CONSTRUCTION OF NEW SOLITON-LIKE SOLUTIONS FOR THE (2+1) DIMENSIONAL KADOMTSEV-PETVIASHVILI EQUATION

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Abstract. In this paper, we present another extended tanh method for obtaining explicit exact soliton-like solutions of nonlinear partial differential equations. By using this method via symbolic computation system, some families of new soliton-like solutions of the (2+1) dimensional Kadomtsev-Petviashvili equation are obtained.

1. Introduction

Seeking exact solutions for nonlinear partial differential equations (NLPDEs) has long been one of the central theme of perpetual interest in Mathematics and Physics. The effort in finding these solutions is important for the understanding of most nonlinear physical phenomena, thus they may give more insight into the physical aspects of the problems. The availability for these exact solutions for these nonlinear equations can greatly facilitate the verification of numerical solvers and the stability analysis of the solutions. In the past decades, there has been significant progression in the development of these methods. Such as inverse scattering method [1,2], Darboux transformation [3-10], Hirota bilinear method [11-13], tanh method [14-16], sine-cosine method [17,18], homogeneous balance method [19-23] and extended tanh method [24-26]. Generally speaking, these methods can be classified into two classes, one is called the direct method that is applied to seek exact solutions by using some ansatz, for example

$$U(x,y,t) = \sum_{i=0}^{n} a_i f^i(\xi), \quad \xi = lx + dy + ct, \tag{1}$$

with $f(\xi) = \tanh(\xi)$, $sech(\xi)$, $\tan(\xi)$, $\sec(\xi)$, $sn(\xi)$, $cn(\xi)$, $dn(\xi)$ etc.(l, d and c are constants). The other one is called the indirect method that is applied to seek exact solutions by using the following transformations:

$$U(x,y,t) = F(\phi_1, \phi_2, ...), \quad G(\phi_1, \phi_2, ...) = 0., \tag{2}$$

If one can determine the functions ϕ_i from $G(\phi_1, \phi_2, ...) = 0$, then he can obtain the solutions from the transformation $U = F(\phi_1, \phi_2, ...)$. The key idea of these generalized indirect methods are to use the solutions of an auxiliary ordinary differential equation to replace $\tanh(\xi)$ in tanh-function method. In the present paper, we shall use this idea to solve the Kadomtsev-Petviashvili equation (KP equation) following the line in [27-30] and using the new auxiliary ordinary differential equation presented in [31-32].

2. The further extended tanh method

For a given NLPDE with three independent variables (x,y,t) and dependent variable U, we seek its solutions in the form

$$U(x,y,t) = \sum_{i=0}^{n} a_i(y,t)\phi^i(\xi(x,y,t)), \tag{3}$$

with

$$(\frac{d\phi}{d\xi})^2 = a\phi^2(\xi) + b\phi^3(\xi) + c\phi^4(\xi), \tag{4}$$

Where a, b, c, are constants. To determine U explicitly, we take the following four steps.
Step 1. Determine n by balancing the highest order derivative terms and the highest nonlinear terms in the given NLPDE.
Step 2. Substituting (3) and (4) into the given NLPDE and collecting coefficients of all powers of ϕ^k, $x^i\phi^m$, and $x^j\phi^s\sqrt{a\phi(\xi)^2 + b\phi^3(\xi) + c\phi^4(\xi)}$ (k=0,1,...; i=1,2,...; m=1,2,...; j=0,1,...; s=1,2,...) then setting each coefficient to zero, to derive a set of partial differential equations of a_i (i=0,1,...n) and ξ.
Step 3. Solving the system of partial differential equations obtained in step 2 for a_i and ξ.
Step 4. As (4) possesses the general solution

$$\phi(\xi) = \begin{cases} \frac{-ab\ sech^2(\frac{\sqrt{a}}{2}\xi)}{b^2 - ac(1-\tanh(\frac{\sqrt{a}}{2}\xi))^2} & : \quad a > 0 \\ \frac{2a\ sech(\sqrt{a}\xi)}{\sqrt{b^2-4ac} - b\ sech(\sqrt{a}\xi)}, & : \quad b^2 - 4ac > 0, \quad a > 0, \end{cases} \tag{5}$$

substituting a_i, ξ and (5) into (3) to obtain the solutions of the NLPDE in concern.

