ABUNDANT FAMILIES OF JACOBI ELLIPTIC FUNCTION-LIKE SOLUTIONS OF THE DISPERSIVE LONG WAVE AND GENERALIZED HIROTA EQUATIONS VIA A NEW MAPPING METHOD

EMMANUEL YOMBA,

MAX-PLANCK INSTITUT FÜR MATHEMATIK,
VIVATSGASSE 7,
BONN D-53111,
GERMANY.

DEPARTMENT OF PHYSICS,
FACULTY OF SCIENCES,
UNIVERSITY OF NGAOUNDÉRÉ,
BO. POX. 454 NGAOUNDÉRÉ, CAMEROON.

Abstract. By using the solutions of the elliptic equation of first kind involving three parameters, an extended mapping method is described to construct the exact solutions for nonlinear partial differential equations (NLPDEs). By this method, the (2+1)-dimensional dispersive long wave equation and the (2+1)-dimensional generalized Hirota equation are investigated. New exact solutions for these NLPDEs are explicitly obtained with the aid of symbolic computation system-Mathematica. Rational-like, and many Jacobi elliptic function-like solutions are obtained. When the module $m \to 1$ or 0, the corresponding soliton-like and triangular function-like solutions are obtained.

1. Introduction

The effort in finding exact solutions, to the nonlinear partial differential equations (NLPDEs) is an important and interesting subject in Mathematical-Physics. In this study, one of the fundamental objects is the travelling wave solutions. Of particular interest are soliton-like solutions, because the waveforms can change in different mechanisms and it usually has travelling wave solutions. Many powerful methods have been presented, some of the most important one are tanh-function method [1-3], sech-function method [4,5], Jacobi elliptic function-method [6-8], extented Jacobi elliptic function [9] and mapping method [10,11]. In the mapping method, the elliptic function, triangular and soliton solutions can, if they exist, be obtained simultaneously to the NLPDE in concern [10,11]. Thus many of tedious and repetitive calculations may be avoided. Now, motivated by the desire to improve the mapping method, a further extension of this method is presented. By this new method, we seek Jacobi elliptic function-like, combined Jacobi elliptic function-like, kink bell soliton-like, formal soliton-like, combined formal soliton-like, triangular function-like and rational-like solutions of the (2+1)-dimensional dispersive long wave equation and the (2+1)-dimensional generalized Hirota equation. The rest of this paper is organized as follows. In section 2, we describe the improved extended mapping method and in section 3, we apply it to solve the above mentioned equations. Finally, some conclusions are given in section 4.

2. The further extended mapping method

For a given NLPDE, say in three independent variables X= (x,y,t) and dependent variable u, we seek its solutions in the form

$$U(x,y,t) = \sum_{i=0}^{n} A_i(X)\phi^i(\xi(X)), \tag{1}$$

while ϕ satisfies the first kind elliptic equation

$$(\frac{d\phi}{d\xi})^2 = r + a\phi^2(\xi) + b/(2)\phi^4(\xi),$$
$$\xi = p(y,t)x + q(y,t), \tag{2}$$

Where r, a and b, are constants. To determine U explicitly, we take the following five steps.

Step 1. Determine n which is fixed by balancing the highest order derivative terms and the highest nonlinear terms in the given NLPDE.

Step 2. Substituting (1) and (2) into the given NLPDE and collecting coefficients of

$x^j\phi^k\sqrt{r+a\phi^2(\xi)+b/(2)\phi^4(\xi)}$, $x^l\phi^s$ (j=0,1,...; k= 0,1,...; l= 0,1,...; s=1,2,...), then setting each coefficient to zero to derive a set of partial differential equations of $A_i(i=0,1...,n)$ and ξ.

Step 3. Solving the system of partial differential equations obtained in step 2, yields the A_i and ξ functions.

Step 4. (2) possesses the general solutions

$$\phi(\xi)=\begin{cases} sn\,\xi;\quad cd\,\xi=cn\,\xi/dn\,\xi & : \; r=1,\;\; a=-(m^2+1),\;\; b=2m^2;\\ cn\,\xi & : \; r=1-m^2,\;\; a=2m^2-1,\;\; b=-2m^2;\\ dn\,\xi & : \; r=m^2-1,\;\; a=2-m^2,\;\; b=-2;\\ ns\,\xi=1/sn\,\xi;\quad dc\,\xi=dn\,\xi/cn\,\xi & : \; r=m^2,\;\; a=-(m^2+1),\;\; b=2;\\ nc\,\xi=1/cn\,\xi & : \; r=-m^2,\;\; a=2m^2-1,\;\; b=2(1-m^2);\\ nd\,\xi=1/dn\,\xi & : \; r=-1,\;\; a=2-m^2,\;\; b=2(m^2-1);\\ cs\,\xi=cn\,\xi/sn\,\xi & : \; r=1-m^2,\;\; a=2-m^2,\;\; b=2;\\ sc\,\xi=sn\,\xi/cn\,\xi & : \; r=1,\;\; a=2-m^2,\;\; b=2(1-m^2);\\ sd\,\xi=sn\,\xi/dn\,\xi & : \; r=1,\;\; a=2m^2-1,\;\; b=2m^2(m^2-1);\\ ds\,\xi=dn\,\xi/sn\,\xi & : \; r=m^2(m^2-1),\;\; a=2m^2-1,\;\; b=2;\\ ns\,\xi\pm cs\,\xi & : \; r=1/4,\;\; a=(1-2m^2)/2,\;\; b=1/2;\\ nc\,\xi\pm sc\,\xi & : \; r=(1-m^2)/4,\;\; a=(1+m^2)/2,\;\; b=(1-m^2)/2;\\ ns\,\xi+ds\,\xi & : \; r=m^4/4,\;\; a=(m^2-2)/2,\;\; b=1/2;\\ sn\,\xi\pm i\,cn\,\xi & : \; r=m^2/4,\;\; a=(m^2-2)/2,\;\; b=m^2/2,\;\; i^2=-1;\\ C/\xi & : \; r=0,\;\; a=0\;\; b=2, \end{cases} \quad (\mathbf{3})$$

