27. MATHEMATISCHE ARBEITSTAGUNG

1987

Max-Planck-Institut
für Mathematik
Gottfried-Claren-Str. 26
5300 Bonn 3

Mathematisches Institut
der Universität Bonn
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MPI 87 - 23

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Programm der Mathematischen Arbeitstagung 1987 (I)

Freitag, den 12.6.1987

16.30 - 17.30 Uhr	M.F. ATIYAH: Invariants of 3- and 4-dimensional manifolds

Samstag, den 13.6.1987

10.00 - 11.00 Uhr	J. TITS: Monster and moonshine
11.45 - 12.45 Uhr	N. SMALE: Bridge principle for minimal submanifolds
17.00 - 18.00 Uhr	A. FLOER: Instantons and homology 3-spheres

Sonntag, den 14.6.1987

10.00 - 10.15 Uhr	Festlegung der nächsten Vorträge
10.15 - 11.15 Uhr	F. HIRZEBRUCH: Introduction to elliptic genera
12.00 - 13.00 Uhr	L. ILLUSIE: Frobenius and the degeneration of the Hodge spectral sequence
17.00 - 18.00 Uhr	Th. FRIEDRICH: Riemannian manifolds with small eigenvalue of the Dirac operator

Die Vorträge finden alle im "*Großen Hörsaal*", Wegelerstraße 10, statt.

Erfrischungspausen mit Tee: Samstag 11.00 - 11.45 Uhr und 16.00 - 17.00 Uhr, Sonntag 11.15 - 12.00 Uhr und 16.00 - 17.00 Uhr, vor dem Großen Hörsaal.

Post liegt während der Teepausen aus. Alle Teilnehmer mögen sich bitte in die *Teilnehmerlisten* eintragen. Teilnehmerlisten und *Informationen* liegen vor dem Großen Hörsaal aus.

Für *Diskussionen* stehen das Haus Wegelerstraße 10 und das Max-Planck-Institut zur Verfügung.

Den *Tagungsbeitrag* bitte während der Teepausen an Herrn *Winter* bezahlen.

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Programm der Mathematischen Arbeitstagung 1987 (II)

Montag, den 15.6.1987

9.45 - 10.45 Uhr	D. EPSTEIN: Finite state automata, groups and hyperbolic geometry
13.00 Uhr	Schiffsausflug nach Bad Breisig. Abfahrt pünktlich um 13.00 Uhr mit Motorschiff "Carmen Silva", Landebrücke der Personenschiffahrt "Siebengebirge" (nahe Alter Zoll). Rückkehr ca. 19.30 Uhr

Dienstag, den 16.6.1987

10.00 - 10.15 Uhr	Festlegung der nächsten Vorträge
10.15 - 11.15 Uhr	A. WILES: On the Tate-Safarevic group
12.00 - 13.00 Uhr	D. BURGHELEA: Free loop spaces and automorphisms of manifolds
17.00 - 18.00 Uhr	D. ZAGIER: Elliptic curves and Fermat's last theorem

Mittwoch, den 17.6.1987

10.00 - 11.00 Uhr	Y. MIYAOKA: The structure of algebraic threefolds

Die Vorträge finden alle im "*Großen Hörsaal*", Wegelerstraße 10, statt.

Erfrischungspausen mit Tee: Dienstag, 11.15 - 12.00 Uhr vor dem Großen Hörsaal, 15.30 - 16.30 Uhr im Max-Planck-Institut.

Post liegt während der Teepausen vor dem Großen Hörsaal aus.

Alle Tagungsteilnehmer mögen sich bitte in die *Teilnehmerlisten* eintragen. *Teilnehmerlisten und Informationen* liegen vor dem Großen Hörsaal aus.

Den *Tagungsbeitrag* bitte während der Teepausen an *Herrn Winter* bezahlen.

Alle Tagungsteilnehmer mit ihren Damen oder Herren sind herzlich zum *Empfang des Rektors* eingeladen. Zeit: Dienstag, 16.6.1987, 20.00 Uhr. Ort: Festsaal der Universität, Hauptgebäude; Eingang von der Straße "Am Hof" durch das Tor gegenüber Buchhandlung Röhrscheid.

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Programm der Mathematischen Arbeitstagung 1987 (III)

Mittwoch, den 17.6.1987

11.45 - 12.45 Uhr	M. SAITO: Mixed Hodge modules
17.00 - 18.00 Uhr	St. RALLIS: L-functions and oscillator representations

Donnerstag, den 18.6.1987

10.00 - 11.00 Uhr	D. GROMOLL: Lower curvature bounds and topological finiteness
11.45 - 12.45 Uhr	K. UHLENBECK: Moment maps in infinite dimensions
17.00 - 18.00 Uhr	Th. ZINK: Points on Shimura varieties and Dieudonné modules

Freitag, den 19.6.1987

16.30 - 17.30 Uhr	B. MEEKS: The geometry of constant mean curvature surfaces and of minimal surfaces (Kolloquiumsvortrag)

Der *Kolloquiumsvortrag* findet im "*Kleinen Hörsaal*", Wegelerstr. 10, statt. Alle *anderen Vorträge* finden im "*Großen Hörsaal*" statt.

Post liegt während der Teepausen aus.

Erfrischungspausen mit Tee: Mittwoch und Donnerstag, 11.00 - 11.45 Uhr und 16.00 - 17.00 Uhr vor dem Großen Hörsaal.

Informationen liegen vor dem Großen Hörsaal aus.

Den *Tagungsbeitrag* bitte während der Teepausen an *Herrn Winter* bezahlen.

Titel: INVARIANTS OF 3 and 4-DIMENSIONAL MANIFOLDS

Autor: MICHAEL ATIYAH

Adresse: Mathematical Institute, OXFORD UNIVERSITY

§1 Donaldson Invariants

At the last Arbeitstagung I described the new invariants of differentiable 4-manifolds introduced by Donaldson, using $SU(2)$-instantons. Roughly speaking one "counts" the number of instantons. Moreover Donaldson showed that his invariants were essentially trivial for connected sums but non-trivial for algebraic surfaces.

Now if a simply-connected 4-manifold X has quadratic form $A = A_1 \oplus A_2$, then X can be decomposed accordingly as a "generalized connected sum" of X_1 and X_2 along a <u>homology</u> 3-sphere Σ. The non-triviality of the Donaldson invariants

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Autor:

Adresse:

should somehow be related to the non-trivial $\pi_1(\Sigma)$. This is the purpose of the present lecture

§2 Instantons for manifolds with boundary

If X^+ has boundary Σ, instantons on X^+ will have boundary values on Σ. The natural space of boundary values is the space of connections modulo gauge transformations on Σ, say $\mathcal{C}(\Sigma)$. All boundary values of instantons on X^+ define a submanifold $\mathfrak{M}^+ \subset \mathcal{C}(\Sigma)$ which can be called a "Hardy cycle" (for the non-linear analogue of Hardy spaces of functions on S^1). Similarly if X^- has boundary $-\Sigma$ (opposite orientation) we get a Hardy cycle $\mathfrak{M}^- \subset \mathcal{C}(\Sigma)$. The set

(1) $\quad M = \mathfrak{M}^+ \cap \mathfrak{M}^-$

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represents instantons on the closed manifold

$$X = X^{+} \cup_{\Sigma} X^{-}$$

When this is finite its cardinality is one of the Donaldson invariants of X. To make good sense of the intersection (1) we need a homology theory for "middle-dimensional" cycles of the infinite dimensional manifold 𝒞. This is the theory developed by A. Floer described in his lecture and summarized below

§ 3 Floer Homology

On $\mathcal{C}(\Sigma)$ there is natural function f with values in R/Z : the Chern-Simons function. Its gradient field is $A \longmapsto *F_A$ given by the dual of the curvature. Its zeros are given by $F=0$ i.e. flat connections or representations $\pi_1(\Sigma) \longrightarrow SU(2)$. Our new

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builds the appropriate Morse homology by the method of Witten. The main difference here from conventional Morse theory is that the Hessian of f is a Dirac type operator with infinite $+$ and $-$ spectrum. However one can define relative Morse indices by spectral flow methods. The homology groups one gets are only indexed by $\mathbb{Z}$ mod 8 because spectral flow round a generator of $\pi_1(B) = \mathbb{Z}$ gives 8.

If we stretch $X = X_+ \cup_\Sigma X_-$ along Σ (long thin tube) this pushes M^+ and M^- towards critical points of f and enables Donaldson to define the intersection number by using Floer homology.

Titel: Monster and Moonshine

Autor: J. Tits

Adresse: Collège de France, 11, pl. Marcelin-Berthelot, 75231 Paris Cedex 05.

Let M be the Griess-Fischer "Monster" group. This lecture is an introduction to the construction by I. Frenkel, J. Lepowsky and A. Meurman of a graded M-module $(V_i)_{-1 \le i \le \infty}$ with $\sum \dim V_i \cdot q^i = J(q) - 744$, where $j(z) = J(e^{2\pi i z})$ is the modular ~~function~~ invariant.

Ref. I. Frenkel, J. Lepowsky and A. Meurman, A natural representation of the Fischer-Griess Monster with the modular function J as character, Proc. Nat. Acad. Sci. U.S.A. 81 (1984), 3256-3260.

Title : A Bridge Principle for Minimal and Constant Mean Curvature Submanifolds

Author : Nathan Smale
Address : (Starting Aug. 1) M.S.R.I. 100 Centennial Way, Berkeley CA., U.S.A.

The bridge principle is a classical problem in minimal surface theory. The basic idea is that if one cuts small arcs out of the boundaries of two stable minimal surfaces, and joins the boundaries together by two nearby curves, then the resulting configuration should span a stable minimal surface that is close to the original two joined by a thin strip. This principle has been used heuristically to produce minimal surfaces having certain desired properties. In 1950 Courant, [C], mentioned this principle in his book, and claimed that it could be proven by the methods in it. In the 1950's, Kruskal, [Kr], published a proof of a version of this principle. Then in the 1970's, Meeks and Yau, [M, Y], gave a proof for compact, stable orientable surfaces in $\mathbb{R}^3$. I'm going to talk a result, [Sm], that is essentially a generalization of the Meeks-Yau theorem. The dimension and codimension can be arbitrary, stability is not assumed, and the result also holds for hypersurfaces of constant mean curvature.

