The minimal 5-Representation of Lyons´sporadic group

by

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1. Introduction

The Lyons simple group was first constructed by Sims <1971> as a permutation group on 8835156 points. This representation is, because of its size, not very useful for calculating in the group.

In this paper we obtain a representation of the Lyons group of dimension 111 over F_5.

After finding three generators, the group $G_2(5)$ is identified as a subgroup, and then the matrices are proved to generate the Lyons group by checking the relations Sims used in his original construction.

2. The generating matrices

The method used to calculate generating matrices is illustrated in fig. 1. Here

- a is of order 3 and acts as (123) on A_{11};
- b is of order 11 and acts as (123456789XE);
- c is the required extra generator of $S_3 \times M_{11}$;
- d is some word in a and b.

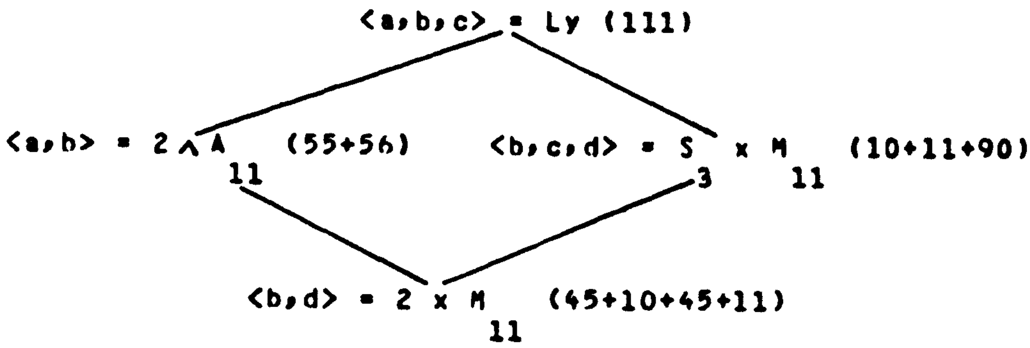

Fig. 1

a and b were calculated in 55- and 56-dimensional representations separately. No attempt was made to obtain the "correct" representations, but the ends justified the means!

The element d was then computed in the 55- and 56-dimensional representations, and the bases of these spaces were then changed so that <b,d> visibly decomposed, and the (45,45) submatrices in the 55- and 56-spaces were identical. The matrices shown below are then readily built.

$$
a = \begin{pmatrix} & & 0 & 0 \\ \multicolumn{2}{c}{a_{55}} & & \\ & & 0 & 0 \\ 0 & 0 & & \\ & & \multicolumn{2}{c}{a_{56}} \\ 0 & 0 & & \end{pmatrix} \tag{2.1}
$$

$$
b = \begin{pmatrix} b_{45} & 0 & 0 & 0 \\ 0 & b_{10} & 0 & 0 \\ 0 & 0 & b_{45} & 0 \\ 0 & 0 & 0 & b_{11} \end{pmatrix} \tag{2.2}
$$

$$
c = \begin{pmatrix} 2I_{45} & 0 & 2I_{45} & 0 \\ 0 & I_{10} & 0 & 0 \\ I_{45} & 0 & 3I_{45} & 0 \\ 0 & 0 & 0 & 4I_{11} \end{pmatrix} \tag{2.3}
$$

where I_n is the (n,n) identity matrix and a_{55}, a_{56}, b_{10}, b_{11}, b_{45} are given in table 1.

