DEGENERATE HYPERSURFACES WITH A TWO-PARAMETRIC FAMILY OF AUTOMORPHISMS

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ABSTRACT. We give a complete classification of Levi-degenerate hypersurfaces of finite type in $\mathbb{C}^2$ with two-dimensional symmetry groups. Our analysis is based on the classification of two-dimensional Lie algebras and an explicit description of isotropy groups for such hypersurfaces, which follows from the construction of Chern-Moser type normal forms at points of finite type, developed in [11].

1. Introduction

Automorphism groups are important geometric invariants, providing a natural way to classify various geometric structures. The problem of classifying real hypersurfaces in complex space according to the dimensions of their symmetry groups can be traced back to É. Cartan, who in [4] listed all homogeneous hypersurfaces in $\mathbb{C}^2$. At the same time, he determined the dimensions of their symmetry groups.

Cartan excluded Levi flat manifolds, which have no local invariants. Owing to their homogeneity, Cartan's hypersurfaces are necessarily Levi-nondegenerate everywhere. In the non-homogeneous case the hypersurface may contain Levi degenerate points, which may be of finite or infinite type.

The notion of finite type plays an important role in the study of bounded domains in complex space and their holomorphic mappings. We note that, if the boundary of a bounded domain is real-analytic, then all boundary points are of finite type. Thus, any such domain, which is not strictly pseudoconvex, necessarily contains Levi degenerate points of finite type.

There are numerous results on extension of holomorphic mappings from a domain of finite type to its boundary and from (parts of) the boundary to the domain. This allows one to pass between domains and their boundaries back and forth (e.g. [2], [6], [14]).

The local geometry at Levi non-degenerate points was studied by Chern and Moser [5]. Their results provided the crucial step towards a complete classification of local symmetry groups, which was achieved by Beloshapka [3] and Kruzhilin and Loboda [13]. For Levi degenerate hypersurfaces of finite type in $\mathbb{C}^2$ the corresponding results were derived in [11], [12].

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Our aim is to use those tools alongside with the classical methods of Lie and Cartan to understand global symmetries of bounded domains in $\mathbb{C}^2$.

In recent years variations of this problem have been studied intensively. In complex dimension two, the works of Isaev-Krantz and Isaev ([7], [8], [9]) give a complete description of hyperbolic manifolds with symmetry groups of dimension three or higher. Note that dealing with 1-dimensional symmetry groups presents no difficulty, so the only remaining interesting case would be to classify hypersurfaces with 2-dimensional symmetry groups. This is the problem that we study in this paper.

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2. Preliminaries

In this section, we consider a real-analytic hypersurface $M \subseteq \mathbb{C}^2$ and a point $p \in M$ of finite type k.

For local description of M in a neighbourhood of p, we will use local holomorphic coordinates (z, w) centered at p, where $z = x + \mathrm{i}\, y,\ w = u + \mathrm{i}\, v$, and such that the hyperplane $\{v = 0\}$ is tangent to M at p. M is then described near p as the graph of a uniquely determined real valued function

$$v = \Phi(z, \bar{z}, u).$$

Recall that $p \in M$ is a point of finite type if and only if there exist local holomorphic coordinates such that M is given by

$$v = P_k(z, \bar{z}) + o(|z|^k + |u|), \tag{1}$$

where

$$P_k(z, \bar{z}) = \sum_{j=1}^{k-1} a_j z^j \bar{z}^{k-j} \tag{2}$$

is a nonzero real-valued homogeneous polynomial of degree k without harmonic terms.

For $k = 2$, M is Levi-nondegenerate in a neighbourhood of p, and $P_2(z, \bar{z}) = |z|^2$. The classical work of Chern and Moser provides a complete normal form for this class of hypersurfaces (see [5]). As an immediate consequence, this gives an estimate of the dimension of the local isotropy group, but not its exact value.

Let $\mathrm{Aut}(M, p)$ denote the local isotropy group of a hypersurface M at a point p. The analysis of such groups in the non-degenerate case was completed by Beloshapka [3] and by Kruzhilin and Loboda [13].

Throughout this paper we consider the degenerate case, and assume that $k > 2$. We use the equation (2) to define two important integer-valued invariants. The first one, denoted by e, is the smallest integer such that $a_e \neq 0$.

For $e < \frac{k}{2}$, we define the second invariant as follows. Let $e = m_0 < m_1 < \cdots < m_s < \frac{k}{2}$ be the indices in (2) for which $a_{m_i} \neq 0$. The second invariant, denoted by d, is the greatest common divisor of the numbers $k-2m_0, k-2m_1, \dots, k-2m_s$.

