Abstracts of the Conference "Partial Differential Equations"

M. Demuth
B.-W. Schulze

Max-Planck-Arbeitsgruppe
Partielle Differentialgleichungen
und Komplexe Analysis
FB Mathematik
Universität Potsdam
Am Neuen Palais 10
O - 1571 Potsdam

Germany

Max-Planck-Institut für Mathematik
Gottfried-Claren-Straße 26
D-5300 Bonn 3

Germany

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Partial Differential Equations

Abstracts

M. Demuth

B.-W. Schulze

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0-1571 Potsdam
Germany

Preface

The present volume contains the abstracts of the lectures held on the international conference

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organized by the Max-Planck-Arbeitsgruppe "Partielle Differentialgleichungen und Komplexe Analysis" at the department of mathematics of the Potsdam University. The conference was also supported by the Potsdam University and the Sonderforschungsbereich 288 "Geometry and Quantum Physics" in Berlin. The Max-Planck research group is a result of a restructuring of mathematics of the former Karl-Weierstrass-Institute of Mathematics in Berlin, Bereich "Reine Mathematik", who organized in the past a series of conferences (Ludwigsfelde, 1976; Reinhardsbrunn, 1985; Holzhau, 1988; Breitenbrunn, 1990; Lambrecht 1991) in the same spirit as the one in Potsdam, namely to bring together specialists in analysis, mathematical physics and geometry and to point out interactions and common aspects in the recent development of these fields.

The interested reader can use the enclosed address list to contact the authors.

Potsdam, 30. November 1992

M. Demuth B.-W. Schulze

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MATHEMATICAL FUNCTIONAL INTEGRATION METHODS IN PARTIAL DIFFERENTIAL EQUATIONS, SOME NEW DEVELOPMENTS

S. Albeverio (Ruhr-Universität Bochum)

We briefly discuss two topics, namely oscillatory integrals in infinite dimensions, on one hand, and probabilistic integrals and stochastic p.d.e., on the other hand.

1. Oscillatory integrals in infinite dimensions and applications to quantum mechanics versus classical mechanics

We briefly recall how one can define oscillatory integrals on Hilbert spaces in such a way that a method of stationary phase holds for them, with applications to the study of solutions of Schrödinger equation.The study of such oscillatory integrals was initiated by K. Ito and continued by several mathematicians, see e.g. [1]. They are linear continuous functionals sharing many properties of integrals in the sense of measure theory (except for full σ-additivity). Recently the method of stationary phase for them, initiated in the 70's by R. Høegh-Krohn, J. Rezende and myself, has been advanced to an effective handling of degenerate cases as well, see [1]-[2]. Explicit asymptotic expansions in powers of the small relevant parameter h are obtained, with control on remainders and leading terms exhibiting negative powers of the form $h^{-\mu}$, with μ a positive rational number (Coxeter number) associated with unfolding of the phase. In applications to Schrödinger equation, with potentials V of the form a quadratic term plus a (bounded) nonlinearity, the initial condition can be of the form $e^{\frac{i}{\hbar}\psi}\varphi(\psi, \varphi$ smooth)(h Planck's constant) or of the δ-form (fundamental solutions). The expansions are around classical orbits. We also discuss singularities and an explicit asymptotic expansion in h for the "Schrödinger theta function" $\Theta(t) = Tr\ e^{-\frac{i}{\hbar}tH_h} = \sum e^{-\frac{i}{\hbar}\ \lambda_n(h)}(\lambda_n(h)$: eigenvalues of the Hamiltonian $H_h = -\frac{h^2}{2}\Delta + V$ in $L^2(\mathbb{R}^d, dx)$), in terms of classical periodic orbits (the leading term gives a mathematical realization of "Gutzwiller's trace formula", of importance in the study of "classical chaos" versus "quantum chaos"). These results obtained recently [2] extend previous ones in [3]. We also point out applications of infinite dimensional oscillatory integrals to the computation of topological invariants from a Chern-Simons (topological) field model (mathematical realization, in the abelian case, of a conjecture of Atiyah-Witten) [4].

2. Probabilistic integrals and stochastic p.d.e., with applications to classical fields versus quantum fields

We mention briefly how probabilistic techniques permit to handle, at least partly, similar problems for quantum fields associated with classical fields like the one given by a non linear Klein-Gordon equation. We show in particular how the quantized solution φ of such an equation, first with "regularized nonlinearity", can be given as $\varphi(t,y) = e^{itH_\mu}\varphi(0,y)e^{-itH_\mu}$, with$(t,y) = x \in \mathbb{R} \times \mathbb{R}^{d-1}$ the space time point. Here $\varphi(0,y)$ is the (generalized) random field $X(0,y;0)$, with $X(x;\tau), \tau \in \mathbb{R}_+$ a diffusion (generalized random field) associated with the classical Dirichlet form (on $\mathcal{S}(\mathbb{R}^d))\frac{1}{2}\int \nabla u \nabla v d\mu_E$ in $L^2(\mu_E)$, μ_E being a certain probability measure on $\mathcal{S}(\mathbb{R}^d)$ (characterized by the action functional associated with the classical field). μ is the invariant measure for the process $\tau \to X(.;\tau)$ and $-H_\mu$ is the generator associated with the classical Dirichlet form $\frac{1}{2}\int \nabla u \nabla v d\mu$ in $L^2(\mu)$ (μ is supported in $\mathcal{S}(\mathbb{R}^{d-1})$). $X(.;\tau)$ solves a parabolic stochastic p.d.e. These results, initiated in the mid 70's by R. Høegh-Krohn and myself, have been greatly advanced recently, see [5] and M. Röckner's lecture at this conference. For $d \leq 2$ and special nonlinearities the regularization can be removed maintaining non triviality. φ is then a relativistic (local) field (over d-dimensional space-time). $X(x;0)$ is a Markov field, homogenous (i.e. invariant in the probabilistic sense) with respect to the Euclidean group over $\mathbb{R}^d$.To handle the case $d = 4$ an alternative construction has been developed, involving solving stochastic p.d.e. with non gaussian noise, see [6]. We also mention a recent strong result on essential self-adjointness of generators of classical infinite dimensional Dirichlet forms [7]. Somewhat similar methods can be used for hyperbolic stochastic p.d.e. obtained by adding a space-time noise to non linear Klein-Gordon equations [8].

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EXPANSION IN EIGENVECTORS OF MULTIPARAMETER SPECTRAL DIFFERENTIAL PROBLEMS

Yu. M. Berezansky (Institute of Mathematics, Ukrainian Acad. Sci., Kiev)

The multiparameter spectral problem for two ordinary differential operators L_1, L_2 has the form:

$$\begin{aligned}(L_1, x_1\varphi)(x_1, x_2) &= (\lambda_1 b_{11}(x_1) + \lambda_2 b_{12}(x_1))\varphi(x_1, x_2)\\ (L_2, x_2\varphi)(x_1, x_2) &= (\lambda_1 b_{21}(x_2) + \lambda_2 b_{22}(x_2))\varphi(x_1, x_2)\end{aligned}$$

Here b_{jk} are fixed functions, φ is an eigenfunction and $\lambda = (\lambda_1, \lambda_2)$ is an eigenvalue. Thus the last two equations are connected only by means of the two spectral parameters λ_1, λ_2. The talk contains some theorems about expansions in this type of eigenfunction for general n–parameter self-adjoint spectral problems. The results include operators with continuous spectrum and general elliptic partial differential operators, they are published in [1]–[3].

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PROBLEMS IN PERTUBATION SPECTRAL THEORY RELATED TO THE PERIODIC SCHRÖDINGER OPERATOR

M. Sh. Birman (Dep. of Math. Physics, University St. Petersburg)

In $L^2(\mathbb{R}^d)$, $d > 1$, the operator $H_0 = -div(a\ grad) + p(x)$ is considered with $a : \mathbb{R}^d \to \mathbb{R}^d$, $0 < a = const$; $p(x+n) = p(x)$, $n \in \mathbb{Z}^d$. For a perturbed operator one takes $H = H_0 + q(x)$ where the impurity potential $q(x)$ has the asymptotics

$$q(x) \sim b(\theta) \mid x \mid^{-p}, \quad \theta = x \mid x \mid^{-1}, \quad p > 1, \tag{1}$$

as $\mid x \mid \to \infty$. Under these conditions there is a unitary scattering matrix $S = S(E)$ for the pair H_0, H with $S - I$ compact. Eigenvalues of $S(E)$ can be written in the form $e^{\mp 2i\delta_m^\pm}$, $0 < \delta_m^\pm \leq \frac{\pi}{2}$. The scattering phases $\delta_m^\pm(E)$ tend to zero as $m \to \infty$. The problem is to find asymptotics of $\delta_m^\pm$ as $m \to \infty$. The analogous problem for $p(x) = 0$ was solved in [1]. In the same paper a generalization is given for an unperturbed operator of the form $F(-i\nabla)$ and for $H = F(-i\nabla) + q$ where the condition (1) is satisfied. In [1] it is shown that in the latter case

$$\lim_{m\to\infty} m^\gamma \delta_m^\pm = g_\pm, \quad \gamma = \frac{p-1}{d-1}. \tag{2}$$

There is an explicit expression for $g_\pm$ (see [1]) as integrals over the surface $G(E) = \{k \in \mathbb{R}^d : F(k) = E\}$. Here one supposes that the surface $G(E)$ does not contain critical points, $\nabla F(k) \neq 0$.

