \to \mathbb{R}$ so that the 2-form*

$$\Omega = -dd^{\mathbb{C}}\phi$$

is strictly positive when restricted to any complex line ξ_p in T_pH and choose a Hermitian metric on X. Then for all $\epsilon > 0$ sufficiently small the function

$$f = \epsilon\,\phi + dist_H^2$$

is J-convex on some open neighbourhood of H.

Proof. We first add small collars to X and H. Then in exponential coordinates on a tubular neighbourhood of $N \cong H \times (-\epsilon, \epsilon)$ the map $\psi = \mathrm{dist}_H^2$ has the form

$$\psi(h, t) = t^2.$$

Since the metric is J-invariant and the levels of the exponential map are orthogonal to $\frac{\partial}{\partial t}$ it follows that $-dd^{\mathbb{C}}\psi(X, JX) \geq 0$ and is strictly positive for all vectors that do not lie in the maximal complex subspace $TH \cap J(TH)$. Thus by our assumptions on ϕ the function $f = \epsilon\,\phi + \mathrm{dist}_H^2$ will be J-convex for all ϵ sufficiently small. □

We will also need to smooth J-convex fillings with corners. To this end we have the following Proposition, which only applies to the case where J is *integrable.*

Proposition 2.8 ([1], Ch. 3, [16], Satz 4.2)**.** *Let f_1, f_2 be two smooth J-convex functions, with J integrable. Then $f = max\{f_1, f_2\}$ can be C^0-approximated by a smooth J-convex function g. If f is smooth on a neighbourhood of a compact set K we may assume that $g|_K = f|_K$. Furthermore, if Y is a smooth vector field with $Y.f_1, Y.f_2 > 0$, then the same holds for g.*

Finally we need Eliashberg's result on filling spheres in the boundary of a J-convex filling (see also [18]). For this we shall assume that the characteristic foliation on the dividing sphere is standard, which can always be achieved after a suitable C^0-small isotopy (cf. [4]).

Theorem 2.9 ([3], Th. 4.1)**.** *Let (X, ω, J) be a J-convex filling of (M, ξ) and let $S^2 \subset \partial X$ be an embedded sphere whose characteristic foliation is diffeomorphic to the characteristic foliation of the unit sphere in $(\mathbb{R}^3, \xi_{st})$. Then after a C^2-small perturbation S^2 can be filled by holomorphic disks, that is there is a proper embedding $B^3 \to X$ which is J-holomorphic when restricted to a disk coming from the standard horizontal foliation.*

With these preliminaries we may now prove Theorem 2.3.

Proof of Theorem 2.3. Let $S \hookrightarrow \partial X$ be an embedded 2-sphere in the boundary of a J-convex filling, whose characteristic foliation may be assumed to be standard. Then by Theorem 2.9 we may fill S with a ball $h : B \to X$ that is foliated by J-holomorphic disks. We let $(z, t) \in B \subset \mathbb{C} \times \mathbb{R}$ be coordinates on B. Then the composition of h^{-1} with the real valued function

$$\phi(z, t) = |z|^2$$

is J-convex on all complex lines in B. By Lemma 2.7, for ϵ sufficiently small the function

$$f = \epsilon\, \phi + \mathrm{dist}_B^2$$

is then J-convex on a regular neighbourhood $N = B \times [-\delta, \delta]$ of B. We then cut open X along B and glue in copies of $B \times [0, \delta]$ and $B \times [-\delta, 0]$ respectively to obtain a filling $\widehat{X}$, which has piecewise J-convex boundary homeomorphic to $M_1 \sqcup M_2$. The non-smooth points of the boundary occur along the boundaries of the 3-balls $B_\pm = B \times \{\pm\delta\}$. Let $f_\pm$ be J-convex functions defining $B_\pm$ and g a J-convex function defining ∂X. Then if J is integrable near $\partial B_\pm$ we may apply Proposition 2.8 twice to obtain smoothings of $f_1 = max\{f_+, g\}$ and $f_2 = max\{f_-, g\}$ near $\partial B_\pm$. Furthermore, by choosing a vector field Y that is transverse to $B_\pm$ and ∂X, we may assume that the level sets of these smoothings are also transverse to Y so that the boundary of the resulting J-convex filling is again diffeomorphic to $M_1 \sqcup M_2$.

