Integral motives and special values of zeta functions

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The theme explored here is that special values of zeta functions are given by Euler-characteristics. This was begun with the work of S. Lichtenbaum in the 1970's; he conjectured that the values of zeta functions of number fields at negative integers are given by ℓ-adic Euler characteristics. Bayer-Neukirch showed that this follows from the Main Conjecture of Iwasawa theory which was subsequently proved by Mazur-Wiles. Recently, Lichtenbaum (The Weil-étale topology) has conjectured an Euler-characteristic formula for the values at non-positive integers of the zeta function of any quasiprojective scheme over Spec $\mathbb{Z}$.

We shall consider his conjecture for varieties over finite fields and more generally for motives over finite fields. The main theorem is a proof of his conjecture for motives over finite fields.

We actually construct an integral category of mixed motives over a finite field; this $\mathbb{Z}$-linear category has many nice properties (finitely generated Hom-groups, noetherian, Tannakian, etc.). The point crucial for us is that it includes the p-part in characteristic p. Prior work of Schneider and Zarhin addresses the non-p part of the conjecture, but the p-part is quite mysterious and special. The construction of the integral category is adelic; it depends on a deep theorem of Gabber which itself uses the work of Deligne (Weil I, II).

In work in progress, we construct the triangulated category (with t-structure) of mixed motives over a finite field; it is not the derived category of its heart. Lichtenbaum's conjecture can be verified for any object in this category. As there is a natural functor from the category of abelian varieties over finite fields to this triangulated category, this proves the Lichtenbaum conjecture for any variety whose motive is generated by abelian varieties. By the Tate conjecture, this includes all varieties over finite fields.