In the present paper it is shown that for the periodic Schrödinger operator the asymptotics (2) are valid too, and the problem may be reduced to the one considered in [1]. Let $E_l(k)$, $l = 1, 2, \ldots$ be branches of a band function for the periodic operator. Suppose that there are no critical points on $G(E) = \{k : E_l(k) = E\}$ for some l. Then (2) is valid for the scattering phases with the same $g_\pm$ as in [1] for $F(k) = E_l(k)$. If the equation $E_l(k) = E$ is solvable for several l, then the phases decompose into series and every series has the asymptotics of the form (2). Thus, the expression for $g_\pm$ contains only the periodic operator band function.

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A C^*-ALGEBRA PROOF OF THE MOURRE ESTIMATE FOR N-BODY HAMILTONIANS

A. Boutet de Monvel (Université Paris VII)

We have proposed to consider the problem of "generalized non-relativistic N-body theory" from another point of view. It is clear that almost all Hamiltonians describing interesting physical models have a "many-channel structure". This always happens when the system has a "subsystem structure", or has several channels, and this is the case for most models of solid state physics, nuclear theory and quantum field theory. So the first problem is to give a precise mathematical meaning to the concept "many-channel Hamiltonians". For the case of system with a finite number of degrees of freedom, my proposal is to define this class of operators by the property to be affiliated to a C^*-algebra which has a special structure. It is graded with respect to a finite partially ordered set $\mathcal{L}$. I have shown that a large part of the usual non-relativistic N-body theory can be developed for Hamiltonians affiliated to graded C^*-algebras: Weinberg–von Winter equation, and HVZ theorem are very natural in this context; the same thing happens with the Mourre theory if the action of the group of automorphisms induced by the conjugate operator is compatible with the grading. We describe the essential spectrum (relative to the natural maximal ideal).
We showed that for any $\mathcal{L}$-graded C^*-algebra $\mathcal{C}$ the application

$$\mathcal{C} \ni S \to (\mathcal{P}_E(S))_{|E|=2} \in \widetilde{\mathcal{C}}$$

is a $*$ -homomorphism of $\mathcal{C}$ onto a C^* -subalgebra of $\widetilde{\mathcal{C}}$ with kernel equal to $\mathcal{C}(\sup \mathcal{L})$. In particular the quotient map is an isometric $*$-homomorphism of the quotient C^* -algebra $\mathcal{C}/\mathcal{C}(\sup \mathcal{L})$ onto a C^*-subalgebra of $\widetilde{\mathcal{C}}$, which provides a canonical C^*-embedding

$$\mathcal{C}/\mathcal{C}(\sup \mathcal{L}) \hookrightarrow \widetilde{\mathcal{C}} = \bigoplus_{|E|=2} \mathcal{C}_E.$$

Using the fact that the spectrum of an element $(S_E)_{|E|=2}$ of $\widetilde{\mathcal{C}}$ is equal to the union of the spectra of its components S_E , we immediately get a new and very neat proof of the HVZ theorem:

$$\mathcal{C}\text{-}\sigma_{\text{ess}}(E) = \bigcup_{|E|=2} \sigma(H_E),$$

Also if $\{\mathcal{W}_\tau\}_{\tau\in I\!R}$ is a C_0-group of automorphisms of $\mathcal{C}$ compatible with the grading ($\mathcal{W}_\tau \mathcal{C}(E) \subset \mathcal{C}(E)$ for each $E \in \mathcal{L}, \tau \in I\!R$), and if H is a self-adjoint operator affiliated to $\mathcal{C}$ of class $C^1_u(\mathcal{A})$ then each H_E is of class $C^1_u(\mathcal{A})$, formally we have $\mathcal{W}_\tau = e^{\mathcal{A}\tau}$. Define $\widehat{\rho}_H$ relative to the $*$ -ideal $\mathcal{C}(\sup \mathcal{L})$. Then we have

$$\widehat{\rho}_H = \min_{|E|=2} \rho_{H_E}$$

The last point, and probably the most important one, is that an obvious generalization can be made to the case when fermionic degrees of freedom are present . In fact CCR (or Weyl) algebra could be replaced by the CAR (or Clifford) algebra, or more generally one may consider a supersymmetric situation (a $\mathbb{Z}_2$ - graded symplectic space and the Weyl-Clifford algebra constructed over it). Finally, we give a new proof of the Mourre estimate, which does not involve any geometric object (such as the partition of unity in the configuration space).
Also I showed the interactions may depend on the intercluster momentum and may be very singular (even hard-core). For this operator we describe the essential spectrum, the absence at singularly continuous spectrum and an optimal form of the limiting absorption principle.

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ASYMPTOTICS OF A PERTURBED HARMONIC OSCILLATOR

L. Boutet de Monvel (Université de Paris VI)

This is a résumé of a work [BBL] with Anne Boutet de Monvel-Berthier and Gilles Lebeau. We consider a perturbed harmonic oscillator $H = H_0 + V(x)$ on $\mathbb{R}^n$, with $H_0 = \frac{1}{2}(-\Delta + q(x))$ a harmonic oscillator (q a real $\gg 0$ quadratic form, e.g. $q = \sum(\lambda_j x_j)^2, \lambda_j > 0$), and V a smooth function with bounded imaginary part. It is immediate that H and H_0 have the same domain in L^2, so their eigenvalues have the same order of magnitude, and the traces $Tr\ e^{itH_0}, Tr\ e^{itH}$ are well defined as distributions . The distribution $Tr\ e^{itH_0}$ is easy to compute (it is the sum of a geometric series) and its singularities are located at the lengths of closed integral curves of the Hamiltonian field $\sum \xi_j \partial/\partial x_j - \lambda_j \partial/\partial \xi_j$. By comparison to a more classical situation one expects the singularities of $Tr\ e^{itH}$ to be located at the same points.
In the "classical" setting [Ch], [DG], H_0 is replaced by a 1st order elliptic pseudodifferential operator A on a compact manifold, and the perturbation H by $A+R$ with R a lower degree real operator. If the perturbation R is mild enough (e.g. $R \in OPS^m_\delta$ with $m + \delta < 1/2$), and R has bounded imaginary part) the "classical pseudodifferential calculus" gives the answer: we have $Tr\ e^{itH} = Tr\ e^{itH_0}\ P$, where P is a pseudodifferential operator in a good Hörmander class [Hö1], whose symbol can be calculated by pseudodifferential calculus (transfer equations of the WBK method).
Here an adequate pseudodifferential calculus for the harmonic oscillator is the Bernstein calculus, in which one assigns the weight 1/2 to both x_j and $\partial/\partial x_j$, so that H_0 appears as an elliptic operator of degree 1. For this calculus the relevant symbol spaces are the S^m_δ (symbols of degree m and irregularity δ; $a(x,\xi) \in S^m_\delta(\mathbb{R}^{2n})$ if for any $(x\xi)$ derivation index α we have $|\partial^\alpha_{x\xi} a| \le c^{st}(1 + x^2 + \xi^2)^{m+(\delta-1/2)|\alpha|}$. There is a notion of elliptic operator (e.g. H_0 is elliptic of degree 1, $\delta = 0$), of Sobolev spaces ($H^S = (H_0)^{-S}L^2, H^\infty$ is the Schwartz space of rapidly decreasing functions), and of wavefront sets and microlocalisation; in fact the whole situation is microlocally isomorphic to the classical one (cf [B]). For the perturbed harmonic oscillator $H = H_0 + V(x)$, even in the simplest case where V is periodic, we are outside of the case where the pseudodifferential or WKB calculus can work (the irregularity δ is $\ge 1/2$ and the asymptotic expansions one would like to construct diverge). Following Beals [B] we introduce modified spaces of symbols $\sum^m_\delta$: an operator A belongs to $OP\sum^m_\delta$ if it is of degree m, ie. continuous $H^s \to H^{s-m}$, all s, and if for any $Q_1, \dots Q_k \in OPS^1_0$ ("classical" of degree 1) the k-th bracket $[Q_k, \dots [Q_1, A] \dots]$ (which is the substitute for a k-th derivative) is of degree $\le m + k\delta$.
We again look at the scattering operator $P = e^{itH_0}e^{itH}$. P satisfies the differential equation

$$\frac{dP}{dt} = iV_t P \text{ with } V_t = e^{-itH_0}V\ e^{itH_0} \quad (P(0) = Id)$$

Using this one can show easily by functional calculus that P belongs to $OP\sum^0_{\gamma+\delta}$ if $V(x) \in S^\gamma_\delta$ with $\gamma + \delta < 1 (\gamma \ge 0,\ \delta \ge 1/2)$ and V has a bounded imaginary part (this does not cover all cases where V is $o(x^2)$ but at least covers the case where V and its derivatives are not too large at ∞, in particular when they are all bounded, e.g. V periodic). In that case P is a microlocal operator, as any operator in a class $OP\sum^m_\delta, \delta<1$, so the distribution kernels of e^{itH_0} and $e^{itH} = e^{itH_0}P$ have the same wavefront. It then follows that for the

trace distributions we have $\mathrm{WF}(Tre^{itH}) \subset \mathrm{WF}(Tre^{itH_0})$ since these traces are integrals of restrictions to the diagonal, and this enters neatly in the usual wavefront calculus, (taking into account the modified Bernstein-type definition of WF in the space directions).
Thus we have proved that the singular set of the distribution $Tr\ e^{itH}$ is still contained in the set of lengths of closed orbits of the Hamiltonian field, as in the unperturbed case, although the usual asymptotics do not apply (note however that this method only gives an inclusion and does not give a precise description of the singularities or their asymptotics, as it is possible in the classical case).