The polynomial P_k need not be determined uniquely by the form (2). In order to make it unique, we impose the following normalization conditions. We require that $a_e = 1$, and

$$\arg a_{m_{i+1}} \in [0, \frac{2\pi}{q_i}) \tag{3}$$

for $0 \le i \le s-1$, where

$$q_i = \frac{\gcd(k-2m_0, k-2m_1, \dots, k-2m_i)}{\gcd(k-2m_0, k-2m_1, \dots, k-2m_{i+1})}.$$

This determines P_k uniquely.

The model hypersurface M_H associated with M at p is defined using the normalized leading homogeneous polynomial,

$$M_H = \{(z,w) \in \mathbb{C}^2 \mid v = \sum_{j=1}^{k-1} a_j z^j \bar{z}^{k-j}\}. \tag{4}$$

In particular, when the leading polynomial is of circular form, we denote the model by

$$O_k = \{(z,w) \in \mathbb{C}^2 \mid v = |z|^k\}. \tag{5}$$

Another exceptional model is the tubular hypersurface

$$T_k = \{(z,w) \in \mathbb{C}^2 \mid v = \frac{1}{k}(z+\bar{z})^k\}. \tag{6}$$

It was proved in [11] that if $e < \frac{k}{2}$, the local isotropy group of M_H is generated by weighted dilations and rotations in the complex tangential variable. Hence the elements of $\mathrm{Aut}(M_H, p)$ are of the form

$$z^* = \lambda \mathrm{e}^{\mathrm{i}\,\theta} z, \qquad w^* = \lambda^k w,$$

where $\mathrm{e}^{\mathrm{i}\,\theta}$ is a d-th root of unity and $\lambda > 0$ for k even or $\lambda \in \mathbb{R}\setminus\{0\}$ for k odd. It follows that $\mathrm{Aut}(M_H, p) = \mathbb{R}^+ \oplus \mathbb{Z}_d$ for k even and $\mathrm{Aut}(M_H, p) = \mathbb{R}^* \oplus \mathbb{Z}_d$ for k odd.

The local isotropy group of O_k is three dimensional, consisting of transformations of the form

$$f(z,w) = \frac{\lambda \mathrm{e}^{\mathrm{i}\,\theta} z}{(1+\mu w)^{\frac{1}{e}}}, \qquad g(z,w) = \frac{\lambda^k w}{1+\mu w}, \tag{7}$$

with $\lambda > 0$, and $\theta, \mu \in \mathbb{R}$.

We write

$$\Phi(z, \bar{z}, u) = P_k(z, \bar{z}) + F(z, \bar{z}, u),$$

where

$$F(z,\bar z,u) = \sum_{j,l} F_{jl}(u) z^j \bar z^l,$$

with $F_{jl}(u) = \sum_m a_{jlm} u^m$.

In [11], three different complete normal forms are constructed, depending on the model. Two for the exceptional models, O_k and T_k, and the third one for the generic model, covering all remaining cases.

An important difference between the Chern-Moser normal form and normal forms for Levi-degenerate hypersurfaces, is that normal forms and transformations to these forms are given by formal power series which not necessarily converge. The fact that a normal form construction solves the local equivalence problem is due to the essential result of M.S.Baouendi, P.Ebenfelt and L.P.Rothschild ([BER]), that any formal equivalence between two finite type hypersurfaces has to converge.

We subject the hypersurface to a general formal power series transformation,

$$z^* = z + f(z,w), \qquad w^* = w + g(z,w), \tag{8}$$

preserving the above form of M, and denote $v^* = F^*(z^*, \bar z^*, u^*)$ the defining equation in the new coordinates. As usual, when there is no danger of confusion, stars will be dropped immediately after the transformation.

Theorem 1 ([11]). *If $e = \frac{k}{2}$, there exists a transformation of the form (8), such that in the new coordinates the defining equation satisfies the following normal form conditions*

$$\begin{aligned} F_{j0} &= 0, \qquad j = 0,1,\dots, \\ F_{e,e+j} &= 0, \qquad j = 0,1,\dots, \\ F_{2e,2e} &= 0, \\ F_{3e,3e} &= 0, \\ F_{2e,2e-1} &= 0. \end{aligned} \tag{9}$$

Normal coordinates, i.e. those in which the conditions (9) hold, are determined uniquely up to the action of the local isotropy group (7).