where C is a constant. The Jacobi elliptic functions $sn\,\xi=sn\,(\xi|m)$, $cn\,\xi=cn\,(\xi|m)$, and $dn\,\xi=dn\,(\xi|m)$, where $m(0<m<1)$ is the modulus of elliptic function . Moreover we know that these Jacobi functions [12,13] have close relations

$$\begin{aligned} & sn^2\xi+cn^2\xi=1,\;\; dn^2\,\xi+m^2sn^2\,\xi=1,\;\; ns^2\,\xi=1+cs^2\,\xi,\\ & ns^2=m^2+ds^2\,\xi,\;\; nc^2\,\xi=1+sc^2\,\xi,\;\; nd^2\,\xi=1+m^2sd^2\,\xi,\\ & m^2cn^2\,\xi=1+(m^2-1)nd^2\,\xi,\;\; dn^2\,\xi=m^2cn^2\,\xi+(1-m^2),\\ & dc^2\,\xi+1=m^2+(1-m^2)nc^2\,\xi,\;\; (m^2-1)sd^2\,\xi+1=cd^2\,\xi,\\ & (1-m^2)sc^2\,\xi+1=dc^2\,\xi,\;\; m^2-1+ds^2\xi=cs^2\,\xi. \end{aligned} \quad (\mathbf{4})$$

Their first-order derivatives are listed below:

$$\begin{aligned} & \tfrac{d}{d\xi}sn\,\xi=cn\,\xi dn\,\xi,\;\; \tfrac{d}{d\xi}cn\,\xi=-sn\,\xi dn\,\xi,\;\; \tfrac{d}{d\xi}dn\,\xi=-m^2cn\,\xi sn\,\xi,\\ & \tfrac{d}{d\xi}ns\,\xi=-ns\quad \xi cs\,\xi dn\,\xi,\;\; \tfrac{d}{d\xi}cs\,\xi=-ns\,\xi ds\,\xi,\;\; \tfrac{d}{d\xi}ds\,\xi=-ns\,\xi cs\,\xi,\\ & \tfrac{d}{d\xi}sc\,\xi=nc\,\xi dc\,\xi,\;\; \tfrac{d}{d\xi}nc\,\xi=sc\,\xi dc\,\xi,\;\; \tfrac{d}{d\xi}sd\,\xi=cd\,\xi nd\,\xi,\\ & \tfrac{d}{d\xi}nd\,\xi=m^2sd\,\xi cd\,\xi,\;\; \tfrac{d}{d\xi}nc\,\xi=sc\,\xi dc\,\xi,\;\; \tfrac{d}{d\xi}dc\,\xi=(1-m^2)sc\,\xi nc\,\xi. \end{aligned} \quad (\mathbf{5})$$

By using the results obtained above steps we can derive many doubly periodic solutions of (2). In addition, we see that other solutions are obtained from solutions (3) in case of degeneracies:
When m$\rightarrow$ 0, the Jacobi elliptic functions degenerate to the trigonometric function solutions of the given NLPDE i.e.
$sn\,\xi \rightarrow \sin\,\xi,\;\; cn\,\xi \rightarrow \cos\,\xi,\;\; dn\,\xi \rightarrow 1.$ (**6**)
When m$\rightarrow$ 1, the Jacobi elliptic functions degenerate to the solitonic solutions of the given NLPDE i.e.
$sn\,\xi \rightarrow \tanh\,\xi,\;\; cn\,\xi \rightarrow sech\,\xi,\;\; dn\,\xi \rightarrow sech\,\xi.$ (**7**)

Step 5. Substituting A_i, ξ and (3) or (6) or (7) into (1) to obtain Jacobi elliptic function-like, combined Jacobi elliptic function-like, rational-like solutions or triangular function-like solutions or formal soliton-like, combined formal soliton-like and kink bell soliton-like solutions respectively.

Remark 1.
From the following application, it is easily seen that our algorithm is more powerful than the method presented recently by Peng [10,11]. When A_i in (1) are constant, the method reduces to be the method previously presented by the above author. It is important to note that many solutions of Eq.(3) have been omitted in [11].