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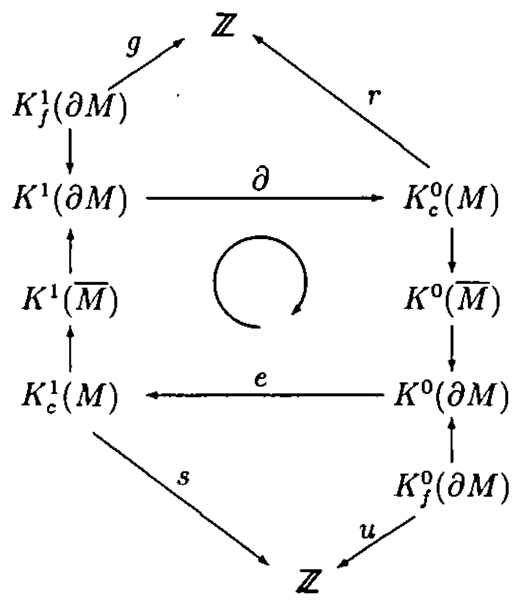
ON CONSTRUCTIONS MAKING DIRAC OPERATORS INVERTIBLE AT INFINITY

Ulrich Bunke (MPI für Mathematik, Bonn)

Let $\overline{M}$ be the compactification of a complete Riemannian manifold by the Higson–corona. There is a long exact sequence of K–theory leading to the diagram



Here K^*_f is the finite K–theory, r is the relative index pairing and s is the spectral flow pairing. r and g are constructed using a graded Dirac operator on M while S and u are defined using an ungraded one. u and g are defined by the index of certain Callias type operators.

Another construction employs a real $Cl_n^{\mathbb{R}}$–equivariant Dirac operator D over a complete Riemannian open manifold. Let $f : \partial M \to S^{k-1}$, $k \le n$ and $\tilde{f} \in C(\overline{M}) \otimes \mathbb{R}^k \cap C^\infty(M) \otimes \mathbb{R}^k$ be a lift of f. We form $A = D + \varepsilon\tilde{f}$. Assume that $0 \in \mathbb{R}^k$ is regular for $\tilde{f}$ and $N := \tilde{f}^{-1}(0)$. Then

$$[ker\ A] = \alpha(N) \in KO_{n-k}(\mathbb{R})$$

If N is compact and open, $\alpha(N) \neq 0$, then there is no metric on $N \times \mathbb{R}^k$ with uniform positive scalar curvature at infinity quasiisometric to the product metric. This generalizes theorems of M. Lesch and J. Roe.

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A GLOBAL ATTRACTING SET FOR THE KURAMOTO–SIVASHINSKY EQUATION

P.Collet, J.-P. Eckmann, H. Epstein and J. Stubbe (Université de Genève)

We prove new bounds on the Kuramoto–Sivashinsky equation

$$\partial_t U(x,t) + \partial_x^4 U(x,t) + \partial_x^2 U(x,t) + U(x,t)\partial_x U(x,t) = 0.$$

The interest in this equation is based on its relation as a phase equation for hydrodynamic problems. We consider it on a bounded interval $[-\frac{L}{2}, \frac{L}{2}]$ with periodic boundary conditions. Since the 'original equation' is for the integral, $H(x,t) = \int_0^x dy\, U(y,t)$, we always require

$$\int_{-\frac{L}{2}}^{\frac{L}{2}} dx\, U(x,t) = 0.$$

In 1985, Nicolaenko, Scheurer and Temam (Physica D16, 155–183) showed that if the initial data are in $\mathcal{L}^2$, and are antisymmetric with respect to the origin, then the evolution leaves them in $\mathcal{L}^2$, forever, and there is a global attracting set in $\mathcal{L}^2$ whose diameter is bounded where the bound depends on the size L of the system.
By extending their method we obtain a more stringent bound and we drop the requirement of antisymmetry.
The following bound holds for all periodic solutions with initial data in $\mathcal{L}^2$:

$$\limsup_{t\to\infty} \int_{-\frac{L}{2}}^{\frac{L}{2}} dx\, U^2(x,t) \le const \cdot L^{\frac{16}{5}}.$$

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ON TRANSPORT AND SPECTRAL PROPERTIES FOR THE SCHRÖDINGER EQUATION WITH STRONGLY FLUCTUATING POTENTIALS

J.M. Combes (Université de Toulon)

The connections between long-time behaviour of solutions for the Schrödinger equations and spectral properties of the time evolution operator are revisited. Interest in this discussion is motivated by recent investigations of quantum models occuring in solid state physics which exhibit unusual spectra (e.g. dense point, Cantor set) and unusual dynamics e.g. diffusive or subdiffusive. It is argued that in such situations the usual description of such connections in terms of the RAGE theorem or it's refinements is not well adapted due to the non-stability of the orthogonal decomposition associated to the components of the spectral family; this non-stability manifests itself in particular for perturbations which are local and thus should not affect the transport properties of the system.
Recent work by I. Guarneri [BBL], T. Geisel et al. [Ch] show that the generalized dimension of the spectral measure is deeply related to the long-time behaviour of correlation functions. We present their results in the light of Strichartz' analysis of Fourier asymptotics of fractal measures [DG]. Guarneri's lower bounds for lattice dynamics are extended to $\mathbb{R}^d$; these bounds read:

$$\frac{1}{T}\int_0^T < \psi_t, |x|^2\psi_t > dt \geq c\, T^{\alpha/d}$$

where α is the dimension of the spectral measure associated to ψ_0 and ψ_t is the solution at time t of the Schrödinger equation with the initial value ψ_0.
These bounds are confronted to conditions obtained by P. Hislop and the author under which fluctuating potentials exhibit absence of diffusion or of absolutely continuous spectrum. In this latter case these conditions generalize results of Simon–Spencer to continuous systems in any dimension: they imply in turn absence of diffusion only for $d \leq 2$ in agreement with Guarneri's lower bounds.

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QUANTITATIVE SEMICLASSICAL ERROR ESTIMATES BETWEEN SCHRÖDINGER AND DIRICHLET SEMIGROUPS

M. Demuth (Max-Planck-Arbeitsgruppe, Universität Potsdam)
F. Jeske (Universität Bochum)
W. Kirsch (Universität Bochum)
I. McGillivray (SFB 288, Technische Universität Berlin)

We consider Schrödinger operators $\hbar^2 H_0 + W$, $H_0 = -\Delta$, in $L^2(\mathbb{R}^d)$. The potential is given in the form $W = V + U$, where V is supposed to be in Kato's class. U is a positive potential barrier, i.e. we assume there is a region Γ in $\mathbb{R}^d$ such that $U(x) \geq U_0$, $x \in \Gamma$, where U_0 is a (large) positive value. Γ can be unbounded so that N-particle situations or periodic potentials used in solid state physics are included.
The Schrödinger semigroup $\exp\{-t\hbar^2\ (H_0 + \frac{1}{\hbar^2}V + \frac{1}{\hbar^2}U)\}$ is compared with the Dirichlet semigroup, the generator of which, denoted by $(\hbar^2 H_0 + V)_{\mathbb{R}^d \setminus \Gamma}$, is the Friedrichs extension of $\hbar^2 H_0 + V$ restricted to $L^2(\mathbb{R}^d \setminus \Gamma) \cap$ dom $(H_0 + V)$. It models the system with infinitely large U_0.
We give a quantitative estimate for the norm resolvent difference of the form

$$\| \, (H_0 + \tfrac{1}{\hbar^2}V + \tfrac{1}{\hbar^2}U + \tfrac{a}{\hbar^2})^{-1} - ((H_0 + \tfrac{1}{\hbar^2}V)_{\mathbb{R}^d \setminus \Gamma} + \tfrac{a}{\hbar^2})^{-1} \, \| \leq c \cdot d(\hbar),$$

if $\hbar$ is small. The value of $d(\hbar)$ depends on the size of Γ. If $\partial\Gamma$ is uniformly Lipschitz continuous and satisfies a cone condition then $d(\hbar) < \hbar^\alpha$, $0 < \alpha < 1$. The power α depends on the geometry of $\partial\Gamma$. This dependence is given. In the simplest case, where Γ is the halfspace, $d(\hbar) = \hbar^{\frac{2}{3}}$. The general case and details are formulated in [1].