When $M_H = T_k$, we have the same result with the following normal form conditions:

$$\begin{aligned} F_{j0} &= 0, \qquad j = 1,2,\dots, \\ F_{k-1+j,1} &= 0, \qquad j = 0,1,\dots, \end{aligned} \tag{10}$$

and

$$F_{2k-2,2} = \mathrm{Re}\ F_{k-2,1} = \mathrm{Re}\ F_{k,k-1} = 0. \tag{11}$$

Again, normal coordinates are determined uniquely up to the action of the group $\mathrm{Aut}(T_k, 0)$.

Now, let M_H be a generic model, i.e. $e < \frac{k}{2}$ and M_H is different from T_k. Denote $F_{k-1}(u) = (F_{1,k-2}(u), F_{2,k-3}(u), \dots, F_{k-2,1}(u))$. In this case the normal form conditions are:

$$
\begin{aligned}
F_{j0} &= 0, \qquad j = 1, 2, \dots, \\
F_{k-e+j,e} &= 0, \qquad j = 0, 1, \dots, \\
F_{2k-2e,2e} &= (F_{k-1}, P_z) = 0,
\end{aligned}
\tag{12}
$$

where

$$
(F_{k-1}, P_z) = \sum_{j=1}^{k-2} (j+1) F_{j,k-1-j} \bar{a}_{j+1}. \tag{13}
$$

The corresponding normal coordinates are unique up to the action of the group $\mathrm{Aut}(M_H, 0)$.

The normal form construction is used to obtain the following full classification of local isotropy groups for Levi-degenerate hypersurfaces of finite type.

Theorem 2 ([12]). *For a given hypersurface exactly one of the following possibilities occurs.*

(1) $\mathrm{Aut}(M,p)$ *has real dimension 3. This occurs if and only if M is equivalent to O_k.*
(2) $\mathrm{Aut}(M,p)$ *is noncompact of real dimension 1, isomorphic to $\mathbb{R}^+ \oplus \mathbb{Z}_m$. This occurs if and only if M is a model hypersurface with $l < \frac{k}{2}$ and $m = d$.*
(3) $\mathrm{Aut}(M,p)$ *is compact of real dimension 1, isomorphic to S^1. This occurs if and only if the defining equation of M in normal coordinates has form*
$$v = G(|z|^2, u).$$
(4) $\mathrm{Aut}(M,p)$ *is finite, isomorphic to $\mathbb{Z}_m$ for some $m \in \mathbb{Z}$. This occurs in all remaining cases.*

3. Two non-singular vector fields

Now we consider Levi-degenerate hypersurfaces of finite type with two-dimensional symmetry groups.

Let X, Y be two holomorphic vector fields such that $\mathrm{Re}\, X$ and $\mathrm{Re}\, Y$ generate a two-parametric group of automorphisms. We assume that $X(0)$ and $Y(0)$ are linearly independent (over $\mathbb{R}$). Otherwise we could replace one of the vector fields by a singular one.

We call a vector field non-contact at 0 if $X(0) \notin T_0^{1,0}M$. Now, if both X, Y were non-contact at 0 we could replace one of them by a contact vector field. On the other hand, X, Y can't be both contact as a consequence of the finite type condition. In fact, we have

Lemma 1. *If X, Y generate a two-parametric family of infinitesimal automorphisms of a finite type hypersurface at 0 then they can't be both contact at 0.*

Proof. Let M be of finite type at 0 and X, Y be two holomorphic contact at 0 vector fields with

$$[X,Y] = \alpha X + \beta Y$$

such that $\operatorname{Re} X$ and $\operatorname{Re} Y$ annihilate the defining equation of M. Without loss of generality we may assume that $T_0^C M = \{w = 0\}$, $X = \frac{\partial}{\partial z}$ and $Y = f\frac{\partial}{\partial z} + g\frac{\partial}{\partial w}$ where $f(0) = \mathrm{i}$ and $g(0) = 0$. Then the defining function of M takes the form

$$v = G(y,u)$$

where $w = u + \mathrm{i}\, v$ and $z = x + \mathrm{i}\, y$ and $G(0) = 0$, $dG(0) = 0$.

If $\beta = 0$ the commutator relation between X and Y yields

$$f = \alpha z + c(w), \quad g = d(w)$$

with $c(0) = \mathrm{i}$ and $d(0) = 0$.

Let y^k be the lowest order pure term in y in the expansion of G. Such term must exist, since otherwise M would contain the complex line $\{w = 0\}$. From $dG(0) = 0$ it follows that $k > 1$. Then

$$\operatorname{Re} Y(v - G(y,u))$$

contains a term ky^{k-1} that cannot be compensated by any other term, thus contradicting to Y being an infinitesimal automorphism.

