ON SOME SYSTEMS OF DIFFERENCE EQUATIONS. Part 2.

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Table of contents

§2.0. Foreword.

Here I continue translation in English of my paper [59].

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§2.1. Introduction.

Let

$$|z| \geq 1, -3\pi/2 < \arg(z) \leq \pi/2, \log(z) = \ln(|z|) + i\arg(z).$$

Then $\log(-z) = \log(z) - i\pi$, if $\Re(z) > 0$ and $\log(z) = \log(-z) - i\pi$, if $\Re(z) < 0$. The followimg functions are considered in [75]:

$$f_{l,1}(z,\nu) = \sum_{k=0}^{\nu}(-1)^{(\nu+k)l}(z)^k \binom{\nu}{k}^{2+l}\binom{\nu+k}{\nu}^{2+l}, \tag{1}$$

where $l = 0,\ 1,\ 2,\ \nu \in [0,+\infty) \cap \mathbb{Z}$ (see (2) in [75]);

$$f_{l,2}(z,\nu) = \sum_{t=1+\nu}^{+\infty} z^{-t}(R(t,\nu))^{2+l} = \tag{2}$$

$$\sum_{t=1}^{+\infty} z^{-t}(R(t,\nu))^{2+l},$$

where $l = 0,\ 1,\ 2,\ \nu \in [0, +\infty) \cap \mathbb{Z}$,

(3)
$$R(t, \nu) = \frac{\prod_{j=1}^{\nu} (t - j)}{\prod_{j=0}^{\nu} (t + j)}.$$

(see (7), (8) and (4) in [75]);

(4)
$$f_{l,3}^{\vee}(z, \nu) = f_{l,3}(z, \nu) = (\log(z)) f_{l,2}(z, \nu) + f_{l,4}(z, \nu)$$

where

(5)
$$f_{l,4}(z, \nu) = - \sum_{t=1+\nu}^{+\infty} z^{-t} \left(\frac{\partial}{\partial t} (R^{2+l}) \right) (t, \nu) =$$

$$- \sum_{t=1}^{+\infty} z^{-t} \left(\frac{\partial}{\partial t} (R^{2+l}) \right) (t, \nu),$$

$l = 0,\ 1,\ 2,\ \nu \in [0, +\infty) \cap \mathbb{Z}$, (see (9), (13) and (12) in the first part);

(6)
$$f_{l,5}(z, \nu) =$$

$$2^{-1} (\log(z))^2 f_{l,2}(z, \nu) + (\log(z)) f_{l,4}(z, \nu) + f_{l,6}(z, \nu) =$$
$$= -2^{-1} (\log(z))^2 f_{l,2}(z, \nu) + +(\log(z)) f_{l,3}(z, \nu) + f_{l,6}(z, \nu),$$

where $l = 1,\ 2,\ \nu \in [0, +\infty) \cap \mathbb{Z}$,

(7)
$$f_{l,6}(z, \nu) = 2^{-1} \sum_{t=\nu+1}^{\infty} z^{-t} \left(\left(\frac{\partial}{\partial t} \right)^2 (R^{2+l}) \right) (t, \nu) =$$

$$2^{-1} \sum_{t=1}^{\infty} z^{-t} \left(\left(\frac{\partial}{\partial t} \right)^2 (R^{2+l}) \right) (t, \nu),$$

(8)
$$f_{l,5}^{\vee}(z, \nu) = -i\pi f_{l,3}(z, \nu) + f_{l,5}(z, \nu)$$

where $l = 1,\ 2,\ \nu \in [0, +\infty) \cap \mathbb{Z}$, (see (14), (16)-(19) in [75]);

(9)
$$f_{l,7}(z, \nu) =$$

$$6^{-1} (\log(z))^3 f_{l,2}(z, \nu) + 2^{-1} (\log(z))^2 f_{l,4}(z, \nu) + (\log(z)) f_{l,6}(z, \nu) + f_{l,8}(z, \nu) =$$
$$-3^{-1} (\log(z))^3 f_{l,2}(z, \nu) + 2^{-1} (\log(z))^2 f_{l,3}(z, \nu) + f_{l,8}(z, \nu) +$$
$$(\log(z)) (f_{l,5}(z, \nu) + 2^{-1} (\log(z))^2 f_{l,2}(z, \nu) - (\log(z)) f_{l,3}(z, \nu)) =$$
$$6^{-1} (\log(z))^3 f_{l,2}(z, \nu) - 2^{-1} (\log(z))^2 f_{l,3}(z, \nu) + (\log(z)) f_{l,5}(z, \nu) + f_{l,8}(z, \nu),$$

where where $l = 2,\ \nu \in [0, +\infty) \cap \mathbb{Z}$,

$$(10) \qquad f_{l,8}(z,\nu) = -6^{-1} \sum_{t=\nu+1}^{\infty} z^{-t} \left(\left(\frac{\partial}{\partial t} \right)^3 (R^{2+l}) \right) (t,\nu) =$$

$$-6^{-1} \sum_{t=1}^{\infty} z^{-t} \left(\left(\frac{\partial}{\partial t} \right)^3 (R^{2+l}) \right) (t,\nu),$$

$$(11) \qquad f^{\vee}_{l,7}(z,\nu) = f_{l,7}(z,\nu) + (2\pi^2/3) f_{l,3}(z,\nu).$$

where $l = 2$, $\nu \in [0, +\infty) \cap \mathbb{Z}$, (see (21), (23)-(26) in [75]). Let

$$\mathfrak{K}_0 = \{1,\, 2,\, 3\},\ \mathfrak{K}_1 = \{1,\, 2,\, 3,\, 5\},\ \mathfrak{K}_2 = \{1,\, 2,\, 3,\, 5,\, 7\},$$

$$(12) \qquad f^{\vee}_{l,k}(z, -\nu-1) = f^{\vee}_{l,k}(z,\nu),$$

where $\nu \in [0, +\infty) \cap \mathbb{Z}$, $l = 0,\, 1,\, 2$, $k \in \mathfrak{K}_, l$; so these functions are defined for all the $\nu \in \mathbb{Z}$, and (12) holds for all the $\nu \in \mathbb{Z}$. Let further $\nu \in [0, +\infty) \cap \mathbb{Z}$,

$$(13) \qquad P^*_0(\nu; w) = \nu^3 - 4\nu^2 w + 8\nu w^2 - 12 w^3,\ Q^*_0(\nu, w) =$$

$$16\nu + 12w,$$

$$(14) \qquad P^*_1(\nu; w) = -\nu^5 + 6\nu^4 w - 18\nu^3 w^2 + 38\nu^2 w^3 - 66\nu w^4 +$$

$$102 w^5,\ Q^*_1(\nu; w) = -146\nu^2 - 240\nu w - 102 w^2,$$

$$(15) \qquad P^*_2(\nu; w) = \nu^7 - 8\nu^6 w + 32\nu^5 w^2 - 88\nu^4 w^3 + 192\nu^3 w^4 -$$

$$360\nu^2 w^5 + 608\nu w^6 - 952 w^7,\ Q^*_2(\nu; w) = 1408\nu^3 + 3640\nu^2 w + 3200\nu w^2 + 952 w^3,$$

$$(16) \qquad D_l(z,\nu,w) = z(w-\nu)^{2+l}(w+\nu+1)^{2+l} - w^{4+2l},$$

where $l = 0,\, 1,\, 2$, $\nu \in \mathbb{Z}$, and w is independent variable. Clearly,

$$(17) \qquad D_l(z,\nu,w) = D_l(z,-\nu-1,w)$$

for $l = 0,\, 1,\, 2$, $\nu \in \mathbb{Z}$. Let further $\delta := z\frac{\partial}{\partial z}$. Then (see [75])

$$(18) \qquad D_l(z,\nu,\delta) f^{\vee}_{l,k}(z,\nu) = 0,$$

where $\nu \in \mathbb{Z}$, $l = 0,\, 1,\, 2$, $k \in \mathfrak{K}_l$, $|z| > 1$. Further we have (see [75])

$$(19) \qquad (\delta + \nu + 1)^{2+l} f^{\vee}_{l,k}(z,\nu) = (\delta - \nu - 1)^{2+l} f^{\vee}_{l,k}(z,\nu+1),$$

where $\nu \in [0, +\infty) \cap \mathbb{Z}$, $l = 0,\, 1,\, 2$, $k \in \mathfrak{K}_l$, $|z| > 1$,

$$(20) \qquad (\delta + \nu)^{2+l} f^{\vee}_{l,k}(z,\nu-1) = (\delta - \nu)^{2+l} f^{\vee}_{l,k}(z,\nu),$$

where $\nu \in \cap\mathbb{Z}$, $l = 0,\, 1,\, 2$, $k \in \mathfrak{K}_l$, $|z| > 1$. Finally the following equalities was established in [75]:

$$(21) \qquad \nu^{3+2l} f^{\vee}_{l,k}(z,\nu-1) -$$

$$(P^*_l(\nu;\delta) + zQ^*_l(\nu;\delta+1)(\delta-\nu)^{2+l}) f^{\vee}_{l,k}(z,\nu) = 0,$$

where $\nu \in \mathbb{Z}$, $l = 0,\, 1,\, 2$, $k \in \mathfrak{K}_l$, $|z| > 1$,

$$(22) \qquad (-(\nu+1))^{3+2l} f^{\vee}_{l,k}(z,\nu+1) -$$

$$(P^*_l(-\nu-1;\delta) + zQ^*_l(-\nu-1;\delta+1)(\delta+\nu+1)^{2+l}) f^{\vee}_{l,k}(z,\nu) = 0,$$

where $\nu \in \mathbb{Z}$, $l = 0,\, 1,\, 2$, $k \in \mathfrak{K}_l$, $|z| > 1$.

§2. Pass to the system of difference equtions. The case $|z| > 1$.

Let

$$\mu = \nu(\nu + 1) \tag{23}$$

where $\nu \in \mathbb{Z}$. Then in view of (16

$$D_l(z, \nu, w) = z(w^2 + w - \mu)^{2+l} - w^{4+2l}, \tag{24}$$

where $l = 0,\ 1,\ 2,$

$$D_0(z, \nu, w) = z\mu^2 - 2zw\mu - 2zw^2\mu + zw^2 + 2zw^3 + (z-1)w^4,$$

$$D_1(z, \nu, w) = -z\mu^3 + 3zw\mu^2 + 3zw^2\mu^2 -$$
$$3zw^2\mu - 6zw^3\mu - 3zw^4\mu + zw^3 + 3zw^4 + 3zw^5 + (z-1)w^6,$$
$$D_2(z, \nu, w) = z\mu^4 - 4zw\mu^3 - 4zw^2\mu^3 +$$
$$6zw^2\mu^2 + 12zw^3\mu^2 + 6zw^4\mu^2 -$$
$$4zw^3\mu - 12zw^4\mu - 12zw^5\mu - 4zw^6\mu +$$
$$zw^4 + 4zw^5 + 6zw^6 + 4zw^7 + (z-1)w^8.$$

Let

$$b_{0,1}(z;\nu) = -(z-1)^{-1}z\mu^2 = -(z-1)^{-1}z(\nu^2 + 2\nu^3 + \nu^4),$$
$$b_{0,2}(z;\nu) = -(z-1)^{-1}2z(-\mu) = (z-1)^{-1}2z(-\nu - \nu^2),$$
$$b_{0,3}(z;\nu) = -(z-1)^{-1}z(1-2\mu) = -(z-1)^{-1}z(1 - 2\nu - 2\nu^2),$$
$$b_{0,4}(z;\nu) = -(z-1)^{-1}2z,$$
$$b_{1,1}(z;\nu) = -(z-1)^{-1}z(-\mu^3) = -(z-1)^{-1}z(-\nu^3 - 3\nu^4 - 3\nu^5 - \nu^6),$$
$$b_{1,2}(z;\nu) = -(z-1)^{-1}3z\mu^2 = -(z-1)^{-1}z(3\nu^2 + 6\nu^3 + 3\nu^4),$$
$$b_{1,3}(z;\nu) = -(z-1)^{-1}z(3\mu^2 - 3\mu) = -(z-1)^{-1}z(-3\nu + 6\nu^3 + 3\nu^4),$$
$$b_{1,4}(z;\nu) = -(z-1)^{-1}z(1-6\mu) = -(z-1)^{-1}z(1 - 6\nu - 6\nu^2),$$
$$b_{1,5}(z;\nu) = -(z-1)^{-1}z(3-3\mu) = -(z-1)^{-1}z(3 - 3\nu - 3\nu^2),$$
$$b_{1,6}(z;\nu) = -(z-1)^{-1}3z,$$
$$b_{2,1}(z;\nu) = -(z-1)^{-1}z(\mu^4) = -(z-1)^{-1}z(\nu^4 + 4\nu^5 + 6\nu^6 + 4\nu^7 + \nu^8),$$
$$b_{2,2}(z;\nu) = -(z-1)^{-1}z(-4\mu^3) = -(z-1)^{-1}z(-4\nu^3 - 12\nu^4 - 12\nu^5 - 4\nu^6),$$
$$b_{2,3}(z;\nu) = -(z-1)^{-1}z(-4\mu^3 + 6\mu^2) = -(z-1)^{-1}z\times$$
$$(-4\nu^3 - 12\nu^4 - 12\nu^5 - 4nu^6 + 6\nu^2 + 12\nu^3 + 6\nu^4) =$$
$$-(z-1)^{-1}z(6\nu^2 + 8\nu^3 - 6\nu^4 - 12\nu^5 - 4\nu^6),$$
$$b_{2,4}(z;\nu) = -(z-1)^{-1}z(-4\mu + 12\mu^2) = -(z-1)^{-1}z(-4\nu + 8\nu^2 + 24\nu^3 + 12\nu^4),$$
$$b_{2,5}(z;\nu) = -(z-1)^{-1}z(1 - 12\mu + 6\mu^2) = -(z-1)^{-1}z(1 - 12\nu - 6\nu^2 + 12\nu^3 + 6\nu^4),$$
$$b_{2,6}(z;\nu) = -(z-1)^{-1}z(4 - 12\mu) = -(z-1)^{-1}z(4 - 12\nu - 12\nu^2),$$

$$b_{2,7}(z;\nu) = -(z-1)^{-1}z(6-4\mu) = -(z-1)^{-1}z(6-4\nu-4\nu^2),$$
$$b_{2,8}(z;\nu) = -(z-1)^{-1}4z.$$

Let $l = 0, 1, 2$ and $B_l(z;\nu)$ denotes $(4+2l)\times(4+2l)$- matrix having the following propeties:

a) it has on the intersection of its first $3+2l$ rows and last $3+2l$ columns the unit $(3+2l)\times(3+2l)$-matrix,

b) its elements on the intersection of its first $3+2l$ rows and first column is equal to 0; the unit $(3+2l)\times(3+2l)$-matrix,

c) the k-th element of its last row is equal to $b_{l,k}$ for $k = 1, ..., 4+2l$.