Theorem A: Let N_1 and N_2 be compact, n-dimensional minimal (or constant mean curvature) submanifolds immersed in $\mathbb{R}^N$, and without Jacobi fields. Here $n \geq 2$, $N > n$. Assume also that they are C^4 up to their boundaries which are also C^4. Let γ be any path connecting the boundaries of N_1 and N_2 and let $\mathcal{N}$ be any tubular neighborhood of γ. Then there exists a bridge (a diffeomorphic copy of $[0,1] \times S^{n-2}$) inside $\mathcal{N}$ connecting the boundaries of N_1 and N_2, and a minimal (constant mean curvature) submanifold whose boundary is the resulting configuration, and is close to N_1 joined to N_2 by a thin strip (a diffeomorphic copy of $[0,1]\times B^{n-1}$).

The approach used here, suggested by Rick Schoen, is to construct a family of approximate solutions M^ε, and then find small perturbations of these, in the normal direction, which are minimal. (For simplicity I will only discuss the minimal case). To obtain M^ε we glue N_1 to N_2 by a strip of diameter ε, inside $\mathcal{N}$. This is done in such a way that $|A|$ is bounded independent of ε, where A = 2nd fundamental form of M^ε. This condition limits somewhat the applications of the theorem. Now, we consider nearby submanifolds (with the same boundary) as sections of the normal bundle NM^ε. That is, if u is a section of NM^ε which is zero on ∂M^ε, and if $f: M^\varepsilon \to \mathbb{R}^N$ is an immersion, then $f+u: M^\varepsilon \to \mathbb{R}^N$ parametrizes a nearby submanifold. Since we will be making estimates on solutions of elliptic equations, the appropriate class of sections to consider will be $C_0^{2,\alpha}(NM^\varepsilon)$, that is $C^{2,\alpha}$ sections which are zero on ∂M^ε. Let $\mathcal{C} \subseteq C_0^{2,\alpha}$ be the open subset of sections whose C^1 norms are sufficiently small to guarantee that $f+u$ defines an immersion, and for $u \in \mathcal{C}$ let $H(u)$ = mean curvature of $f+u$. Thus, we want to solve $H(u)=0$ ..... for some $u \in \mathcal{C}$ and show that u is small. From the analysis point of view, H is not a nice operator since it doesn't take sections of NM^ε into sections of NM^ε. However, we can project

$H(u)$ orthogonally back onto NM^ε. Call this projection $H^\perp(u)$. If u is small eneough in C^1 norm, then $H(u)=0$ $\Longleftrightarrow$ $H^\perp(u)=0$. Thus our goal is to solve $H^\perp(u)=0$ for some $u\in\mathcal{C}$ (a non linear elliptic system) and show that u is small. Now, $H^\perp : \mathcal{C} \subseteq C_0^{2,\alpha} \to C^{0,\alpha}$ is a smooth mapping. Making a Taylor expansion about zero we get
$$H^\perp(u) = H_0 + Lu + E(u)$$

where
$$H_0 = H^\perp(0) = \text{mean curvature of } M^\varepsilon$$
$$Lu = \left.\frac{dH^\perp}{dt}(tu)\right|_{t=0} = \Delta u + Bu \text{ is the Jacobi operator}$$

and $E(u)$ is a quadratic and cubic expression in u, ∇u, and $\nabla^2 u$. Since N_1 and N_2 have no Jacobi fields, one can show that M^ε has no Jacobi fields. In fact the spectrum of L is bounded away from zero for all ε sufficiently small. Therefore L is invertible by standard elliptic theory and $L^{-1} : C^{0,\alpha} \to C_0^{2,\alpha}$ is a continuous operator. Applying L^{-1} to the equation
$$H^\perp(u) = H_0 + Lu + E(u) = 0$$
gives us the fixed point problem
$$u = -L^{-1}H_0 - L^{-1}E(u) \equiv T(u)$$
Here $T : C_0^{2,\alpha} \to C_0^{2,\alpha}$ is a continuous nonlinear map. So we want to solve $Tu = u$ for some $u \in \mathcal{C}$. This will be done by use of the Schauder fixed point theorem which states that a contious mapping of a Banach Space

that preserves a convex, compact set must have a fixed point. In our problem, the appropriate sets to look at are
$\mathcal{K}(\varepsilon,\sigma,\alpha) = \{ u \in C_0^{2,\alpha} : |u|_{H^1} \le \varepsilon^{\frac{n-1}{2}-\sigma}, |u|_{H^2} \le \varepsilon^{\frac{n^2-n-4}{2n}-\sigma},$
$|u|_{C^0} \le \varepsilon^{2-\frac{2}{n}-\sigma}, |u|_{C^{2,\alpha}} \le \varepsilon^{-\frac{2}{n}-\sigma-\alpha} \}$ (for $n \ge 4$).

Note that $\mathcal{K}$ is convex, and compact in $C_0^{2,\beta}$ if $\beta < \alpha$. Then Theorem A is implied by the Schauder fixed point theorem and

<u>Theorem B</u>: $\exists\ \bar\alpha, \bar\sigma$ and $\bar\varepsilon$ such that for all $\varepsilon < \bar\varepsilon$
$$T : \mathcal{K}(\varepsilon,\bar\sigma,\bar\alpha) \to \mathcal{K}(\varepsilon,\bar\sigma,\bar\alpha).$$

Note that $Tu = v_1 + v_2$ where $Lv_1 = -H_0$ and $Lv_2 = -E(u)$. So proving Theorem B involves estimating v_1 and v_2 in the H^1, H^2, C^0 and $C^{2,\alpha}$ norms. In order to make these estimates we need the following

<u>Lemma</u>: a) $|w|_{H^1} \le c|Lw|_{L^2}$

b) $|w|_{C^0} \le c\left[|Lw|_{L^{n/2+\sigma}} + |w|_{L^{n/2+\sigma}} \right]$

c) $|w|_{C^{2,\alpha}} \le c\left[\varepsilon^{-\alpha}|Lw|_{C^{0,\alpha}} + \varepsilon^{-2-\alpha}|w|_{C^0} \right]$

a) essentially follows from the fact that the spectrum of L is bounded away from zero, and standard Hilberts space theory. One proves b) by using the Sobolev inequality and an iteration argument. To prove c) one redoes the standard Schauder linear theory taking into account the parameter ϵ and how it affects the estimates.

Proving Theorem B then involves applying the Lemma to v_1 and v_2. Essentially, v_1 is small since H_0 is small in any integral norm, being supported on the strip. The smallness of u (being in $\mathcal{H}$) implies a certain smallness of $\mathcal{E}(u)$ which is then used to estimate v_2.

The method of constructing approximate solutions and then perturbing them to get true solutions has been used successfully in other geometric problems. See for example [T], [Sc] and [Ka].

Nathan Smale

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Titel: Instantons and Homology 3-Spheres
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The general aim of the research described here is to explore possible applications of instanton moduli spaces to the study of 3-manifolds. In particular, we define for any closed 3-manifold with $H_1(M,\mathbb{Z}) = 0$ an abelian group $I^* = \bigoplus I^p$ graded by $p \in \mathbb{Z}_8$.

Let P be a principal SU_2-bundle over a closed 3-manifold M. We can associate to P its adjoint bundle $ad(P)$, which is a vector bundle with fibre the Lie algebra su_2 of SU_2. All such bundles are topologically trivial, i.e. $P = M \times SU_2$ and $ad(P) = M \times su_2$. We consider on P the set $\mathcal{A}$ of connections, which in the trivial case can be identified with $\Omega^1 \otimes su_2$ (Ω^1 is the set of smooth 1-forms on M). To each $A \in \mathcal{A}$ we assign its curvature $F_A \in \Omega^2 \otimes \mathfrak{g}$, which in local terms is given by

$$(F_A)_{ij} = \nabla_i A_j - \nabla_j A_i - [A_i, A_j].$$

The curvature is gauge equivariant in the sense that $F_{g(A)} = g F_A g^{-1}$, where $g : M \longrightarrow SU_2$ is a gauge transformation and $g(A) = gAg^{-1} + (dg)g^{-1}$. Connections A with $F_A = 0$ are called flat. They can be constructed from representations of $\pi_1(M)$ in SU_2 in such a way that we have a bijection

$$\mathcal{R} := \{A \mid F_A = 0\}/\mathcal{G} = \mathrm{Hom}(\pi_1(M), SU_2)/SU_2.$$

Here, SU_2 acts on the homomorphism space by the adjoint representation, and a flat connection is irreducible (i.e. its only symmetry group is the center $g \equiv \pm 1$) iff the same is true for the corresponding homomorphism. One then easily verifies that in the case $H_1(M, \mathbb{Z}) = 0$, all flat connections are irreducible except for the trivial one, which we will from now on omit in $\mathcal{R}$.