If $\beta \neq 0$ then after replacing Y by $Y + \alpha X$ the commutator relation becomes $[X,Y] = \beta Y$ and

$$f = c(w)\,\mathrm{e}^{\beta z}, \quad g = d(w)\,\mathrm{e}^{\beta z}$$

with $c(0) = \mathrm{i}$ and $d(0) = 0$.

Aa before, consider the lowest order pure term y^k in the expansion of G. Again, in

$$\operatorname{Re} Y(v - G(y,u))$$

a term ky^{k-1} is produced and cannot be compensated, thus contradicting to Y being an infinitesimal automorphism. □

Now we will assume that X is non-contact and Y is contact at 0. Hence $X(0)$ and $Y(0)$ are linearly independent over $\mathbb{C}$.

By choosing a suitable basis of the two dimensional Lie algebra spanned by X, Y we may reduce the problem to the 3 cases

(1) $[X,Y] = 0$
(2) $[X,Y] = X$
(3) $[Y,X] = Y$.

We have

Theorem 3. *If M is a germ of a real-analytic hypersurface at 0 in $\mathbb{C}^2$ and X, Y are two holomorphic vector fields that generate a 2-dimensional Lie algebra (over $\mathbb{R}$) of infinitesimal automorphisms of M and such that X is non-contact and Y is contact at 0 then there exist local coordinates such that one of the following is true*

(1) $X = \frac{\partial}{\partial w}$, $Y = \frac{\partial}{\partial z}$ *and* M *has a defining equation of the form*

$$v = g(y),$$

i.e. M *is tubular.*

(2) $X = \frac{\partial}{\partial w}$, $Y = \frac{\partial}{\partial z} + w\frac{\partial}{\partial w}$ *and* M *has a defining equation of the form*

$$v = \mathrm{e}^x\, g(y).$$

(3) $X = z\frac{\partial}{\partial z} + \frac{\partial}{\partial w}$, $Y = \frac{\partial}{\partial z}$ *and* M *has a defining equation of the form*

$$v = g(\mathrm{e}^{-u}\, y).$$

Proof. If $[X, Y] = 0$ then there are coordinates such that $X = \frac{\partial}{\partial w}$, $Y = \frac{\partial}{\partial z}$. It follows that the defining equation

$$v = G(x, y, u)$$

is invariant with respect to the flow of $\operatorname{Re} X = \frac{\partial}{\partial u}$ and $\operatorname{Re} Y = \frac{\partial}{\partial x}$ and therefore does not depend on x and u.

If $[X, Y] = X$ then, by the Lemma 2 below there are coordinates such that $X = \frac{\partial}{\partial w}$, $Y = \frac{\partial}{\partial z} + w\frac{\partial}{\partial w}$. It follows that the defining equation does not depend on u and is invariant with respect to the flow of Y

$$z \mapsto z + t$$
$$w \mapsto \mathrm{e}^t\, w,$$

hence it satisfies

$$\mathrm{e}^{-t}\, G(x + t, y) = G(x, y).$$

Therefore the defining equation is

$$v = \mathrm{e}^x\, g(y)$$

where g is a smooth function with $g(0) = 0$.

If $[Y, X] = Y$, by the Lemma 2 below there are coordinates such that $X = z\frac{\partial}{\partial z} + \frac{\partial}{\partial w}$, $Y = \frac{\partial}{\partial z}$. Then the defining equation becomes

$$v = g(\mathrm{e}^{-u}\, y),$$

where g is a smooth function with $g(0) = 0$. □

Lemma 2. *If* X, Y *are germs of holomorphic vector fields as above with* $[X, Y] = X$, *then there exist local coordinates such that*

$$X = \frac{\partial}{\partial w}, \quad Y = \frac{\partial}{\partial z} + w\frac{\partial}{\partial w}.$$

Proof. $[X,Y]=X$. First we choose coordinates such that $X=\dfrac{\partial}{\partial w}$. Then in $Y=A\dfrac{\partial}{\partial z}+B\dfrac{\partial}{\partial w}$ the function A depends only on z and $B=w+b(z)$ where b depends only on z. Using the transformations

$$\begin{aligned} z &\mapsto \phi(z) \\ w &\mapsto w+\psi(z) \end{aligned}$$

that preserve $X=\dfrac{\partial}{\partial w}$ we may achieve $Y=\dfrac{\partial}{\partial z}+w\dfrac{\partial}{\partial w}$. □

4. Transitive automorphism and rotation

Lemma 3. *If $X=\mathrm{i}\,z\dfrac{\partial}{\partial z}$ and Y is a germ of a non-singular, holomorphic vector field, such that X,Y span a 2-dimensional Lie algebra then $[X,Y]=0$ and there exist local coordinates such that $Y=\dfrac{\partial}{\partial w}$.*

Proof. Let $Y=f\dfrac{\partial}{\partial z}+g\dfrac{\partial}{\partial w}$.