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A TRACE FORMULA FOR SCHRÖDINGER OPERATORS AND QUANTIZATION ON SYMPLECTIC MANIFOLDS

B.V. Fedosov (Institute of Physics and Technology, Moscow)

For $a(x) \in C_0^\infty(\mathbb{R}^{2n})$ and $H(x,h) \sim H_0(x) + hH_1(x) + \ldots$ not depending on x at ∞ with real leading term $H_0(x)$, consider an expression

$$f(t,h) = tr\ \hat{a}\ e^{-\frac{i}{h}\hat{H}t};$$

where $\hat{a}$, $\hat{H}$ are Weyl pseudo-differential operators, corresponding to the symbols a, H. By a trace formula we mean an asymptotic expansion of f when $h \to 0$ and $t > 0$ is fixed.

THEOREM: *Let $\varphi(x,t)$ be a hamiltonian flow in $\mathbb{R}^{2n}$ corresponding to $H_0(x)$, M^0 be the set of fixed points of φ.*

1. *If supp $a \cap M^0 = \emptyset$ then $f = O(h^\infty)$*
2. *If supp $a \cap M^0$ is a finite set of non-degenerate fixed points, then*
   $f = \sum_{x \in M^0} \frac{e^{-\frac{i}{h}(H_0(x)+hH_1(x))t}}{\sqrt{det(1-\varphi'(x,t))}} a(x) + O(h).$

A similar theorem is proved in more general 'global' situations, when $\mathbb{R}^{2n}$ is replaced by a symplectic manifold (M,ω), admitting the quantization procedure.
As a consequence a theorem of Atiyah–Bott–Lefschetz type is obtained. Another interesting example is the Weyl character formula for classical groups.

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HEAT CONTENT ASYMPTOTICS

P.B. Gilkey (University of Oregon)

Let M be a compact Riemannian manifold with smooth boundary ∂M and let D be a second order operator on the space of smooth sections to a smooth vector boundle V. If $f_1 \in C^\infty(V)$ and $f_2 \in C^\infty(V^*)$, we define

$$\beta(f_1, f_2, D, \mathcal{B})(t) = \int_M e^{-tD_\mathcal{B}} f_1 \cdot f_2$$

where we impose suitable boundary conditions $\mathcal{B}$ to define the heat operator $D_\mathcal{B}$. For example, if $D = \Delta_0$ is the scalar Laplacian, if $\mathcal{B}$ is Dirichlet boundary conditions, and if $f_1 = f_2 = 1$, then β represents the total heat energy content of M with initial constant temperature and Dirichlet boundary conditions.
As $t \to 0^+$, there is an asymptotic series

$$\beta \sim \sum_{n=0}^{\infty} \beta_n(f_1, f_2, D, \mathcal{B}) t^{\frac{n}{2}}$$

where the β_n are locally computable. We compute β_n for $n \leq 4$ for Dirichlet, for $n \leq 6$ for Neumann, and for $n \leq 3$ for mixed boundary conditions. This is joint work with M. Van den Berg and S. Desjardins.

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LONG RANGE SCATTERING FOR THE NON- LINEAR SCHRODINGER (NLS) AND HARTREE EQUATION

J. Ginibre (Université de Paris Sud)
(in collaboration with T. Ozawa)

We study the theory of scattering and more precisely the problem of the existence of modified wave operators (W.O.) for the NLS equation

$$i\partial_t u = -(1/2)\Delta u + \lambda |u|^{p-1} u$$

and for the Hartree equation

$$i\partial_t u = -(1/2)\Delta u + \lambda(|x|^{-\gamma} * |u|^2)u$$

in space-time $\mathbb{R}^{n+1}$ in the Coulomb-like limiting case $p - 1 = 2/n$ (resp. $\gamma = 1$). In the same way as for the linear Schrödinger equation, the ordinary wave operators are expected to exist if the interaction decreases sufficiently fast at infinity in space, namely for $p - 1 > 2/n$ (resp. $\gamma > 1$). They are proved not to exist in the opposite case $p-1 \leq 2/n$ (resp. $\gamma \leq 1$). In that case one should define modified W.O. by replacing the free dynamics by a modified asymptotic dynamics in their definition. The present lecture is devoted to that problem in the Coulomb-like limiting case $p - 1 = 2/n$ (resp. $\gamma = 1$). We propose several modified asymptotic dynamics inspired by the linear case and taking advantage of the special commutation properties of the free Schrödinger group. We reduce the existence proof of the modified W.O. to solving an integral equation for the Cauchy problem with infinite initial time. We solve that equation by a contraction method for large times and a standard continuation to finite times, thereby proving the existence of modified W.O. defined on a dense set (in L^2) of suitably small asymptotic states, for the NLS equation with $p - 1 = 2/n$ in dimension $1 \leq n \leq 3$ and for the Hartree equation with $\gamma = 1$ in dimension $n \geq 2$. The modified W.O. satisfy the standard intertwining property.

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FRÉCHET ALGEBRAS IN THE PSEUDODIFFERENTIAL ANALYSIS AND AN APPLICATION TO THE PROPAGATION OF SINGULARITIES

B. Gramsch (Fachbereich Mathematik, Universität Mainz)

For a theory of analytic Fredholm functions in algebras of pseudo-differentialoperators and of operators on singular spaces it is convenient to consider, following order reduction, unital symmetric Fréchet algebras A continuously embedded in a C^*-algebra B with the properties: I) $A \cap B^{-1} = A^{-1}$ (spectral invariance); II) A is a countable projective limit of Banach algebras (cf. Operator Th. vol. 57, Birkhäuser 1992, 71 - 98, and the literature there). The properties I and II are invariant with respect to countable intersections in B. This leads to the possibility of localizations of I) and II) on singular spaces. E. Schrohe (Hab.-Thesis, Mainz 91/92) and K. Lorentz (Thesis, Mainz 91/92) made substantial contributions to this program on Ψ^*-algebras. The properties I and II help to overcome problems concerning the non linear functional analysis for the implicit function theorem in Fréchet spaces arising for Fredholm functions and the Oka principle (cf. e.g. Gromov 1989, Leiterer 1990). Now we can prove using only I and II for the set M of semi-Fredholm operators with kernel dimension $\nu < \infty$ in A

$$\pi_0(\mathcal{H}(\Omega,M)) \cong \pi_0(\mathcal{C}(\Omega, M)) \qquad \text{(Oka)},$$

where Ω is a Stein manifold, $\mathcal{H}$ the space of holomorphic and $\mathcal{C}$ the space of continuous mappings, π_0 denotes the set of connected components. The notion of a locally A-rational homogeneous space is very useful to treat special submanifolds of Fréchet spaces (cf. Math. Ann. 269 (1984) 27-71). Many open problems arise for the Oka principle connected to Fréchet spaces and Fréchet-Lie groups (e.g. boundary behavior; without I or II). For connections to the meromorphic inversion and the division problems for operator valued distributions see Gramsch, Kaballo, Integr. Eq. Op. Th. 12 (1989). For a "non commutative" approach to Hörmander's result on the propagation of singularities we use ideas of Helton (1977) and also of Cordes (1968, 1986) and M. Taylor (1976). Let X be a Banach space with dense inclusions $E \subset X \subset F$ for the locally convex vector spaces E and F. Let A be a subalgebra of $\mathcal{L}(X) \cap \mathcal{L}(E) \cap \mathcal{L}(F)$ with unit $e = \mathrm{Id}$ and B the norm closure of A in $\mathcal{L}(X)$; let $\pi : B \longrightarrow B/I$ be the homomorphism for the closed two-sided ideal I of B. With respect to the duality of the Banach spaces $(B/I)'$ and B/I we define the set $\mathfrak{M}$ of extreme points

$$\mathfrak{M} := \operatorname{extr}\left\{\mu \in (B/I)' : \|\mu\| \le 1,\ \mu(\pi(e)) = 1\right\}$$

and relative to A, $E \subset X \subset F$ and I the generalized wave front $\tilde{W}F(u)$ for $u \in F$ with $J_u = \left\{a \in A : a\,u \in E\right\}$

$$\tilde{W}F(u) := \left\{m \in \mathfrak{M} : \langle m, M_u \rangle = 0\right\}$$