Recently, Casson defined a new topological invariant for homology 3-spheres by "counting" the elements of $\mathcal{Z}$ with an appropriate sign. (In the following, we make the simplifying assumption that $\mathcal{Z}$ is a finite set). According to C. Taubes, this $\mathbb{Z}$-valued invariant can be interpreted as the Euler characteristic of the vector field $V(A) = *F_A$ on $\mathcal{A}/\mathcal{G}$. Here, $*: \Omega^2 \otimes su_2 \to \Omega^1 \otimes su_2 = T\mathcal{A}$ is the Hodge duality isomorphism. It is well known that this vector field is actually the gradient field of a function (the Chern-Simons function) on the universal covering

$$\widetilde{\mathcal{A}/\mathcal{G}} = \mathcal{A}/\mathcal{G}_0 \; ; \; \mathcal{G}_0 = \{ g : M \to SU_2 \,/\, g \text{ has degree } 0 \}$$

of $\mathcal{A}/\mathcal{G}$. For a gradient field, an invariant stronger than the Euler characteristic can be defined through a construction which in finite dimensions is due to E. Witten: For two elements of $\mathcal{Z}$, define the space $\mathcal{M}(A_1, A_2)$ of trajectories of the vector field $*F_A$

connecting A_1 and A_2. This space can be identified with the space of anti self dual connection classes on the cylinder $M\times\mathbb{R}$ with appropriate asymtotics at the ends. These spaces have been studied by C. Taubes. Applying a perturbation, we can assume that $\mathcal{M}(A_1,A_2)$ is a smooth manifold of dimension

$$\dim \mathcal{M}(A_1,A_2)=\mu(A_1)-\mu(A_2),$$

where $\mu:\tilde{\mathcal{Z}}\to\mathbb{Z}$. For $\mu(A_1)-\mu(A_2)=1$, $\mathcal{M}(A_1,A_2)$ is therefore discrete up to translations of trajectories, so that we can define $\langle A_1,\delta A_2\rangle\in\mathbb{Z}$ by counting trajectories with an appropriate sign (Here, we use Uhlenbeck's compactness theorem for instanton spaces to show that $\langle A_1,\delta A_2\rangle$ is finite). Considering $\langle A_1,\delta A_2\rangle$ as matrix elements we obtain an operator $\delta:\tilde{\mathcal{Z}}^*\to\tilde{\mathcal{Z}}^*$ on the free abelian group over $\tilde{\mathcal{Z}}$, which is of degree

$\mathbb{Z}$ if $\tilde{Z}^*$ is graded by means of the "Morse index" μ. It turns out that $\delta\delta = 0$ and that although δ generally depends on the choice of the perturbation that makes the trajectory spaces $\mathcal{M}$ regular, its cohomology

$$I^*(M) = \ker \delta / \operatorname{im} \delta$$

depends only on M. Finally, one can show that the group of covering transformations of $\mathcal{A}/\mathcal{G}$, i.e. the group $\mathcal{G}/\mathcal{G}_0 = \mathbb{Z}$, acts on $I^*(M)$ by degree 8, so that the quotient is graded by $\mathbb{Z}_8$.

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Titel: Introduction to elliptic genera

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The theory of elliptic genera is due to Ochanine, Landweber, Stong. E. Witten has brought in new aspects by using the free loop space of the manifold and the S^1-action on it. I shall use also a paper by D. Zagier. Last fall there was a mini-conference at Princeton. The Proceedings will appear in the Springer lecture notes. I refer to them and shall give no further references.

A power series $Q(x) = 1 + a_2 x^2 + a_4 x^4 + \dots$ with complex coefficients determines a "genus" (ring homomorphism)

$$\varphi : \Omega \otimes \mathbb{Q} = \sum \Omega^{4k} \otimes \mathbb{Q} \longrightarrow \mathbb{C}$$

where Ω is the Thom cobordism ring of oriented differentiable manifolds. If X is a $4k$-dim. manifold, we write the total Pontrjagin class $p(X)$ formally as

$$p(X) = (1+x_1^2) \cdots (1+x_{2k}^2),$$

then

$$\varphi(X) = (Q(x_1) \cdots Q(x_{2k}))[X].$$

Let $\wp(x)$, $x \in \mathbb{C}$, be the Weierstraß $\wp$-function for the lattice $2\pi i(\mathbb{Z} + \mathbb{Z}\tau)$ with base $\omega_1 = 2\pi i$, $\omega_2 = 2\pi i \tau$ $(\tau \in \mathbb{H})$ and put as usual $e_1 = \wp(\omega_1/2)$, $e_2 = \wp(\omega_2/2)$, $e_3 = \wp(\frac{\omega_1+\omega_2}{2})$.

Consider the power series

$$Q(x) = x\sqrt{\wp(x) - e_1}$$

This defines the elliptic genus $\varphi(X)$.

In fact, for any manifold X of dim $4k$ $\varphi(X)$ depends on τ. It is a modular form of $\Gamma_0(2)$ of weight $2k$. If we write $Q(x) = x/f(x)$ with $f = (\wp - e_1)^{-1/2}$, then $\wp'^2 = 4\wp^3 - g_2\wp - g_3 = 4(\wp - e_1)(\wp - e_2)(\wp - e_3)$ implies immediately

$$f'^2 = 1 - 2\delta f^2 + \varepsilon f^4$$

with $\delta = -\frac{3}{2} e_1$, $\varepsilon = (e_1 - e_2)(e_1 - e_3)$ being modular forms of weight 2 and 4 respectively for $\Gamma_0(2)$. We have

$$f(2x) = 2f(x)f'(x) / (1 - \varepsilon f(x)^4)$$

and more generally

$$f(x+y) = (f(x)f'(y) + f(y)f'(x)) / (1 - \varepsilon f(x)^2 f(y)^2)$$

The formula for $f(2x)$ shows by an easy residue calculation that for the quaternionic projective spaces

$$\varphi(P_k(\mathbb{H})) = 0 \quad \text{for } k \text{ odd}$$
$$\varphi(P_k(\mathbb{H})) = \varepsilon^{k/2} \quad \text{for } k \text{ even}.$$

Here one uses $p(P_k(\mathbb{H})) = (1+u)^{2k+2}(1+4u)^{-1}$ where u generates $H^4(P_k(\mathbb{H}), \mathbb{Z})$. (This formula for $p(P_k(\mathbb{H}))$ is an exercise I did 35 years ago.)

We have $\varphi(P_2(\mathbb{C})) = \delta$. If $\delta = \varepsilon$, then φ is up to normalization the signature (characteristic power series $x/\mathrm{tgh}\, x$).

If $\varepsilon = 0$, then φ is (up to normalization) the $\hat{A}$-genus. These 2-cases correspond to the cusps of $\Gamma_0(2)$. Classical formulas (see Hurwitz–Courant for example, observe $\left[\begin{array}{l} q = e^{2\pi i \tau} \\ v = \frac{x}{2\pi i} \end{array}\right]$

$$\sqrt{p - e_1} = \frac{\vartheta'_{11}(0,\tau)}{\vartheta_{10}(0,\tau)} \cdot \frac{\vartheta_{10}(v,\tau)}{\vartheta_{11}(v,\tau)} \quad \text{with}$$

theta functions suitably normalized) give

$$(Q(x)/x) = \frac{1}{2\,\mathrm{tgh}\, x/2} \; \frac{\prod_{n=1}^{\infty}(1+q^n e^x)(1+q^n e^{-x})}{\prod_{n=1}^{\infty}(1-q^n e^x)(1-q^n e^{-x})} : \left(\frac{\eta(2\tau)}{\eta(\tau)^2}\right)^2$$

where $\eta(\tau) = q^{1/24} \cdot \prod_{n=1}^{\infty} (1-q^n)$.

It follows that for a manifold X^{4k} (with k even)

$$\prod_{i=1}^{2k} \left(\frac{x_i}{tgh\, x_i} \prod_{n=1}^{\infty} \frac{(1+q^n e^{2x_i})(1+q^n e^{-2x_i})}{(1-q^n e^{2x_i})(1-q^n e^{-2x_i})} \right) [X]$$

is a modular <u>function</u> for $\Gamma_0(2)$ which can also be written as

$$(*) \qquad \operatorname{sign}\left(X, \prod_{n=1}^{\infty} \Lambda_{q^n} T \cdot \prod_{n=1}^{\infty} S_{q^n} T\right)$$

which is a power series in q with <u>integral</u> coefficients which are signatures with coefficients in complex vector bundles.

(T is the complex extension of the tangent bundle of X and $\Lambda_t T = \sum t^i \Lambda^i T$, $S_t T = \sum t^i S^i T$ where Λ^i are exterior powers, S^i symmetric powers).

According to E. Witten one should look at the free loop space LX with S^1-action. The fixed point set of this action is X (constant loops). A tangent vector at $p_0 \in X$ in LX is a Fourier series

$$a_0/2 + \sum_{k \geq 1} (a_k \cos 2\pi k x + b_k \sin 2\pi k x)$$ with a_k, b_k tangent vectors of X at p_0

A normal vector of X in LX at $p_0 \in X$ is given by the sequence $a_k + i b_k$ $(k \geqq 1)$ of complex numbers and therefore the normal bundle of X in LX is equivariantly
$$\sum_{n=1}^{\infty} q^n T .$$

If LX were a manifold with a signature operator, then $(*)$ would be its equivariant signature according to the Atiyah-Bott-Singer index and fixed point theorem. How to write $(*)$ in the other cusp? We have to look at the characteristic power series $x\sqrt{p(x)-e_2}$ and use the standard product formula for $\vartheta_{01}/\vartheta_{11}$. If we give the cusp the local coordinate q, then $(*)/2^{2k}$ is

$$(*)' \quad q^{-\frac{k}{2}} \cdot \hat{A}\Big(X, \prod_{\substack{n=1\\ n \text{ odd}}}^{\infty} \Lambda_{-q^n} T \cdot \prod_{\substack{n=1\\ n \text{ even}}}^{\infty} S_{q^n} T\Big)$$

which has integral coefficients if X is a Spin-manifold (see, for example, Atiyah's lecture of July 1962 in the Arbeitstagung). Also $(*)'$ is a modular function for $\Gamma_0(2)$.

E. Witten conjectured for a Spin-manifold X:
If S^1 acts on X $(e^{2\pi i t} \in S^1)$, then one can consider the coefficients of $(*)$ and $(*)'$ as functions of t. (Equivariant $\hat{A}$-genus of X with coefficients in a vector bundle.) <u>They are constant</u>. As far as I heard, this was proved by Taubes. It generalizes a result of Atiyah–Hirzebruch on the equivariant $\hat{A}$-genus (1969/70).

E. Witten also introduced the equivariant Dirac operator index of LX which is (it has to do with ϑ_{11}^{-1})

$$q^{-k/6}\hat{A}\left(X^{4k}, \prod_{n=1}^{\infty} S_{q^n}T\right).$$

It is a modular function for $SL_2(\mathbb{Z})$ if $w_2 = 0$ and $p_1 = 0$ (for X). If $\dim X = 24$, then

$$q^{-1}\hat{A}\left(X, \prod_{n=1}^{\infty} S_{q^n}T\right) = \hat{A}(X)(j-744) + \hat{A}(X,T)$$

If $\hat{A}(X) = 1$, then

$$196884 = \hat{A}(X,T) + \hat{A}(X,S^2T).$$

Think about moonshine and look at the lecture of Jacques Tits in this 27th Arbeitstagung!

F. Hirzebruch

Titel: Frobenius and the degeneration of the Hodge spectral sequence

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This is a report on joint work with P. Deligne [D] (see also [O]).