Let further $X_{l,k}(z;\nu)$, where $l = 0, 1, 2$ and $k \in \mathfrak{K}_l$, $|z| > 1$, be the columnn with $4+2l$ elements, i-th of which is equal to $\delta^{i-1}f^{\vee}_{l,k}$ for $i = 1, ..., 4+2l$. Then

(25) $$\delta X_{l,k}(z;-\nu-1) = X_{l,k}(z;\nu),$$

(26) $$\delta X_{l,k}(z;\nu) = B_l(z;\nu)X_{l,k}(z;\nu),$$

where $l = 0, 1, 2$ and $k \in \mathfrak{K}_l$, $|z| > 1, \nu \in \mathbb{Z}$. Let $\tau = \nu + 1/2$; then $tau^2 = \mu + 1/4$. Since, in view of (13) – (15),

$$Q_0^*(\nu, w+1) = 12 + 16\nu + 12w,$$

$$Q_1^*(\nu; w+1) =$$
$$-102 - 240\nu - 146\nu^2 - 204w - 240\nu w - 102w^2,$$
$$Q_2^*(\nu; w+1) = 952 + 3200\nu + 3640\nu^2 + 1408\nu^3 +$$
$$2856w + 6400\nu w + 3640\nu^2 w + 2856w^2 + 3200\nu w^2 + 952w^3,$$

it follows that

$$Q_0^*(\nu, w+1)(w-\nu)^2 =$$
$$12\nu^2 + 16\nu^3 - 24\nu w - 20\nu^2 w - 8\nu w^2 + 12w^3 =$$
$$12(\tau-1/2)^2 + 16(\tau-1/2)^3 - (24(\tau-1/2)-$$
$$20(\tau-1/2)^2)w - 8(\tau-1/2)w^2 + 12w^3 =$$
$$\sum_{k=0}^{3} q^*_{0,k}(\nu)w^k,$$

where

$$q^*_{0,0}(\nu) = -2 - 12\mu + (4+16\mu)\tau,$$
$$q_{0,1}(\nu)^a st = 2 - 20\mu - 4\tau,$$
$$q_{0,2}(\nu)^* = 16 - 8\tau,$$
$$q_{0,3}(\nu)^* = 12,$$

$$Q_1^*(\nu, w+1)(w-\nu)^3 = 102\nu^3 + 240\nu^4 + 146\nu^5 + (-306\nu^2 - 516\nu^3 - 198\nu^4)w +$$
$$(306\nu + 108\nu^2 - 180\nu^3)w^2 + (-102 + 372\nu + 268\nu^2)w^3 +$$
$$(-204 + 66\nu)w^4 - 102w^5 =$$

$$\sum_{k=0}^{5} q^*_{1,k}(\nu) w^k,$$

where

$$q^*_{1,0}(\nu) = -4 - 38\mu - 125\mu^2 + (8 + 60\mu + 146\mu^2)\tau,$$
$$q^*_{1,1}(\nu) = 6 + 72\mu - 198\mu^2 + (-12 - 120\mu)\tau,$$
$$q^*_{1,2}(\nu) = -9 + 378\mu + (18 - 180\mu)\tau,$$
$$q^*_{1,3}(\nu) = -154 + 268\mu + 104\tau,$$
$$q_{1,4}(\nu) = -237 + 66\tau,$$
$$q^*_{1,5}(\nu) = -102,$$
$$Q^*_2(\nu, w+1)(w-\nu)^4 = 952\nu^4 + 3200\nu^5 + 3640\nu^6 + 1408\nu^7 +$$
$$(-3808\nu^3 - 9944\nu^4 - 8160\nu^5 - 1992\nu^6)w +$$
$$(5712\nu^2 + 7776\nu^3 - 904\nu^4 - 2912\nu^5)w^2 +$$
$$(-3808\nu + 4336\nu^2 + 12416\nu^3 + 4360\nu^4)w^3 +$$
$$(952 - 8224\nu - 4824\nu^2 + 2240\nu^3)w^4 +$$
$$(2856 - 5024\nu - 3448\nu^2)w^5 + (2856 - 608\nu)w^6 + (952)w^7 =$$
$$\sum_{k=0}^{7} q^*_{2,k}(\nu) w^k,$$

where

$$q_{2,0}(\nu)^* = -8 - 104\mu - 524\mu^2 - 1288\mu^3 + (16 + 176\mu + 728\mu^2 + 1408\mu^3)\tau,$$
$$q_{2,1}(\nu)^* = 16 + 248\mu + 1492\mu^2 - 1992\mu^3 + (-32 - 432\mu - 2184\mu^2)\tau,$$
$$q_{2,2}(\nu)^* = -28 - 480\mu + 6376\mu^2 +$$
$$(56 + 848\mu - 2912\mu^2)\tau,$$
$$q_{2,3}(\nu)^* = 44 - 5568\mu + 4360\mu^2 + (-88 + 3696\mu)\tau,$$
$$q_{2,4}(\nu)^* = 1532 - 8184\mu + (-1160 + 2240\mu)\tau,$$
$$q_{2,5}(\nu)^* = 3644 - 3448\mu - 1576\tau,$$
$$q_{2,6}(\nu)^* = 3160 - 608\tau,$$
$$q_{2,7}(\nu)^* = 952.$$

Let

$$q^\vee_{0,0}(\nu) = -2 - 12\mu,\ q^\wedge_{0,0}(\nu) = 4 + 16\mu,$$
$$q_{0,1}(\nu)^\vee = 2 - 20\mu,\ q_{0,1}(\nu)^\wedge = -4,$$
$$q_{0,2}(\nu)^\vee = 16,\ q_{0,2}(\nu)^\wedge = -8,$$
$$q_{0,3}(\nu)^\vee = 12,\ q_{0,3}(\nu)^\wedge = 0,$$
$$q^\vee_{1,0}(\nu) = -4 - 38\mu - 125\mu^2,\ q^\wedge_{1,0}(\nu) = 8 + 60\mu + 146\mu^2,$$
$$q^\vee_{1,1}(\nu) = 6 + 72\mu - 198\mu^2,\ q^\wedge_{1,1}(\nu) = -12 - 120\mu),$$
$$q^\vee_{1,2}(\nu) = -9 + 378\mu,\ q^\wedge_{1,2}(\nu) = 18 - 180\mu,$$

$$q_{1,3}^{\vee}(\nu) = -154 + 268\mu,\ q_{1,3}^{\wedge}(\nu) = 104,$$
$$q_{1,4}^{\vee}(\nu) = -237,\ q_{1,4}(\nu)^{\wedge} = 66,$$
$$q_{1,5}^{\vee}(\nu) = -102,\ q_{1,5}^{\wedge}(\nu) = 0,$$
$$q_{2,0}(\nu)^{\vee} = -8-104\mu-524\mu^2-1288\mu^3,\ q_{2,0}(\nu)^{\wedge} = 16+176\mu+728\mu^2+1408\mu^3,$$
$$q_{2,1}(\nu)^{\vee} = 16 + 248\mu + 1492\mu^2 - 1992\mu^3,\ q_{2,1}(\nu)^{\wedge} = -32 - 432\mu - 2184\mu^2,$$
$$q_{2,2}(\nu)^{\vee} = -28 - 480\mu + 6376\mu^2,\ q_{2,2}(\nu)^{\wedge} = 56 + 848\mu - 2912\mu^2,$$
$$q_{2,3}(\nu)^{\vee} = 44 - 5568\mu + 4360\mu^2,\ q_{2,3}(\nu)^{\wedge} = -88 + 3696\mu,$$
$$q_{2,4}(\nu)^{\vee} = 1532 - 8184\mu,\ q_{2,4}(\nu)^{\wedge} = -1160 + 2240\mu,$$
$$q_{2,5}(\nu)^{\vee} = 3644 - 3448\mu,\ q_{2,5}(\nu)^{\wedge} = -1576,$$
$$q_{2,6}(\nu)^{\vee} = 3160,\ q_{2,6}(\nu)^{\wedge} = -608,$$
$$q_{2,7}(\nu)^{\vee} = 952,\ q_{2,7}(\nu)^{\wedge} = 0.$$

Then
$$q_{l,k}(\nu)^{*} = q_{l,k}(\nu)^{\vee} + q_{l,k}(\nu)^{\wedge}\tau$$
for $l = 0,\ 1,\ 2,\ k = 0,\ ...,\ 3 + 2l$. In view of (13) – (15),

(27) $$P_l^{*}(\nu; w) = \sum k = 0^{3+2l} p_{l,k}^{*}(\nu) w^k,$$

for $l = 0,\ 1,\ 2,$ where

$$p_{0,0}^{*}(\nu) = \nu^3 = (\tau - 1/2)^3 =$$
$$-1/8 - (3/2)(\mu + 1/4) + (\mu + 1/4 + 3/4)\tau = -1/2 - (3/2)\mu + (1 + \mu)\tau,$$
$$p_{0,1}^{*}(\nu) = -4\nu^2 = -4(\tau - 1/2)^2 = -2 - 4\mu + 4\tau,$$
$$p_{0,2}^{*}(\nu) = 8\nu = -4 + 8\tau, p_{0,3}^{*}(\nu) = -12,$$
$$p_{1,0}^{*}(\nu) = -\nu^5 = -(\tau - 1/2)^5 = (1/2)(1 + 5\mu + 5\mu^2) + (-1 - 3\mu - \mu^2)\tau,$$
$$p_{1,1}^{*}(\nu) = 6\nu^4 = 3 + 12\mu + 6\mu^2 + (-6 - 12\mu)\tau,$$
$$p_{1,2}^{*}(\nu) = -18\nu^3 = 9 + 27\mu + (-18 - 18\mu)\tau,$$
$$p_{1,3}^{*}(\nu) = 38\nu^2 = 19 + 38\mu - 38\tau,$$
$$p_{1,4}^{*}(\nu) = -66\nu = 33 - 66\tau, p_{1,5}^{*}(\nu) = 102,$$
$$p_{2,0}^{*}(\nu) = \nu^7 = \nu^5\mu - \nu^6 =$$
$$(1/2)(-\mu - 5\mu^2 - 5\mu^3) + (\mu + 3\mu^2 + \mu^3)\tau - \mu\nu^4 + \nu^5 =$$
$$(1/2)(-\mu - 5\mu^2 - 5\mu^3) + (\mu + 3\mu^2 + \mu^3)\tau+$$
$$(-1/2)\mu - 2\mu^2 - \mu^3 + (\mu + 2\mu^2)\tau+$$
$$(-1/2)(1 + 5\mu + 5\mu^2) + (1 + 3\mu + \mu^2)\tau =$$
$$(1/2)(-1 - 7\mu - 14\mu^2 - 7\mu^3) + (1 + 5\mu + 6\mu^2 + \mu^3)\tau,$$
$$p_{2,1}^{*}(\nu) = -8\nu^6 = -4 - 24\mu - 36\mu^2 - 8\mu^3 + (8 + 32 + 24\mu^2)\tau,$$
$$p_{2,2}^{*}(\nu) = 32\nu^5 = -16 - 80\mu - 80\mu^2 + (32 + 96\mu + 32\mu^2)\tau,$$

$$p^*_{2,3}(\nu) = -88\nu^4 = -44 - 176\mu - 88\mu^2 + (88 + 176\mu)\tau,$$
$$p^*_{2,4}(\nu) = 192\nu^3 = -96 - 288\mu + (192 + 192\mu)\tau,$$
$$p^*_{2,5}(\nu) = -360\nu^2 = -180 - 360\mu + 360\tau,$$
$$p^*_{2,6}(\nu) = 608\nu = -304 + 608\tau,$$
$$p^*_{2,7}(\nu) = -952.$$