3. Application

3.1. **New solutions of the (2+1)-dimensional dispersive long wave equation.**

$$\begin{cases} u_{yt} + v_{xx} + \frac{1}{2}(u^2)_{xy} = 0, \\ v_t + (uv + u + u_{xy})_x = 0, \end{cases} \tag{8}$$

. Which was first obtained by Boiti et al as compatibility condition for Lax Pair [14]. The Kac Moody-Virasoro symmetry algebra was given in [15] and the soliton solutions of this equation have been studied [16,17]. The test of Painlevé indicates this equation is not thought of integrable [18]. Now, we apply the further extended mapping method to Eq.(8). By balancing procedure, ansätz (1) becomes

$$u = f + h\phi(\xi),$$

$$v = F + H\phi(\xi) + G\phi^2(\xi), \tag{9}$$

$$\xi = px + q,$$

$\phi(\xi)$ satisfies Eq.(2), while f, h, g, F, H, G, p and q are functions of y and t to be determined later. With the aid of Mathematica, substituting (9) and (2) into (8) and collecting all terms with the same power of $x^j\phi^k\sqrt{r+a\phi^2(\xi)+b/(2)\phi^4(\xi)}$, $x^l\phi^s$ (j=0,1 ; k=0,1,2; l=0,1,2; s=1,...,4), (notice that f, h, g, F, H, G, p and q are independent of x) yield the following system of partial differential equations

$$ahp_yp_t = 0, \tag{10.1}$$

$$bhp_yp_t = 0, \tag{10.2}$$

$$rh^2pp_y = 0, \tag{10.3}$$

$$ah(fpp_y + p_yq_t + p_tq_y) = 0, \tag{10.4}$$

$$2ah^2pp_y = 0, \tag{10.5}$$

$$bh(fpp_y + p_yq_t + p_tq_y) = 0, \tag{10.6}$$

$$\frac{3}{2}bh^2pp_y = 0, \tag{10.7}$$

$$2rGp^2 + rh^2pq_y + f_{yt} = 0, \tag{10.8}$$

$$aHp^2 + afhpq_y + ahq_tq_y + h_{yt} = 0, \tag{10.9}$$

$$2ap(2pG + h^2q_y) = 0, \tag{10.10}$$

$$b(Hp^2 + fhpq_y + hq_tq_y) = 0, \tag{10.11}$$

$$\frac{3}{2}bp(2Gp + h^2q_y) = 0, \tag{10.12}$$

$$p_th_y + h_tp_y + hp_{yt} = 0, \tag{10.13}$$

$$hpf_y + fph_y + q_th_y + fhp_y + h_tq_y + hq_{yt} = 0, \tag{10.14}$$

$$h(2ph_y + hp_y) = 0, \tag{10.15}$$

$$F_t = 0, \tag{10.16}$$

$$H_t + ap^2h_y + 2ahpp_y = 0, \tag{10.17}$$

$$G_t = 0, \tag{10.18}$$

$$bp(ph_y + 2hp_y) = 0, \tag{10.19}$$

$$Hp_t + ahp^2p_y = 0, \tag{10.20}$$

$$2Gp_t = 0, \tag{10.21}$$

$$3bhp^2p_y = 0, \tag{10.22}$$

$$hp + Fhp + fHp + Hq_t + ahp^2q_y = 0, \tag{10.23}$$

$$2(Gfp + Hhp + Gq_t) = 0, \tag{10.24}$$

$$3hp(G+bpq_y)=0. \qquad (\mathbf{10.25})$$

By analysing (10.1)-(10.25), it follows that
$p_y=0,\ \ p_t=0,\ \ h_y=0,$ $\qquad (\mathbf{11})$
Taking into account (11) into (10.1)-(10.25), we get

$$2rGp^2+rh^2pq_y+f_{yt}=0, \qquad (\mathbf{12.1})$$

$$a(Hp^2+afhpq_y+ahq_tq_y)=0, \qquad (\mathbf{12.2})$$

$$2ap(2pG+h^2q_y)=0, \qquad (\mathbf{12.3})$$

$$b(Hp^2+fhpq_y+hq_tq_y)=0, \qquad (\mathbf{12.4})$$

$$hpf_y+h_tq_y+hq_{yt}=0, \qquad (\mathbf{12.5})$$

$$F_t=0, \qquad (\mathbf{12.6})$$

$$H_t=0 \qquad (\mathbf{12.7})$$

$$G_t=0, \qquad (\mathbf{12.8})$$

$$hp+Fhp+fHp+Hq_t+ahp^2q_y=0, \qquad (\mathbf{12.9})$$

$$Gfp+Hhp+Gq_t=0, \qquad (\mathbf{12.10})$$

$$G+bpq_y=0. \qquad (\mathbf{12.11})$$

Then we find from (12.3) and (12.11) that,

$$G=-bpq_y, \qquad (\mathbf{13.1})$$

$$h^2=2bp^2. \qquad (\mathbf{13.2})$$

It is easy to see from (11) and (13.2) that p and h are constants. Substituting (13.1), (12.8) and (13.2) into (12.5) we obtain that

$$q_{yt}=0, \qquad (\mathbf{14.1})$$

$$f_y=0. \qquad (\mathbf{14.2})$$

Then, using (13.1),(13.2), (14.1) and (14.2) into (12.1),(12.2), (12.4),(12.9) and (12.10), we find that

$$H=0, \qquad (\mathbf{15.1})$$

$$pf+q_t=0, \qquad (\mathbf{15.2})$$

$$1+F+apq_y=0, \qquad (\mathbf{15.3})$$

$$F_t=0. \qquad (\mathbf{15.4})$$

Then, Eqs.(14.1),(14.2), (15.2)-(15.4) can be solved as:

$$F = -1 - apK_1(y),$$
$$f = -\frac{1}{p}K_{2t}, \tag{16}$$
$$q = \int K_1(y)dy + K_2(t)$$

