ON CATEGORY WEIGHT AND THE ARNOLD CONJECTURE

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Let (M^{2n}, ω) be a closed symplectic manifold. The well-known Arnold conjecture claims that, for every Hamiltonian symplectomorphism $\phi : M \to M$, the number of fixed points of ϕ is at least the minimal number of critical points of a smooth function on M. Floer and Hofer proved that, under the condition $\pi_2(M) = 0$, the number of fixed points of ϕ is at least the cup-length of M. So, here we have a weak form of the Arnold conjecture.

In the talk we prove, also under the condition $\pi_2 = 0$, the precise estimation from the Arnold conjecture, i.e., that the number of fixed points of ϕ is at least the minimal number of critical points of a smooth function on M. More precisely, we prove that the number of fixed points of ϕ is at least $2n+1$ (provided $\pi_2 = 0$). Since the minimal number of critical points of a function on every closed $2n$-dimensional manifold is at most $2n+1$ (actually, equal to $2n+1$ if $\pi_2(M) = 0$), this implies the conjecture.

The proof uses the analitical results done by Floer and Hofer. We did not develop any new analitical things, but just added some topological arguments. Namely, making purely topological research, I have found a way to compute the Lusternik–Schnirelmann category of some maps and spaces (using the technique of so-called category weight). Then I applied it to estimating of number of rest points of gradient-like flows on "bad" topological spaces. This enabled us to improve Floer–Hofer estimation.

References

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