where M_u denotes the norm closure of $\pi(J_u)$ in B/I and $\langle\cdot,\cdot\rangle$ the dual pairing of (B/I)' and B/I. Instead of $\mathfrak{M}$ we may consider the weak closure of $\mathfrak{M}$.
If the parameterized family α_t of isometric automorphisms of B/I has the left ideal M_u as an invariant set, $\alpha_t(M_u) \subset M_u$, then with $m \in \tilde{W}F(u)$ the generalized bicharacteristic strip $t \longrightarrow \alpha_t'(m)$ is contained in $\tilde{W}F(u)$ for the dual automorphisms $\alpha_t' : (B/I)' \longrightarrow (B/I)'$ of α_t. For an appropriate unbounded operator T on X, $T : D(T) \longrightarrow X$ we have in some cases the following realization of α_t with a strongly continuous group $e(t) = \exp(it\,T)$ on E, X and F. For $u \in D_F(T)$ and $[e(t)u]' = i\,e(t)T\,u$ we obtain $e(-t)a\,e(t)u \in E$ under the assumption $T\,u \in E$ and $a \in J_u$. If we assume

$$\alpha_t(\pi(b)) = \pi\,(e(-t)b\,e(t)),\ b \in B \ \text{ or } \ A\,,$$

we obtain for the singularity $m \in \tilde{W}F(u)$ and for $T\,u \in E$

$$t \longrightarrow \alpha_t'(m) \in \tilde{W}F(u)$$

corresponding to the dual family α_t'. The idea is that (in view of order reduction) the elements T of the "Lie algebra" of the group $G \subset \mathcal{L}(X)^{-1} \cap \mathcal{L}(E)^{-1} \cap \mathcal{L}(F)^{-1}$ for which $G \ni g \longmapsto g^{-1}\,a\,g$ generates an automorphism of A, qualify with $T\,u \in E$ for the above procedure. Using with the properties I and II appropriate specializations on compact manifolds one is in well known situations.

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ON A GEOMETRIC CRITERION FOR THE UNIFORM PARABOLIC HARNACK INEQUALITY ON A COMPLETE RIEMANNIAN MANIFOLD

A.Grigor'yan (Universität Bielefeld)

Let M be a smooth connected non–compact complete n–dimensional Riemannian manifold, $n \geq 2$. Let Δ denote the Laplace–Beltrami operator on M, $B_R(x)$ the geodesic ball of radius R centered at the point $x \in M$.

THEOREM 1 *Suppose that the following conditions (a) and (b) are satisfied for some positive constants A, a, N:*

(a) for any ball $B_R(x)$
$Vol\ B_{2R}(x) \leq A\ Vol\ B_R(x);$

(b) Poincaré inequality:
for any ball $B_{NR}(x)$ and any function $f \in C^\infty(B_{NR}(x))$
$$\int_{B_{NR}(x)} |\nabla f|^2 \geq \frac{a}{R^2} \int_{B_R(x)} (f - \overline{f})^2,$$

where $\overline{f} = \fint_{B_R(x)} f$ and all integrals are taken against the Riemannian volume. Then the uniform Harnack inequality for the heat equation is valid, in particular for any positive solution $u(x,t)$ of the heat equation $u_t - \Delta u = 0$ defined in a cylinder $C_0 = B_R(x_0) \times (0, R^2)$

$$\sup_{C_2} u \leq P \inf_{C_1} u$$

where $C_1 = B_{\frac{R}{2}}(x_0) \times (\frac{3}{4}R^2, R^2)$, $C_2 = B_{\frac{R}{2}}(x_0) \times (\frac{1}{4}R^2, \frac{1}{2}R^2)$ and $P = P(A, a, N)$.

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TUNNELING EFFECT FOR THE KLEIN–GORDON EQUATION

B. Helffer (Ecole Normale Supérieure, Paris)

This is a report on work in progress with B. Paresse.
We consider the Klein–Gordon equation

$$\sqrt{1-h^2\Delta}+V$$

on $L^2(\mathbb{R}^n)$, with $h>0$. We assume $\lim_{|x|\to\infty} V=-1$

$$|D^\alpha V| \leq C_\alpha \quad , \quad \forall \alpha \in \mathbb{N}^n$$

$$V(x', -x_n) = V(x', x_n)$$

and that V has two non–degenerate minima ($\min V < -1$). Then we prove that the splitting between the two smallest eigenvalues λ_2^{KG}, λ_1^{KG} satisfies

$$\lambda_2^{KG}(h) - \lambda_1^{KG}(h) = O_\varepsilon(e^{-\frac{S_0}{h}+\frac{\varepsilon}{h}}); \quad \forall \varepsilon > 0$$

where S_0 is the Agmon distance between the two minima for the Agmon–metric associated to the potential

$$W^{KG} = (1-((1+\min V - V)_+)^2)_+.$$

Moreover we get identity in the case where for example

$$V < 1 + \min V.$$

The question of identity is open when this inequality is not satisfied (see however results by R. Carmona, W. Masters and B. Simon). One problem is the singularity of $\xi \to \sqrt{1+\xi^2}$ at the points where $\xi^2 = -1$.
The comparison with corresponding results for the Dirac operator obtained by X. P. Wang appears to be very interesting.

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REMARKS ON HOLMGREN'S UNIQUENESS THEOREM

L. Hörmander (University of Lund)

1. Holmgren's uniqueness theorem can be regarded as the combination of a microlocal non-characteristic analyticity theorem and the fact that for any distribution (or hyperfunction) u

$$N(\operatorname{supp} u) \subset WF_A(u). \tag{1}$$

Recall (see [1, Chap. 8, 9]) that if F is a closed subset of a C^2 manifold X, then the exterior conormal set $N_e(F) \subset T^*(X) \setminus 0$ is defined as the set of all (x^0, ξ^0) such that $x^0 \in F$ and there exists a real valued function $f \in C^2(X)$ with $df(x^0) = \xi^0 \neq 0$ and

$$f(x) \leq f(x^0) \quad \text{when } x \in F \cap U, \tag{2}$$

for some open neighborhood U of x^0. We write $N(F) = \{(x, \pm\xi); (x, \xi) \in N_e(F)\}$. A stronger result than (1) is due to Kashiwara: If $(x^0, \xi^0) \in N_e(\operatorname{supp} u)$, then

$$(x^0, \xi) \in WF_A(u) \implies (x^0, \xi + t\xi^0) \in WF_A(u) \quad \text{for all } t \in \mathbf{R} \text{ with } \xi + t\xi^0 \neq 0. \tag{3}$$

It can be further improved by taking curvature properties into account. If F is a closed set and $(x^0, \xi^0) \in N_e(F)$, then the set G_{x^0,ξ^0} of all $f \in C^2(X)$ such that $f(x^0) = 0, f'(x^0) = \xi^0$, and (2) holds for some open neighborhood U of x^0, depending on f, is a convex set. For $f_0 \in G_{x^0,\xi^0}$ the invariantly defined set

$$Q_{x^0,\xi^0} = \{(f - f_0)''(x^0); f \in G_{x^0,\xi^0}\} \tag{4}$$

is a convex set in the symmetric tensor product $S^2(T^*_{x^0}(X))$ consisting of quadratic forms on $T_{x^0}(X)$, and it has a closed convex asymptotic cone $\widehat{Q}_{x^0,\xi^0}$ independent of the choice of f_0, containing all negative semidefinite forms. The positive semidefinite forms of maximal rank in $\widehat{Q}_{x^0,\xi^0}$ have the same radical, and the annihilator $\varrho(x^0, \xi^0)$ in $T^*_{x^0}$ contains ξ^0 and $-\xi^0$. It is independent of ξ^0 when ξ^0 is in the relative interior of $\{\xi; (x^0, \xi) \in N_e(F)\}$, and we define $N_{\text{eff}}(F)_{x^0} = \varrho(x^0, \xi^0)$ then. A proof of an *extension of* (1) *to arbitrary* $(x^0, \xi^0) \in N_{\text{eff}}(\operatorname{supp} u)$ was outlined; it can also be used to extend (3) similarly.

2. Counterexamples show that the conclusion of Holmgren's uniqueness theorem is in general false for differential operators with C^∞, non-analytic coefficients. However, Robbiano [3] proved recently a related uniqueness theorem for wave equations $\partial^2 u/\partial t^2 - A(x, D_x)u = 0$ with coefficients independent of t. In [2] we have proved a more precise and general version of this result which shows that for solutions u of second order hyperbolic equations invariant under the flow along a timelike vector field θ the normal set $N(\operatorname{supp} u)$ is located between the characteristic surface and a dilation of it in a fixed ratio $K \in [1, \sqrt{27/23}]$ along the annihilator of θ. It is not known if this is true with $K = 1$ which would mean that the conclusion of Holmgren's theorem is valid for such operators.