Let

$$p^\vee_{0,0}(\nu) = -1/2 - (3/2)\mu,\ p^\wedge_{0,0}(\nu) = (1 + \mu),$$
$$p^\vee_{0,1}(\nu) = -2 - 4\mu,\ p^\wedge_{0,1}(\nu) = 4,$$
$$p^\vee_{0,2}(\nu) = -4,\ p^\wedge_{0,2}(\nu) = 8,$$
$$p^\vee_{0,3}(\nu) = -12,\ p^\wedge_{0,3,}(\nu) = 0,$$
$$p^\vee_{1,0}(\nu) = (1/2)(1 + 5\mu + 5\mu^2),\ p^\wedge_{10,}(\nu) = -1 - 3\mu - \mu^2,$$
$$p^\vee_{1,1}(\nu) = 3 + 12\mu + 6\mu^2,\ p^\wedge_{1,1}(\nu) = -6 - 12\mu,$$
$$p^\vee_{1,2}(\nu) = 9 + 27\mu,\ p^\wedge_{1,2}(\nu) = -18 - 18\mu,$$
$$p^\vee_{1,3}(\nu) = 19 + 38\mu,\ p^\wedge_{1,3}(\nu) = -38,$$
$$p^\vee_{1,4}(\nu) = 33,\ p^\wedge_{1,4}(\nu) = -66,$$
$$p^\vee_{1,5}(\nu) = 102,\ p^\wedge_{1,5}(\nu) = 0,$$
$$p^\vee_{2,0}(\nu) = (1/2)(-1 - 7\mu - 14\mu^2 - 7\mu^3),\ p^\wedge_{2,0}(\nu) = 1 + 5\mu + 6\mu^2 + \mu^3,$$
$$p^\vee_{2,1}(\nu) = -4 - 24\mu - 36\mu^2 - 8\mu^3,\ p^\wedge_{2,1}(\nu) = 8 + 32 + 24\mu^2,$$
$$p^\vee_{2,2}(\nu) = -16 - 80\mu - 80\mu^2,\ p^\wedge_{2,2}(\nu) = 32 + 96\mu + 32\mu^2,$$
$$p^\vee_{2,3}(\nu) = -44 - 176\mu - 88\mu^2,\ p^\wedge_{2,3}(\nu) = 88 + 176\mu,$$
$$p^\vee_{2,4}(\nu) = -96 - 288\mu,\ p^\wedge_{2,4}(\nu) = 192 + 192\mu,$$
$$p^\vee_{2,5}(\nu) = -180 - 360\mu,\ p^\wedge_{2,5}(\nu) = 360,$$
$$p^\vee_{2,6}(\nu) = -304,\ p^\wedge_{2,6}(\nu) = 608,$$
$$p^\vee_{2,7}(\nu) = -952,\ p^\wedge_{2,7}(\nu) = 0.$$

Then

$$p_{l,k}(\nu)^* = p_{l,k}(\nu)^\vee + p_{l,k}(\nu)^\wedge\tau$$

for $l = 0,\ 1,\ 2,\ k = 0,\ ...,\ 3 + 2l$. We denote by

$$\alpha^*_{l,1}(z;\nu),\ \alpha^\vee_{l,1}(z;\nu),\ \alpha^\wedge_{l,1}(z;\nu)$$

for $l = 0,\ 1,\ 2,\ k = 0,\ ...,\ 3 + 2l$ the row with $4 + 2l$ elements, $(k + 1)$-th of which is equal respectively

$$p^*_{l,k}(\nu) + zq^*_{l,k}(\nu),$$
$$p^\vee_{l,k}(\nu) + zq^\vee_{l,k}(\nu),$$
$$p^\wedge_{l,k}(\nu) + zq^\wedge_{l,k}(\nu),$$

where $k = 0, ..., 3 + 2l$. Let

(28) $$\alpha^{*}_{l,i+1}(z;\nu) = \delta\alpha^{*}_{l,i}(z;\nu) + \alpha^{*}_{l,i}(z;\nu)B_l(z;\nu),$$

(29) $$\alpha^{\vee}_{l,i+1}(z;\nu) = \delta\alpha^{*}_{l,i}(z;\nu) + \alpha^{\vee}_{l,i}(z;\nu)B_l(z;\nu),$$

(30) $$\alpha^{\wedge}_{l,i+1}(z;\nu) = \delta\alpha^{\wedge}_{l,i}(z;\nu) + \alpha^{\wedge}_{l,i}(z;\nu)B_l(z;\nu),$$

where $l = 0, 1, 2, i = 0, ..., 3 + 2l$. Clearly,

$$\alpha^{*}_{l,k}(z;\nu) = \alpha^{\vee}_{l,k}(z;\nu) + \alpha^{\wedge}_{l,k}(z;\nu)\tau$$

for $l = 0, 1, 2, k = 0, ..., 3 + 2l$. We denote by

$$a^{*}_{l,i,k}(z;\nu),\ a^{\vee}_{l,i,k}(z;\nu),\ a^{\wedge}_{l,i,k}(z;\nu)$$

for $l = 0, 1, 2, i = 1, ..., 4 + 2l, k = 1, ..., 4 + 2l$ the k-th elements of the rows respecively

$$\alpha^{*}_{l,i}(z;\nu),\ \alpha^{\vee}_{l,i}(z;\nu),\ \alpha^{\wedge}_{l,i}(\nu).$$

Then,

$$a^{\vee}_{0,1,1}(z;\nu) = (1/2)(-5 - 27\mu + (z-1)(-4 - 24\mu)),$$
$$a^{\vee}_{0,1,2}(z;\nu) == (1/2)(-48\mu + (z-1)(4 - 40\mu)),$$
$$a^{\vee}_{0,1,3}(z;\nu) = -4 + z16 = (1/2)(24 + (z-1)32),$$
$$a^{\vee}_{0,1,4}(z;\nu) = -12 + z12 = (1/2)(z-1)24,$$
$$a^{\wedge}_{0,1,1}(z;\nu) = 1 + \mu + z(4 + 16\mu) = 5 + 17\mu + (z-1)(4 + 16\mu),$$
$$a^{\wedge}_{0,1,2}(\nu) = 4 + z(-4) = (z-1)(-4),$$
$$a^{\wedge}_{0,1,3}(z;\nu) = 8 + z(-8) = (z-1)(-8),$$
$$a^{\wedge}_{0,1,4}(z;\nu) = 0,$$
$$a^{\vee}_{1,1,1}(z;\nu) = (1/2)(-7 - 71\mu - 245\mu^2 + (z-1)(-8 - 76\mu - 250\mu^2)),$$
$$a^{\vee}_{1,1,2}(z;\nu) = (1/2)(18 + 168\mu - 384\mu^2 + (z-1)(12 + 144\mu - 396\mu^2)),$$
$$a^{\vee}_{1,1,3}(z;\nu) = (1/2)(810\mu + (z-1)(-18 + 756\mu)),$$
$$a^{\vee}_{1,1,4}(z;\nu) = (1/2)(-270 + 612\mu + (z-1)(-308 + 536\mu)),$$
$$a^{\vee}_{1,1,5}(z;\nu) = (1/2)(-408 + (z-1)(-474\mu)),$$
$$a^{\vee}_{1,1,6}(z;\nu) = -(1/2)(z-1)204$$
$$a^{\wedge}_{1,1,1}(z;\nu) = 7 + 57\mu + 145\mu^2 + (z-1)(8 + 60\mu + 146\mu^2),$$
$$a^{\wedge}_{1,1,2}(z;\nu) = -18 - 132\mu + (z-1)(-12 - 120\mu),$$
$$a^{\wedge}_{1,1,3}(z;\nu) = -198\mu + (z-1)(18 - 180\mu),$$

$$a^{\wedge}_{1,1,4}(z;\nu) = 66 + (z-1)(104),$$
$$a^{\wedge}_{1,1,5}(z;\nu) = (z-1)66,$$
$$a^{\wedge}_{1,1,6}(z;\nu) = 0,$$
$$a^{\vee}_{2,1,1}(z;\nu) = (1/2)(-17 - 215\mu - 1062\mu^2 - 2583\mu^3) +$$
$$(1/2)(z-1)(-16 - 208\mu - 1048\mu^2 - 2576\mu^3),$$
$$a^{\vee}_{2,1,2}(z;\nu) = (1/2)(24 + 448\mu + 2912\mu^2 - 4000\mu^3) +$$
$$(1/2)(z-1)(32 + 496\mu + 2984\mu^2 - 3984\mu^3),$$
$$a^{\vee}_{2,1,3}(z;\nu) = (1/2)(-88 - 1120\mu + 12592\mu^2) +$$
$$(1/2)(z-1)(-56 - 960\mu + 12752\mu^2),$$
$$a^{\vee}_{2,1,4}(z;\nu) = (1/2)(-11488\mu + 8544\mu^2) +$$
$$(1/2)(z-1)(88 - 11136\mu + 8720\mu^2),$$
$$a^{\vee}_{2,1,5}(z;\nu) = (1/2)(2872 - 16944\mu) + (z-1)(3064 - 16368\mu),$$
$$a^{\vee}_{2,1,6}(z;\nu) = (1/2)(6928 - 7616\mu) + (z-1)(7288 - 6896\mu)),$$
$$a^{\vee}_{2,1,7}(z;\nu) = (1/2)(5712) + (z-1)(6320)),$$
$$a^{\vee}_{2,1,8}(z;\nu) = (1/2)(z-1)(1904),$$
$$a^{\wedge}_{2,1,1}(z;\nu) = 17 + 181\mu + 734\mu^2 + 1409\mu^3 +$$
$$(z-1)(16 + 176\mu + 728\mu^2 + 1408\mu^3),$$
$$a^{\wedge}_{2,1,2}(z;\nu) = -24 - 400\mu - 2160\mu^2 + (z-1)(-32 - 432\mu - 2184\mu^2),$$
$$a^{\wedge}_{2,1,3}(z;\nu) = 88 + 944\mu - 2880\mu^2 + (z-1)(56 + 848\mu - 2912\mu^2),$$
$$a^{\wedge}_{2,1,4}(z;\nu) = 3872\mu + (z-1)(-88 + 3696\mu),$$
$$a^{\wedge}_{2,1,5}(z;\nu) = -968 + 2432\mu + (z-1)(-1160 + 2240\mu),$$
$$a^{\wedge}_{2,1,6}(z;\nu) = -1216 + (z-1)(-1576),$$
$$a^{\wedge}_{2,1,7}(z;\nu) = (z-1)(-608),$$
$$a^{\wedge}_{2,1,8}(z;\nu) = 0,$$

and, in view of (28) – (30),

$$a^{\vee}_{0,2,1}(z;\nu) = (1/2)(-4 - 24\mu - 24\mu^2 + (z-1)(-4 - 24\mu - 24\mu^2)),$$
$$a^{\vee}_{0,2,2}(z;\nu) = (1/2)(-1 - 19\mu + (z-1)(-16)),$$
$$a^{\vee}_{0,2,3}(z;\nu) = (1/2)(8 + (z-1)(12 + 8\mu)),$$
$$a^{\vee}_{0,2,4}(z;\nu) = (1/2)(8(z-1)),$$
$$a^{\vee}_{0,3,1}(z;\nu) = (1/2)(-4 - 24\mu - 32\mu^2 + (z-1)(-4 - 24\mu - 32\mu^2)),$$
$$a^{\vee}_{0,3,2}(z;\nu) = (1/2)(-4 - 24\mu - 24\mu^2 + (z-1)(-4 - 24\mu - 24\mu^2)),$$
$$a^{\vee}_{0,3,3}(z;\nu) = (1/2)(3 + 5\mu + (z-1)(4 + 8\mu)),$$
$$a^{\vee}_{0,3,4}(z;\nu) = (1/2)((z-1)(4 + 8\mu)),$$
$$a^{\vee}_{0,4,1}(z;\nu) =$$