31132.1442.431.314331311241..4.31234132222.12.21241421.
4422433..4.321.313.13233441.1......1.423222.3313...4313
443222..33..3313224..4334241231222.414.4..3122424434432
.32..231343.223.32.341.441321414221133332312 22.4213.411
.422234424242.24133.4123.1.2344223314142.1222434443 2.43
24113211.2.24331.313321231243.334.1.333113144.22.43314.
.42..1222433.11441.431133..1131.243342212.2312241.31113
12343411131..2.4.333411222..314232.3.2.13321122 1213.414
233433.3...2432.13.1.3433.2.141142.2243331343421 1424432
32.441.3231.3.24.341.1..3213.1.233224.3.21111314341.1..
222.24.112441344242..4...33.2233.143133.114124324241414
4312334..322.1.343.42221413.444.2411.122.2144.412.4.432
34412242114..31..42324324..114112421112113111343442 43.12
33.244241.342.124213.1124223.1.1412124.223233..44.34143
3.1.344..33231.144121214.44144..144.2..1..144412.14232.
2.2433..1..2.23223.1.42.111.414233124324211 32313.44312.
11.421132434.241.3443.131.2413.1334321112233134343 2422.
211.121.1..14.121413442..344113431..333412.414.131.3..2
324134234211.1.24443312132.2212.3444222411 3...133.34323
3133.1.12131322242.33133423131343.233.414221...13423.1.
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.44.2.2.1.3233.124.34222.112.23.21.2443..14.1..41.14422
41.431341234131.344.4441..2412411221332141213..34434223
12431.2.42444224.4.413343444214..14114.3231.41.3234.44
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..334444.2.421.3.13..2.44332442442343314333131322.33.44
433313.313.213.222312...21.1343123314.131.2121244314211
3.12.14113343442.22.243434332442444.34.1.422443..314134
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4333412.33221442143133.22142.4.31241.341.4.4.2334.41141
.44.241422.42.1124223233134312232.333322441221 14232...4
332..3343.43..234..34.12.41.434.2.242..3441.14.33143.2.
311.3.1412144.33143141.2.12.34..1.12.3.444124123.3443.1
14.411.33.1413..422.3131.112.14333.22.42311123421.32324
11313221.4233.22.214...2.24.41...122244332423141 23421.3
..24.122432.4234223322412231 3.1321333.4242.31223.244143
32114..2342..3121..3..3.343..4111414422..23432222111..2
333.33311111421.221.1234334231 41222.32433.143422.2.2313
11342.1.142133.2331121..23.223422.2342114323111 244342.2
.4.121.43..34144123.122..1.331422.34212414114323 24442.4
42142341.2411221342331213131 34.212.3314.431322113211244
.34..312.1.221133.2....221..342331.141432243 2.122224.14
133142.2234422112244.4.31..224.1.441414.13114432.214312
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3313244.24.14..242111.1411.41.33.414412.1341.3.14222121
44..1323123424243.2...232333.2....411.4.1.3133.33.324..
.112.1112.4.33.223.2.41424123.2213..2.13.2..22442343333
1331..214142.12.32.4411.31433113312241.2.43.4342122.443
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32212.323441...21332342413442222 14.1....2434241..2..1.2
.224232342442234 43.314.42.1333.3412..231443..41112112.2
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.241..2....2.24123424232..1341.2312121231414223221..22.

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3422342411 4.332.32.144414.321.132114.4..33.412323143134 1
12343.32.3433232244..43143122224113122224323 23.13334.34
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1.31.2133413131.32..2311414111.2314342.13.21131.414.4..2
.3434144212233.4432324..2241214.14322..41.44114333234...
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...1....................................
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.....1..................................
..2233313134241 2.4232212122.4..2234.431444..2
.......1................................
........1...............................
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.............1..........................
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..................1.....................
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....................1...................
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...............................1........
................................1.......
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.....................................1..
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....1
3.13.3.42241.1.432..441123433.411.21334144.31
....1
....1
443...33311334333222413243241 1312..4.41433314
....1
.323214243242.2.24121321422 21.14.112113441.23

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323.......
14312.....
2.1.244...
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.3....4423
.111.42112
12444.3224
.221.21433
.323232114
4234111414
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243.......
3.143.....
1221.4....
1.4.2224...
21413.322..
4144234421.
4222.322.23
1.244.133..
34244323221
34.424.4344
.123.2....4
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3. Construction of the $G_2(5)$-invariant subspace U

We shall now prove the following

Main Theorem

The group

$$X = \langle a, b, c \rangle \tag{3.1}$$

is isomorphic to the sporadic group discovered by Lyons <1971>. Furthermore any subgroup of X of type $G_2(5)$ leaves a unique 7-dimensional subspace U of the representation space

$$V = F_5^{111} \tag{3.2}$$

invariant.

Proof

Our first goal will be the determination of a candidate for a subgroup isomorphic to the Chevalley group $G_2(5)$ and the corresponding invariant space U.