Where K_1 is arbitrary function of y, while K_2 is arbitrary function of t. Hence, the solutions of Eq.(8) reads

$$u = [-\frac{1}{p}K_{2t}] \pm \sqrt{2b}p\phi[px + \int K_1(y)dy + K_2(t)], \tag{17}$$
$$v = [-1 - apK_1(y)] - bpK_1(y)\phi^2[px + \int K_1(y)dy + K_2(t)].$$

where p, r, a and b are arbitrary constants, but b will be positif constant and ϕ satisfies Eq.(2). Thanks to the arbitrariness of r, a and b, we may obtain multiple exact solutions of Eq.(8). In the following, we consider several cases by the aid of relation (3) in which only functions with $b > 0$ have been considered

case 1

$$u_1 = [-\frac{1}{p}K_{2t}] \pm 2mp\ sn[px + \int K_1(y)dy + K_2(t)], \tag{18.1}$$
$$v_1 = [-1 + (m^2+1)pK_1(y)] - 2m^2pK_1(y)\ sn^2[px + \int K_1(y)dy + K_2(t)].$$

case 2

$$u_2 = [-\frac{1}{p}K_{2t}] \pm 2mp\ cd[px + \int K_1(y)dy + K_2(t)], \tag{18.2}$$
$$v_2 = [-1 + (m^2+1)pK_1(y)] - 2m^2pK_1(y)\ cd^2[px + \int K_1(y)dy + K_2(t)].$$

case 3

$$u_3 = [-\frac{1}{p}K_{2t}] \pm 2p\ ns[px + \int K_1(y)dy + K_2(t)], \tag{18.3}$$
$$v_3 = [-1 + (m^2+1)pK_1(y)] - 2pK_1(y)\ ns^2[px + \int K_1(y)dy + K_2(t)].$$

case 4

$$u_4 = [-\frac{1}{p}K_{2t}] \pm 2p\ dc[px + \int K_1(y)dy + K_2(t)], \tag{18.4}$$
$$v_4 = [-1 + (m^2+1)pK_1(y)] - 2pK_1(y)\ dc^2[px + \int K_1(y)dy + K_2(t)].$$

case 5

$$u_5 = [-\frac{1}{p}K_{2t}] \pm 2\sqrt{(1-m^2)}p\ nc[px + \int K_1(y)dy + K_2(t)], \tag{18.5}$$

$$v_5 = [-1-(2m^2-1)pK_1(y)] - 2(1-m^2)pK_1(y)\ nc^2[px + \int K_1(y)dy + K_2(t)].$$

case 6

$$u_6 = [-\frac{1}{p}K_{2t}] \pm 2p\ cs[px + \int K_1(y)dy + K_2(t)], \qquad (\mathbf{18.6})$$

$$v_6 = [-1-(2-m^2)pK_1(y)] - 2pK_1(y)\ cs^2[px + \int K_1(y)dy + K_2(t)].$$

case 7

$$u_7 = [-\frac{1}{p}K_{2t}] \pm 2\sqrt{1-m^2}p\ sc[px + \int K_1(y)dy + K_2(t)], \qquad (\mathbf{18.7})$$

$$v_7 = [-1-(2-m^2)pK_1(y)] - 2(1-m^2)pK_1(y)\ sc^2[px + \int K_1(y)dy + K_2(t)].$$

case 8

$$u_8 = [-\frac{1}{p}K_{2t}] \pm 2p\ ds[px + \int K_1(y)dy + K_2(t)], \qquad (\mathbf{18.8})$$

$$v_8 = [-1-(2m^2-1)pK_1(y)] - 2pK_1(y)\ ds^2[px + \int K_1(y)dy + K_2(t)].$$

case 9

$$u_9 = [-\frac{1}{p}K_{2t}] \pm p\{ns[px + \int K_1(y)dy + K_2(t)] \pm cs[px + \int K_1(y)dy + K_2(t)]\}, \quad (\mathbf{18.9})$$

$$v_9 = [-1-\frac{(1-2m^2)}{2}pK_1(y)] - \frac{1}{2}pK_1(y)\{ns[px+\int K_1(y)dy+K_2(t)] \pm cs[px+\int K_1(y)dy+K_2(t)]\}^2.$$

case 10

$$u_{10} = [-\frac{1}{p}K_{2t}] \pm \sqrt{1-m^2}p\{nc[px + \int K_1(y)dy + K_2(t)] \pm sc[px + \int K_1(y)dy + K_2(t)]\}, \qquad (\mathbf{18.10})$$

$$v_{10} = [-1-\tfrac{(1+m^2)}{2}pK_1(y)] - \tfrac{(1-m^2)}{2}pK_1(y)\{nc[px+\int K_1(y)dy+K_2(t)] \pm sc[px+\int K_1(y)dy+K_2(t)]\}^2.$$

case 11

$$u_{11} = [-\frac{1}{p}K_{2t}] \pm p\{ns[px + \int K_1(y)dy + K_2(t)] + ds[px + \int K_1(y)dy + K_2(t)]\}, \quad (\mathbf{18.11})$$

$$v_{11} = [-1-\frac{(m^2-2)}{2}pK_1(y)] - \frac{1}{2}pK_1(y)\{ns[px+\int K_1(y)dy+K_2(t)] + ds[px+\int K_1(y)dy+K_2(t)]\}^2.$$

case 12

$$u_{12} = [-\frac{1}{p}K_{2t}] \pm mp\{sn[px + \int K_1(y)dy + K_2(t)] \pm i\ cn[px + \int K_1(y)dy + K_2(t)]\}, \qquad (\mathbf{18.12})$$

$$v_{12} = [-1+\frac{(2-m^2)}{2}pK_1(y)] - \frac{m^2}{2}pK_1(y)\{sn[px+\int K_1(y)dy+K_2(t)] \pm i\ cn[px+\int K_1(y)dy+K_2(t)]\}^2.$$