We claim that J can be chosen to be integrable on neighbourhoods of $\partial B_\pm$. For by ([3], Lemma 2.1) the symplectic form can be modified on a collar of the boundary so that it has the form $\Omega = \omega + C\, d(t\alpha)$, where α is any contact form on the boundary and C is an arbitrarily large, positive constant. Since the characteristic foliation on the filling sphere S is standard, the contact form may be chosen so that $d(t\alpha)$ agrees with the standard Stein fillable contact structure near S. It follows that we can find a J that tames Ω and is integrable on suitable neighbourhoods of $\partial B_\pm$.

Thus, we may smooth to get a J-convex filling of $M_1 \sqcup M_2$. *A priori* this filling may be connected, however this cannot happen if $H_3(X) = 0$. For if B were non-seperating, then the union of M_1 with a ball deleted and B would be a closed embedded 3-manifold $\widehat{M_1}$ representing a non-trivial class in $H_3(X)$, which is zero by assumption. Thus we conclude that B is separating and we obtain the desired decomposition of X as a boundary connected sum. □

Remark 2.10. If one does not assume that $H_3(X) = 0$, then the 3-ball in the above proof may not separate and one obtains a connected convex filling of the union of M_1 and M_2. This is also the case if the one considers weak fillings, since any weak filling can be made J-convex for a suitable choice of almost complex structure. Furthermore, by capping off one of the boundary components as in [6] one deduces that if $M_1 \# M_2$ is weakly fillable then each of the summands is also weakly fillable. The converse is also true as one sees by attaching a symplectic one handle to the disjoint union of two weak fillings (cf. [17]). Thus the map on isotopy classes of weakly fillable contact structures

$$\pi_0(Weak(M_1)) \times \pi_0(Weak(M_2)) \to \pi_0(Weak(M_1 \# M_2))$$

given by connect sum is bijective. Similarly, the map given by connect sum is bijective for strongly fillable contact structures.

3. Non-Stein Exact Symplectic Fillings

We use a construction due to McDuff and Mitsumatsu to construct many examples of exact symplectic fillings $(X, d\alpha)$ with $H_3(X) \neq 0$. Since the third homology is non-trivial, these fillings, though exact, cannot be Stein. The starting point for the construction is an exact symplectic filling of the form $(M \times [0,1], d\lambda)$ both of whose ends are convex, which can, for example, be obtained by considering compact quotients of $PSL(2,\mathbb{R})$.

Example 3.1 ([13], [15]). Let $\mathfrak{psl}_2$ denote the lie algebra of $PSL(2,\mathbb{R})$ and choose the following basis:

$$h = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},\ l = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\ k = \frac{1}{2}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}.$$

We identify $\mathfrak{psl}_2^*$ with the space of left-invariant 1-forms on $PSL(2,\mathbb{R})$ and define a linking pairing by

$$LK(\alpha, \beta) = \alpha \wedge d\beta.$$

With respect to this pairing the ordered basis $\{h^*, l^*, k^*\}$ is orthogonal and

$$LK(h^*, h^*) = LK(l^*, l^*) = -1 \text{ and } LK(k^*, k^*) = 1.$$

Any non-zero 1-form $\alpha \in \mathfrak{psl}_2^*$ then defines a positive resp. negative contact structure or a taut foliation, depending on whether $LK(\alpha, \alpha)$ is positive resp. negative or zero. We let Γ be a co-compact lattice in $PSL(2,\mathbb{R})$ and consider $M = PSL(2,\mathbb{R})/\Gamma$. If we set

$$\lambda = tk^* + (1-t)h^*$$

on $M \times [0,1]$, then the pair $(M \times [0,1], d\lambda)$ is a symplectic filling with convex ends.