Let

$$H = \langle a, b \rangle = 2 \wedge A_{11} . \tag{3.3}$$

The involution

$$z = \mathrm{diag}(1^{55}, 4^{56}) \qquad (3.4)$$

is central in H. Indeed, z can be expressed as a word in the generators a and b:

$$z = (a.b^{3})^{11} . \qquad (3.5)$$

Among the 8835156 $G_2(5)$-subgroups in Ly there are exactly 2772 containing z. Let G_o be one of these.

The centralizer of z in G_o is the intersection of the full centralizer H with G_o. This group is of type

$$(1/2).2\wedge(S_5 x S_5) \qquad (3.6)$$

From the permutation character on the points of the Tits geometry (Tits <1980>) we derive that there is exactly one $G_2(5)$ containing this group. We now consider its commutator group, S. It is a central product of two factors, S_1 and S_2, each of which is an $SL_2(5)$.

In $G_2(5)$ we can describe S_1 and S_2 as the point stabilizer of the fixed space of z in the natural 8-dimensional representation on the Cayley algebra over the field F_5 and the unique other $SL_2(5)$ in S.

The character tables of $G_2(5)$ and Ly show that the elements of order 3 in S_1 and S_2 are of type 3B and 3A, respectively (in $G_2(5)$ as well as in Ly).

Because 3A-elements which lie in H correspond to single 3-cycles while 3B-elements are products of two or three such cycles, we deduce that S_1 transitively permutes 6 of the 11 letters, leaving the remaining ones fixed. On the other hand, S_2 acts trivially on the 6-set and permutes the rest. We thus get a (6,5)-partition of the 11 letters.

There are exactly 462 (6,5)-partitions each of which corresponds to 6 possibilities of choosing S_1. Hence the 462.6 = 2772 groups conjugate to S_1 in H are in 1-1-correspondence with the $G_2(5)$'s containing z.

This justifies to define

$$S_1 = \langle \alpha, \beta \rangle \tag{3.7}$$

and

$$S_2 = \langle \gamma, \delta \rangle \tag{3.8}$$

where

$$\alpha = b^{-2}.a.b^{2}.a; \tag{3.9}$$

$$\beta = b^{-4}.a^{-1}.b^{4}.a.b^{-2}.a.b^{2}.a^{-1}.b^{-4}.a.b.a^{-1}.b^{3}; \tag{3.10}$$

$$\gamma = b^{5}.a.b^{-5}; \tag{3.11}$$

$$\delta = h^{3}.a.h^{-3}. \tag{3.12}$$

The generators of S correspond to the following elements of $2\wedge A_{11}$:

$$\alpha = -(12345); \tag{3.13}$$

$$\beta = (26)(45); \tag{3.14}$$

$$\gamma = +(789); \tag{3.15}$$

$$\delta = +(9XE). \tag{3.16}$$

By Vermutung 2 of Meyer / Neutsch <1984>, $G_2(5)$ and hence $S_1 \vee S_2$ should leave a 7-dimensional subspace U invariant which splits into the eigenspaces U_+ and U_- for the eigenvalues +1 and -1 of z ($\dim U_+ = 3$, $\dim U_- = 4$). The representation of $G_2(5)$ on U is isomorphic to the standard representation on the pure octaves over F_5, so

$$U_+ \leq Fix(S_1). \tag{3.17}$$

Because

$$\dim Fix(S_1) = 3, \tag{3.18}$$

even

$$U_+ = Fix(S_1). \tag{3.19}$$

This implies the uniqueness of U (the second 7-dimensional constituent of the restriction of the 5-character on V to $G_2(5)$ occurs only as a factor space).

Next we have to construct U_-.

To achieve this, we observe that in the Cayley algebra there are exactly 144 nonzero octaves of norm 0 in U_-. They are permuted transitively by $S_1 v S_2$. Thus a Sylow-5-subgroup of $S_1 v S_2$ fixes at least one of them.

The vectors invariant under this Sylow group form a 5-dimensional space intersecting 3-dimensionally the eigenspace of z for eigenvalue -1.

Among the 31 one-dimensional subspaces of the intersection there is exactly one the image of which under $S_1 v S_2$ is 4-dimensional. This space therefore must be U_-.

A basis of U is given in table 2.