case 13

$$u_{13} = [-\frac{1}{p}K_{2t}] \pm 2p\frac{C}{[px + \int K_1(y)dy + K_2(t)]}, \quad \mathbf{(18.13)}$$

$$v_{13} = -1 - 2pK_1(y)\frac{C^2}{[px + \int K_1(y)dy + K_2(t)]^2}.$$

Remark 2

When m→ 0, these solutions (18.1)-(18.12) degenerate as triangular function-like solutions of Eq.(8), namely

$$u_{14} = [-\frac{1}{p}K_{2t}] \pm 2p\ csc[px + \int K_1(y)dy + K_2(t)], \quad \mathbf{(19.1)}$$

$$v_{14} = [-1 + pK_1(y)] - 2pK_1(y)\ csc^2[px + \int K_1(y)dy + K_2(t)].$$

$$u_{15} = [-\frac{1}{p}K_{2t}] \pm 2p\ sec[px + \int K_1(y)dy + K_2(t)], \quad \mathbf{(19.2)}$$

$$v_{15} = [-1 + pK_1(y)] - 2pK_1(y)\ sec^2[px + \int K_1(y)dy + K_2(t)].$$

$$u_{16} = [-\frac{1}{p}K_{2t}] \pm 2p\cot[px + \int K_1(y)dy + K_2(t)], \quad \mathbf{(19.3)}$$

$$v_{16} = [-1 - 2pK_1(y)] - 2pK_1(y)\cot^2[px + \int K_1(y)dy + K_2(t)].$$

$$u_{17} = [-\frac{1}{p}K_{2t}] \pm 2p\tan[px + \int K_1(y)dy + K_2(t)], \quad \mathbf{(19.4)}$$

$$v_{17} = [-1 - 2pK_1(y)] - 2pK_1(y)\tan^2[px + \int K_1(y)dy + K_2(t)].$$

$$u_{18} = [-\frac{1}{p}K_{2t}] \pm p\{sec[px+\int K_1(y)dy+K_2(t)] \pm \tan[px+\int K_1(y)dy+K_2(t)]\}, \quad \mathbf{(19.5)}$$

$$v_{18} = [-1-\frac{1}{2}pK_1(y)]-\frac{1}{2}pK_1(y)\{sec[px+\int K_1(y)dy+K_2(t)]\pm\tan[px+\int K_1(y)dy+K_2(t)]\}^2.$$

$$u_{19} = [-\frac{1}{p}K_{2t}] \pm p\{csc[px+\int K_1(y)dy+K_2(t)] \pm \cot[px+\int K_1(y)dy+K_2(t)]\}, \quad \mathbf{(19.6)}$$

$$v_{19} = [-1-\frac{1}{2}pK_1(y)]-\frac{1}{2}pK_1(y)\{csc[px+\int K_1(y)dy+K_2(t)]\pm\cot[px+\int K_1(y)dy+K_2(t)]\}^2.$$

Remark 3

When m→ 1, these solutions (18.1)-(18.12) degenerate as formal soliton-like, combined formal soliton-like and kink bell soliton-like solutions of Eq.(8), namely

$$u_{20} = [-\frac{1}{p}K_{2t}] \pm 2p\tanh[px + \int K_1(y)dy + K_2(t)], \tag{20.1}$$

$$v_{20} = [-1 + 2pK_1(y)] - 2pK_1(y)\tanh^2[px + \int K_1(y)dy + K_2(t)].$$

$$u_{21} = [-\frac{1}{p}K_{2t}] \pm 2p\coth[px + \int K_1(y)dy + K_2(t)], \tag{20.2}$$

$$v_{21} = [-1 + 2pK_1(y)] - 2pK_1(y)\coth^2[px + \int K_1(y)dy + K_2(t)].$$

$$u_{22} = [-\frac{1}{p}K_{2t}] \pm 2p\ csch^2[px + \int K_1(y)dy + K_2(t)], \tag{20.3}$$

$$v_{22} = [-1 - pK_1(y)] - 2pK_1(y)\ csch[px + \int K_1(y)dy + K_2(t)].$$

$$u_{23} = [-\frac{1}{p}K_{2t}] \pm p\{\coth[px + \int K_1(y)dy + K_2(t)] \pm\ csch[px + \int K_1(y)dy + K_2(t)]\}, \tag{20.4}$$

$$v_{23} = [-1+\frac{1}{2}pK_1(y)]-\frac{1}{2}pK_1(y)\{\coth[px+\int K_1(y)dy+K_2(t)]\pm\ csch[px+\int K_1(y)dy+K_2(t)]\}^2.$$

$$u_{24} = [-\frac{1}{p}K_{2t}] \pm p\{\tanh[px + \int K_1(y)dy + K_2(t)] \pm i\ sech[px + \int K_1(y)dy + K_2(t)]\}, \tag{20.5}$$

$$v_{24} = [-1+\frac{1}{2}pK_1(y)]-\frac{1}{2}pK_1(y)\{\tanh[px+\int K_1(y)dy+K_2(t)]\pm i\ sech[px+\int K_1(y)dy+K_2(t)]\}^2.$$

3.2. The (2+1)-dimensional generalized Hirota equation

. Consider the (2+1)-dimensional generalized Hirota equation

$$\begin{cases} iu_t + u_{xy} + iu_{xxx} + uv - i|u|^2u_x = 0, \\ 3v_x + (|u|^2)_y = 0, \end{cases} \tag{21}$$