$$(1/2)(-4-24\mu-36\mu^2-8\mu^3+(z-1)(-4-24\mu-8\mu^3)),$$

$$a^{\vee}_{0,4,2}(z;\nu)=$$

$$(1/2)(-8-40\mu-40\mu^2+(z-1)(-8-40\mu-40\mu^2)),$$

$$a^{\vee}_{0,4,3}(z;\nu)=(1/2)(-4-16\mu-8\mu^2+(z-1)(-4-16\mu-8\mu^2)),$$

$$a^{\vee}_{0,4,4}(z;\nu)=(1/2)(-1-3\mu),$$

$$a^{\wedge}_{0,2,1}(z;\nu)=4+16\mu+(z-1)(4+16\mu),$$

$$a^{\wedge}_{0,2,2}(z;\nu)=1+17\mu+(z-1)(4+16\mu),$$

$$a^{\wedge}_{0,2,3}(z;\nu)=-8+(z-1)(-12),$$

$$a^{\wedge}_{0,2,4}(z;\nu)=8(z-1),$$

$$a^{\wedge}_{0,3,1}(z;\nu)=4+16\mu+8\mu^2+(z-1)(4+16\mu+8\mu^2),$$

$$a^{\wedge}_{0,3,2}(z;\nu)=4+16\mu+(z-1)(4+16\mu),$$

$$a^{\wedge}_{0,3,3}(z;\nu)=-3+\mu+(z-1)(-4),$$

$$a^{\wedge}_{0,3,4}(z;\nu)=(z-1)(-4),$$

$$a^{\wedge}_{0,4,1}(z;\nu)=4+16\mu+12\mu^2+(z-1)(4+16\mu+12\mu^2),$$

$$a^{\wedge}_{0,4,2}(z;\nu)=8+24\mu+8\mu^2+(z-1)(8+24\mu+8mu^2),$$

$$a^{\wedge}_{0,4,3}(z;\nu)=4+8\mu+(z-1)(4+8\mu),$$

$$a^{\wedge}_{0,4,4}(z;\nu)=1+\mu,$$

$$a^{\vee}_{1,2,1}(z;\nu)=$$

$$(1/2)(-8-76\mu-250\mu^2-204\mu^3+(z-1)(-8-76\mu-250\mu^2-204\mu^3)),$$

$$a^{\vee}_{1,2,2}(z;\nu)=(1/2)(5+73\mu-29\mu^2+(z-1)(4+68\mu-34\mu^2)),$$

$$a^{\vee}_{1,2,3}(z;\nu)=(1/2)(312\mu+228\mu^2+(z-1)(-6+288\mu+216\mu^2)),$$

$$a^{\vee}_{1,2,4}(z;\nu)=(1/2)(-104+122\mu+(z-1)(-122+68m),$$

$$a^{\vee}_{1,2,5}(z;\nu)=(1/2)(-132+(z-1)(-170-76\mu)),$$

$$a^{\vee}_{1,2,6}(z;\nu)=(1/2)(z-1)(-66),$$

$$a^{\vee}_{1,3,1}(z;\nu)=$$

$$(1/2)(-8-76\mu-250\mu^2-270\mu^3+(z-1)(-8-76\mu-250\mu^2-270\mu^3)),$$

$$a^{\vee}_{1,3,2}(z;\nu)=(1/2)(-4-8\mu-86\mu^2-204\mu^3+(z-1)(-4-8\mu-86\mu^2-204\mu^3)),$$

$$a^{\vee}_{1,3,3}(z;\nu)=(1/2)(-1+163\mu+385\mu^2+(z-1)(-2+158\mu+380\mu^2)),$$

$$a^{\vee}_{1,3,4}(z;\nu)=(1/2)(-56-16\mu+228\mu^2+(z-1)(-62-40m+216m^2),$$

$$a^{\vee}_{1,3,5}(z;\nu)=(1/2)(-76-152\mu+(z-1)(-94-206\mu)),$$

$$a^{\vee}_{1,3,6}(z;\nu)=(1/2)(z-1)(-38-76\mu),$$

$$a^{\vee}_{1,4,1}(z;\nu)=(1/2)(-8-76\mu-250\mu^2-308\mu^3-76\mu^4+$$

$$(1/2)(z-1)(-8-76\mu-250\mu^2-308\mu^3-76\mu^4),$$

$$a^{\vee}_{1,4,2}(z;\nu)=$$

$$(1/2)(-12-84\mu-222\mu^2-246\mu^3+(z-1)(-12-84\mu-222\mu^2-246\mu^3)),$$

$$a^{\vee}_{1,4,3}(z;\nu)=(1/2)(-6+36\mu+180\mu^2+24\mu^3+(z-1)(-6+36\mu+180\mu^2+24\mu^3)),$$

$$a^{\vee}_{1,4,4}(z;\nu)=(1/2)(-25-29\mu+145\mu^2+(z-1)(-26-34m+140m^2),$$

$$a^{\vee}_{1,4,5}(z;\nu)=(1/2)(-36-108\mu+(z-1)(-42-132\mu-12\mu^2)),$$

$$a^{\vee}_{1,4,6}(z;\nu)=(1/2)(z-1)(-18-54\mu),$$

$$a^{\vee}_{1,5,1}(z;\nu)=(1/2)(-8-76\mu-250\mu^2-326\mu^3-130\mu^4+$$
$$(1/2)(z-1)(-8-76\mu-250\mu^2-326\mu^3-130\mu^4),$$

$$a^{\vee}_{1,5,2}(z;\nu)=(1/2)(-20-160\mu-418\mu^2-392\mu^3-76\mu^4)+$$
$$(1/2)(z-1)(-20-160\mu-418\mu^2-392\mu^3-76\mu^4)),$$

$$a^{\vee}_{1,5,3}(z;\nu)=$$
$$(1/2)(-18-102\mu+150\mu^2-60\mu^3+(z-1)(-18-102\mu-150\mu^2-60\mu^3)),$$

$$a^{\vee}_{1,5,4}(z;\nu)=$$
$$(1/2)(-14-52\mu-4\mu^2+24\mu^3+(z-1)(-14-52m-4m^2+24\mu^3)),$$

$$a^{\vee}_{1,5,5}(z;\nu)=(1/2)(-13-53\mu-29\mu^2+(z-1)(-14-58\mu-34\mu^2)),$$

$$a^{\vee}_{1,5,6}(z;\nu)=(1/2)(z-1)(-6-24\mu-12\mu^2),$$

$$a^{\vee}_{1,6,1}(z;\nu)=(1/2)(-8-76\mu-250\mu^2-332\mu^3-154\mu^4-12\mu^5$$
$$(1/2)(z-1)(-8-76\mu-250\mu^2-332\mu^3-154\mu^4-12\mu^5),$$

$$a^{\vee}_{1,6,2}(z;\nu)=(1/2)(-28-236\mu-650\mu^2-646\mu^3-170\mu^4)+$$
$$(1/2)(z-1)(-28-236\mu-650\mu^2-646\mu^3-170\mu^4)),$$

$$a^{\vee}_{1,6,3}(z;\nu)=$$
$$(1/2)(-38-280\mu-622\mu^2-416\mu^3-40\mu^4+$$
$$(1/2)(z-1)(-38-280\mu-622\mu^2-416\mu^3-40\mu^4),$$

$$a^{\vee}_{1,6,4}(z;\nu)=$$
$$(1/2)(-26-166\mu-286\mu^2+108\mu^3+(z-1)(-26-166\mu-286\mu^2+108\mu^3)),$$

$$a^{\vee}_{1,6,5}(z;\nu)=$$
$$(1/2)(-10-56\mu-74\mu^2-12\mu^3+(z-1)(-10-56\mu-74\mu^2-12\mu^3)),$$

$$a^{\vee}_{1,6,6}(z;\nu)=(1/2)(-1-5\mu-5\mu^2+(z-1)(-2-10\mu-10\mu^2),$$

$$a^{\vee}_{2,2,1}(z;\nu)=(1/2)(-16-208\mu-1048\mu^2-2576\mu^3-1904\mu^4)+$$
$$(1/2)(z-1)(-16-208\mu-1048\mu^2-2576\mu^3-1904\mu^4),$$

$$a^{\vee}_{2,2,2}(z;\nu)=(1/2)(15+281\mu+1922\mu^2+1049\mu^3)+$$
$$(1/2)(z-1)(16+288\mu+1936\mu^2+1056\mu^3),$$

$$a^{\vee}_{2,2,3}(z;\nu)=(1/2)(-32-512\mu+4240\mu^2+3616\mu^3+$$
$$(1/2)(z-1)(-24-464\mu+4312\mu^2+3632\mu^3),$$

$$a^{\vee}_{2,2,4}(z;\nu)=(1/2)(-4640\mu-1536\mu^2+(z-1)(32-4480\mu-1376\mu^2)),$$

$$a^\vee_{2,2,5}(z;\nu) = (1/2)(1160 - 5008\mu - 2880\mu^2 + (z-1)(1248 - 4656\mu - 2704\mu^2)),$$
$$a^\vee_{2,2,6}(z;\nu) = (1/2)(2544 - 992\mu + (z-1)(2736 - 416\mu)),$$
$$a^\vee_{2,2,7}(z;\nu) = (1/2)(1824 + (z-1)(2184 + 720\mu)),$$
$$a^\vee_{2,2,8}(z;\nu) = (1/2)(z-1)(608),$$
$$a^\vee_{2,3,1}(z;\nu) = (1/2)(-16 - 208\mu - 1048\mu^2 - 2576\mu^3 - 2512\mu^4$$
$$+(1/2)(z-1)(-16 - 208\mu - 1048\mu^2 - 2576\mu^3 - 2512\mu^4),$$
$$a^\vee_{2,3,2}(z;\nu) = (1/2)(80\mu + 888\mu^2 + 912\mu^3 - 1904\mu^4)+$$
$$(1/2)(z-1)(80\mu + 888\mu^2 + 912\mu^3 - 1904m^4),$$
$$a^\vee_{2,3,3}(z;\nu) = (1/2)(-9 - 183\mu + 2856\mu^2 + 7113\mu^3)+$$
$$(1/2)(z-1)(-8 - 176\mu + 2600\mu^2 + 7120\mu^3),$$
$$a^\vee_{2,3,4}(z;\nu) = (1/2)(-2560\mu - 4432\mu^2 + 3616\mu^3)+$$
$$(1/2)(z-1)(8 - 2512\mu - 4360\mu^2 + 3632\mu^3),$$
$$a^\vee_{2,3,5}(z;\nu) = (1/2)(640 - 2000\mu - 7888\mu^2)+$$
$$(1/2)(z-1)(672 - 1840\mu - 7728\mu^2),$$
$$a^\vee_{2,3,6}(z;\nu) = (1/2)(1464 + 1872\mu - 2880\mu^2)+$$
$$(1/2)(z-1)(1552 + 2224\mu - 2704\mu^2),$$
$$a^\vee_{2,3,7}(z;\nu) = (1/2)(1080 + 2160\mu + (z-1)(1272 + 2736\mu)),$$
$$a^\vee_{2,3,8}(z;\nu) = (1/2)(z-1)(360 + 720\mu),$$
$$a^\vee_{2,4,1}(z;\nu) = (1/2)(-16 - 208\mu - 1048\mu^2 - 2576\mu^3 - 2872\mu^4 - 720\mu^5)+$$
$$(1/2)(z-1)(-16 - 208\mu - 1048\mu^2 - 2576\mu^3 - 2872m^4 - 720\mu^5),$$
$$a^\vee_{2,4,2}(z;\nu) = (1/2)(-16 - 128\mu - 160\mu^2 - 224\mu^3 - 1536\mu^4)+$$
$$(1/2)(z-1)(-16 - 128\mu - 160\mu^2 - 224\mu^3 - 1536\mu^4),$$
$$a^\vee_{2,4,3}(z;\nu) = (1/2)(-8 - 96\mu + 1328\mu^2 + 5152\mu^3 + 976\mu^4)+$$
$$(1/2)(z-1)(-8 - 96\mu + 1328\mu^2 + 5152\mu^3 + 976\mu^4),$$
$$a^\vee_{2,4,4}(z;\nu) = (1/2)(-1 - 1225\mu - 3214\mu^2 + 2105\mu^3)+$$
$$(1/2)(z-1)(-1248\mu - 3200\mu^2 + 2112\mu^3),$$
$$a^\vee_{2,4,5}(z;\nu) = (1/2)(312 - 800\mu - 5680\mu^2 - 704\mu^3)+$$
$$(1/2)(z-1)(320 - 752\mu - 5608\mu^2 - 688\mu^3),$$
$$a^\vee_{2,4,6}(z;\nu) = (1/2)(752 + 1664\mu - 1952\mu^2)+$$
$$(1/2)(z-1)(784 + 1824\mu - 1792\mu^2),$$
$$a^\vee_{2,4,7}(z;\nu) = (1/2)(576 + 1728\mu) + (z-1)(664 + 2080\mu + 176\mu^2)),$$
$$a^\vee_{2,4,8}(z;\nu) = (1/2)(z-1)(192 + 576\mu\mu),$$
$$a^\vee_{2,5,1}(z;\nu) = (1/2)(-16 - 208\mu - 1048\mu^2 - 2576\mu^3 - 3064\mu^4 - 1296\mu^5)+$$
$$(1/2)(z-1)(-16 - 208\mu - 1048\mu^2 - 2576\mu^3 - 3064\mu^4 - 10296\mu^5),$$

