Holomorphic Discs in the Space of Oriented Lines via Mean Curvature Flow and Applications

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We introduce a metric $\mathbb{G}$ on the space $\mathbb{L}$ of oriented geodesics in Euclidean 3-space. It is of index $(2,2)$. Together with the natural complex structure $\mathbb{J}$ due to Nigel Hitchin and the classical symplectic structure Ω on this space, this endows $\mathbb{L}$ with the structure of a neutral Kähler surface. The geometry of this space captures the geometry of C^1-smooth surfaces in Euclidean 3-space via a correspondance that associates with S the family $\Sigma \subset \mathbb{L}$ of oriented Euclidean-normal lines to S.

Our results are as follows.

1. We establish long-time existence for those solutions of mean curvature flow for spacelike surfaces in $(\mathbb{L}, \mathbb{G})$ that remain in a fixed compact subset of $\mathbb{L}$.

2. Given a Lagrangian surface $\Sigma \subset \mathbb{L}$, we consider mean curvature flow for spacelike surfaces Σ_t in $\mathbb{L}$ subject to three boundary conditions:

a) $\partial\Sigma_t \subset \Sigma$ for all t,
b) the angle between Σ_t and Σ remains constant in time,
c) $T\partial\Sigma_t$ is holomorphic as $t \to \infty$.

In this situation, we prove that there exist times $t_j \to \infty$ such that Σ_{t_j} converges to a holomorphic curve Σ_{t_∞}.

3. We prove that the existence of Σ_{t_∞} as above implies a bound on the relative first chern class of the pair $(\mathbb{L}, \Sigma)$ along the boundary of Σ_{t_∞}. This in turn implies a local index bound on the index of an isolated umbilic point of S. Here S arises as an integral surface of the family of lines that correspond to Σ in Euclidean 3-space.