3. The soliton-like solutions for KP equation

In this section, by the use of the auxiliary ordinary differential equation (4), we investigate a typical breaking equation

$$(U_t + 6UU_x + U_{xxx})_x + 3\sigma^2 U_{yy} = 0, \tag{6}$$

Called Kadomtsev-Petviashvili equation (KP equation), where $\sigma^2 = \pm$, which is of considerable importance both in Physics and Mathematics. The KP equation arises in many physical applications including weakly two-dimensional long waves in shallow water [33]. Its mathematical significance is related to the fact that it is an integrable soliton equation which has been solved in [31,34]. By balancing the highest-order contributions from both the linear and nonlinear terms in Eq.(6), we obtain n=2 in step 2. There fore, we assume the solutions of Eq.(6) in the form:

$$U(x, y, t) = f + h\phi(\xi) + g\phi^2(\xi), \tag{7}$$

Where $f = f(y,t), g = g(y,t), h = h(y,t)$ and $\xi = p(y,t)x + q(y,t)$ are all differential functions and $\phi(\xi)$ satisfies Eq.(4). Substituting Eq.(7) along with Eq.(4) into Eq.(6) and collecting coefficients of polynomials of $\phi^k, x^i\phi^m, \phi^l\sqrt{a\phi(\xi)^2 + b\phi^3(\xi) + c\phi^4(\xi)}$ and

$x\phi^l\sqrt{a\phi(\xi)^2+b\phi^3(\xi)+c\phi^4(\xi)}$ (k=0,...,6; i=1,2; m=1,...,4; l=0,1) with the aid of Mathematica (Note: f, g, h, p,q are independent of x), then setting each coefficient to zero, we can deduce the following set of over-determined partial differential equations with respect to unknown derivative functions (f,g,h,p,q). (Note: in the rest of this paper, p_y denotes $\partial p(y,t)/(\partial y)$, and so on).

$$
\begin{gathered}
2ahp_y^2=0,\\
(8ag+3bh)p_y^2=0,\\
2(5bg+2ch)p_y^2=0,\\
12cgp_y^2=0,\\
2ah(pp_t+6\sigma^2p_yq_y)=0,\\
(8ag+3bh)(pp_t+6\sigma^2p_yq_y)=0,\\
2(5bg+2ch)(pp_t+6\sigma^2p_yq_y)=0,\\
12cg(pp_t+6\sigma^2p_yq_y)=0,\\
6\sigma^2f_{yy}=0,\\
ah(6fp^2+ap^4+pq_t+3\sigma^2q_y^2)+3\sigma^2h_{yy}=0, \qquad (\mathbf{8})\\
48afgp^2+18bfhp^2+24ap^2h^2+32a^2gp^4+15abhp^4+p(8agp+3bh)q_t+3\sigma^2(8ag+3bh)q_y^2+6\sigma^2g_{yy}=0,\\
60bfgp^2+24cfhp^2+108ahgp^2+30bh^2p^2+130abgp^4+15b^2hp^4+40achp^4+2(pq_t+3\sigma^2q_y^2)(5bg+2ch)=0,\\
72cfgp^2+96ag^2p^2+126bghp^2+36ch^2p^2+105b^2gp^4+240acgp^4+60bchp^4+12cgpq_t+36c\sigma^2gq_y^2=0,\\
108bg^2p^2+144cghp^2+336bcgp^4+48c^2hp^4=0,\\
120cgp^2(g+2cp^2)=0,\\
ph_t+hp_t+3\sigma^2hq_{yy}+6\sigma^2h_yq_y=0,\\
pg_t+gp_t+6\sigma^2g_yq_y+3\sigma^2gq_{yy}=0,\\
2h_yp_y+hp_{yy}=0,\\
gp_{yy}+2g_yp_y=0.
\end{gathered}
$$

Solving the set of partial equations we found three cases:

Case 1, $p_y=0, p_t=0, g=-2cp^2, f_{yy}=0$ Then Eq.(8) can degenerate to

$$
\begin{gathered}
h+bp^2=0,\\
q_{yy}=0,\\
b^2-4ac=0, \qquad (\mathbf{9})\\
ap^4+6p^2f+pq_t+3\sigma^2q_y^2=0,
\end{gathered}
$$

which can be solved to get the solution

$$\begin{aligned} b^2 &= 4ac, \\ h &= -bp^2, \\ q &= f_{11}(t)y + f_{21}(t), \\ f &= f_{31}(t)y + f_{41}(t), \end{aligned} \tag{10}$$

with

$$f_{11} = -6p \int f_{31}(t)dt + K_{11}, \tag{11}$$

$$f_{21} = -ap^3t - 6p \int f_{41}(t)dt - \frac{3\sigma^2}{p} \int \left[K_{11} - 6p \int f_{31}(t)dt\right]^2 dt + K_{21}.$$

Where f_{31}, f_{41} are arbitrary functions of time, p, K_{11}, K_{21}, a, c are arbitrary constants. Thus we obtain the following soliton like solutions

$$U(x,y,t) = [f_{31}(t)y + f_{41}(t)] + bp^2\{\frac{ab\ sech^2(\frac{\sqrt{a}}{2}(px+f_{11}(t)y+f_{21}(t)))}{b^2-ac(1-\tanh(\frac{\sqrt{a}}{2}(px+f_1(t)y+f_2(t))))^2}\} - 2cp^2\{\frac{ab\ sech^2(\frac{\sqrt{a}}{2}(px+f_{11}(t)y+f_{21}(t)))}{b^2-ac(1-\tanh(\frac{\sqrt{a}}{2}(px+f_{11}(t)y+f_{21}(t))))^2}\}^2, \quad a > 0. \tag{12}$$

where f_{11} and f_{21} verify Eq.(11).