Table 2

Basis of U

```
1...2312342.3..124243.31434.14322.223232.2114233124233...................................................
.1.1424234.22434341.42221321323242.14.143212.2.33334332..................................................
..14331.4.311341.21242..42.41.3..3131224322.2413114133...................................................
.......................................................1.3.4.21.3224.14.4224.14...33411143231341443.12443
........................................................1..4.2.21223.41433422212234222....23.11.14132223
..........................................................11......44432.14..3..241434144221.1431332231412
............................................................124.3.132..3.44423.33..2..131.1441.3.31111434
```

4. A subgroup isomorphic to $G_2(5)$

The intersection of G_0 and H is a maximal subgroup of G_0.

For the purpose of constructing G_0 it will suffice to find an element in G_0 which does not commute with z. By trial and error we find that

$$\varepsilon = a^{-1}.b.a.c.a^{-1}.b^{-1}.a \qquad (4.1)$$

satisfies this condition.

Let

$$G = \langle \alpha, \beta, \gamma, \delta, \varepsilon \rangle. \qquad (4.2)$$

It is easy to see that

$$G|_U \cong G_2(5). \qquad (4.3)$$

Now we can show the group generated by S_1 and

$$\mu = \varepsilon.\beta.\alpha^4.\beta.\alpha.\beta.\varepsilon.\alpha^2.\beta^3.\alpha^3.\varepsilon.\gamma.\delta^2.\gamma.\delta^2.\gamma \qquad (4.4)$$

to be isomorphic to $SL_3(5)$.

We next have to give the generators of the root subgroups for the Chevalley group $G_2(5)$. For any root r of the G_2-system and any t in F_5 let us denote the associated root element by $X_r(t)$. We choose in accordance with the restriction to U:

$$X_a(1) = n^{-1}.(\delta.\gamma)^{-2}.(\gamma.\delta)^2.(\delta.\gamma)^2.n; \qquad (4.5)$$

$$X_{-a}(1) = n^{-1}.(\delta.\gamma).(\gamma.\delta)^{-2}.(\delta.\gamma)^{-1}.n; \qquad (4.6)$$

$$X_{b}(1) = (\alpha^{-1}.\beta)^{2}; \qquad (4.7)$$

$$X_{-b}(1) = \alpha.\beta^{2}; \qquad (4.8)$$

where

$$n = \pi^{2}.(\omega.\sigma)^{4}.\pi^{2} \qquad (4.9)$$

with

$$\sigma = \mu^{4}.\beta^{2}.\mu, \qquad (4.10)$$

$$\tau = (\alpha^{4}.\mu.\beta.\alpha.\mu)^{5}, \qquad (4.11)$$

$$\pi = (\tau.\alpha.\tau)^{2}. \qquad (4.12)$$

We now determine the root elements $X_r(t)$ by the formulas of Humphreys <1975, ch. 33.5>. (Humphreys prefers the notation $\varepsilon_r(t)$ instead of $X_r(t)$).

These elements obey all of the Chevalley relations.

By the theorem of Steinberg <1957>, cf. also Carter <1972>, we deduce from this:

$$\langle X_r(t);\ r \text{ root},\ t \in F_5 \rangle \cong G_2(5). \qquad (4.13)$$

Furthermore

$$\alpha = X_{b}(2).X_{-b}(-1).X_{b}(2); \tag{4.14}$$

$$\beta = X_{b}(2).X_{-b}(-1); \tag{4.15}$$

$$\gamma = X_{2a+b}(-2).X_{-2a-b}(-2).X_{2a+b}(1); \tag{4.16}$$

$$\delta = X_{2a+b}(2).X_{-2a-b}(1); \tag{4.17}$$

$$\varepsilon = M.N.Q.N^{-1}.M^{-1}; \tag{4.18}$$

where

$$M = X_{3a+2b}(-2).X_{-b}(1).X_{-3a-b}(1).X_{3a+b}(2).$$

$$.X_{-3a-b}(-1).X_{b}(1).X_{-3a-2b}(2); \tag{4.19}$$

$$N = X_{-2a-b}(1).X_{2a+b}(2).X_{-2a-b}(-1); \tag{4.20}$$

$$Q = X_{-3a-2b}(-2).X_{3a+b}(2).X_{-3a-b}(2).X_{3a+b}(-1).$$

$$.X_{-3a-b}(2).X_{3a+b}(2).X_{-b}(1). \tag{4.21}$$

The formulas necessary to evaluate the root elements $X_r(t)$ can be found in the book of Humphreys <1975>.