Which was derived by Maccari recently and it was shown that the equation is integrable in Lax sense [19]. If we take x=y, then the system (21) is reduced to the(1+1)-dimensional Hirota equation [20]. Exact solution of Eq.(21) was derived in [17]. We apply our proposed method to construct its series of new exact solutions as follows. To extend our proposed method to Eq.(21), we introduced the transformation

$$u = e^{i\theta}U(x,y,t), \quad v = V(x,y,t), \quad \theta = ex+dy+st, \qquad \mathbf{(22)}$$

and reduce Eq.(21) to the following system

$$\begin{cases} U_t + eU_y + NU_x - U^2U_x + U_{xxx} = 0, \\ MU + U_{xy} - 3eU_{xx} + UV + eU^3 = 0 \\ 3V_x + (U^2)_y = 0, \end{cases} \qquad \mathbf{(23)},$$

Where

$$M = e^3 - ed - s, \qquad N = d - 3e^2$$

We expand the solutions of Eq.(23) in the form

$$U(x,y,t) = \sum_{i=0}^{n} A_i(X)\phi^i(\xi(X)), \qquad \mathbf{(24)}$$

$$V(x,y,t) = \sum_{i=0}^{l} B_i(X)\phi^i(\xi(X),$$

while ϕ satisfies the first kind elliptic equation (2). Balancing the highest derivative with the highest nonlinear terms in Eq.(23) n=1, l=2 and thus we have:

$$U(x,y,t) = f(y,t) + h(y,t)\phi(\xi),$$

$$V(x,y,t) = F(y,t) + H(y,t)\phi(\xi) + G(y,t)\phi^2(\xi), \qquad \mathbf{(25)}$$

$$\xi = p(y,t)x + q(y,t)$$

where ϕ satisfies (2). Inserting (25) and (2) into (21) with the aid of Mathematica, we obtain the following set of equations

$$f(M + ef^2 + F) = 0, \qquad \mathbf{(26.1)}$$

$$Mh + 3ef^2h + Fh + fH - 3aehp^2 + ahpq_y = 0, \qquad \mathbf{(26.2)}$$

$$fG + 3eh^2f + hH = 0, \qquad \mathbf{(26.3)}$$

$$h(G + eh^2 - 3ebp^2 + bpq_y) = 0, \qquad \mathbf{(26.4)}$$

$$ahpp_y = 0, \qquad \mathbf{(26.5)}$$

$$hbpp_y = 0, \qquad \mathbf{(26.6)}$$

$$ph_y + hp_y = 0, \qquad \mathbf{(26.7)}$$

$$f_t + ef_y = 0, \qquad \mathbf{(26.8)}$$

$$h_t + eh_y = 0, \qquad \mathbf{(26.9)}$$

$$h(p_t + ep_y) = 0, \qquad \mathbf{(26.10)}$$

$$h(Np - f^2p + ap^3 + q_t + eq_y) = 0, \qquad (\mathbf{26.11})$$

$$-2h^2fp = 0, \qquad (\mathbf{26.12})$$

$$ph(3bp^2 - h^2) = 0, \qquad (\mathbf{26.13})$$

$$2ff_y = 0, \qquad (\mathbf{26.14})$$

$$2(fh_y + hf_y) = 0, \qquad (\mathbf{26.15})$$

$$2hh_y = 0, \qquad (\mathbf{26.16})$$

$$2hfp_y = 0, \qquad (\mathbf{26.17})$$

$$2h^2p_y = 0, \qquad (\mathbf{26.18})$$

$$3Hp + 2fhq_y = 0, \qquad (\mathbf{26.19})$$

$$2(3pG + h^2q_y) = 0. \qquad (\mathbf{26.20})$$

From (26.5)- (26.10), (26.12), (26.14) - (26.18) we find

$$h_y = 0, \quad p_y = 0, \quad f = 0 \;\; h_t = 0, \quad p_t = 0, \quad f_y = 0, \quad f_t = 0, \qquad (\mathbf{27})$$

Using this Eq.(27), we obtain from (26.19),

$$H = 0. \qquad (\mathbf{28})$$

It occur from relations (27) that p, h are constants. The rest of equations degenerates to

$$h(M + F - 3aep^2 + apq_y) = 0, \qquad (\mathbf{29.1})$$

$$h(G + eh^2 - 3bep^2 + bpq_y) = 0, \qquad (\mathbf{29.2})$$

$$h(Np + ap^3 + q_t + eq_y) = 0, \qquad (\mathbf{29.3})$$

$$ph(h^2 - 3bp^2) = 0, \qquad (\mathbf{29.4})$$

$$(3Gp + h^2q_y) = 0. \qquad (\mathbf{29.5})$$

Which can be solved as

$$h^2 = 3bp^2, \quad b > 0, \qquad (\mathbf{30.1})$$

$$G = -bpq_y, \qquad (\mathbf{30.2})$$

$$F = -M + 3aep^2 - apq_y, \qquad (\mathbf{30.3})$$

$$q_t + eq_y + Np + ap^3 = 0. \qquad (\mathbf{30.4})$$

By integrating (30.4) we have

$$q(y,t) = -(\frac{N}{e}p + \frac{a}{e}p^3)y + C_1(t - \frac{y}{e}). \qquad (\mathbf{31})$$

where C_1 is a free constant. Thus,

$$G = bp[\frac{N}{e}p + \frac{a}{e}p^3 + \frac{C_1}{e}], \qquad (\mathbf{32.1})$$

$$F = -M + 3aep^2 + ap[\frac{N}{e}p + \frac{a}{e}p^3 + \frac{C_1}{e}], \qquad (\mathbf{32.2})$$