$$a^{\vee}_{2,5,2}(z;\nu) = (1/2)(-32 - 336\mu - 1208\mu^2 - 2032\mu^3 - 2104\mu^4 - 720\mu^5)+$$
$$(1/2)(z-1)(-32 - 336\mu - 1208\mu^2 - 2032\mu^3 - 2104\mu^4 - 720\mu^5),$$
$$a^{\vee}_{2,5,3}(z;\nu) = (1/2)(-24 - 224\mu + 16\mu^2 + 2240\mu^3 + 1744\mu^4)+$$
$$(1/2)(z-1)(-24 - 224\mu + 16\mu^2 + 2240\mu^3 + 1744\mu^4),$$
$$a^{\vee}_{2,5,4}(z;\nu) = (1/2)(-8 - 576\mu - 1872\mu^2 + 352\mu^3 + 976\mu^4)+$$
$$(1/2)(z-1)(-8 - 576\mu - 1872\mu^2 + 352\mu^3 + 976\mu^4),$$
$$a^{\vee}_{2,5,5}(z;\nu) = (1/2)(127 - 279\mu - 3062\mu^2 - 2039\mu^3)+$$
$$(1/2)(z-1)(128 - 272\mu - 3048\mu^2 - 2032\mu^3),$$
$$a^{\vee}_{2,5,6}(z;\nu) = (1/2)(328 + 1024\mu - 560\mu^2 - 704\mu^3)+$$
$$(1/2)(z-1)(336 + 1072\mu - 488\mu^2 - 688\mu^3),$$
$$a^{\vee}_{2,5,7}(z;\nu) = (1/2)(264 + 1056\mu + 528\mu^2 + (z-1)(296 + 1216\mu + 688\mu^2)),$$
$$a^{\vee}_{2,5,8}(z;\nu) = (1/2)(z-1)(88 + 352\mu + 176\mu^2),$$
$$a^{\vee}_{2,6,1}(z;\nu) = (1/2)(-16-208\mu-1048\mu^2-2576\mu^3-3152\mu^4-1648\mu^5-176\mu^6)+$$
$$(1/2)(z-1)(-16 - 208\mu - 1048\mu^2 - 2576\mu^3 - 3152\mu^4 - 1648\mu^5 - 176\mu^6),$$
$$a^{\vee}_{2,6,2}(z;\nu) = (1/2)(-48 - 544\mu - 2256\mu^2 - 4256\mu^3 - 3760\mu^4 - 1312\mu^5)+$$
$$(1/2)(z-1)(-48 - 544\mu - 2256\mu^2 - 4256\mu^3 - 3760\mu^4 - 1312\mu^5),$$
$$a^{\vee}_{2,6,3}(z;\nu) = (1/2)(-56 - 560\mu - 1720\mu^2 - 1552\mu^3 - 8\mu^4 - 16\mu^5)+$$
$$(1/2)(z-1)(-56 - 560\mu - 1720\mu^2 - 1552\mu^3 - 8\mu^4 - 16\mu^5),$$
$$a^{\vee}_{2,6,4}(z;\nu) = (1/2)(-32 - 448\mu - 1504\mu^2 - 928\mu^3 + 608\mu^4)+$$
$$(1/2)(z-1)(-32 - 448\mu - 1504\mu^2 - 928\mu^3 + 608\mu^4),$$
$$a^{\vee}_{2,6,5}(z;\nu) = (1/2)(32 - 144\mu - 1400\mu^2 - 1680\mu^3 - 80\mu^4)+$$
$$(1/2)(z-1)(32 - 144\mu - 1400\mu^2 - 1680\mu^3 - 80\mu^4),$$
$$a^{\vee}_{2,6,6}(z;\nu) = (1/2)(111 + 441\mu - 30\mu^2 - 615\mu^3)+$$
$$(1/2)(z-1)(112 + 448\mu - 16\mu^2 - 608\mu^3),$$
$$a^{\vee}_{2,6,7}(z;\nu) = (1/2)(96 + 480\mu + 480\mu^2)+$$
$$(1/2)(z-1)(104 + 528\mu + 552\mu^2 + 16\mu^3),$$
$$a^{\vee}_{2,6,8}(z;\nu) = (1/2)(z-1)(32 + 160\mu + 160\mu^2),$$
$$a^{\vee}_{2,7,1}(z;\nu) = (1/2)(-16-208\mu-1048\mu^2-2576\mu^3-3184\mu^4-1808\mu^5-336\mu^6)+$$
$$(1/2)(z-1)(-16 - 208\mu - 1048\mu^2 - 2576\mu^3 - 3184\mu^4 - 1808\mu^5 - 336\mu^6),$$
$$a^{\vee}_{2,7,2}(z;\nu) = (1/2)(-64-752\mu-3304\mu^2-6704\mu^3-6272\mu^4-2320\mu^5-176\mu^6)+$$
$$(1/2)(z-1)(-64 - 752\mu - 3304\mu^2 - 6704\mu^3 - 6272\mu^4 - 2320\mu^5 - 176\mu^6),$$
$$a^{\vee}_{2,7,3}(z;\nu) = (1/2)(-104 - 1104\mu - 4168\mu^2 - 6640\mu^3 - 4088\mu^4 - 688\mu^5)+$$
$$(1/2)(z-1)(-104 - 1104\mu - 4168\mu^2 - 6640\mu^3 - 4088\mu^4 - 688\mu^5),$$
$$a^{\vee}_{2,7,4}(z;\nu) = (1/2)(-88 - 880\mu - 2968\mu^2 - 3760\mu^3 - 1320\mu^4 - 16\mu^5)+$$

$$(1/2)(z-1)(-88-880\mu-2968\mu^2-3760\mu^3-1320\mu^4-16\mu^5),$$

$$a^{\vee}_{2,7,5}(z;\nu)=(1/2)(-32-368\mu-1336\mu^2-1648\mu^3-432\mu^4)+$$

$$(1/2)(z-1)(-32-368\mu-1336\mu^2-1648\mu^3-432\mu^4),$$

$$a^{\vee}_{2,7,6}(z;\nu)=(1/2)(16+48\mu-136\mu^2-368\mu^3-80\mu^4)+$$

$$(1/2)(z-1)(16+48\mu-136\mu^2-368\mu^3-80\mu^4),$$

$$a^{\vee}_{2,7,7}(z;\nu)=(1/2)(23+137\mu+202\mu^2+41\mu^3)+$$

$$(1/2)(z-1)(24+144\mu+216\mu^2+48\mu^3),$$

$$a^{\vee}_{2,7,8}(z;\nu)=(1/2)(z-1)(8+48\mu+72\mu^2+16\mu^3),$$

$$a^{\vee}_{2,8,1}(z;\nu)=$$

$$(1/2)(-16-208\mu-1048\mu^2-2576\mu^3-3192\mu^4-1856\mu^5-408\mu^6-16\mu^7)+$$

$$(1/2)(z-1)(-16-208\mu-1048\mu^2-2576\mu^3-3192\mu^4-1856\mu^5-408\mu^6-16\mu^7),$$

$$a^{\vee}_{2,8,2}(z;\nu)=(1/2)(-80-960\mu-4352\mu^2-9248\mu^3-9264\mu^4-3840\mu^5-448\mu^6)+$$

$$(1/2)(z-1)(-80-960\mu-4352\mu^2-9248\mu^3-9264\mu^4-3840\mu^5-448\mu^6),$$

$$a^{\vee}_{2,8,3}(z;\nu)=$$

$$(1/2)(-168-1856\mu-7520\mu^2-13600\mu^3-10600\mu^4-2816\mu^5-112\mu^6)+$$

$$(1/2)(z-1)(-168-1856\mu-7520\mu^2-13600\mu^3-10600\mu^4-2816\mu^5-112\mu^6),$$

$$a^{\vee}_{2,8,4}(z;\nu)=(1/2)(-192-1952\mu-7040\mu^2-10688\mu^3-6208\mu^4-896\mu^5)+$$

$$(1/2)(z-1)(-192-1952\mu-7040\mu^2-10688\mu^3-6208\mu^4-896\mu^5),$$

$$a^{\vee}_{2,8,5}(z;\nu)=(1/2)(-128-1200\mu-3848\mu^2-4848\mu^3-1992\mu^4-112\mu^5)+$$

$$(1/2)(z-1)(-128-1200\mu-3848\mu^2-4848\mu^3-1992\mu^4-112\mu^5),$$

$$a^{\vee}_{2,8,6}(z;\nu)=(1/2)(-48-416\mu-1184\mu^2-1216\mu^3-320\mu^4)+$$

$$(1/2)(z-1)(-48-416\mu-1184\mu^2-1216\mu^3-320\mu^4),$$

$$a^{\vee}_{2,8,7}(z;\nu)=(1/2)(-8-64\mu-160\mu^2-128\mu^3-16\mu^4)+$$

$$(1/2)(z-1)(-8-64\mu-160\mu^2-128\mu^3-16\mu^4),$$

$$a^{\vee}_{2,8,8}(z;\nu)=(1/2)(-1-7\mu-14\mu^2-7\mu^3),$$

$$a^{\wedge}_{1,2,1}(z;\nu)=8+60\mu+146\mu^2+(z-1)(8+60\mu+146\mu^2),$$

$$a^{\wedge}_{1,2,2}(z;\nu)=-5-63\mu+145\mu^2+(z-1)(-4-60\mu+146\mu^2)),$$

$$a^{\wedge}_{1,2,3}(z;\nu)=-312\mu+(z-1)(6-300\mu),$$

$$a^{\wedge}_{1,2,4}(z;\nu)=104-198\mu+(z-1)(122-180m),$$

$$a^{\wedge}_{1,2,5}(z;\nu)=132+(z-1)(170),$$

$$a^{\wedge}_{1,2,6}(z;\nu)=(z-1)(66),$$

$$a^{\wedge}_{1,3,1}(z;\nu)=8+60\mu+146\mu^2+66\mu^3+(z-1)(8+60\mu+146\mu^2+66\mu^3),$$

$$a^{\wedge}_{1,3,2}(z;\nu)=4+94\mu^2+(z-1)(4+94\mu^2)),$$

$$a^{\wedge}_{1,3,3}(z;\nu) = 1 - 165\mu - 53\mu^2 + (z-1)(2 - 162\mu - 52\mu^2),$$
$$a^{\wedge}_{1,3,4}(z;\nu) = 56 - 96\mu + (z-1)(62 - 84\mu),$$
$$a^{\wedge}_{1,3,5}(z;\nu) = 76 + (z-1)(94 + 18\mu),$$
$$a^{\wedge}_{1,3,6}(z;\nu) = (z-1)(38),$$
$$a^{\wedge}_{1,4,1}(z;\nu) = 8 + 60\mu + 146\mu^2 + 104\mu^3 + (z-1)(8 + 60\mu + 146\mu^2 + 104\mu^3),$$
$$a^{\wedge}_{1,4,2}(z;\nu) = 12 + 60\mu + 126\mu^2 + 66\mu^3 + (z-1)(12 + 60\mu + 126\mu^2 + 66\mu^3),$$
$$a^{\wedge}_{1,4,3}(z;\nu) = 6 - 48\mu - 72\mu^2 + (z-1)(6 - 48\mu - 72\mu^2),$$
$$a^{\wedge}_{1,4,4}(z;\nu) = 25 - 21\mu - 53\mu^2 + (z-1)(26 - 18\mu - 52\mu^2),$$
$$a^{\wedge}_{1,4,5}(z;\nu) = 36 + 36\mu + (z-1)(42 + 48\mu),$$
$$a^{\wedge}_{1,4,6}(z;\nu) = (z-1)(18 + 18\mu),$$
$$a^{\wedge}_{1,5,1}(z;\nu) = 8 + 60\mu + 146\mu^2 + 122\mu^3 + 18\mu^4 +$$
$$(z-1)(8 + 60\mu + 146\mu^2 + 122\mu^3 + 18\mu^4),$$
$$a^{\wedge}_{1,5,2}(z;\nu) = 20 + 120\mu + 218\mu^2 + 116\mu^3 + (z-1)(20 + 1200\mu + 218mu^2 + 116\mu^3),$$
$$a^{\wedge}_{1,5,3}(z;\nu) = 18 + 66\mu + 54\mu^2 + 12\mu^3 + (z-1)(18 + 66\mu + 54\mu^2 + 12\mu^3),$$
$$a^{\wedge}_{1,5,4}(z;\nu) = 14 + 24\mu - 16\mu^2 + (z-1)(14 + 24\mu - 16\mu^2),$$
$$a^{\wedge}_{1,5,5}(z;\nu) = 13 + 27\mu + \mu^2 + (z-1)(14 + 30\mu + 2\mu^2),$$
$$a^{\wedge}_{1,5,6}(z;\nu) = (z-1)(6 + 12\mu),$$
$$a^{\wedge}_{1,6,1}(z;\nu) = 8 + 60\mu + 146\mu^2 + 128\mu^3 + 30\mu^4 +$$
$$(z-1)(8 + 60\mu + 146\mu^2 + 128\mu^3 + 30\mu^4),$$
$$a^{\wedge}_{1,6,2}(z;\nu) = 28 + 180\mu + 346\mu^2 + 202\mu^3 + 18\mu^4 +$$
$$(z-1)(28 + 180\mu + 346\mu^2 + 202\mu^3 + 18\mu^4),$$
$$a^{\wedge}_{1,6,3}(z;\nu) = 38 + 204\mu + 290\mu^2 + 92\mu^3 + (z-1)(38 + 204\mu + 290\mu^2 + 92\mu^3),$$
$$a^{\wedge}_{1,6,4}(z;\nu) = 26 + 114\mu + 110\mu^2 + 12\mu^3 + (z-1)(26 + 114\mu + 110\mu^2 + 12\mu^3),$$
$$a^{\wedge}_{1,6,5}(z;\nu) = 10 + 36\mu + 22\mu^2 + (z-1)(10 + 36\mu + 22\mu^2),$$
$$a^{\wedge}_{1,6,6}(z;\nu) = 1 + 3\mu + \mu^2 + (z-1)(2 + 6\mu + 2\mu^2),$$
$$a^{\wedge}_{2,2,1}(z;\nu) =$$
$$16 + 176\mu + 728\mu^2 + 1408\mu^3 + (z-1)(16 + 176\mu + 728\mu^2 + 1408\mu^3),$$
$$a^{\wedge}_{2,2,2}(z;\nu) = -15 - 251\mu - 1450\mu^2 + 1409\mu^3 +$$
$$(z-1)(-16 - 256\mu - 1456\mu^2 + 1408\mu^3),$$
$$a^{\wedge}_{2,2,3}(z;\nu) = 32 + 448\mu - 5072\mu^2 + (z-1)(24 + 416\mu - 5096\mu^2),$$
$$a^{\wedge}_{2,2,4}(z;\nu) = 4640\mu - 2880\mu^2 + (z-1)(-32 + 4544\mu - 2912\mu^2),$$
$$a^{\wedge}_{2,2,5}(z;\nu) = -1160 + 6112\mu + (z-1)(-1248 + 5936\mu),$$
$$a^{\wedge}_{2,2,6}(z;\nu) = -2544 + 2432\mu + (z-1)(-2736 + 2240\mu),$$
$$a^{\wedge}_{2,2,7}(z;\nu) = -1824 + (z-1)(-2184\mu),$$