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Box 118, S-221 00 Lund, Sweden
E-mail address: lvh@maths.lth.se

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THE INTERBAND LIGHT ABSORPTION COEFFICIENT FOR HEAVILY DOPED SEMICONDUCTORS

W. Kirsch (Universität Bochum)
(joint work with L. Pastur and B. Koruzhenko, Kharkov, Ukraine)

We define and investigate the interband light absorption coefficient (ILAC) for semiconductors with random impurities. The semiconductor is described in a two-band model by two Schrödinger operators $H^{\pm} = -\Delta \pm V_W$, where V_W is a random (ergodic) potential. The interband light absorption coefficient $a(\lambda)$ measures the amount of light of a given energy λ that can be absorbed by the semiconductor per unit volume. It is defined as a thermodynamic limit of a function of operators $H^{\pm}$ restricted to bounded domains of $\mathbb{R}^d$. This function involves both the eigenvalues and the eigenfunctions of the restricted operators. The Laplace transform of the limit $a(\lambda)$ can be expressed through certain (double) Wiener integrals. This expression is used to investigate the asymptotic behaviour of the ILAC for large or small energies.

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SOME PRODUCT FORMULAS WITH ERROR ESTIMATES

S.T. Kuroda (Gakushuin University of Tokyo)

This research is motivated by a study of explicit methods in numerical analysis for computing the solution $\exp(-itH)u$ of the Schrödinger equation. In general terms the method may be formulated as $\exp(-itH)u = V(t/n)^n u + R_n(t)u$, where $V(t)$ is a small time approximation of $\exp(-itH)$ and $R_n(t)u$ is an error term.
When $H = A + B$, the usefulness of the Lie-Trotter approximation $V_1(t) = \exp(-itA)$ $\exp(-itB)$ or its symmetrized form $V_2(t) = \exp(-itA/2)\exp(-itB)\exp(-itA/2)$ is emphasized by several authors. Provided that A and B are bounded, the order of error is $R_n(t) = O(1/n)$ for V_1 and $= O(1/n^2)$ for V_2.
In this talk the following two questions are discussed.

(1) Are there any product formulas of symmetrized type which give an error of higher order? As an answer a (unique) $O(1/n^4)$ formula is explicitly computed. The formula contains seven exponential factors.

(2) How to estimate the error, knowing the order of error? In the case of bounded generators A, B, estimates in the operator norm are easily obtained. In the unbounded case the order of error depends on how "regular" the initial vector u is. The "regularity" is expressed by the condition that u belongs to the domain of a fractional power of $A + B$.

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THE TIME-ASYMPTOTIC BEHAVIOUR OF SOLUTIONS OF INITIAL VALUE PROBLEMS IN ELASTICITY FOR MEDIA WITH CUBIC SYMMETRY

R. Leis (Universität Bonn)

Considerable progress has been made proving the existence of global in time smooth solutions for nonlinear wave equations and small data during recent years. To do so, on the one hand one uses local existence theorems for the nonlinear equation, and on the other hand estimates on the spreading of the solutions of the linearized equation. The idea then is to combine both, choose the data small enough such that the linear solution lives long enough until the linear spreading effect takes over and finally prevents the solution from exploding.

Little work has been done for equations with underlying anisotropic media. Therefore estimates of this kind for solutions to linear equations of elasticity and media with cubic symmetry are presented in the lecture. The free space problem can be completely discussed whereas in case of exterior boundary value problems an additional damping term has to be inserted.

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RADIATION CONDITIONS FOR ELLIPTIC PROBLEMS IN DOMAINS WITH NON-SMOOTH BOUNDARY

B.A. Plamenevsky (Inst. of Telecommunications, St. Petersburg)

The present talk is a survey of a number of papers by S.A. Nazarov and B.A. Plamenevsky. Correct statements of elliptic boundary value problems involving radiation conditions on singularities of boundary are discussed. One introduces the notions of outgoing and incoming waves.(In the classical situation where the Sommerfeld and Mandelstam principles are applicable, these notions agree with definitions adopted in those principles). Such a classification of waves allows one to give a statement of the problem with natural radiation conditions and establish its solvability. The motivation of such considerations is the intention to develop a rigorous theory of the "threshold effect" and the Wood anomalies in the elasticity theory.

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THE SPHERICALLY SYMMETRIC VLASOV-EINSTEIN SYSTEM

G. Rein (Universität München)
(joint work with A.D. Rendal)

Consider a large ensemble of mass points interacting by a selfconsistent gravitational field, e.g. the stars in a galaxy. Such a system is usually described by a distribution function on phase space, satisfying a continuity equation, the Vlasov equation, coupled to a field equation for the gravitational field. In the Newtonian setting this leads to the Vlasov-Poisson system. The relativistic analogue is the Vlasov-Einstein system where gravity is described by Einstein's field equation of general relativity. We have the following results on this system:

Global existence of solutions of the spherically symmetric Vlasov-Einstein system with small initial data (to appear in Commun. in Math. Phys.),

The Newtonian limit of the spherically symmetric Vlasov-Einstein system (to appear in Commun. in Math. Phys.),

Smooth static solutions of the spherically symmetric Vlasov-Einstein system (to appear in Ann. d' Inst. H. Poincaré, Phys. Theor.).

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DIRICHLET FORMS, ANALYTIC CONTRACTION SEMIGROUPS, AND CORRESPONDING MARKOV PROCESSES

M. Röckner (Universität Bonn)

In the first part of the talk an overview of the general theory of (non-symmetric) Dirichlet forms on general state spaces was given. In particular, the corresponding semigroups $(T_t)_{t>0}$, $(\hat{T}_t)_{t>0}$, resolvents $(G_\alpha)_{\alpha>0}$, $(\hat{G}_\alpha)_{\alpha>0}$ and generators L, $\hat{L}$ were characterized. Subsequently, an analytic characterization of the class of Dirichlet forms which are associated with pairs of right continuous strong Markov processes was presented. Finally, examples on finite and infinite dimensional state spaces were described.
In the second part of the talk a solution to the problem of 'Dirichlet (or Markov) uniqueness' for generalized Schrödinger operators on $\mathbb{R}^d$ was given. This result extends to corresponding infinite dimensional situations.

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THE REDUCED WAVE EQUATION WITH TWO UNBOUNDED MEDIA

Yoshimi Saitō (University of Alabama, Birmingham)

Consider the self-adjoint operator H defined by

$$\begin{cases} D(H) &= H^2(I\!R^N) \subset \mathcal{H} = L_2(I\!R^N, \mu(x)dx), \\ Hu &= -\frac{1}{\mu(x)}\Delta, \end{cases}$$

where $H^2(I\!R^N)$ is the Sobolev space of the second order, and

$$\mu(x) = \begin{cases} \mu_1 & (x \in \Omega_1), \\ \mu_2 & (x \in \Omega_2), \end{cases}$$

$\mu_1, \mu_2 > 0$, $\mu_1 \neq \mu_2$ with

$$\begin{cases} \Omega_1 = & \{x = (x_1, ..., x_N) \in I\!R^N \quad | \; x_N > \varphi(x_1, ..., x_{N-1})\}, \\ \Omega_2 = & \{x = (x_1, ..., x_N) \in I\!R \quad | \; x_N < \varphi(x_1, ..., x_{N-1})\}. \end{cases}$$

The separating surface

$$S = \{x = (x_1, ..., x_N) \in I\!R^N \; | \; x_N = \varphi(x_1, ..., x_{N-1})\}$$

is assumed to have a 'cone-like' shape such that its vertex is at the origin and its axis is the positive (or negative) x_N-axis. We shall discuss the limiting absorption principle for the operator H with emphasis on finding the appropriate boundary condition at ∞ (the radiation condition) for H.

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ELLIPTIC PAIRS AND INDEX THEOREMS

J.-P. Schneiders, P. Schapira (Université Paris Nord)

An elliptic pair $(\mathcal{M}, F)$ on a complex analytic manifold X is the data of bounded complex of coherent $\mathcal{D}_X$-modules $\mathcal{M}$ and a bounded complex of $\mathbb{R}$-constructible sheaves F such that

$$char\ \mathcal{M} \cap SSF \subset T^*_X X$$

Let X be a complexification of a real analytic manifold M and let $\mathcal{M}$ be a $\mathcal{D}_X$-module which represents a classical system of equations on X which is elliptic along M. Then $(\mathcal{M}, \mathbb{C}_M)$ is an elliptic pair. Let X be an arbitrary complex analytic manifold and let $\mathcal{M}$ be a coherent $\mathcal{D}_X$-module, U an open subset of X with non characteristic boundary, F an $\mathbb{R}$-constructible sheaf and J a coherent $\mathcal{O}_X$-module then the pairs $(\mathcal{M}, \mathbb{C}_X)$, $(J \otimes_{\mathcal{O}_X} \mathcal{D}_X, \mathbb{C}_X)$, $(\mathcal{O}_X, F)$, $(\mathcal{M}, \mathbb{C}_U)$ are elliptic.
The extension to elliptic pairs of the classical elliptic regularity allows us to prove that the solution complex $RHom_{\mathcal{D}_X}(\mathcal{M} \otimes F, \mathcal{O}_X)$ of an elliptic pair has finite dimensional cohomology if $supp\ \mathcal{M} \cap supp\ F$ is compact and to identify its dual with the solution complex of the dual pair. These results are also true in a relative situation. To any elliptic pair $(\mathcal{M}, F)$ we associate by a diagonal construction a microlocal Euler class $\mu\ eu(\mathcal{M}, F) \in H^{2\kappa}_{char\ \mathcal{M}+SSF}(T^*X; \mathbb{C})$. The integral of this class along $T^*_X X$ gives us the index $\chi(RHom_{\mathcal{D}_X}(\mathcal{M} \otimes F, \mathcal{O}_X))$. We can prove that $\mu\ eu(\mathcal{O}_X, F)$ is the characteristic cycle of F defined by $\mathcal{M}$. Kashiwara and that $\mu\ eu(\mathcal{M}, F) = \mu\ eu(\mathcal{M}) * \mu\ eu(\mathcal{O}_X, F)$. Assuming the right $\mathcal{D}_X$ module $\mathcal{M}$ endowed with a good filtration we define the microlocal Chern character of $\mathcal{M}$ to be

$$\mu\ ch(\mathcal{M}) = ch_{char\mathcal{M}}(\mathcal{O}_{T^*X} \underset{\pi^{-1}gr\mathcal{D}_X}{\otimes} \pi^{-1}gr\ \mathcal{M}) \cup \pi^* td(X)$$

and conjecture that $\mu\ eu(\mathcal{M}) = \mu\ ch^{2\kappa}(\mathcal{M})$.