Let X be a smooth, proper algebraic variety over a field k. Denote by $\Omega^{\bullet}_{X/k} = (\mathcal{O}_X \xrightarrow{d} \Omega^1_{X/k} \longrightarrow \dots \longrightarrow \Omega^{\dim X}_{X/k})$ the De Rham complex of X/k. The Hodge groups $H^j(X,\Omega^i_{X/k})$ and the De Rham groups $H^n_{DR}(X/k) := H^n(X;\Omega^{\bullet}_{X/k})$ are related by the <u>Hodge spectral sequence</u>:

$$(1) \qquad E_1^{ij} = H^j(X,\Omega^i_{X/k}) \Rightarrow H^*_{DR}(X/k) .$$

In particular, we have

$$(2) \qquad \sum_{i+j=n} h^{ij} \geq h^n$$

where $h^{ij} = \dim_k H^j(X,\Omega^i)$, $h^n = \dim_k H^n_{DR}(X/k)$, and (1) degenerates at E_1 if and only if (2) is an equality for all n. The following result is well known:

<u>Theorem</u> 1. Assume $\operatorname{char}(k) = 0$. Then (1) degenerates at E_1. The classical proof uses Hodge theory. We will explain an elementary algebraic proof, based on the following result [D]:

<u>Theorem</u> 2. Assume k perfect, $\operatorname{char}(k) = p > 0$. Assume:

(i) X is liftable to $W_2(k)$ (ii) $\dim X < p$. Then (1) degenerates at E_1 .

Standard arguments show that th.2 implies th.1 .

Theorem 2 is a corollary of the following stronger result:

<u>Theorem</u> 2'. Let k be as in th.2 , let X/k be a smooth scheme (not necessarily proper). Assume X is liftable to $W_2(k)$. Then there exists an isomorphism in $D(X',\mathcal{O}_{X'})$

$$\tau_{<p}F_*\Omega_{X/k} \cong (\mathcal{O}_{X'} \xrightarrow{0} \Omega^1_{X'} \xrightarrow{0} \cdots \xrightarrow{0} \Omega^{p-1}_{X'}) = \bigoplus_{i<p} \Omega^i_{X'}\,[-i]$$

where $F : X \longrightarrow X'$ is the relative Frobenius map, given by the commutative diagram with Cartesian square

$$\begin{tikzcd} X \arrow[d] & X' \arrow[l] \arrow[d] \arrow[dl, phantom, "\square"] & X \arrow[l, "F"'] \arrow[dl] \arrow[ll, bend right, "F_X"'] \\ S & S \arrow[l, "F_S"'] & \end{tikzcd}$$

(S = Spec k , F_X = (resp F_S) = absolute Frobenius endomorphism of X(resp. S)) ,

and $\tau_{<n}L$, for a complex L , is the truncation $(\ldots \longrightarrow L^{n-2} \longrightarrow \operatorname{Ker} d \longrightarrow 0)$.

The proof consists in constructing, for a given lifting $\tilde{X}$ of X on $W_2(k)$, a map

$$\varphi_{\widetilde{X}} : \bigoplus_{i<p} \Omega^i_{X'}[-i] \longrightarrow F_*\Omega^\bullet_{X/k}$$

in $D(X')$ inducing the Cartier isomorphism $C^{-1}:\Omega^i_{X'} \to \mathcal{H}^i F_*\Omega^\bullet_X$ on $\mathcal{H}^i$. In the case where F lifts to $\widetilde{F} : \widetilde{X} \to \widetilde{X}' = \widetilde{X} \otimes_{W_2(k)} (W_2(k),\sigma)$, $\frac{\widetilde{F}^*}{p}$ gives $\varphi^1_{\widetilde{X}}$. In the general case, one can glue the maps $\frac{\widetilde{F}^*_i}{p}$ where $\widetilde{F}_i$ is a lifting of F on U_i, $\mathcal{U} = (U_i)$ an open cover of X, by means of the "homotopies" $h_{ij} = \frac{\widetilde{F}^*_j - \widetilde{F}^*_i}{p} : \Omega^1_{X'} \longrightarrow F_*\mathcal{O}_X$, on $U_i \cap U_j$. Once $\varphi^1_{\widetilde{X}}$ is defined, one constructs $\varphi^i_{\widetilde{X}}$ by using the multiplicative structure of the De Rham complex.

From th.2' one can deduce the following vanishing result:

<u>Theorem</u> 3 (Raynaud). Let X/k be as in th.2. Let L be an ample line bundle on X. Assume X of pure dimension $d \leq p$. Then:

$$H^j(X, L \otimes \Omega^i_X) = 0 \qquad \text{for} \quad i+j > d .$$

By the same standard arguments as above, one deduces from th.3 the classical Kodaira-Akizuki-Nakano vanishing theorem.

<u>Remarks</u> 1) In th.2', the liftability assumption is essential: the obstruction to lifting X (or X') coincides, up to sign, with the class in $\mathrm{Ext}^2(\mathcal{H}^1,\mathcal{H}^0)$ of the complex $\tau_{<2}F_*\Omega^\bullet_X$.

2) For $\dim X = p$, and X liftable to $W_2(k)$, one can

show that

$$F_*\Omega^{\cdot}_X \cong \oplus\, \Omega^i_{X'}[-i] \quad \text{in} \quad D(X') .$$

However it is not known whether this still holds for $\dim X > p$. The first unknown case is that of a quadric of dimension 3 in char.2 .

3) There are several variants and generalizations of th.2' : $\Omega^{\cdot}_X$ replaced by $\Omega^{\cdot}_X(\log D)$, for D a divisor with normal crossings, k replaced by a general basis $S/\mathbb{F}_p$, with a flat lifting $\tilde{S}$ mod p^2 , etc., which lead to further degeneration theorems.

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Riemannian manifolds with small eigenvalue of the Dirac operator

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Let M^n be a compact Riemannian manifold and denote by D the Dirac operator acting on sections of the spinor bundle S. In 1980 (see [1]) we proved for the first eigenvalue λ_1 of D the inequality

$$\lambda_1^2 \geq \frac{1}{4} \frac{n \cdot R_0}{n-1}$$

where R_0 is the minimum of the scalar curvature R. Moreover, if $\pm \frac{1}{2} \sqrt{\frac{n \cdot R_0}{n-1}}$ is an eigenvalue and ψ is the corresponding eigenspinor, then ψ satisfies the stronger equation

$$\nabla_X \psi = \mp \frac{1}{2} \sqrt{\frac{R_0}{n(n-1)}} \; X \cdot \psi$$

(see [1]) and these spinors are sometimes called Killing spinors. In 1986 O. Hijazi (see [7]) obtained the inequality

$$\lambda_1^2 \geq \frac{1}{4} \frac{n}{n-1} \mu_1$$

where μ_1 is the first eigenvalue of the Yamabe operator $4\,\frac{n-1}{n-2}\,\Delta+R$ (see also [11] for a general approach). However, if this lower bound is an eigenvalue then the scalar curvature R is constant and the eigenspinor ψ is a Killing spinor. Thus, there arises the interesting problem to classify all those Riemannian manifolds M^n admiting a Killing spinor. The existence of such a spinor imposes algebraic conditions on the Weyl tensor of the space (see [2]) as well as on the covariant derivative of the curvature tensor and the harmonic forms on M^n (see [7]); in particular M^n must be an Einstein manifold.

In dimension four the only possible manifold is $M^4 = S^4$ (see [2]).In dimension five we have an Einstein metric on the Stiefel manifold $V_{4,2}$ admiting a Killing spinor (see [1]). In 1985 we constructed in a common paper with R. Grunewald (see [3])Einstein metrics on $P^3(\mathbb{C})$ and $F(1,2)$ with Killing spinors. These two spaces are twistor spaces over S^4 and $P^2(\mathbb{C})$ respectively and one obtains the mentioned metric from the standard Kähler metric by scaling in the direction of the fibres of the twistor projection. Moreover, R. Grunewald constructed on $S^3 \times S^3$ a left - invariant Einstein metric with Killing spinors (see [6]). On the squashed seven-sphere there exists also a Killing spinor (see [12]).

We now study the above mentioned classification problem in the case of 5-dimensional Einstein spaces M^5. If ψ is a Killing spinor then there exists a vector field ζ such that

$$\zeta \cdot \psi = i\psi .$$

Consider the 1-form $\eta(X) = \frac{1}{i} < X \cdot \psi, \psi >$ as well as the endomrophism $\varphi : TM^5 \longrightarrow TM^5$ given by $\varphi = -\nabla\zeta$.

Theorem 1 (see [4]): Let $(M,^5 g)$ be an Einstein space with scalar curvature $R = 20$ and Killing spinor ψ . Then $(M^5;\varphi,\zeta,\eta,g)$ is an Einstein-Sasaki manifold.

Theorem 2 (see [4]): Let $(M^5;\varphi,\xi,\eta,g)$ be a simply-connected Einstein-Sasaki manifold with spin-structure. Then M^5 admits Killing spinors.

Theorem 3 (see [4]): Let (M^5,g) be a compact Einstein space with Killing spinor ψ and scalar curvature $R = 20$. Suppose in additions that the associated contact structure is regular. Then there are three possibilities:

1) M^5 is isometric to S^5 or to the homogeneous space S^5/Z_3 .

2) M^5 is isometric to the Stiefel manifold $V_{4,2}$ or $V_{4,2}|Z_2$ with the Einstein metric considered in [1].

3) M^5 is the simply-connected S^1-fibre bundle with Chern class $c_1^* = c_1(P_k)$ over a del Pezzo surface P_k $(3 \leq k \leq 8)$.

The question whether the last case is actually possible is essentially equivalent to the problem whether the surfaces P_k

admit Einstein-Kähler structures. There is a one-to-one correspondence between Killing spinors on M^5 and Einstein-Kähler metrics on P_k .

Finally I want to discuss some results obtained by K.D. Kirchberg from Berlin. O. Hijazi (see [7]) observed that on a Kähler manifold $\pm\frac{1}{2}\sqrt{\frac{n\cdot R}{n-1}}$ cannot be an eigenvalue of the Dirac operator. This leads to the conjecture that in the Kähler case it may be possible to obtain a better lower bound for the eigenvalues of D .