Case 2: $p_y = 0, p_t = 0, h = 0, f_{yy} = 0$. In this subcase, Eq.(8) is reduced to

$$\begin{aligned} b &= 0, \\ g + 2cp^2 &= 0, \\ q_{yy} &= 0, \\ 4ap^4 + 6p^2f + pq_t + 3\sigma^2q_y^2 &= 0, \end{aligned} \tag{13}$$

which can be solved to get the solution

$$\begin{aligned} g &= -2cp^2, \\ q &= f_{12}(t)y + f_{22}(t), \\ f &= f_{32}(t)y + f_{42}(t), \end{aligned} \tag{14}$$

with

$$f_{12} = -6p \int f_{32}(t)dt + K_{12}, \tag{15}$$

$$f_{22} = -4ap^3t - 6p \int f_{42}(t)dt - \frac{3\sigma^2}{p} \int \left[K_{12} - 6p \int f_{32}(t)dt\right]^2 dt + K_{22}.$$

Where f_{32}, f_{42} are arbitrary functions of time, p, K_{12}, K_{22}, a, c are arbitrary constants. So, Eq.(6) possesses the soliton-like solutions

$$U(x,y,t) = [f_{32}(t)y+f_{42}(t)]-2cp^2\left\{\frac{a\ sech(\sqrt{a}(px+f_{12}(t)y+f_{22}(t)))}{\sqrt{-ac}}\right\}, \quad a>0, \quad c<0. \qquad \textbf{(16)}$$

with f_{12}, f_{22} satisfying Eq.(15).

Case 3. $p_y = 0, p_t = 0, g = 0, f_{yy} = 0$. In this subcase, Eq.(8) is reduced to

$$\begin{aligned} c &= 0, \\ 2h + bp^2 &= 0, \\ q_{yy} &= 0, \\ ap^4 + 6p^2 f + pq_t + 3\sigma^2 q_y^2 &= 0. \end{aligned} \qquad \textbf{(17)}$$

These equations can be solved as

$$\begin{aligned} h &= -\frac{b}{2}p^2, \\ q &= f_{13}(t)y + f_{23}(t), \\ f &= f_{33}(t)y + f_{43}(t), \end{aligned} \qquad \textbf{(18)}$$

with

$$f_{13} = -6p\int f_{33}(t)dt + K_{13}, \qquad \textbf{(19)}$$

$$f_{23} = -ap^3t - 6p\int f_{43}(t)dt - \frac{3\sigma^2}{p}\int\left[K_{13} - 6p\int f_{33}(t)dt\right]^2 dt + K_{23},$$

where f_{33}, f_{43} are arbitrary functions of time, p, K_{13}, K_{23}, a, and b are arbitrary constants. So Eq.(6) admits the following two forms of solitons like.

$$U_1(x,y,t) = [f_{33}(t)y+f_{43}(t)]-\frac{b}{2}p^2\left\{\frac{-a}{b}\ sech^2\left(\frac{\sqrt{a}}{2}(px+f_{13}(t)y+f_{23}(t))\right)\right\}, \ a>0, \qquad \textbf{(20)}$$

$$U_2(x,y,t) = [f_{33}(t)y+f_{43}(t)]-\frac{b}{2}p^2\left\{\frac{sech(\sqrt{a}(px+f_{13}(t)y+f_{23}(t)))}{|b|-b\ sech(\sqrt{a}(px+f_{13}(t)y+f_{23}(t)))}\right\}, \ a>0,$$

with f_{31}, f_{23} satisfying Eq.(19). It is easy to show that the solutions (48)-(51) obtained in [31] are special case of our solutions. It is obvious that we have obtained some new soliton like that are not obtained in the KP equation.

In summary, we have obtained some new soliton-like solutions for the integrable KP equations in (2+1) dimensional spaces by using the auxiliary equation proposed by Sirendaorezi and Jiong [31,32] and the improved extended tanh-method. From the solutions obtained by the restriction on $\xi(x,y,t)$ as merely a linear function of x, y, t and the restriction on coefficients a_i as constants are removed. The method, proposed in this paper for single equation, may be extended to find explicit exact soliton-like solutions of other NLPDE and coupled NLPDE.

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