$$a^{\wedge}_{2,2,8}(z;\nu) = (z-1)(-608),$$
$$a^{\wedge}_{2,3,1}(z;\nu) = 16 + 176\mu + 728\mu^2 + 1408\mu^3 + 608\mu^4 +$$
$$(z-1)(16 + 176\mu + 728\mu^2 + 1408\mu^3 + 608\mu^4),$$
$$a^{\wedge}_{2,3,2}(z;\nu) = -80\mu - 728\mu^2 + 384\mu^3 +$$
$$(z-1)(-80\mu - 728\mu^2 + 384\mu^3),$$
$$a^{\wedge}_{2,3,3}(z;\nu) = 9 + 165\mu - 2898\mu^2 - 1023\mu^3 +$$
$$(z-1)(8 + 160\mu - 2904\mu^2 - 1024\mu^3),$$
$$a^{\wedge}_{2,3,4}(z;\nu) = 2560\mu - 688\mu^2 + (z-1)(-8 + 2528\mu - 712\mu^2),$$
$$a^{\wedge}_{2,3,5}(z;\nu) = -640 + 3280\mu + 768\mu^2 + (z-1)(-672 + 3184\mu + 736\mu^2),$$
$$a^{\wedge}_{2,3,6}(z;\nu) = -1464 + 1056\mu + (z-1)(-1552 + 880\mu),$$
$$a^{\wedge}_{2,3,7}(z;\nu) = -1080 - (z-1)(-1272 - 192\mu),$$
$$a^{\wedge}_{2,3,8}(z;\nu) = (z-1)(-360),$$
$$a^{\wedge}_{2,4,1}(z;\nu) = 16 + 176\mu + 728\mu^2 + 1408\mu^3 + 968\mu^4 +$$
$$(z-1)(16 + 176\mu + 728\mu^2 + 1408\mu^3 + 968\mu^4),$$
$$a^{\wedge}_{2,4,2}(z;\nu) = 16 + 96\mu + 352\mu^3 + 608\mu^4$$
$$(z-1)(16 + 96\mu + 352\mu^3 + 608\mu^3),$$
$$a^{\wedge}_{2,4,3}(z;\nu) = 8 + 80\mu - 1472\mu^2 - 2080\mu^3 +$$
$$(z-1)(8 + 80\mu - 1472\mu^2 - 2080\mu 3),$$
$$a^{\wedge}_{2,4,4}(z;\nu) = 1 + 1253\mu + 710\mu^2 - 1023\mu^3 +$$
$$+(z-1)(1248\mu + 704\mu^2 - 1024\mu^3),$$
$$a^{\wedge}_{2,4,5}(z;\nu) = -312 + 1424\mu + 2208\mu^2 +$$
$$(z-1)(-320 + 1392\mu + 2184\mu^2),$$
$$a^{\wedge}_{2,4,6}(z;\nu) = -752 - 160\mu + 768\mu^2 + (z-1)(-784 - 256\mu + 736\mu^2),$$
$$a^{\wedge}_{2,4,7}(z;\nu) = -576 - 576\mu + (z-1)(-664 - 752\mu),$$
$$a^{\wedge}_{2,4,8}(z;\nu) = (z-1)(-192 - 192\mu),$$
$$a^{\wedge}_{2,5,1}(z;\nu) = 16 + 176\mu + 728\mu^2 + 1408\mu^3 + 1160\mu^4 + 192\mu^5 +$$
$$(z-1)(16 + 176\mu + 728\mu^2 + 1408\mu^3 + 1160\mu^4 + 192\mu^5),$$
$$a^{\wedge}_{2,5,2}(z;\nu) = 32 + 272\mu + 728\mu^2 + 992\mu^3 + 808\mu^4$$
$$(z-1)(32 + 272\mu + 728\mu^2 + 992\mu^3 + 808\mu^4),$$
$$a^{\wedge}_{2,5,3}(z;\nu) = 24 + 176\mu - 320\mu^2 - 1344\mu^3 - 160\mu^4 +$$
$$(z-1)(24 + 176\mu - 320\mu^2 - 1344\mu^3 - 160\mu^4),$$
$$a^{\wedge}_{2,5,4}(z;\nu) = 8 + 560\mu + 768\mu^2 - 800\mu^3 +$$
$$+(z-1)(8 + 560\mu + 768\mu^2 - 800\mu^3),$$
$$a^{\wedge}_{2,5,5}(z;\nu) = -127 + 533\mu + 1742\mu^2 + 129\mu^3$$

$$(z-1)(-128+528\mu+1736\mu^2+128\mu^3),$$
$$a^{\wedge}_{2,5,6}(z;\nu)=-328-368\mu+640\mu^2+(z-1)(-336-400\mu+616\mu^2),$$
$$a^{\wedge}_{2,5,7}(z;\nu)=-264-528\mu+(z-1)(-296-624\mu-32\mu^2),$$
$$a^{\wedge}_{2,5,8}(z;\nu)=(z-1)(-88-176\mu),$$
$$a^{\wedge}_{2,6,1}(z;\nu)=16+176\mu+728\mu^2+1408\mu^3+1248\mu^4+368\mu^5+$$
$$(z-1)(16+176\mu+728\mu^2+1408\mu^3+1248\mu^4+368\mu^5),$$
$$a^{\wedge}_{2,6,2}(z;\nu)=48+448\mu+1456\mu^2+2048\mu^3+1264\mu^4+192\mu^5+$$
$$(z-1)(48+448\mu+1456\mu^2+2048\mu^3+1264\mu^4+192\mu^5),$$
$$a^{\wedge}_{2,6,3}(z;\nu)=56+448\mu+936\mu^2+352\mu^3-56\mu^4+$$
$$(z-1)(56+448\mu+936\mu^2+352\mu^3-56\mu^4),$$
$$a^{\wedge}_{2,6,4}(z;\nu)=32+384\mu+800\mu^2-32\mu^3-160\mu^4+$$
$$(z-1)(32+384\mu+800\mu^2-32\mu^3-160\mu^4),$$
$$a^{\wedge}_{2,6,5}(z;\nu)=-32+208\mu+920\mu^2+384\mu^3+$$
$$(z-1)(-32+208\mu+920\mu^2+384\mu^3),$$
$$a^{\wedge}_{2,6,6}(z;\nu)=-111-219\mu+246\mu^2+129\mu^3+$$
$$+(z-1)(-112-224\mu+240\mu^2+128mu^3),$$
$$a^{\wedge}_{2,6,7}(z;\nu)=-96-288\mu-96\mu^2+(z-1)(-104-320\mu-120\mu^2),$$
$$a^{\wedge}_{2,6,8}(z;\nu)=(z-1)(-32-96\mu-32\mu^2),$$
$$a^{\wedge}_{2,7,1}(z;\nu)=16+176\mu+728\mu^2+1408\mu^3+1280\mu^4+464\mu^5+32\mu^6+$$
$$(z-1)(16+176\mu+728\mu^2+1408\mu^3+1280\mu^4+464\mu^5+32\mu^6),$$
$$a^{\wedge}_{2,7,2}(z;\nu)=64+624\mu+2184\mu^2+3328\mu^3+2128\mu^4+432\mu^5+$$
$$(z-1)(64+624\mu+2184\mu^2+3328\mu^3+2128\mu^4+432\mu^5),$$
$$a^{\wedge}_{2,7,3}(z;\nu)=104+896\mu+2584\mu^2+2848\mu^3+1016\mu^4+64\mu^5+$$
$$(z-1)(104+896\mu+2584\mu^2+2848\mu^3+1016\mu^4+64\mu^5),$$
$$a^{\wedge}_{2,7,4}(z;\nu)=88+704\mu+1736\mu^2+1344\mu^3+168\mu^4+$$
$$(z-1)(88+704\mu+1736\mu^2+1344\mu^3+168\mu^4),$$
$$a^{\wedge}_{2,7,5}(z;\nu)=32+304\mu+792\mu^2+544\mu^3+32\mu^4+$$
$$(z-1)(32+304\mu+792\mu^2+544\mu^3+32\mu^4),$$
$$a^{\wedge}_{2,7,6}(z;\nu)=-16-16\mu+136\mu^2+128\mu^3+$$
$$(z-1)(-16-16\mu+136\mu^2+128mu^3),$$
$$a^{\wedge}_{2,7,7}(z;\nu)=-23-91\mu-66\mu^2+\mu^3+(z-1)(-24-96\mu-72\mu^2),$$
$$a^{\wedge}_{2,7,8}(z;\nu)=(z-1)(-8-32\mu-24\mu^2)$$
$$a^{\wedge}_{2,8,1}(z;\nu)=16+176\mu+728\mu^2+1408\mu^3+1288\mu^4+496\mu^5+56\mu^6+$$
$$(z-1)(16+176\mu+728\mu^2+1408\mu^3+1288\mu^4+496\mu^5+56\mu^6),$$

$$a^{\wedge}_{2,8,2}(z;\nu) = 80 + 800\mu + 2912\mu^2 + 4704\mu^3 + 3280\mu^4 + 800\mu^5 + +32\mu^6 +$$
$$(z-1)(80 + 800\mu + 2912\mu^2 + 4704\mu^3 + 3280\mu^4 + 800\mu^5 + +32\mu^6),$$
$$a^{\wedge}_{2,8,3}(z;\nu) = 168 + 1520\mu + 4816\mu^2 + 6336\mu^3 + 3160\mu^4 + 400\mu^5 +$$
$$(z-1)(168 + 1520\mu + 4816\mu^2 + 6336\mu^3 + 3160\mu^4 + 400\mu^5),$$
$$a^{\wedge}_{2,8,4}(z;\nu) = 192 + 1568\mu + 4288\mu^2 + 4480\mu^3 + 1472\mu^4 + 64\mu^5 +$$
$$(z-1)(192 + 1568\mu + 4288\mu^2 + 4480\mu^3 + 1472\mu^4 + 64\mu^5 +),$$
$$a^{\wedge}_{2,8,5}(z;\nu) = 128 + 944\mu + 2216\mu^2 + 1792\mu^3 + 344\mu^4 +$$
$$(z-1)(128 + 944\mu + 2216\mu^2 + 1792\mu^3 + 344\mu^4),$$
$$a^{\wedge}_{2,8,6}(z;\nu) = 48 + 320\mu + 640\mu^2 + 384\mu^3 + 32\mu^4 +$$
$$(z-1)(48 + 320\mu + 640\mu^2 + 384\mu^3 + 32\mu^4),$$
$$a^{\wedge}_{2,8,7}(z;\nu) = 8 + 48\mu + 80\mu^2 + 33\mu^3 +$$
$$(z-1)(8 + 48\mu + 80\mu^2 + 32\mu^3),$$
$$a^{\wedge}_{2,8,8}(z;\nu) = 1 + 5\mu + 6\mu^2 + \mu^3.$$