Theorem 4 (see [9]): Let λ be eigenvalue of the Dirac operator on a closed Kähler spin manifold M^n with positive scalar curvature, then

$$\lambda^2 \geq \frac{1}{4}\,\frac{n+2}{n}\,R_0 \ .$$

If $\pm\frac{1}{2}\sqrt{\frac{n+2}{n}\,R_0}$ itself is an eigenvalue of D , then M^n is an Einstein-Kähler space of odd complex dimension $m = \frac{n}{2}$.

The corresponding eigenspinor ψ satisfies the stronger equation $\nabla_X\psi = \mathrm{const}(X + (-1)^k i\, J(X) i^2)\psi$ with $m = 8l-2k-1$, $k \in \{0,1,2,3\}$ and J is the compex structure of M .

Theorem 5 (see [10]): Let M^6 be a closed Kähler spin manifold of positive scalar curvature for which $\sqrt{\frac{R_0}{3}}$ is an eigenvalue of the Dirac operator. Then M^6 is analytically isometric either to the projective space $P^3(\mathbb{C})$ or to the flag manifold $F(1,2)$.

The idea of the proof: each non-trivial solution of the equation $D\psi = \sqrt{R/3}\,\psi$ determines a folation of M^6 such that any leave is a totally geodesic 2-sphere and the map $\tau : M^6 \longrightarrow M^6$ which coincides on the leaves with the antipodal map is antiholomorphic. Then M^6 becomes the twistor space of certain 4-dimensional Riemannian manifold and one can apply the result of N. Hitchin [8] and Th. Friedrich/ H. Kurke [5] concerning the classification of Kählerian twistor spaces.

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Titel: Finite State Automata, Groups and Hyperbolic Geometry
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This work is the result of collaboration between Cannon (Provo, Utah), Epstein (Warwick), Holt (Warwick), Paterson (Warwick) and Thurston (Princeton).

Let G be a group and I a finite set of generators closed under $x \mapsto x^{-1}$. There are various ways of associating a language over I with G.

Example 1. The language of shortest words:
e.g. the set of reduced words in a free group.
This language has good properties in many of the groups in which we are interested. It may have terrible properties (for example if the word problem is not soluble in G).

Example 2. G is free abelian of rank 3 on generators x, y, z. We take $I = \{x, y, z, X, Y, Z\}$ where $X = x^{-1}$, $Y = y^{-1}$, $Z = z^{-1}$. The language is the set of all words of the form $x^i y^j z^k$, $i, j, k \in \mathbb{Z}$, where $x^{-2} y^0 z$ is more correctly written as XXz.

Example 3. Heisenberg group $x, y, z = [x,y]$, $[x,z] = [y,z] = e$.
Same language as in Example 2 $\{x^i y^j z^k\}$.

Languages are classified according to the type of machine that accepts them. We work with the simplest kind of machine, namely a finite state automaton. So we restrict ourselves to triples (G, I, L) where L is recognized by an FSA.

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Examples 2 and 3 show us that (I, L) is not sufficient to understand a group. To understand G we need more data.

Def. A group G is said to be automatic if it has an automatic structure. An automatic structure on G is a set of generators I (finite), closed under inverse, and finite state automata, $I = \{x_1, \ldots, x_n\}$

$W, E, M_1, \ldots, M_n$, where

$L(W) \subset I^*$, $L(E) \subset I^* \times I^*$ and $L(M_i) \subset I^* \times I^*$ for $1 \le i \le n$,

We assume the following axioms are satisfied:

1) $L(W) \xrightarrow{\pi} G$ is onto

2) $(w_1, w_2) \in L(E) \iff \begin{cases} w_1 \in L(W), \; w_2 \in L(w) \\ \pi w_1 = \pi w_2 \end{cases}$

3)$_i$ $(w_1, w_2) \in L(M_i) \iff \begin{cases} w_1 \in L(W), \; w_2 \in L(W) \\ \pi(w_1 x_i) = \pi(w_2). \end{cases}$

Theorem The property of having an automatic structure is invariant under change of generators.

Theorem Automatic $\Rightarrow$ finitely presented

Theorem M compact, all sectional curvatures < 0 implies $\pi_1 M$ automatic

Theorem M geometrically finite, curvature $\equiv -1$ $\Rightarrow \pi_1 M$ automatic

Theorem M flat Riemannian complete $\Rightarrow \pi_1 M$ automatic

Theorem ~~M~~ G finite $\Rightarrow$ automatic

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Theorem G_1, G_2 automatic $\Rightarrow G_1 \times G_2$ automatic

Theorem G_1, G_2 automatic, $H \subset G_1$, $H \subset G_2$, H finite or H cofinite in each $G_i \Rightarrow G_1 *_H G_2$ automatic

Theorem H.N.N extension of automatic group over finite or cofinite subgroup is automatic

Theorem class of automatic groups is closed under subgroup of finite index, supergroup of finite index, quotient by finite subgroup, inverse of last operation.

Given a set of finite state automata, $W, E, M_1, \ldots, M_n$ over I with $|I| = n$, one can give axioms which are equivalent to saying that this is the automatic structure of an automatic group. These axioms can be expressed with reference only to the automata $W, E, M_1, \ldots, M_n$ and without mentioning the group e.g.

$$(\forall w_1, w_2, w_3)\big((w_1, w_2) \in L(E) \text{ and } (w_2, w_3) \in L(E) \Rightarrow (w_1, w_3) \in L(E)\big)$$

We have axioms to say that $(w_1, w_2) \in L(E)$ is an equivalence relation. We define the set G to be $L(W)$ modulo this equivalence.

We have an action of x_i, given by M_i, and axioms to say this is well-defined on G. We have axioms to say that x_i induces a bijection of G.

We have axioms which ensure that every

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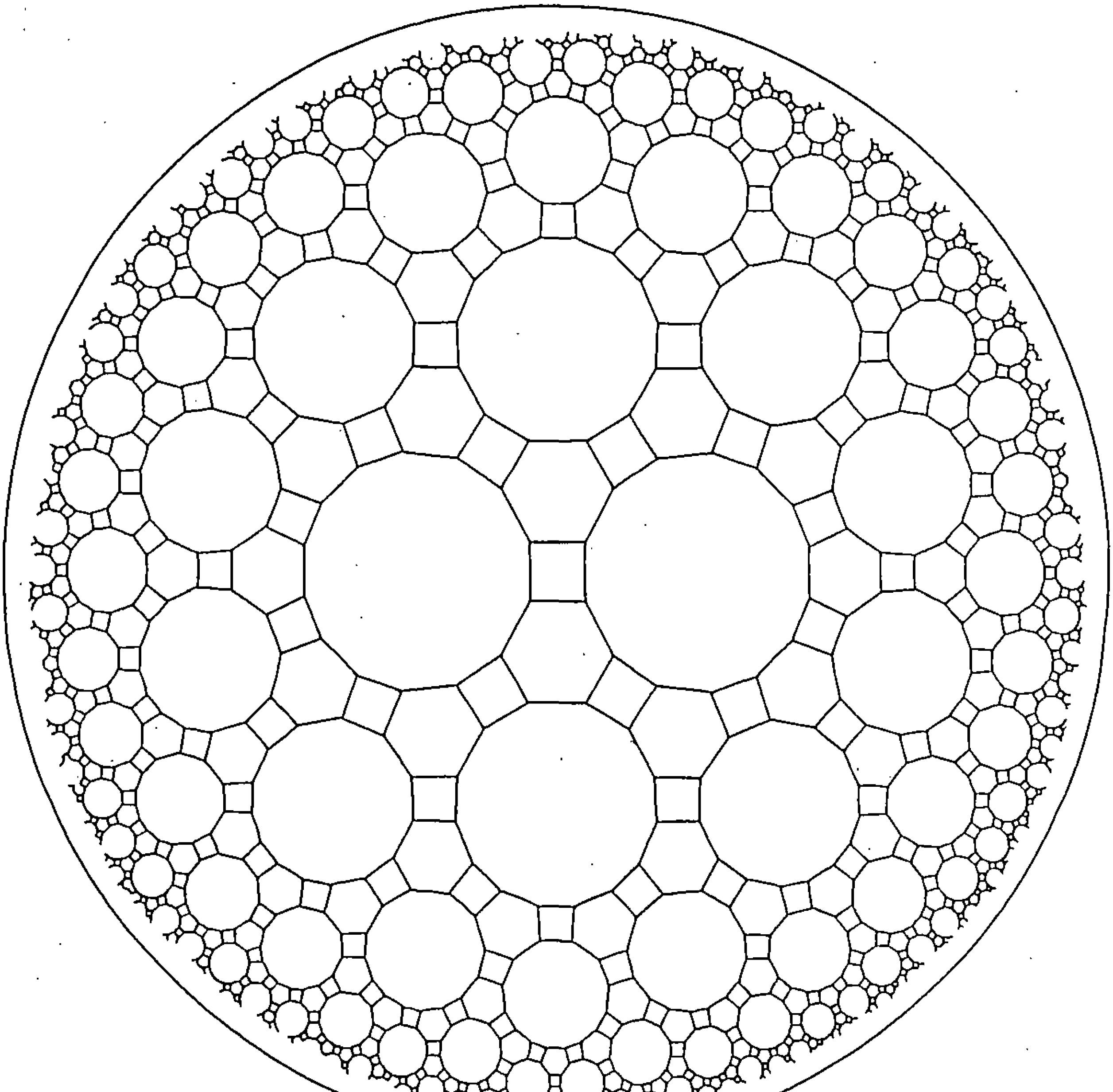
prefix of every word in L(W) defines an element of G. Then, choosing the null-prefix to be the identity element, G becomes a group.

These axioms involve only the operators NOT, AND, OR, FOR ALL, THERE EXISTS. Such operators can be defined in a constructive way on finite state automata. Therefore we can check under automation whether the above axioms are satisfied.

The theory described has been used to increase the speed of drawing certain pictures in hyperbolic space by a factor of several hundred. It is hoped to use it in developing (with Thurston) a data-base of 3-dimensional manifolds. In such a project, the ability to compute quickly with groups will be important.

Reference J. Cannon, Geometriae Dedicata around 1982

This paper was the starting point for the present theory.



Cayley graph of the (2,3,7) group.