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BOUNDARY VALUE PROBLEMS ON NONCOMPACT MANIFOLDS

Elmar Schrohe (Universität Mainz and Max-Planck-Arbeitsgruppe, Universität Potsdam)

After earlier work by Vishik and Eskin, Boutet de Monvel's calculus, established in 1971, showed a new way of treating boundary value problems by pseudodifferential methods. In particular, it gave necessary and sufficient conditions for the Fredholm property of boundary value problems on smooth compact manifolds.
It has been an open problem to treat the noncompact case. For a large class of noncompact manifolds, a solution is presented. It starts with the construction of a Boutet de Monvel type calculus for a class of "weighted" symbols. On $\mathbb{R}^n$, these symbol classes were introduced by Shubin, Parenti, and Cordes. For $m = (m_1, m_2) \in \mathbb{R}^2$ one requires that

$$|D_\xi^\alpha D_x^\beta p(x,\xi)| \leq C_{\alpha\beta} < \xi >^{m_1-|\alpha|} < x >^{m_2-|\beta|} .$$

These symbols can be transferred to manifolds with a compatible structure (Schrohe 1987).
The main results are the following:

(1) The algebra $\mathcal{G}$ of operators of order and type zero is a spectrally invariant Fréchet subalgebra of $\mathcal{L}(H)$, H a suitable Hilbert space, i.e.

$$\mathcal{G} \cap \mathcal{L}(H)^{-1} = \mathcal{G}^{-1}.$$

It is a Ψ^*-subalgebra of $\mathcal{L}(H)$ in the sense of Gramsch (1984).

(2) Focusing on the elements of order and type zero is no restriction since there are order reducing operators within the calculus.

(3) There is a necessary and sufficient criterion for the Fredholm property of boundary value problems, based on the invertibility of symbols modulo lower order symbols.

(4) There is a functional calculus for the elements of $\mathcal{G}$ in several complex variables.

(5) There is a Fedosov type index formula on the half-space $\mathbb{R}^n_+$.

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ASYMTOTICS FOR ELLIPTIC OPERATORS CLOSE TO HIGHER CORNERS

B.-W. Schulze (Max-Planck-Arbeitsgruppe, Universit"at Potsdam)

The asymptotics of solutions of elliptic equations on manifolds with a piece-wise smooth geometry may be understood as a sort of elliptic regularity. For corners of higher orders, understood as singular sets of stratified spaces, this type of result seems to be inaccessible without a systematic approach in terms of pseudo-differential operators with a sufficiently rich symbolic structure. It turns out that hierarchies of leading symbols, associated with the "lower-dimensional skeletons", partially being operator-valued, are the appropriate structure for the concept of ellipticity. The ellipticity is defined as invertibility of all components. Parametrices follow by inverting symbols. They can be described now up to second order corners (locally being cones with bases having conical singularities, again) in such a precise manner that the asymptotics actually follow by applying a parametrix from the left, using the mapping properties of operators in the calculus between weighted Sobolev spaces with asymptotics. The asymptotics for second order corners contain a cone part with double asymptotics in the two singular directions $\mathbb{R}_+ \times \mathbb{R}_+ \ni (t,r)$, for $t \to 0, r \to 0$ and Mellin edge asymptotics on the one-dimensional edges, emanated from the corners.
The latter part corresponds to a reformulation of the edge calculus, first being given in the operator convention from the Fourier transform, into a Mellin set-up, which can similarly be established as Mellin operator conventions in the cone theory. This also determines the nature of corner Sobolev spaces for $t \to 0, s \to 0$. The first part of the asymptotics contains global data in terms of points of non-bijectivity of an action globally along the base of the corner which has conical points. This is an occasion, where the aspect of cone operator pseudo-differential algebras is necessary, also in parameter-dependent form with the parameter in the complex Mellin plane, further holomorphic and meromorphic families and the invertibility within that class.
Technique and results may be found in

B.-W. Schulze: Pseudo-differential operators on manifolds with singularities. North Holland 1991,

B.-W. Schulze: The Mellin pseudo-differential calculus on manifolds with corners. Teubner-Texte zur Mathematik vol. 131 Leipzig 1992, pp. 208-289

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MATHEMATICAL QUESTIONS OF QUANTUM MECHANICS

I.M. Sigal (University of Toronto)

In my talk I reviewed some recent results and open problems in the rigorous treatment of Quantum Mechanics of many particle systems. I discussed two groups of problems: (a) Scattering Theory and (b) Bound State Problem. In the first case I reported on new results on the central problem of asymptotic completeness (work with A. Soffer) and in the second case, on new results on the problem of asymptotics of ground states of large atoms and molecules (work with V. Ivrii). Unfortunately, because of lack of time I had to omit the discussion of the problems of non-linear perturbation of quantum systems (paper to appear in the CMI).

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FEYNMAN-KAC SEMIGROUPS AND HARMONIC FUNCTIONS

J.A. van Casteren (University of Antwerp)

Let E be a locally compact second countable Hausdorff space and let K_0 be the generator of a Feller semigroup $\{\exp(-tK_0) : t \geq 0\}$ in $C_\infty(E)$. Suppose that there exists a Radon measure m and a symmetric continuous function $p_0 : (0,\infty) \times E \times E \to [0,\infty)$ with the following properties:

(i) The identity of Chapman-Kolmogorov is valid.

(ii) It is symmetric: $p_0(t,x,y) = p_0(t,y,x), t > 0, x,y \in E$;

(iii) It is continuous in the sense that for all open subsets U of E and for all $x \in U$, the following identity is valid: $\lim_{t\downarrow 0} \int_U p_0(t,x,y)dm(y) = 1$;

(iv) If $0 \leq f \leq 1,\ f \in C_\infty(E)$, then $0 \leq \int p_0(t,x,y)f(y)dm(y) \leq 1,\ t > 0, x \in E$;

(v) The semigroup $\{\exp(-tK_0) : t \geq 0\}$ is $L^1 - L^\infty$–smoothing.

In addition let $V : E \to [-\infty,\infty]$ be a so-called Kato-Feller potential, i.e. suppose that for every compact subset K of E the following identity is valid:

$$\lim_{t\downarrow 0} \sup_{x\in E} \int_0^t ds \int_E dm(y) p_0(s,x,y)(V_-(y) + 1_K(y)V_+(y)) = 0.$$

Moreover let Γ be closed subset of E, that may be considered as a singularity region carrying an infinitely high potential barrier. Some estimates are given for the trace and Hilbert-Schmidt norms of operator of the form

$$D_\Sigma(t) = \exp(-t(K_0 + V)) - J^* \exp(-t(K_0 + V)_\Sigma)J$$

in terms of (generalized harmonic functions) h_{a+V}, defined by

$$h_{a+V}(x) = E_x\Big(\exp\Big(-\int_0^S (a + V(X(u)))du\Big) : S < \infty\Big).$$

Here S is the hitting time of the singularity region $\Gamma = E\backslash\Sigma$:

$$S = \inf\{s > 0 : X(s) \in E\backslash\Gamma\}$$

and the Dirichlet semigroup $\{\exp(-t(K_0 + V)_\Sigma) : t \geq 0\}$ is defined by

$$[\exp(-t(K_0 \dot{+} V)_\Sigma)f](x) = E_x\Big(\exp\Big(-\int_0^t V(X(u))du\Big) f(X(t)) : S > t\Big).$$

The following result can be proved.

Proposition. (a) If, for some $a \in I\!R$, the integral $\int h_{a+V}(x)^{1/2}dm(x)$ is finite, then the operators $D_\Sigma(t),\ t > 0$, are of trace class.

(b) If, for a sufficiently large, the function h_{a+V} belongs to $L^2(E,m)$, then the operators $D_\Sigma(t)$, $t > 0$, are Hilbert-Schmidt.