Other examples of symplectic structures on $M \times [0,1]$ with convex ends are given by T^2-bundles over S^1 with Anosov monodromy or by smooth volume preserving Anosov flows (cf. [15]). It is now easy to construct examples of non-Stein exact fillings: one simply attaches a symplectic 1-handle to $(M \times [0,1], d\lambda)$ with ends in each component of the boundary.

Proposition 3.2. *There exist exact, non-Stein symplectic fillings.*

Proof. Let $(M \times [0,1], d\lambda) = (X, \omega)$ be an exact symplectic filling with convex ends. Attach a symplectic 1-handle to obtain a filling of the connected sum $(\overline{M}, \xi_0)\#(M, \xi_1)$ (cf. [17]). We denote this new filling $\widetilde{X} = X \cup e_1$, where e_1 denotes a topological 1-handle. The symplectic form on $\widetilde{X}$ restricts to ω on X. Thus by the long exact sequence in cohomology of the pair $(\widetilde{X}, X)$ we see that $\widetilde{X}$ is an exact filling and the hypersurface $M \times \frac{1}{2} \subset \widetilde{X}$ is non-separating, whence $H_3(\widetilde{X}) \neq 0$ and $\widetilde{X}$ cannot be Stein. □

The manifolds $(N, \xi) = (\overline{M}, \xi_0)\#(M, \xi_1)$ in Proposition 3.2 are always exactly fillable, but their natural fillings are not Stein. This raises the question of whether they are always Stein fillable or not. Or equivalently whether $(\overline{M}, \xi_0)$ and (M, ξ_1) are always Stein fillable. We will answer this question, by considering various Brieskorn spheres, which can be realised as finite covers of compact quotients of $PSL(2,\mathbb{R})$.

4. Brieskorn Spheres and $PSL(2,\mathbb{R})$-structures

We consider the Brieskorn spheres $\overline{\Sigma}(2,3,6n+5)$ taken with the opposite orientation to that given by their description as the link of the complex singularity $z_1^2+z_2^3+z_3^{6n+5}=0$. These are Seifert fibred homology spheres, whose quotient orbifolds are hyperbolic for any natural number n.

The manifold $\overline{\Sigma}(2,3,6n+5)$ admits a contact structure that is tangential to the Seifert fibration (cf. [14], p. 1764). This contact structure has the property that it is isotopic to its conjugate and we note this in the following proposition.

Proposition 4.1. *For any natural number n the manifold $\overline{\Sigma}(2,3,6n+5)$ admits a tangential contact structure η_{tan}. Moreover, any tangential contact structure is isotopic to its conjugate and is universally tight.*

Proof. We let B denote the quotient orbifold of $\overline{\Sigma}(2,3,6n+5)$ given by the Seifert fibration. A tangential contact structure η_{tan} induces a fibrewise cover $\overline{\Sigma}(2,3,6n+5)\to ST^*B$ to the unit cotangent bundle of the orbifold B so that η_{tan} is the pullback of the canonical contact structure ξ_{can} on ST^*B by ([14], Proposition 8.9). By assumption B is a hyperbolic orbifold and hence ST^*B is a compact quotient of $PSL(2,\mathbb{R})$ by a discrete lattice. Furthermore ξ_{can} comes from a left-invariant contact structure on $PSL(2,\mathbb{R})$, which is the kernel of some left-invariant 1-form, where we have identified $PSL(2,\mathbb{R})$ with $ST^*\mathbb{H}^2$ using the action of $PSL(2,\mathbb{R})$ on $\mathbb{H}^2$ via Möbius transformations.