From (4.5)-(4.9) and (4.13)-(4.21) we get

$$G \cong G_2(5). \tag{4.22}$$

5. The generators of the Sims-presentation

In the representation of G on U we can find elements A, B, C, D corresponding to the matrices given in Sims <1972>:

$$A = \delta.\gamma^2.\delta^2.\beta^3\alpha^2.\beta.\pi^2.\tau^3.\delta.\gamma.\delta; \qquad (5.1)$$

$$B = \delta.\gamma.\delta.\gamma; \qquad (5.2)$$

$$C = \omega^2.\delta^2.\pi.\alpha.\beta^2.\pi^2.\beta.\omega^4.\delta; \qquad (5.3)$$

$$D = \alpha^{-1}.\beta.\alpha^{-1}.\beta.\alpha.\beta. \qquad (5.4)$$

To complete Sims' system of generators we need another element Z in the group $X = \langle a,b,c \rangle$ which satisfies Sims' relations.

From one of them, namely

$$Z^2 = A^2, \qquad (5.5)$$

we are provided with

$$Z^4 = A^4 = z. \qquad (5.6)$$

Hence Z is an element of $H = \langle a,b \rangle$.

A, B and D are also in H. They belong to the permutations

$$A = (2346)(7X), \qquad (5.7)$$

$$B = (79E8X), \qquad (5.8)$$

$$D = (15)(24). \qquad (5.9)$$

Using some of the relations we see that Z is one of the preimages of the two permutations

$$(2346)(19) \qquad (5.10)$$

and

$$(2346)(59). \qquad (5.11)$$

There are 4 possibilities for Z in all. Exactly one of them, namely

$$Z = b^5.a.b^4.a.b^9.a.b^2.a^2.b^7.a^2.b^5.a.b^4.a^2.b^9.a^2.b^2.a.b^8 \qquad (5.12)$$

belonging to the permutation (19)(2346) fulfills all of the defining relations for Ly as given by Sims <1972>.

This yields:

$$Y^* = \langle A,B,C,D,Z \rangle \cong Ly. \qquad (5.13)$$

On the other hand, we have

$$b = \alpha.\beta.Z^{-1}.\beta.\alpha^{-1}.\delta^{-1}.Z.\gamma.\delta.Z^{-1}.\delta.\alpha.\beta.Z.\beta.\delta \qquad (5.14)$$

and from the formulas for γ and ε we get

$$a = b^{-5}.\gamma.b^5 \qquad (5.15)$$

as well as

$$c = a^{-1} \cdot b^{-1} \cdot a \cdot \varepsilon \cdot a^{-1} \cdot b \cdot a \qquad (5.16)$$

Thus the generators of X lie in X^* and vice versa.

This completes the proof of the main theorem.

Because of the maximality of $G \cong G_2(5)$ in $X \cong Ly$ we finally derive

$$\mathrm{Stab}_X(U) = G. \qquad (5.17)$$

6. Concluding remarks: the Tits geometry of Ly

In this paper we constructed a 111-dimensional faithful representation of the Lyons group over F_5, which by an earlier investigation (Meyer / Neutsch <1984>) is the unique minimal 5-representation (up to conjugation).

Furthermore it is easy to see that in any characteristic other than 5 there is no nontrivial representation of degree ≤ 111.

With the results given here we can now complete the Brauer character φ of this representation (cf. Meyer / Neutsch <1984>):

x_{Ly}	67A	67B	67C
$\varphi(x)$	$-1-C_{1}$	$-1-C_{29}$	$-1-C_{37}$

(6.1)

Concerning the Tits geometry only very few facts are known:

First of all there is an invariant quadratic form F on V which is unique up to a scalar factor.

We may attach to the points of the geometry 7-dimensional totally F-isotropic subspaces of V (the Ly-conjugates of U). Two points which are collinear or coplanar (both conditions are equivalent) correspond to mutually orthogonal 7-spaces.

This allows to define lines and planes as the subspaces generated by all the points (7-spaces) which are incident with the object given.

Lines and planes are 24- and 35-dimensional totally isotropic spaces, respectively.

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