We conjecture that in assertion (a) the condition that the integral $\int h_{a+V}(x)^{1/2}dm(x)$ be finite, may be replaced with an L^1-condition on h_{a+V}. In [2] Stollmann obtains quite similar results. However, he uses techniques from operator ideals, whereas in the proofs we employ more straight-forward estimates. In [4] and in [3] some other closely related results were announced. As a corollary we have the following.

Corollary. If the functions $P_{(.)}(S < 1)$ and $V_-(.)^{1/2}P_{(.)}(S < 1)$ both belong to $L^2(E,m)$, then there exists $a \in \mathbb{R}$ such that h_{a+V}, is also a member of $L^2(E,m)$.

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PERTURBATION OF DIRICHLET FORMS BY MEASURES

J. Voigt (Universität Oldenburg)

The presented results were obtained jointly with P. Stollmann.
Let X be a locally compact space, B the σ-algebra of Borel sets, m a Borel-Radon measure on X. Let h be a regular Dirichlet form in $L_2(m)$, H the corresponding self-adjoint operator.
The object is to present a class of Borel measures μ such that "$H+\mu$" can be defined, and moreover to discuss properties of the semigroup $(e^{-t(H+\mu)}; t \geq 0)$. The operator "$H+\mu$" will be defined as corresponding to the form $h+\mu$, where the form μ is defined as

$$\mu[u,v] = \int u\bar{v}d\mu,$$

on a suitable domain.
1. The suitable class for obtaining a closed form $h+\mu$ - and thus a self-adjoint operator - is

$$M_0 := \{\mu : B \to [0,\infty)\ \sigma - \text{additive; } \operatorname{cap}(B) = 0 \Rightarrow \mu(B) = 0\}$$

2. In order to obtain a form $h-\mu$ such that the corresponding self-adjoint operator has "good" properties the measure μ should belong to a suitably defined "Kato class".
3. If $\mu_+ \in M_0$, μ_- in the "Kato class", then $h - \mu_- + \mu_+$ is a closed form, with corresponding self-adjoint operator H_μ, enjoying the following properties:
(a) $(e^{-tH_\mu};\ t \geq 0)$ acts as a C_0-semigroup on $L_p(\mu)$, for all $p \in [1,\infty]$.
(b) If $e^{-tH} : L_1(m) \to L_\infty(m)$ for all $t > 0$, then the same is true for e^{-tH_μ}.
(c) If $(e^{-tH};\ t \geq 0)$ extends to a holomorphic semigroup on $L_1(m)$ then the same is true for $(e^{-tH_\mu};\ t \geq 0)$.

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Adress-list of Participants

S. Albeverio	Mathematisches Institut Universität Bochum Universitätsstr. 150 W-4630 Bochum
M. Baranowsky	FB Mathematik Technische Universität Str. des 17. Juni 136 W-1000 Berlin 12
S. Behm	Max-Planck-Arbeitsgruppe "Partielle Differentialgleichungen und Komplexe Analysis" FB Mathematik, Universität Potsdam Am Neuen Palais 10 0-1571 Potsdam
E. Behrends	Institut für Mathematik I Freie Universität Berlin Arnimallee 2-6 W-1000 Berlin 33
Yu. M. Berezansky	Institute of Mathematics Ukrainian Academy of Sciences Repin Str. 3 252601 Kiev 4 Ukraine
M.S. Birman	Dep. of Mathematical Physics State University of St. Petersburg ul. Uljanowa 198904 St. Petersburg Russia

A. Boutet de Monvel — U.F.R. de Mathématiques
Université Paris VII
2, place Jussieu
75251 Paris Cedex 05
France

L. Boutet de Monvel — - Mathématique-
Université de Paris VI
4, place de Jussieu
75232 Paris Cedex 05
France

T. Buchholz — FB Mathematik
Universität Potsdam
Am Neuen Palais 10
0-1571 Potsdam

U. Bunke — Max-Planck-Institut für Mathematik
Gottfried-Claren-Str. 26
W-5300 Bonn 3

J.M. Combes — Dep. de Mathématiques
Centre Université de Toulon
83130 La Garde
France

M. Demuth — Max-Planck-Arbeitsgruppe
"Partielle Differentialgleichungen und Komplexe Analysis"
FB Mathematik, Universität Potsdam
Am Neuen Palais 10
0-1571 Potsdam

H.D. Doebner — Institut für Theoretische Physik
Technische Universität Clausthal
W-3392 Clausthal-Zellerfeld

C. Dorschfeldt — Max-Planck-Arbeitsgruppe
"Partielle Differantielgleichungen und Komplexe Analysis"
FB Mathematik, Universität Potsdam
Am Neuen Palais 10
0-1571 Potsdam

B.V. Fedosov
Moscow Institute of Physics and Technology
Dolgoprudny
Moscow region 141700
Russia

P.B. Gilkey
Dep. of Mathematics
University of Oregon
Eugene OR 97403-1222
USA
z.Z. Max-Planck-Institut für Mathematik, Bonn

J. Ginibre
Lab. de Phys. Theor.
et Hautes Energies
Université de Paris Sud
91405 Orsay Cedex
France

B. Gramsch
FB Mathematik
Universität Mainz
Saarstr. 21
W-6500 Mainz

A. Grigor'yan
Fakultät für Mathematik
Universität Bielefeld
Universitätsstr.
W-4800 Bielefeld

B. Helffer
Dep. de Mathématiques et d' Informatique
Ecole Normale Superieure
45, rue d' Ulm
75230 Paris Cedex 05
France

T. Hirschmann	Max-Planck-Arbeitsgruppe "Partielle Differentialgleichungen und Komplexe Analysis" FB Mathematik, Universität Potsdam Am Neuen Palais 10 0-1571 Potsdam
Hochmuth	Inst. für Mathematik I Freie Universität Arnimallee 2-6 W-1000 Berlin 33
L. Hörmander	Dep. of Mathematics Lunds University Box 118 22100 Lund Sweden
D. Hundertmark	Inst. für Mathematik Ruhr-Universität Bochum Postfach 102148 W-4630 Bochum
W. Kirsch	Inst. für Mathematik Ruhr-Universität Bochum Postfach 102148 W-4630 Bochum
S.T. Kuroda	Dep. of Mathematics Faculty of Sciences Gakushuin University 1-5-1 Mejiro, Toshima-ku Tokyo 171 Japan
R. Leis	Inst. für Angewandte Mathematik Universität Bonn Wegeler Str. 10 W-5300 Bonn
I.S. Louhivaara	Inst. für Mathematik I Freie Universität Berlin Arnimallee 2-6 W-1000 Berlin 33
M. Lesch	Inst. für Mathematik Universität Augsburg Universitätsstr. 8 W-8900 Augsburg

H. Neidhardt	FB Mathematik Technische Universität Berlin Str. des 17. Juni 136 W-1000 Berlin 12
B.A. Plamenevsky	Math. Dep. of Electrotechn. Inst. of Telecommunications 61, ul. Mojki 191065 St. Petersburg Russia
G. Rein	Mathematisches Inst. Universität München Theresienstr. 39 W-8000 München 2
M. Röckner	Inst. für Angewandte Mathematik Universität Bonn Wegeler Str. 6 W-5300 Bonn 1
Y. Saito	University of Alabama Birmingham UAB-station 35294 Birmingham, Alabama USA
J.-P. Schneiders	Dept. de Mathématiques CSP Univ. Paris Nord 93430 Villetaneuse France
E. Schrohe	Max-Planck-Arbeitsgruppe "Partielle Differentialgleichungen und Komplexe Analysis" FB Mathematik, Universität Potsdam Am Neuen Palais 10 0-1571 Potsdam

B.-W. Schulze	Max-Planck-Arbeitsgruppe "Partielle Differentialgleichungen und Komplexe Analysis" FB Mathematik, Universität Potsdam Am Neuen Palais 10 0-1570 Potsdam
S. Seifarth	Inst. für Angewandte Analysis und Stochastik Hausvogteiplatz 5-7 0-1086 Berlin
I.M. Sigal	Dept. of Mathematics University of Toronto Toronto Canada
F. Steiner	II. Inst. für Theoretische Physik Universität Hamburg Luruper Chaussee 149 W-2000 Hamburg 50
P. Stollmann	FB Mathematik Universität Frankfurt Robert-Mayer-Str. 6-8 W-6000 Frankfurt 1
J. Stubbe	Dep. of Theoretical Mathematics University of Geneva 1211 Geneva 4 Switzerland
J. van Casteren	Dep. of Mathematics and Computer Science University of Antwerp Universitetsplein 1 2610 Wilrijk/Antwerp Belgium
J. Voigt	FB Mathematik Universität Oldenburg Ammerländer Heerstr. 114-118 W-2900 Oldenburg
T. Weidl	Dep. of Mathematical Physics State University of St. Petersburg ul. Uljanowa 1 Stary Peterhof, 198904 St. Petersburg Russia