Using the notation of Example 3.1, any left-invariant 1-form that is tangential lies in the span of h^* and l^*, since k generates the circle action. Moreover, since the linking form is negative definite on the span of h^* and l^*, any non-zero form that is tangential determines a contact structure. Hence the space of tangential $PSL(2,\mathbb{R})$-invariant contact structures is connected, so in particular ξ_{can} is isotopic to its conjugate and the same then holds for η_{tan} by taking pullbacks. In general any tangential contact structure can be perturbed to a horizontal contact structure, which is then universally tight by ([14], Theorem A). □

Ghiggini has shown that $\overline{\Sigma}(2,3,6n+5)$ admits a contact structure which is strongly fillable, but admits no Stein fillings, when n is even ([9], Theorem 1.5). The only properties of the contact structures used in Ghiggini's proof of non-Stein fillability is that they are isotopic to their conjugates and that their d_3-invariant is $-\frac{3}{2}$. However, it follows from the classification of [10] that all tight contact structures on $\overline{\Sigma}(2,3,6n+5)$ satisfy this latter constraint, thus in view of Proposition 4.1 we deduce the following.

Theorem 4.2 ([9])**.** *If n is even, then a tangential contact structure η_{tan} on $\overline{\Sigma}(2,3,6n+5)$ does not admit any Stein fillings.*

Remark 4.3. One can show that the contact structure η_{tan} corresponds to $\eta_{n-1,0}$ in terms of the classification of tight contact structures on $\overline{\Sigma}(2,3,6n+5)$ given in [10]. In this way, one can remove the assumption that n is even in Theorem 4.2 by ([12], Theorem 1.8).

With these preliminaries we may now construct examples of exactly fillable contact structures that admit no Stein fillings.

Theorem 4.4. *There exist infinitely many exactly fillable contact structures that do not admit Stein fillings.*

Proof. We consider the fibrewise covering $\overline{\Sigma}(2,3,6n+5) \to ST^*B$ given in the proof of Proposition 4.1. Since ST^*B admits a $PSL(2,\mathbb{R})$-structure, the product $ST^*B \times [0,1]$ can be made into an exact symplectic filling with convex ends as in Example 3.1 and by taking pullbacks the same is true of $\overline{\Sigma}(2,3,6n+5) \times [0,1]$.

We let $\xi = \xi_0 \# \xi_1$ be the contact structure on $\overline{\Sigma}(2,3,6n+5)\#\Sigma(2,3,6n+5)$ given by attaching a contact 1-handle as in Proposition 3.2 and note that ξ_0 is tangential by construction. Then by Remark 2.5 we have that ξ is Stein fillable if and only if ξ_0 and ξ_1 are Stein fillable. However, if n is even the contact structure ξ_0 is not Stein fillable by Theorem 4.2 and it follows that ξ is exactly fillable, but not Stein fillable. □

Remark 4.5. In fact, the argument used to show that η_{tan} is not Stein fillable actually shows that it is not even exactly fillable ([9], p. 1685). This then exhibits the failure of the analogue of Corollary 2.4 for exactly fillable contact structures.

Since the contact structure η_{tan} is isotopic to a deformation of a taut foliation, we deduce the following as a corollary.

Corollary 4.6. *There exist infinitely contact structures that are deformations of taut foliations on homology spheres, which are not Stein fillable.*

The non-Stein fillable contact structures described above were defined as pullbacks of tangential contact structures. This is completely analogous to the examples of Eliashberg in [3], who showed that the pullbacks of the standard contact structure on $T^3 = ST^*T^2$ under suitable coverings admit weak, but not strong, symplectic fillings. In view of this, one might make the following conjecture, which would provide a very large class of symplectically fillable contact structures without Stein fillings.

Conjecture 4.7. *Let M be a Seifert fibred space given as a fibrewise d-fold cover of a compact quotient of $PSL(2,\mathbb{R})$ with $d > 1$. Then the pullback of the canonical contact structure is not Stein fillable.*

This conjecture is already interesting for S^1-bundles over higher genus surfaces, whose contact structures were classified in [11], although the question of which are Stein fillable appears to be open. We remark that in the case of coverings of ST^*T^2 the obstruction to strong filling can be seen as given by the Giroux torsion of the contact structure (cf. [8]). Thus one might hope that there is some similar type of obstruction for covers of the unit cotangent bundle of a higher genus surface or on more general Seifert fibred spaces.

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