Then, taking into account (22),(25), (31), (32.1) and (32.2), the solutions of Eq.(21) are recovered as:

case 1 $r = 1, \ \ a = -(m^2+1), \ \ b = 2m^2$

$$u_1 = he^{i\theta}\ sn[px + q(y,t)], \qquad (\mathbf{33.1})$$

$$v_1 = F + G\ sn^2[px + q(y,t)];$$

case 2 $r = 1, \ \ a = -(m^2+1), \ \ b = 2m^2$

$$u_2 = he^{i\theta}\ cd[px + q(y,t)], \qquad (\mathbf{33.2})$$

$$v_2 = F + G\ cd^2[px + q(y,t)];$$

case 3 $r = m^2, \ \ a = -(m^2+1), \ \ b = 2$

$$u_3 = he^{i\theta}\ ns[px + q(y,t)], \qquad (\mathbf{33.3})$$

$$v_3 = F + G\ ns^2[px + q(y,t)];$$

case 4 $r = m^2, \ \ a = -(m^2+1), \ \ b = 2$

$$u_4 = he^{i\theta}\ dc[px + q(y,t)], \qquad (\mathbf{33.4})$$

$$v_4 = F + G\ dc^2[px + q(y,t)];$$

case 5 $r = -m^2, \ \ a = 2m^2 - 1, \ \ b = 2(1-m^2)$

$$u_5 = he^{i\theta}\ nc[px + q(y,t)], \qquad (\mathbf{33.5})$$

$$v_5 = F + G\ nc^2[px + q(y,t)];$$

case 6 $r = 1 - m^2, \ \ a = 2 - m^2, \ \ b = 2$

$$u_6 = he^{i\theta}\ cs[px + q(y,t)], \qquad (\mathbf{33.6})$$

$$v_6 = F + G\ cs^2[px + q(y,t)];$$

case 7 $r = 1, \ \ a = 2 - m^2, \ \ b = 2(1-m^2)$

$$u_7 = he^{i\theta}\ sc[px + q(y,t)], \qquad (\mathbf{33.7})$$

$$v_7 = F + G\ sc^2[px + q(y,t)];$$

case 8 $r = m^2(m^2-1), \;\; a = 2m^2-1, \;\; b = 2$

$$u_8 = he^{i\theta}\; ds[px + q(y,t)], \tag{33.8}$$

$$v_8 = F + G\; ds^2[px + q(y,t)];$$

case 9 $r = 1/4, \;\; a = (1-2m^2)/2, \;\; b = 1/2$

$$u_9 = he^{i\theta}\{ns[px + q(y,t)] \pm cs[px + q(y,t)]\}, \tag{33.9}$$

$$v_9 = F + G\{ns[px + q(y,t)] \pm cs[px + q(y,t)]\}^2;$$

case 10 $r = (1-m^2)/4, \;\; a = (1+m^2)/2, \;\; b = (1-m^2)/2$

$$u_{10} = he^{i\theta}\{nc[px + q(y,t)] \pm sc[px + q(y,t)]\}, \tag{33.10}$$

$$v_{10} = F + G\{nc[px + q(y,t)] \pm sc[px + q(y,t)]\}^2;$$

case 11 $r = m^4/4, \;\; a = (m^2-2)/2, \;\; b = 1/2$

$$u_{11} = he^{i\theta}\{ns[px + q(y,t)] + ds[px + q(y,t)]\}, \tag{33.11}$$

$$v_{11} = F + G\{ns[px + q(y,t)] + ds[px + q(y,t)]\}^2;$$

case 12 $r = m^2/4, \;\; a = (m^2-2)/2, \;\; b = m^2/2$

$$u_{12} = he^{i\theta}\{sn[px + q(y,t)] \pm i\; cn[px + q(y,t)]\}, \tag{33.12}$$

$$v_{12} = F + G\{sn[px + q(y,t)] \pm i\; cn[px + q(y,t)]\}^2;$$

case 13 $r = 0, \;\; a = 0, \;\; b = 2$

$$u_{13} = he^{i\theta}\frac{C}{\{[px + q(y,t)]\}}, \tag{33.13}$$

$$v_{13} = F + G\frac{C^2}{[px + q(y,t)]^2}.$$

In all these solutions, θ, h, q, G and F are given by (22), (30.1), (31), (32.1) and (32.2) respectively for the corresponding values of r, a and b above. The given solutions satisfy b positif.

Remark 4

When $m \to 0$, The solutions (33.1)-(33.12) degenerate as triangular function-like solutions as follows

$$u_{14} = he^{i\theta}\; csc[px + q(y,t)], \tag{34.1}$$

$$v_{14} = F + G\; csc^2[px + q(y,t)];$$

$$u_{15} = he^{i\theta}\; sec[px + q(y,t)], \tag{34.2}$$

$$v_{15} = F + G\ sec^2[px + q(y,t)];$$

$$u_{16} = he^{i\theta} \cot[px + q(y,t)], \qquad (\mathbf{34.3})$$

$$v_{16} = F + G \cot^2[px + q(y,t)];$$

$$u_{17} = he^{i\theta} \tan[px + q(y,t)], \qquad (\mathbf{34.4})$$

$$v_{17} = F + G \tan^2[px + q(y,t)];$$

$$u_{18} = he^{i\theta}\{\ csc[px + q(y,t)] \pm \cot[px + q(y,t)]\}, \qquad (\mathbf{34.5})$$

$$v_{18} = F + G\{\ csc[px + q(y,t)] \pm \cot[px + q(y,t)]\}^2;$$

$$u_{19} = he^{i\theta}\{\ sec[px + q(y,t)] \pm \tan[px + q(y,t)]\}, \qquad (\mathbf{34.6})$$

$$v_{19} = F + G\{\ sec[px + q(y,t)] \pm \tan[px + q(y,t)]\}^2;$$

where $\theta,\ h,\ q(y,t),\ G$ and F are defined by (22), (30.1), (31), (32.1) and (32.2) respectively