Titel: On some ℓ-adic representations

Autor: A. Wiles

Adresse: Princeton

Let F be a totally real algebraic number field. Suppose that f is a Hilbert modular form of character χ, level $\mathfrak{c}$ and weight k. Suppose further that $T(\mathfrak{a})f = c(\mathfrak{a}, f)f$ for all ideals $\mathfrak{a}$ of $\mathcal{O}_F$, the ring of integers of F, where $T(\mathfrak{a})$ is the usual Hecke operator. Let K_f be the field generated over $\mathbb{Q}$ by the $\{c(\mathfrak{a}, f)\}$ as $\mathfrak{a}$ runs through the ideals of $\mathcal{O}_F$. It is known to be a number field. The following conjecture is well-known:

<u>Conjecture</u> For each prime λ of $\mathcal{O}_f$; the ring of integers of K_f, there is a continuous representation

$$\rho_{f,\lambda} : \mathrm{Gal}(\bar{F}/F) \longrightarrow GL_2(K_{f,\lambda})$$

which is unramified outside the primes dividing $\mathfrak{c}(N\lambda)$ and which for $\mathfrak{p} \nmid \mathfrak{c}(N\lambda)$ has the property that the characteristic polynomial of $\mathrm{Frob}_{\mathfrak{p}}$ is $x^2 - c(\mathfrak{p}, f)x + \chi(\mathfrak{p})N\mathfrak{p}^{k-1}$.

This has been proved for $F = \mathbb{Q}$ (Eichler-Shimura for $k=2$, Deligne for $k>2$, Serre-Deligne for $k=1$) and $[F:\mathbb{Q}]$ odd (Ohta and Rogawski-Tunnell using results of Shimura and Jacquet-Langlands). Some cases where $[F:\mathbb{Q}]$ is even were also ~~pess~~ proved.

Let us say that f is <u>ordinary</u> at λ if $c(\mathfrak{p}, f) \not\equiv 0 \ (\lambda)$ for each $\mathfrak{p} \mid N\lambda$.

<u>Theorem</u> $\rho_{f,\lambda}$ exists if f is ordinary at λ.

Titel: On some ℓ-adic representations

Autor: A. Wiles

Adresse: Princeton.

This theorem includes some crucial cases not covered by the earlier work. In the special case $k=1$ f is ordinary at all $\lambda \nmid \mathfrak{N}$ and we can prove the generalization of the Serre-Deligne theorem giving a representation in $GL_2(\mathbb{C})$. (This was done for $[F:\mathbb{Q}]$ odd by Rogawski-Tunnell.

The main application of the above theorem ~~was~~ is to prove the Iwasawa conjecture for all totally real F, for all χ, and for all odd p. This extends an earlier joint result with Mazur for the case $F=\mathbb{Q}$. Certain standard consequences of this are then proved, for example the prime-to-2 part of the Birch-Tate conjecture giving the order of $K_2 \mathcal{O}_F$. This was previously known for F real abelian after the above joint work with Mazur. (That it was a consequence of the Iwasawa conjecture was previously proved by Tate and Coates).

Titel: FREE LOOP SPACES and AUTOMORPHISMS OF MANIFOLDS

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§0

By the free loop space of a connected ANR, X, I mean the space X^{S^1} of continuous maps from S^1 to X endowed with the compact-open topology and with the obvious $O(2)$ action. $O(2)$ = isometries of S^1.

One denotes by $X^{S^1}/\!\!/S^1$ the space $X^{S^1} \times_{S^1} EO(2)$; the deck transformation in the covering $X^{S^1}/\!\!/S^1 \to X^{S^1} \times_{O(2)} EO(2)$ provides an involution $\tau: X^{S^1}/\!\!/S^1 \to X^{S^1}/\!\!/S^1$. Each point $x \in X$ provides a crossection $i_x: BS^1 = {}_{S^1}\backslash EO(2) \to X^{S^1}/\!\!/S^1$ for the projection $X^{S^1}/\!\!/S^1 \to {}_{S^1}\backslash EO(2) = BS^1$

One defines:

$$H^*_{S^1}(X^{S^1}; R) = H^*(X^{S^1}/\!\!/S^1; R),$$

$$\tilde{H}^*_{S^1}(X^{S^1}; R) = \tilde{H}^*(X^{S^1}/\!\!/S^1 / i_x(BS^1); R)$$

$\pm\tilde{H}^*_{S^1}(X^{S^1}; R)$ = the eigenspace corresponding to the eigenvalue $\begin{matrix}+1\\-1\end{matrix}$ of the involution induced by τ on $\tilde{H}^*_{S^1}(X^{S^1}; \mathbb{Q})$.

§1 The rational homotopy type of $X^{S^1}/\!\!/S^1$.

Let $(\Lambda[x_\alpha], d)$ be the Sullivan minimal model of the 1-connected space X ($\deg x_\alpha = n_\alpha$). Let $\Lambda[x_\alpha, \bar{x}_\alpha]$ resp. $\Lambda[x_\alpha, \bar{x}_\alpha, u]$ be the free commutative graded algebras generated by x_α, $\bar{x}_\alpha$ ($\deg \bar{x}_\alpha = n_\alpha - 1$) resp. by x_α, $\bar{x}_\alpha$, u ($\deg u = 2$), and β resp. δ be the differentials of degree -1 resp $+1$ in $\Lambda[x_\alpha, \bar{x}_\alpha]$ defined by the formulas $\beta(x_\alpha) = \bar{x}_\alpha$ resp. $\delta(x_\alpha) = dx_\alpha$ $\delta(\bar{x}_\alpha) = -\beta(dx_\alpha)$ ~~and~~ let D be the differential of degree $+1$ in $\Lambda[x_\alpha, \bar{x}_\alpha, u]$ defined by the formula $D(x_\alpha) = dx_\alpha + u\beta(x_\alpha)$, $D(\bar{x}_\alpha) = -\beta\, d(x_\alpha)$, $D(u) = 0$.

<u>Theorem 1</u>: (1) ([V-S]) $(\Lambda[x_\alpha, \bar{x}_\alpha], \delta)$ is the minimal model of X^{S^1}.

(2) ([V-B]) $(\Lambda[x_\alpha, \bar{x}_\alpha, u], D)$ is the minimal model for $X^{S^1} /\!/ S^1$.

(3) If $* > 0$ $H^*(\Lambda[x_\alpha, \bar{x}_\alpha, u], D) = H^*(\mathrm{Im}\beta, \delta)$, $(\mathrm{Im}\beta, \delta) = \bigoplus_{r \geq 0} (\mathrm{Im}\beta(r), \delta)$ with $\mathrm{Im}\beta(n) =$ = the span of the elements $\beta(x)$, x a monomial in $\Lambda[x_\alpha, \bar{x}_\alpha]$ which involves exactly n $\bar{x}_\alpha$'s.

(4) $H^*(\mathrm{Im}\beta(0), \delta) = H^{*+1}(\Lambda[x_\alpha], d)$

Theorem 1 provides good methods to calculate $H^*_{S^1}(X^{S^1}; Q)$. Here is a table with the Poincaré series $P_{H^*(X^{S^1})}(t) = \sum t^r \dim H^r(X^{S^1}; \mathbb{Q})$ resp.

$P_{H^*_{S^1}(X^{S^1})}(t) = \sum t^r \dim \tilde{H}^r_{S^1}(X^{S^1}; \mathbb{Q})$ for

some familiar spaces

X	$P_{H^*(X^{S^1})}(t)$	$P_{H^*_{S^1}(X^{S^1})}(t)$
CP^n	$\frac{t+1}{1-t^2}$	$\frac{t}{1-t^2} + \frac{1}{1-t^2} = \frac{t+1}{1-t^2}$
QP^n	$\frac{t^3(1-t^{4n})}{(1-t^4)(1-t^{2(2n+1)})} + \frac{1}{1-t^4} - \frac{t^{4n+4}(1-t^2)}{(1-t^4)(1-t^{4n+2})}$	$\frac{t^3(1-t^{4n})}{(1-t^4)(1-t^{4n+2})} + \frac{1}{1-t^2}$
S^{2p+1}	$\frac{1+t^{2p+1}}{1-t^{2p}}$	$\frac{t^{2p}}{1-t^{2p}} + \frac{1}{1-t^2}$
S^{2p}	$\frac{t^{2p-1}}{1-t^{2(2p-1)}} + \frac{1}{1-t^{2p}} - \frac{t^{4p}(1-t^{4p-2})}{(1-t^{2p})(1-t^{4p-2})}$	$\frac{t^{2p-1}}{1-t^{4p-2}} + \frac{1}{1-t^2}$

§2 Automorphisms of manifolds

Let M^n be a compact connected manifold with nonempty boundary, $H(M^n)$ the topological monoid of the homotopy equivalences $h: (M^n, \partial M) \to (M^n, \partial M)$ so that $h/\partial M = id$ equipped with the compact open topology and $D(M^n) = \{h \in H(M^n) / h \text{ diffeomorphism}\}$

equipped with the C^∞-topology. Let $H(M)/\!\!/_{D(M)}$ denote the homotopy quotient

Theorem 2: If M^n is 1-connected and $* \leq n/3 - 6$ one has the following short exact sequence

$$0 \rightarrow \tilde{H}^{S^1}_{\varepsilon(n)}(M^{S^1}; Q) \rightarrow \pi_*(H(M)/\!\!/_{D(M)}) \otimes Q \rightarrow \widetilde{KO}^{-*}(M) \otimes Q \rightarrow 0$$

where $\varepsilon(n) = \begin{cases} +1 & \text{if } n \text{ is odd} \\ -1 & \text{if } n \text{ is even} \end{cases}$

The present proof of this theorem is based on Igusa's stability theorem for concordances, [B-L], the Waldhausen result about the relation between stabilized concordances and algebraic K-theory and on the following theorem

Theorem 3: If X is 1-connected then

$\tilde{H}^{S^1}_*(X^{S^1}; Q) \cong \tilde{K}_{*+1}(Z[\Omega X]) \otimes Q$ with $Z[\Omega X]$ denoting the "simplicial group ring" of ΩX.

In this generality this theorem is proved in $[B]_1$ + [B, F]. The case $X = \Sigma Y$ was implicit in the work of Hsiang-Staffeldt [H-S].
This theorem was extended to a relative version (where X is replaced by a 2-connected map $f: X \rightarrow Y$) in [G]. My initial proof uses the work of Loday Quillen on Connes' cyclic homology but the proof of [G] does not.