We denote by

$$A^{*}_{l}(z;\nu),\ A^{\vee}_{l}(z;\nu),\ A^{\wedge}_{l}(z;\nu)$$

for $l = 0,\ 1,\ 2$ the $(4+2l)\times(4+2l)$-matrix, such that its element in i-th row and k-th column is equal respectively to the k-th elements of the rows respecively

$$a^{*}_{l,i,k}(z;\nu),\ a^{\vee}_{l,i,k}(z;\nu),\ a^{\wedge}_{l,i,k}(z;\nu)$$

for $i = 1,\ ...,\ 4+2l$, $k = 1,\ ...,\ 4+2l$. Clearly,

$$A^{*}_{l}(z;\nu) = A^{\vee}_{l}(z;\nu) + A^{\wedge}_{l}(z;\nu)\tau \tag{31}$$

for $l = 0, 1, 2$. Let

$$U^{\vee}_{l}(z,\mu) = \sum_{k=0}^{3+2l} \mu^k (R^{\vee}_{l}(k)) + (z-1)V^{\vee}_{l}(k),$$

$$U^{\wedge}_{l}(z,\mu) = \sum_{k=0}^{2+2l} \mu^k (R^{\wedge}_{l}(k)) + (z-1)V^{\wedge}_{l}(k)),$$

where $l = 0,\ 1,\ 2,$

$$R^{\vee}_{0}(0) = \begin{pmatrix} -5 & 0 & 24 & 0 \\ -4 & -1 & 8 & 0 \\ -4 & -4 & 3 & 0 \\ -4 & -8 & -4 & -1 \end{pmatrix},\ V^{\vee}_{0}(0) = \begin{pmatrix} -4 & 4 & 32 & 24 \\ -4 & -4 & 12 & 8 \\ -4 & 0 & 4 & 4 \\ -4 & -8 & -4 & 0 \end{pmatrix},$$

$$R^{\vee}_{0}(1) = \begin{pmatrix} -27 & -48 & 0 & 0 \\ -24 & -19 & 0 & 0 \\ -24 & -24 & 5 & 0 \\ -24 & -40 & -16 & -3 \end{pmatrix},\ V^{\vee}_{0}(1) = \begin{pmatrix} -24 & -40 & 0 & 0 \\ -24 & -16 & 8 & 0 \\ -24 & -24 & 8 & 8 \\ -24 & -40 & -16 & 0 \end{pmatrix},$$

$$R_0^\vee(2) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ -24 & 0 & 0 & 0 \\ -32 & -24 & 0 & 0 \\ -36 & -40 & -8 & 0 \end{pmatrix}, \; V_0^\vee(2) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ -24 & 0 & 0 & 0 \\ -32 & -24 & 0 & 0 \\ -36 & -40 & -8 & 0 \end{pmatrix},$$

$$R_0^\vee(3) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -8 & 0 & 0 & 0 \end{pmatrix}, \; V_0^\vee(3) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -8 & 0 & 0 & 0 \end{pmatrix},$$

$$R_0^\wedge(0) = \begin{pmatrix} 5 & 0 & 0 & 0 \\ 4 & 1 & -8 & 0 \\ 4 & 4 & -3 & 0 \\ 4 & 8 & 4 & 1 \end{pmatrix}, \; V_0^\wedge(0) = \begin{pmatrix} 4 & -4 & -8 & 0 \\ 4 & 0 & -12 & -8 \\ 4 & 4 & -4 & -4 \\ 4 & 8 & 4 & 0 \end{pmatrix},$$

$$R_0^\wedge(1) = \begin{pmatrix} 17 & 0 & 0 & 0 \\ 4 & 1 & -8 & 0 \\ 16 & 16 & 1 & 0 \\ 16 & 24 & 8 & 1 \end{pmatrix}, \; V_0^\wedge(1) = \begin{pmatrix} 16 & 16 & 0 & 0 \\ 16 & 16 & 0 & 0 \\ 16 & 16 & 0 & 0 \\ 16 & 24 & 8 & 0 \end{pmatrix},$$

$$R_0^\wedge(2) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 8 & 0 & 0 & 0 \\ 12 & 8 & 0 & 0 \end{pmatrix}, \; V_0^\wedge(2) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 8 & 0 & 0 & 0 \\ 12 & 8 & 0 & 0 \end{pmatrix}$$

$$R_1^\vee(0) = \begin{pmatrix} -7 & 18 & 0 & -270 & -408 & 0 \\ -8 & 5 & 0 & -104 & -132 & 0 \\ -8 & -4 & -1 & -56 & -76 & 0 \\ -8 & -12 & -6 & -25 & -36 & 0 \\ -8 & -20 & -18 & -14 & -13 & 0 \\ -8 & -28 & -38 & -26 & -10 & -1 \end{pmatrix},$$

$$V_1^\vee(0) = \begin{pmatrix} -8 & 12 & -18 & -308 & -474 & -204 \\ -8 & 4 & -6 & -122 & -170 & -66 \\ -8 & -4 & -2 & -62 & -94 & -38 \\ -8 & -12 & -6 & -26 & -42 & -18 \\ -8 & -20 & -18 & -14 & -14 & -6 \\ -8 & -28 & -38 & -26 & -10 & -2 \end{pmatrix},$$

$$R_1^\vee(1) = \begin{pmatrix} -71 & 168 & 810 & 612 & 0 & 0 \\ -76 & 73 & 312 & 122 & 0 & 0 \\ -76 & -8 & 163 & -16 & -152 & 0 \\ -76 & -84 & 36 & -29 & -108 & 0 \\ -76 & -160 & -102 & -52 & -53 & 0 \\ -76 & -236 & -280 & -166 & -56 & -5 \end{pmatrix},$$

$$V_1^\vee(1) = \begin{pmatrix} -76 & 144 & 756 & 536 & -474 & 0 \\ -76 & 68 & 288 & 68 & -76 & 0 \\ -76 & -8 & 158 & -40 & -206 & -76 \\ -76 & -84 & 36 & -34 & -132 & -54 \\ -76 & -160 & -102 & -52 & -58 & -24 \\ -76 & -236 & -280 & -166 & -56 & -10 \end{pmatrix},$$

$$R_1^{\vee}(2) = \begin{pmatrix} -245 & -384 & 0 & 0 & 0 & 0 \\ -250 & -29 & 228 & 0 & 0 & 0 \\ -250 & -86 & 385 & 228 & 0 & 0 \\ -250 & -222 & 180 & 145 & 0 & 0 \\ -250 & -418 & -150 & -4 & -29 & 0 \\ -250 & -650 & -622 & -286 & -74 & -5 \end{pmatrix},$$

$$V_1^{\vee}(2) = \begin{pmatrix} -250 & -396 & 0 & 0 & 0 & 0 \\ -250 & -34 & 216 & 0 & 0 & 0 \\ -250 & -86 & 380 & 216 & 0 & 0 \\ -250 & -222 & 180 & 140 & -12 & 0 \\ -250 & -418 & -150 & -4 & -34 & -12 \\ -250 & -650 & -622 & -286 & -74 & -10 \end{pmatrix},$$

$$R_1^{\vee}(3) = V_1^{\vee}(3) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ -204 & 0 & 0 & 0 & 0 & 0 \\ -270 & -204 & 0 & 0 & 0 & 0 \\ -308 & -246 & 24 & 0 & 0 & 0 \\ -326 & -392 & -60 & 24 & 0 & 0 \\ -332 & -646 & -416 & -108 & -12 & 0 \end{pmatrix},$$

$$R_1^{\vee}(4) = V_1^{\vee}(4) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -76 & 0 & 0 & 0 & 0 & 0 \\ -130 & -76 & 0 & 0 & 0 & 0 \\ 154 & -170 & -40 & 0 & 0 & 0 \end{pmatrix},$$

$$R_1^{\vee}(5) = V_1^{\vee}(5) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -12 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$R_1^{\wedge}(0) = \begin{pmatrix} 7 & -18 & 0 & 66 & 0 & 0 \\ 8 & -5 & 0 & 104 & 132 & 0 \\ 8 & 4 & 1 & 56 & 76 & 0 \\ 8 & 12 & 6 & 25 & 36 & 0 \\ 8 & 20 & 18 & 14 & 13 & 0 \\ 8 & 28 & 38 & 26 & 10 & 1 \end{pmatrix},$$

$$V_1^{\wedge}(0) = \begin{pmatrix} 8 & -12 & 18 & 104 & 66 & 0 \\ 8 & -4 & 6 & 122 & 170 & 0 \\ 8 & 12 & 6 & 26 & 42 & 18 \\ 8 & 12 & 6 & 25 & 36 & 0 \\ 8 & 20 & 18 & 14 & 14 & 6 \\ 8 & 28 & 38 & 26 & 10 & 2 \end{pmatrix},$$

$$R_1^{\wedge}(1) = \begin{pmatrix} 57 & -132 & -198 & 0 & 0 & 0 \\ 60 & -63 & -312 & -198 & 0 & 0 \\ 60 & 0 & -165 & -96 & 0 & 0 \\ 60 & 60 & -48 & -21 & 36 & 0 \\ 60 & 120 & 66 & 24 & 27 & 0 \\ 60 & 180 & 204 & 114 & 36 & 3 \end{pmatrix},$$

$$V_1^{\wedge}(1) = \begin{pmatrix} 60 & -120 & -180 & 0 & 0 & 0 \\ 60 & -60 & -300 & -180 & 0 & 0 \\ 60 & 0 & -162 & -84 & 18 & 0 \\ 60 & 60 & -48 & -18 & 48 & 18 \\ 60 & 120 & 66 & 24 & 30 & 12 \\ 60 & 180 & 204 & 114 & 36 & 6 \end{pmatrix},$$

$$R_1^{\wedge}(2) = \begin{pmatrix} 145 & 0 & 0 & 0 & 0 & 0 \\ 146 & 145 & 0 & 0 & 0 & 0 \\ 146 & 94 & -53 & 0 & 0 & 0 \\ 146 & 126 & -72 & -53 & 0 & 0 \\ 146 & 218 & 54 & -16 & 1 & 0 \\ 146 & 346 & 290 & 110 & 22 & 1 \end{pmatrix},$$

$$V_1^{\wedge}(2) = \begin{pmatrix} 146 & 0 & 0 & 0 & 0 & 0 \\ 146 & 146 & 0 & 0 & 0 & 0 \\ 146 & 94 & -52 & 0 & 0 & 0 \\ 146 & 126 & -72 & -52 & 0 & 0 \\ 146 & 218 & 54 & -16 & 2 & 0 \\ 146 & 346 & 290 & 110 & 22 & 2 \end{pmatrix},$$

$$R_1^{\wedge}(3) = V_1^{\wedge}(3) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 66 & 0 & 0 & 0 & 0 & 0 \\ 104 & 66 & 0 & 0 & 0 & 0 \\ 122 & 116 & 12 & 0 & 0 & 0 \\ 128 & 202 & 92 & 12 & 0 & 0 \end{pmatrix},$$

$$R_1^{\wedge}(4) = V_1^{\wedge}(4) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 18 & 0 & 0 & 0 & 0 & 0 \\ 30 & 18 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$R_2^{\vee}(0) = \begin{pmatrix} -17 & 24 & -88 & 0 & 2872 & 6928 & 5712 & 0 \\ -16 & 15 & -32 & 0 & 1160 & 2544 & 1824 & 0 \\ -16 & 80 & -9 & 0 & 640 & 1464 & 1080 & 0 \\ -16 & -16 & -8 & -1 & 312 & 752 & 576 & 0 \\ -16 & -32 & -24 & -8 & 127 & 328 & 264 & 0 \\ -16 & -48 & -56 & -32 & 32 & 111 & 96 & 0 \\ -16 & -64 & -104 & -88 & -32 & 16 & 23 & 0 \\ -16 & -80 & -168 & -192 & -128 & -48 & -8 & -1 \end{pmatrix},$$

$$V_2^{\vee}(0) = \begin{pmatrix} -16 & 32 & -56 & 88 & 3064 & 7288 & 6320 & 1904 \\ -16 & 16 & -24 & 32 & 1248 & 2736 & 2184 & 608 \\ -16 & 0 & -8 & 8 & 672 & 1552 & 1272 & 360 \\ -16 & -16 & -8 & 0 & 320 & 784 & 664 & 192 \\ -16 & -32 & -24 & -8 & 128 & 336 & 296 & 88 \\ -16 & -48 & -56 & -32 & 32 & 112 & 104 & 32 \\ -16 & -64 & -104 & -88 & -32 & 16 & 24 & 8 \\ -16 & -80 & -168 & -192 & -128 & -48 & -8 & 0 \end{pmatrix},$$

$$R_2^{\vee}(1) = \begin{pmatrix} -215 & 448 & -1120 & -11488 & 16944 & -7616 & 0 & 0 \\ -208 & 281 & -512 & -4640 & -5008 & -992 & 0 & 0 \\ -208 & 80 & -183 & -2560 & -2000 & 1872 & 2160 & 0 \\ -208 & -128 & -96 & -1225 & -800 & 1664 & 1728 & 0 \\ -208 & -336 & -224 & -576 & -279 & 1024 & 1256 & 0 \\ -208 & -544 & -560 & -448 & -144 & 441 & 480 & 0 \\ -208 & -752 & -1104 & -880 & -368 & 48 & 137 & 0 \\ -208 & -960 & -1856 & -1952 & -1200 & -416 & -64 & -7 \end{pmatrix},$$

$$V_2^{\vee}(1) = \begin{pmatrix} -208 & 496 & -960 & -11136 & 16368 & -6896 & 0 & 0 \\ -208 & 288 & -464 & -4480 & -4656 & -416 & 720 & 0 \\ -208 & 80 & -176 & -2512 & -1840 & 2244 & 2736 & 720 \\ -288 & -128 & -96 & -1248 & -752 & 1824 & 2080 & 576 \\ -208 & -336 & -224 & -576 & -272 & 1072 & 1216 & 352 \\ -208 & -544 & -560 & -448 & -144 & 448 & 528 & 160 \\ -208 & -752 & -1104 & -880 & -368 & 48 & 144 & 48 \\ -208 & -960 & -1856 & -1952 & -1200 & -416 & -64 & 0 \end{pmatrix},$$