Remark 5

When $m \to 1$, The solutions (33.1)-(33.12) degenerate as formal soliton-like, combined formal soliton-like and kink bell soliton-like solutions of Eq.(21) as follows

$$u_{20} = he^{i\theta} \tanh[px + q(y,t)], \qquad (\mathbf{35.1})$$

$$v_{20} = F + G \tanh^2[px + q(y,t)];$$

$$u_{21} = he^{i\theta}\ csch[px + q(y,t)], \qquad (\mathbf{35.2})$$

$$v_{21} = F + G\ csch^2[px + q(y,t)];$$

$$u_{22} = he^{i\theta} \coth[px + q(y,t)], \qquad (\mathbf{35.3})$$

$$v_{22} = F + G \coth^2[px + q(y,t)];$$

$$u_{23} = he^{i\theta}\{\coth[px + q(y,t)] \pm\ csch[px + q(y,t)]\}, \qquad (\mathbf{35.4})$$

$$v_{23} = F + G\{\coth[px + q(y,t)] \pm\ csch[px + q(y,t)]\}^2;$$

$$u_{24} = he^{i\theta}\{\tanh[px + q(y,t)] \pm i\ sech[px + q(y,t)]\}, \qquad (\mathbf{35.5})$$

$$v_{24} = F + G\{\tanh[px + q(y,t)] \pm i\ sech[px + q(y,t)]\}^2;$$

where $\theta,\ h,\ q(y,t),\ G$ and F are defined by (22), (30.1), (31), (32.1) and (32.2) respectively

4. conclusion

In summary, using a modification of extended mapping method, we present explicit abundant solutions namely: Jacobi elliptic function-like, combined Jacobi elliptic function-like, rational-like, triangular function-like, formal soliton-like, combined formal soliton-like and kink bell soliton-like solutions of the (2+1)-dimensional dispersive long wave and the (2+1)-dimensional generalized Hirota equations. Because this method is quite powerful and unified method, we find new and more general solutions. These solutions include the previously known travelling wave solutions found by extended tanh-function method, sech-function method, Jacobi elliptic-function method. By using this method, many tedious and repetitive calculations have been avoided. Our method is similar to that of Peng [11] in the sense that, it use the elliptic equation of the first kind involving three parameters instead of specific functions as in the previous methods. In the constrast to the method of Peng [11], we are able to go beyong the constant coefficients for the series by introducing coefficient functions. Thus, our method can used to obtain more types of solutions to the NLPDEs. Presently, we are making the method computerizable, which allow us to perform complicated and tedious algebraic calculation on computer.

Acknowledgement.
This research was carried out during the author's stay at the Max–Planck Institut für Mathematik in Bonn. I thank this institution for the generous hospitality and support.

References

[1] W. Malfliet; Am.J. Phys. 60 650 (1992).
[2] E.J. Parkes; J. Phys. A 27 L497 (1994).
[3] E.J. Parkes, B.R.Duffy; Phys. Lett. A 229 217 (1997).
[4] B.R. Duffy, E.J. Parkes; Phys.Lett. A 214 271 (1996).
[5] E.J. Parkes, Z. Zhu, B.R. Duffy, H.C. Huang; Phys. Lett. A 248 219 (1998).
[6] S.K. Liu, Z. Fu, S.D. Liu, Q. Zhao; Phys. Lett. A 289 69 (2001).
[7] Z. Fu, S.K. Liu, S.D. Liu, Q. Zhao; Phys. Lett. A 290 72 (2001).
[8] E.J. Parkes, B.R.Duffy, P.C. Abbott; Phys.Lett. A 295 280 (2002).
[9] Z.Y. Yan; Commun. Theor. Phys. 38 143 (2002).
[10] Y.Z. Peng; Chin.J. Phys.41 103 (2003).
[11] Y.Z. Peng; Phys. Lett. A 314 401 (2003).
[12] K. Chamdrasekharan, Elliptic functions. Berlin: Springer-Verlag, 1985.
[13] F. Bowman, Introduction to Elliptic functions with applications, Universities, London, 1959.

[14] M. Boiti, J.J.P. Leon, F. Pempinelli; Inverse Probl. 3 371 (1987).

[15] B. Paquin, P. Wintermitz; Physica D 46 122 (1990).

[16] M.L. Wang, Y.B. Zhou, Z.B. Li; Phys. Lett. A 216 67 (1996).

[17] E. Fan; Chaos, Solitons and Fractals 16 819 (2003).

[18] S.Y. Lou; J. Math. Phys. 33 4300 (1992).

[19] A Maccari; J. Math. Phys. 39 6547 (1998).

[20] R. Hirota; J. Math. Phys. 14 805 (1973).

E-mail address: eyomba@yahoo.com