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The recent works [C C G H] and [O] provide an analogous of Theorem 3 for $X = \Sigma Y$. In this analogous $\tilde{K}_*(Z[\Omega X])\otimes Q$ is replaced by $\tilde{K}_*(\lim \Omega^k \Sigma^k \Omega X) = \tilde{A}_*(X)$ and $\tilde{H}^{S^1}_*(X^{S^1}; Q)$ by $\Omega^{fr}_*(X^{S^1}/\!/S^1 / BS^1)$

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Titel: Elliptic curves: recent developments
(different from title in program!)

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In the last two years, several beautiful results in the theory of elliptic curves have been discovered; the lecture gave a survey of some of these.

Background: An elliptic curve over a field K is a (smooth projective) curve E of genus 1 together with a point $0 \in E$, all defined over K. By the Riemann-Roch theorem, E is an algebraic group and $E(K)$, the set of points of E with coordinates in K, an abelian group. For the same reason, $\mathrm{End}_{\bar{K}}(E)$, the set of morphisms $E \to E$ over K preserving 0, is a ring under addition and composition of endomorphisms. An important invariant of E is the j-invariant $j(E) \in K$, defined as $\frac{6912A^3}{4A^3+27B^2}$ if E is given by a Weierstrass equation $y^2 = x^3+Ax+B$; this invariant classifies E up to $\bar{K}$-isomorphism.

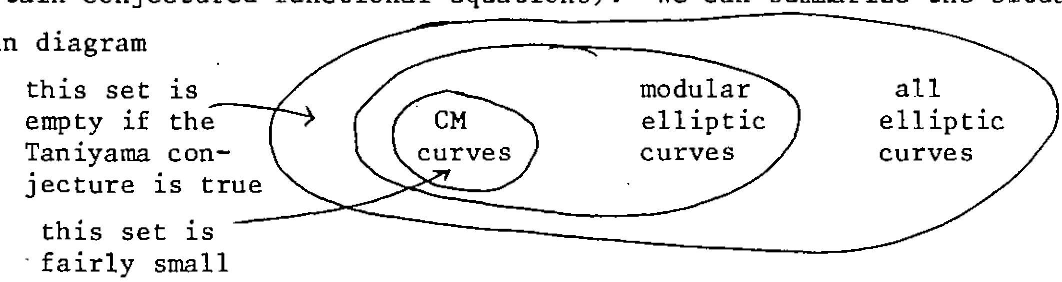
The cases of main interest are $K = \mathbb{Q}$ (or a number field), $K = \mathbb{Z}/p\mathbb{Z}$ (or a finite field), and the relation between these two.

K=$\mathbb{Q}$: The group $E(\mathbb{Q})$ is finitely generated (Mordell), i.e. has the structure $\mathbb{Z}^r + F$ with $r \geq 0$ and F finite abelian. The rank r is the most important invariant of E. It can take values at least as large as 14 (Mestre). It is generally believed that r is "almost always" 0 or 1; however, recent calculations by Kramarz and myself indicate that 12% of the curves $x^3+y^3=m$ (m cubefree) may have rank ≥ 2. The group F has the structure $\mathbb{Z}/a\mathbb{Z}$ or $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2b\mathbb{Z}$ (easy) with $a \leq 10$, $a=12$, or $b \leq 4$ (Mazur, 1977). The ring $\mathrm{End}_{\bar{\mathbb{Q}}}(E)$ is isomorphic to $\mathbb{Z}$ or to an order $\mathcal{O}_{-d} = \mathbb{Z} + \mathbb{Z}\frac{d+\sqrt{-d}}{2}$ in an imaginary quadratic field. The latter case, complex multiplication, is relatively rare: it happens only if $j(E)$ has one of the values 0, 1728, −3375, 8000, ..., −262537412640768000 corresponding to $d = 3, 4, 7, 8, \ldots, 163$ with class number $h(-d)=1$. Over $\bar{\mathbb{Q}}$, CM by $\mathcal{O}_{-d}$ occurs iff $j(E)$ is a root of a polynomial $h_{-d}(X)$ of degree $h(-d)$ with integer coefficients, e.g. $h_{-7}(X) = X + 3375$, $h_{-15}(X) = X^2 + 191025X - 121287375$.

K = $\mathbb{Z}/p\mathbb{Z}$: The group $E(\mathbb{Z}/p\mathbb{Z})$ is finite (cyclic or a product of two cyclic groups) of order $p+1-a$ with $|a| < 2\sqrt{p}$ (Hasse, 1933); all a occur (Honda). $\mathrm{End}_{\bar{\mathbb{F}}_p}(E)$ is either an order in the imaginary quadratic field $\mathbb{Q}(\sqrt{a^2-4p})$ (the ordinary case) or else has rank 4 and is an order in the quaternion algebra over $\mathbb{Q}$ ramified at p and ∞ (the supersingular case), depending whether $p \nmid a$ or $p \mid a$ (i.e., for $p \geq 5$, whether $a \neq 0$ or $a=0$).

Relationship between them: If E is an elliptic curve over $\mathbb{Q}$, then by reduction mod p one gets an elliptic curve over $\mathbb{Z}/p\mathbb{Z}$ for almost all primes p.

Write $\#E(\mathbb{Z}/p\mathbb{Z}) = p+1-a_p(E)$ with $|a_p(E)|<2\sqrt{p}$. The question is what collections $\{a_p(E)\}$ can arise and how they are related to the arithmetic of E over $\mathbb{Q}$. The information is encoded by the <u>L-series</u> of E over $\mathbb{Q}$; this is a Dirichlet series $L_E(s)$ having an Euler product with Euler factor $(1-a_p(E)p^{-s}+p^{1-2s})^{-1}$ for p with good reduction (i.e. the equation of E still defines an elliptic curve mod p) and $(1\pm p^{-s})^{-1}$ or 1 for p of bad reduction (the equation becomes singular). We call E <u>modular</u> if there is a cuspidal modular form f (on some group $\Gamma_0(N)$) of weight 2 with a Fourier development $\sum a_n q^n$ $(q=e^{2\pi i z})$ with $a_n \in \mathbb{Z}$ multiplicative (i.e. a Hecke eigenform) such that $a_p(E)=a_p$ for all good p. Results of Eichler and Shimura imply that every cusp form f with the properties mentioned corresponds to an elliptic curve; the Taniyama or Taniyama-Weil conjecture is that all elliptic curves over $\mathbb{Q}$ arise in this way (Weil proved that this is true if $L_E(s)$ satisfies certain conjectured functional equations). We can summarize the situation by a Venn diagram



If E is modular, then $L_E(s)$ has an analytic continuation and hence a well-defined order of vanishing ρ at $s=1$. The <u>Birch-Swinnerton-Dyer conjecture</u> says that $r=\rho$ and moreover that the first non-zero derivative at $s=1$, $L^{(\rho)}(1)$, equals a certain known positive number times the order of Ш, the <u>Shafarevich group</u>. The group Ш consists of the isomorphism classes of curves of genus 1 over $\mathbb{Q}$ which are isomorphic to E over $\mathbb{R}$ and $\mathbb{Q}_p$ for all p; it is trivial if every such curve has a point over $\mathbb{Q}$, but until last year was not known to be finite for a single elliptic curve E.

<u>Results</u>: (1) In the last couple of years there have been many applications of the group law on elliptic curves over $\mathbb{Z}/p\mathbb{Z}$ to questions of factorization of large numbers and primality testing. The original idea was due to H. Lenstra. A typical application (Pomerance, 1987) is "prime certification": a number $p>34$ is prime iff there exist integers C,A such that the point $P=(0,0)$ on the elliptic curve $Cy^2=x^3+Ax^2+x$ satisfies $2^kP=0$ (where 2^k is the smallest power of 2 bigger than $2\sqrt{p}$) but $2^{k-1}P\neq 0$ in $E(\mathbb{Z}/q\mathbb{Z})$ for any divisor q of p (this can be checked without knowing the factorization of p by doing a g.c.d. calculation of the denominator of the coordinates of P with p); thus the primality of p can be verified in $O(\log p)$ steps, i.e. as many as one needs to write p down.

(2) Let E be an elliptic curve over $\mathbb{Q}$. If E has CM by $\mathcal{O}_{-d}$, then the

reduction of E (mod p) is supersingular for all good p with $(\frac{-d}{p})=-1$, i.e. for half of all primes. But if E doesn't have CM, the supersingular primes are much rarer, e.g. only 27 up to 31500 for the curve $y^2+y=x^3-x^2-10x-20$ with j-invariant $-2^{12}31^3/11^5$ (Lehmer). Conjecturally the number of supersingular primes $<x$ grows asymptotically like (const.) $\frac{\sqrt{x}}{\log x}$ (Lang-Trotter), but it was not known that there were infinitely many supersingular primes for a single non-CM curve. Last year, Noam Elkies, a graduate student at Harvard, found a very elementary proof that there are infinitely many for <u>every</u> E. The idea is similar to Euclid's proof that there are infinitely many primes. Roughly, if $p_1,\ldots,p_n$ are the supersingular primes already found, you choose a prime p with $(p_i/p)=1$ for all i and some other congruence conditions and show that if p is sufficiently large then the numerator of $h_{-p}(j(E))$ or $h_{-4p}(j(E))$ is a quadratic non-residue of p and hence contains a prime factor p_{n+1} with $(p_{n+1}/p)=-1$, and this is another supersingular prime for E by virtue of what was said above.

(3) The two main results which were previously known in the direction of the Birch-Swinnerton-Dyer conjecture were the theorem of Coates and Wiles (1977) that for CM curves E, $\rho=0$ implies $r=0$ and the theorem of Gross and myself (1983) that for modular elliptic curves E, $\rho=1$ implies $r\geq 1$. Now Karl Rubin has proved that for CM curves, $\rho=1$ implies $r\leq 1$ (and hence r=1). More important, he proved the finiteness of Ш for certain CM curves (namely, those whose L-series over the field of complex multiplication does not vanish at s=1). Moreover, his proof shows that the prime factors of |Ш| can be only those predicted by the Birch-Swinnerton-Dyer conjecture, and possibly 2 (or 3 if the CM field is $\mathbb{Q}(\sqrt{-3})$). In some cases one can calculate Ш completely, e.g. |Ш| = 1, 4, 9 for the curves $y^2=x^3-x$, $y^2=x^3+17x$, and $y^2=x^3-2^83^45^2$. The main new ingredient in Rubin's proof was a construction by the South American mathematician Thaine for cyclotomic fields.