$$R_2^{\vee}(2) = \begin{pmatrix} -1062 & 2912 & 12592 & 8544 & 0 & 0 & 0 & 0 \\ -1048 & 1992 & 4240 & -1536 & -2880 & 0 & 0 & 0 \\ -1048 & 888 & 2856 & -4432 & -7888 & -2880 & 0 & 0 \\ -1048 & -160 & 1328 & -3214 & -5680 & -1952 & 0 & 0 \\ -1048 & -1208 & 16 & -1872 & -3062 & -560 & 528 & 0 \\ -1048 & -2256 & -1720 & -1504 & -1400 & -30 & 480 & 0 \\ -1048 & -3304 & -4168 & -2968 & -1336 & -136 & 202 & 0 \\ -1048 & 4352 & -7520 & -7040 & -3848 & -1184 & -160 & -14 \end{pmatrix},$$

$$V_2^{\vee}(2) = \begin{pmatrix} -1048 & 2984 & 12752 & 8720 & 0 & 0 & 0 & 0 \\ -1048 & 1936 & 4312 & -1376 & -2704 & 0 & 0 & 0 \\ -1048 & 888 & 2600 & -4360 & -7728 & -2704 & 0 & 0 \\ -1048 & -160 & 1328 & -3200 & -5608 & -1792 & 176 & 0 \\ -1048 & -1208 & 16 & -1872 & -3048 & -488 & 688 & 176 \\ -1048 & -2256 & -1720 & -1504 & -1400 & -16 & 552 & 160 \\ -1048 & -3304 & -4168 & -2968 & -1336 & -136 & 216 & 72 \\ -1048 & -4352 & -7520 & -7040 & -3848 & -1184 & -160 & 0 \end{pmatrix},$$

$$R_2^{\vee}(3) =$$

$$\begin{pmatrix} -2583 & -4000 & 0 & 0 & 0 & 0 & 0 & 0 \\ -2576 & 1049 & 3616 & 0 & 0 & 0 & 0 & 0 \\ -2576 & 912 & 7113 & 3613 & 0 & 0 & 0 & 0 \\ -2576 & -224 & 5152 & 2105 & -704 & 0 & 0 & 0 \\ -2576 & -2032 & 2240 & 352 & -2039 & -704 & 0 & 0 \\ -2576 & -4256 & -1552 & -928 & -1680 & -615 & 0 & 0 \\ -2576 & -6704 & -6640 & -3760 & -1648 & -368 & 41 & 0 \\ -2576 & -9248 & -13600 & -10688 & -4848 & -1216 & -128 & -7 \end{pmatrix},$$

$$V_2^\vee(3) =$$

$$\begin{pmatrix} -2576 & -3984 & 0 & 0 & 0 & 0 & 0 & 0 \\ -2576 & 1056 & 3632 & 0 & 0 & 0 & 0 & 0 \\ -2576 & 912 & 7120 & 3632 & 0 & 0 & 0 & 0 \\ -2576 & -224 & 5152 & 2112 & -688 & 0 & 0 & 0 \\ -2576 & -2032 & 2240 & 352 & -2032 & -688 & 0 & 0 \\ -2576 & -4256 & -1552 & -928 & -1680 & -608 & 16 & 0 \\ -2576 & -6704 & -6640 & -3760 & -1648 & -368 & 48 & 16 \\ -2576 & -9248 & -13600 & -10688 & -4848 & -1216 & -128 & 0 \end{pmatrix},$$

$$R_2^\vee(4) = V_2^\vee(4) =$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1904 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -2512 & -1904 & 0 & 0 & 0 & 0 & 0 & 0 \\ -2872 & -1536 & 976 & 0 & 0 & 0 & 0 & 0 \\ -3064 & -2104 & 1744 & 976 & 0 & 0 & 0 & 0 \\ -3152 & -3760 & -8 & 608 & -80 & 0 & 0 & 0 \\ -3184 & -6272 & -4088 & -1320 & -432 & -80 & 0 & 0 \\ -3192 & -9264 & -10600 & -6208 & -1992 & -320 & -16 & 0 \end{pmatrix}$$

$$R_2^\vee(5) = V_2^\vee(5) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -720 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1296 & -720 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1648 & -1312 & -16 & 0 & 0 & 0 & 0 & 0 \\ -1808 & -2320 & -688 & -16 & 0 & 0 & 0 & 0 \\ -1856 & -3840 & -2816 & -896 & -112 & 0 & 0 & 0 \end{pmatrix},$$

$$R_2^\vee(6) = V_2^\vee(6) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -176 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -336 & -176 & 0 & 0 & 0 & 0 & 0 & 0 \\ -408 & -448 & -112 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$R_2^{\vee}(7) = V_2^{\vee}(7) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -16 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$R_2^{\wedge}(0) = \begin{pmatrix} 17 & -24 & 88 & 0 & -968 & -1216 & 0 & 0 \\ 16 & -15 & 32 & 0 & -1160 & -2544 & -1824 & 0 \\ 16 & 0 & 9 & 0 & -640 & -1464 & -1080 & 0 \\ 16 & 16 & 8 & 1 & -312 & -752 & -576 & 0 \\ 16 & 32 & 24 & 8 & -127 & -328 & -264 & 0 \\ 16 & 48 & 56 & 32 & -32 & -111 & -96 & 0 \\ 16 & 64 & 104 & 88 & 32 & -16 & -23 & 0 \\ 16 & 80 & 168 & 192 & 128 & 48 & 8 & 1 \end{pmatrix},$$

$$V_2^{\wedge}(0) = \begin{pmatrix} 16 & -32 & 56 & -88 & -1160 & -1576 & -608 & 0 \\ 16 & -16 & 24 & -32 & -1248 & -2736 & -2184 & -608 \\ 16 & 0 & 8 & -8 & -672 & -1552 & -1272 & -360 \\ 16 & 16 & 8 & 0 & -320 & -784 & -664 & -192 \\ 16 & 32 & 24 & 8 & -128 & -336 & -296 & -88 \\ 16 & 48 & 56 & 32 & -32 & -112 & -104 & -32 \\ 16 & 64 & 104 & 88 & 32 & -16 & -24 & -8 \\ 16 & 80 & 168 & 192 & 128 & 48 & 8 & 0 \end{pmatrix},$$

$$R_2^{\wedge}(1) = \begin{pmatrix} 181 & -400 & 944 & 3872 & 2432 & 0 & 0 & 0 \\ 176 & -251 & 448 & 4640 & 6112 & 2432 & 0 & 0 \\ 176 & -80 & 165 & 2560 & 3280 & 1056 & 0 & 0 \\ 176 & 96 & 80 & 1253 & 1424 & -160 & -576 & 0 \\ 176 & 272 & 176 & 560 & 533 & -368 & -528 & 0 \\ 176 & 448 & 448 & 384 & 208 & -219 & -288 & 0 \\ 176 & 624 & 896 & 704 & 304 & -16 & -91 & 0 \\ 176 & 800 & 1520 & 1568 & 944 & 320 & 48 & 5 \end{pmatrix},$$

$$V_2^{\wedge}(1) = \begin{pmatrix} 176 & -432 & 848 & 3696 & 2240 & 0 & 0 & 0 \\ 176 & -256 & 416 & 4544 & 5936 & 2240 & 0 & 0 \\ 176 & -80 & 160 & 2528 & 3184 & 880 & -192 & 0 \\ 176 & 96 & 80 & 1248 & 1392 & -256 & -752 & -192 \\ 176 & 272 & 176 & 560 & 528 & -400 & -624 & -176 \\ 176 & 448 & 448 & 384 & 208 & -224 & -320 & -96 \\ 176 & 624 & 896 & 704 & 304 & -16 & -96 & -32 \\ 176 & 800 & 1520 & 1568 & 944 & 320 & 48 & 0 \end{pmatrix},$$

$$R_2^{\wedge}(2) = \begin{pmatrix} 734 & -2160 & -2880 & 0 & 0 & 0 & 0 & 0 \\ 728 & -1450 & -5072 & -2880 & 0 & 0 & 0 & 0 \\ 728 & -728 & -2898 & -688 & 768 & 0 & 0 & 0 \\ 728 & 0 & -1472 & 710 & 2208 & 768 & 0 & 0 \\ 728 & 728 & -320 & 768 & 1742 & 640 & 0 & 0 \\ 728 & 1456 & 936 & 800 & 920 & 246 & -96 & 0 \\ 728 & 2184 & 2584 & 1736 & 792 & 136 & -66 & 0 \\ 728 & 2912 & 4816 & 4288 & 2216 & 640 & 80 & 6 \end{pmatrix},$$

$$V_2^{\wedge}(2) = \begin{pmatrix} 728 & -2184 & -2912 & 0 & 0 & 0 & 0 & 0 \\ 728 & -1456 & -5096 & -2912 & 0 & 0 & 0 & 0 \\ 728 & -728 & -2904 & -712 & 736 & 0 & 0 & 0 \\ 728 & 0 & -1472 & 704 & 2184 & 736 & 0 & 0 \\ 728 & 728 & -320 & 768 & 1736 & 616 & -32 & 0 \\ 728 & 1456 & 936 & 800 & 920 & 240 & -120 & -32 \\ 728 & 2184 & 2584 & 1736 & 792 & 136 & -72 & -24 \\ 728 & 2912 & 4816 & 4288 & 2216 & 640 & 80 & 0 \end{pmatrix},$$

$$R_2^{\wedge}(3) = \begin{pmatrix} 1409 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1408 & 1409 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1408 & 384 & -1023 & 0 & 0 & 0 & 0 & 0 \\ 1408 & 352 & -2080 & -1023 & 0 & 0 & 0 & 0 \\ 1408 & 992 & -1344 & -800 & 129 & 0 & 0 & 0 \\ 1408 & 2048 & 352 & -32 & 384 & 129 & 0 & 0 \\ 1408 & 3328 & 2848 & 1344 & 544 & 128 & 1 & 0 \\ 1408 & 4704 & 6366 & 4480 & 1792 & 384 & 33 & 1 \end{pmatrix},$$

$$V_2^{\wedge}(3) = \begin{pmatrix} 1408 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1408 & 1408 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1408 & 384 & -1024 & 0 & 0 & 0 & 0 & 0 \\ 1408 & 352 & -2080 & -1024 & 0 & 0 & 0 & 0 \\ 1408 & 992 & -1344 & -800 & 128 & 0 & 0 & 0 \\ 1408 & 2048 & 352 & -32 & 384 & 128 & 0 & 0 \\ 1408 & 3328 & 2848 & 1344 & 544 & 128 & 0 & 0 \\ 1408 & 4704 & 6366 & 4480 & 1792 & 384 & 32 & 0 \end{pmatrix},$$

$$R_2^{\wedge}(4) = V_2^{\wedge}(4) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 608 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 968 & 608 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1160 & 808 & -160 & 0 & 0 & 0 & 0 & 0 \\ 1248 & 1264 & -56 & -160 & 0 & 0 & 0 & 0 \\ 1280 & 2128 & 1016 & 168 & 32 & 0 & 0 & 0 \\ 1288 & 3280 & 3160 & 1472 & 344 & 32 & 0 & 0 \end{pmatrix},$$

$$R_2^{\wedge}(5) = V_2^{\wedge}(5) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 192 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 368 & 192 & 0 & 0 & 0 & 0 & 0 & 0 \\ 464 & 432 & 64 & 0 & 0 & 0 & 0 & 0 \\ 496 & 800 & 400 & 64 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$R_2^{\wedge}(6) = V_2^{\wedge}(6) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 32 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 56 & 32 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

Comparing the above

$$a^*_{l,i,k}(z;\nu),\ a^{\vee}_{l,i,k}(z;\nu),\ a^{\wedge}_{l,i,k}(z;\nu)$$

where $l = 0, 1, 2,\ i = 1,\ ...,\ 4 + 2l,\ k = 1,\ ...,\ 4 + 2l$, with the elements of the matrices respectively $U_l^{\vee}(z,\mu)$, $U_l^{\wedge}(z,\mu)$, we see that

$$A_l^{\vee}(z;\nu) = (1/2)U_l^{\vee}(z;\nu),\ A_l^{\wedge}(z;\nu) = U_l^{\wedge}(z;\nu),$$

and, in view of (27) ,

$$A_l^*(z;\nu) = (1/2)U_l^{\vee}(z;\nu) + U_l^{\wedge}(z;\nu)\tau,$$

for $l = 0, 1, 2$. In view of (21), (26),

$$\nu^{3+2l}X_{l,k}(z;\nu-1) = A_l^*(z;\nu)X_{l,k}(z;\nu), \tag{32}$$

where $l = 0,\ 1,\ 2$ and $k \in \mathfrak{K}_l$, $|z| > 1, \nu \in \mathbb{Z}$. Replacing in (32) ν by $-\nu$ and taking in the account (25), we see that

$$(-\nu)^{3+2l}X_{l,k}(z;\nu) = A_l^*(z;-\nu)X_{l,k}(z;\nu+1), \tag{33}$$

Therefore the following equality must be fufilled:

$$-\nu^{6+4l}E_{3+2l} = A_l^*(z;\nu)A_l^*(z;-\nu), \tag{34}$$

where E_{3+2l} is the $(3 + 2l) \times (3 + 2l)$ unit matrix, $l = 0, 1, 2,\ z \in \mathbb{C},\ \nu \in \mathbb{C}$. This equality opens the possibility for us to check independently the previous calculations.

§2.3. Corrections in the first part of this paper.

The equality (40) in the first part must have the same form, as (14) here.

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