DYNAMICAL SYSTEMS (SPECTRAL THEORY)

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This talk is both information about a recent progress in this area and an announcement of fresh results.

By dynamical system T we mean an invertible measure preserving map (transformation). Iterations of this map (powers) give a subgroup of a group of all measure preserving maps, i.e. an action of $\mathbb{Z}$ by transformations.

Usually (but not always!) different properties for $\mathbb{Z}$ have a more or less natural extension to actions of $\mathbb{R}$ (i.e. flows) or to more large groups.

Spectral invariants of T, i.e. the spectrum of a unitary operator induced on $L_2 : Tf(x) = f(Tx)$ have become classical in the theory of dynamical systems. However we are so far away from any solution of the following problem:

to determine precise conditions on the spectrum of a unitary operator under which it can be realized by a dynamical system.

¿From time to time here was a pulsing activity stimulated by coming of new ideas. We note sixties- seventies when by approximations were solved a few of very important problems. The main goal of my talk is to advertise a new approach, via limit polynomials. This approach gave recently a possibility to solve(Ageev, Ryzhikov) old Rokhlin's problem and to answer some well-known questions, for example, to answer to Katok's question.

As we know, the spectrum of any unitary operator U is completely determined by the spectral multiplicity function $M(U)$ and the measure σ_U of maximal spectral type. Below we will concentrate our attention at $M(T)$. There exist two points of view: to consider the spectrum of T (and $M(T)$) on L_2 and on the main invariant subspace $\{\text{const}\}^{\perp}$. In the first case, the best result reached by the method of approximations was on realizations of any subset A of $\mathbb{N}$ with

$$m_1, m_2 \in A \Rightarrow \operatorname{lcm}(m_1, m_2) \in A$$

as $M(T)$ for some ergodic transformation T. By limit polynomials on subspaces the same was proved (Lemanczyk and ..., Ageev) for all possible subsets of $\mathbb{N}$ (because we consider ergodic transformations, "possible" means any subset of $\mathbb{N}$ with 1).

In the second case, the problem remains unsolved. For typical T, Katok showed by approximations that

$$M(T \times T) \in \{\{2\}, \{2, 4\}\}$$

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and conjectured that

$$M(\underbrace{T\times\cdots\times T}_{n \text{ times}}) = \{n, n(n-1), \ldots, n!\}.$$

This means in particular Rokhlin's problem if $n = 2$.

By limit polynomials, this conjecture was confirmed by Ageev (Ryzhikov for $n = 2$). This approach gives a lot of new information about $M(T)$. However, the folowing extended problem of Rokhlin is still open:

to prove that for any n, there exists a dynamical system T such that $M(T) = \{n\}$ on $\{\text{const}\}^{\perp}$.

Next we study $M(T)$ for mixing T. Staircase constructions of rank 1 transformations are used in search of series of mixing transformations with various multiplicity functions. In particular, we announce that there exist dynamical systems with staircase multiplicity functions. More precisely,

Theorem 0.1. *For any n there exists a mixing transformation T such that*

$$M(T) = \{2, 3, ..., n\} \text{ on } \{const\}^{\perp},$$

this gives

$$M(T) = \{1, 2, 3, ..., n\} \text{ on } L_2.$$

Remark 0.2. For $n = 2$ it was proved by Ryzhikov.

Remark 0.3. Theorem 1 also gives an answer to a question of Robinson about possible values of $m(T) = \max M(T)$, who showed that for any n there exists a mixing T with $n < m(T) < \infty$.