$[X,Y]=X$ requires $z\dfrac{\partial f}{\partial z}-f=z$, which is impossible.

$[X,Y]=\delta Y$ requires $\mathrm{i}\,z\dfrac{\partial f}{\partial z}-\mathrm{i}\,f=\delta f$ with real δ, which is impossible unless $f\equiv 0$.

Hence $[X,Y]=0$. Then

$$Y=zh(w)\frac{\partial}{\partial z}+g(w)\frac{\partial}{\partial w}.$$

Now, $g(0)\neq 0$ (otherwise Y would be singular) and after a coordinate change that preserves X we obtain $Y=\frac{\partial}{\partial w}$. □

As an immediate corollary we get the following result.

Theorem 4. *If M is a germ of a real hypersurface in $\mathbb{C}^2$ and $X=\mathrm{i}\,z\dfrac{\partial}{\partial z}$ and Y is a germ of a non-singular, holomorphic vector field, such that X,Y span a 2-dimensional Lie algebra then in suitable coordinates M has the equation*

$$v=G(|z|^2).$$

5. Transitive automorphism and dilation

A hypersurface M admits a weighted dilation

$$X=z\frac{\partial}{\partial z}+kw\frac{\partial}{\partial w}$$

only if it is a model hypersurface with the equation

$$v=\sum_{j=0}^{k}\alpha_j z^j\,\bar{z}^{k-j}.$$

Such hypersurfaces admit an additional translation $\dfrac{\partial}{\partial w}$ in the u-direction.

Lemma 4. *If $X = z\dfrac{\partial}{\partial z} + kw\dfrac{\partial}{\partial w}$ and Y is a germ of a non-singular, holomorphic vector field, such that X, Y span a 2-dimensional Lie algebra then $[X, Y] = \delta Y$ with $\delta = -1$ or $\delta = -k$ and there exist local coordinates such that $Y = \dfrac{\partial}{\partial z}$ or $Y = \dfrac{\partial}{\partial w}$.*

Proof. Let $Y = f\dfrac{\partial}{\partial z}+g\dfrac{\partial}{\partial w}$. Then $[X, Y] = X$ requires $z\dfrac{\partial f}{\partial z}+kw\dfrac{\partial f}{\partial w}-f = z$, which is impossible.

$[X, Y] = 0$ implies that f is a weighted homogeneous polynomial of degree 1 and g is a weighted homogeneous polynomial of degree k which is impossible if Y is non-singular.

$[X, Y] = \delta Y$ requires

$$z\frac{\partial f}{\partial z} + kw\frac{\partial f}{\partial w} = (\delta + 1)f \text{ and } z\frac{\partial g}{\partial z} + kw\frac{\partial g}{\partial w} = (\delta + k)g,$$

i.e. f is a monomial of weighted degree $\delta+1$ and g is a monomial of weighted degree $\delta+k$. Since one of f and g contains a weight zero term we have either $\delta = -1$ or $\delta = -k$. For $\delta = -k$ the only option is $f = 0, g = 1$, which is the translation in u-direction.

For $\delta = -1$ we get (after rescaling z) $f = 1, g = az^{k-1}$, and after a further transformation

$$\begin{aligned} z &\mapsto z \\ w &\mapsto w - az^k \end{aligned}$$

we get $Y = \dfrac{\partial}{\partial z}$. □

It follows that for $Y = \dfrac{\partial}{\partial z}$ the equation of M is $v = y^k$. Hence we have established the following result.

Theorem 5. *If M is a germ of a degenerate finite type real hypersurface in $\mathbb{C}^2$ and $X = z\dfrac{\partial}{\partial z} + kw\dfrac{\partial}{\partial w}$ and Y is a germ of a non-singular, holomorphic vector field, such that X, Y span a 2-dimensional Lie algebra then in suitable coordinates M has one of the following equations*

$$v = y^k \quad (k > 2)$$

$$v = y^k + \sum_{j=1}^{k-1} \alpha_j z^j \bar{z}^{k-j} \quad (k > 2, \alpha_{n-j} = \bar{\alpha}_j, \exists \alpha_j \neq 0).$$

In the first case M has a 3-parametric family of infinitesimal automorphisms generated by X, $\frac{\partial}{\partial z}$, $\frac{\partial}{\partial w}$ and in the second case M has a 2-parametric family of infinitesimal automorphisms generated by X, $\frac{\partial}{\partial w}$.

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