(4) The proof of my theorem with Gross used so-called <u>Heegner points</u>: if E is modular, then there is a map from the modular curve $X_0(N)=\bar{H}/\Gamma_0(N)$ to E defined over $\mathbb{Q}$, and the images of points in $X_0(N)$ corresponding to CM curves (i.e. of points $\tau\in H$, the upper half-plane, satisfying a quadratic equation over $\mathbb{Q}$) give points in $E(\bar{\mathbb{Q}})$; by adding these points to their conjugates under $\mathrm{Gal}(\bar{\mathbb{Q}}/\mathbb{Q})$ one gets a point in $E(\mathbb{Q})$. The problem is to show that this point is non-zero; in the paper with Gross this was done by relating its size to $L'_E(1)\neq 0$. There is also a purely algebraic approach (Heegner, Birch). Using it, Satgé (1986) proved an analogue of Sylvester's conjecture. This conjecture says that any prime $p\equiv 4$ (mod 9) is a sum of two rational cubes, and is still open. Satgé proved that 2p is a sum of two rational cubes if $p\equiv 2$ (mod 9). The idea is to use modular functions to construct a point on the curve C: $X^3+2Y^3+pZ^3=0$ with coefficients in a number field K of degree $\frac{p+1}{3}$ over $\mathbb{Q}$ (the ring class field of discriminant $-3p^2$).

Now C is not (yet) an elliptic curve over $\mathbb{Q}$, since we have no point 0 over $\mathbb{Q}$, so there is no group law. But we still know when the sum of 3 points is 0 (namely, if they are collinear on the cubic curve C), so we can add n points if $n \equiv 1 \pmod 3$. Since $\frac{p+1}{3}$ is $\equiv 1 \pmod 3$, we can take the trace from $C(K)$ to $C(\mathbb{Q})$ to get a "Heegner point" on $C(\mathbb{Q})$, and this point is automatically non-trivial because there is no trivial point on C over $\mathbb{Q}$. A simple transformation then gives a non-trivial point on the elliptic curve $E: X^3+Y^3+2pZ^3=0$.

(5) Finally, and perhaps most dramatically, Ribet has shown that Fermat's last theorem follows from the Taniyama conjecture. More precisely, if $a^\ell+b^\ell=c^\ell$ is a counterexample to Fermat's last theorem, then the elliptic curve $E_F : y^2 = x(x-a^\ell)(x-c^\ell)$ is a counterexample to the Taniyama conjecture (i.e. belongs to the mysterious outer ring of the Venn diagram on page 2). The index "F" stands for Fermat or for Frey, who first suggested this approach (the curve E_F had been written down previously by Hellegouarch and perhaps Hurwitz). The idea of the proof that E_F cannot be modular is to show that if it were modular, and hence satisfied $a_p(E) = a_p(f)$ for some cusp form $f \in S_2(\Gamma_0(N))$ with N equal to the product of the prime factors of abc, then modulo ℓ one could remove the prime factors of N one at a time to get a new modular form g of weight 2 on $\Gamma_0(2)$ with $a_p(E) \equiv a_p(g) \pmod{\ell}$ for all p; this gives a contradiction because the only modular form of weight 2 and level 2 is an Eisenstein series with $a_p(g) = p+1$, and $a_p(E) \equiv p+1 \pmod{\ell}$ for all p implies that E or an isogenous curve has a rational torsion point of order ℓ, contradicting Mazur's theorem. Ribet's proof is in the area of ℓ-adic representations (cf. Wiles's lecture) and uses detailed knowledge of the algebraic geometry of modular and Shimura curves in characteristic 0 and characteristic p.

In fact, the statement Ribet proved is a very special case of certain conjectures of Serre (first 1974, precise formulation 1986) which predict the existence of a modular form of specified level, weight, and character for <u>any</u> odd ℓ-adic representation $\rho: \mathrm{Gal}(\bar{\mathbb{Q}}/\mathbb{Q}) \longrightarrow GL_2(\bar{\mathbb{F}}_\ell)$. These conjectures include the Taniyama conjecture but make many other verifiable predictions. Dozens of these have been checked by Mestre in the last year. A typical case: the polynomial $x^5+x^4-37x^3+67x^2+21x+1$ has Galois group $A_5 \simeq SL_2(\mathbb{F}_4)$ and discriminant $2^2\cdot 24077$, and gives a 2-adic representation which by Serre's conjecture predicts that there is a modular form (cuspidal, eigenform) in $S_2(\Gamma_0(24077))$ whose Fourier development begins $q+q^2+q^3+q^4+cq^5+\dots \pmod 2$, where $c \in \mathbb{F}_4$ is an element not in $\mathbb{F}_2$; using the "graph method" (Oesterlé-Mestre), Mestre was able to check that there is, indeed, such a form.

Titel: The structure of algebraic 3-folds

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The talk is a review on the current state of the classification theory of 3-dimensional algebraic varieties / $\mathbb{C}$, the foundation of which is now almost completely available. The theory consists of two ingredients, namely minimal models and the base-point freeness of pluricanonical systems.

Theorem 1 (Minimal models; Mori-Reid-Kawamata-Kollár-Shokurov). Let X be an algebraic 3-fold ($/\mathbb{C}$). Then one of the following cases occurs:

a) X is uniruled;

b) X is birational to a minimal 3-fold X_0; i.e. X_0 is normal, projective with only "terminal singularities" and $(K_{X_0} \cdot C) \geq 0$ for $\forall C \subset X_0$.

Corollary (Kawamata-Benveniste) For an algebraic 3-fold X, the canonical ring $R(X)$ is a finitely generated graded $\mathbb{C}$-algebra.

Theorem 2₁ (Non-negativity of the Kodaira dimension) Let X be a minimal 3-fold. Then $\kappa(X) \geq 0$, i.e. $\exists m \in \mathbb{N}$ s.t. $|mK_X| \neq \phi$.

Theorem 2₂ (Base-point freeness). Let X be a minimal 3-fold. Suppose either

a) $[K_X^2] = 0 \in H^4(X, \mathbb{Q})$ or

b) $K_X^3 > 0$

then $\exists\, m \in \mathbb{N}$ s.t. $|mK_X|$ is base pnt free.

The proof of Theorem 1 is done inductively. For the inductive steps we need:

Theorem 1' a) (Mori - Reid - Kawamata - Shokulov - Kollár; in dim n) Let X be a projective variety with only terminal singularities. If there is a curve $C \subset X$ with $K_X \cdot C < 0$, then there is a "contraction map" $\pi : X \longrightarrow X'$ such that

(1) X' is normal, projective & (2) K_X is π-negative.

If π is not birational, then X is uniruled. If π is birational and contracts a divisor to a lower dimensional variety, then X' has again with only terminal singularities and $\rho(X') = \rho(X) - 1$.

b) (Mori, dim 3) Suppose the contraction $\pi : X \longrightarrow X'$ is birational and contracts only curves. Then there is a diagram ("flip")
$$\begin{tikzcd} X \arrow[dr, "\pi"'] & & X_+ \arrow[dl, "\pi_+"] \\ & X' & \end{tikzcd}$$
which is symmetric except that K_{X_+} is π_+-ample (instead of π_+-negative).

c) (Shokulov -Kawamata, dim 3 or 4) The flips terminate after finite steps.

Theorem 2'' follows from Theorem 2', and the latter is essentially a consequence of the "generic semi-positivity of the cotangent bundle" and the "pseudo-effectiveness" of c_2 for a minimal variety (dim n).

More precisely, the following holds

Theorem 3 (a) Let X be a ~~smooth~~, projective n-fold. Then for $\forall \mathcal{F} \subset T_X$, $c_1(\mathcal{F}) H_1 \cdots H_{n-1} \leqq 0$ for $\forall H_i$ ample.

(b) If X is a minimal n-fold and $\pi : \tilde{X} \longrightarrow X$ is a resolution, then $(\pi_* c_2(\tilde{X})) H_1 \cdots H_{n-2} \geqq 0$.

Titel: *Mixed Hodge Modules*

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Let X be a complex algebraic variety (assumed always separated and reduced), and $\mathrm{Perv}(\mathbb{Q}_X)$ the category of $\mathbb{Q}$-perverse sheaves on X^{an} whose cohomology sheaves are algebraically constructible. We associate to X the category of mixed Hodge Modules $MHM(X)$ with an additive functor $\mathrm{rat}: MHM(X) \to \mathrm{Perv}(\mathbb{Q}_X)$ satisfying the following properties:

i) $MHM(X)$ is an abelian category and rat is exact and faithful.

ii) We have the canonical functors f_*, $f_!$, f^*, $f^!$, ψ_g, $\varphi_{g,1}$, $\mathbb{D}$, $\boxtimes$, $\otimes$, $\underline{Hom}$ between $D^b(MHM(X))$'s such that they are compatible with the corresponding functors on the $\mathbb{Q}$-complexes by the functor

$$\mathrm{rat}: D^b(MHM(X)) \xrightarrow{\mathrm{rat}} D^b(\mathrm{Perv}(\mathbb{Q}_X)) \xrightarrow{\mathrm{real}} D^b_c(\mathbb{Q}_X),$$

(cf. [BBD] for the definition of real). Here f is a morphism of algebraic varieties and g is a section of $\mathcal{O}_X$.

iii) The objects and the morphisms of $MHM(X)$ are locally defined; e.g. for an open covering $\{U_i\}$ of X and $\underline{M}_i \in \mathrm{Ob}\,MHM(U_i)$ with isomorphisms $u_{ij}: M_j|_{U_i \cap U_j} \simeq M_i|_{U_i\cap U_j}$ satisfying $u_{ik} = u_{ij}\circ u_{jk}$ on $U_i \cap U_j \cap U_k$, there exists uniquely (up to a unique isomorphims) $\underline{M} \in \mathrm{Ob}\,MHM(X)$ with isomorphisms $\underline{M}|_{U_i} \simeq \underline{M}_i$ compatible with u_{ij}'s. (Here, for an open immersion $j: U \to X$, $j^* = j^!$ are exact and $\underline{M}|_U := j^*\underline{M} = j^!\underline{M}$.)

iv) If X is smooth, $MHM(X)$ is a full subcategory of $MF_hW(\mathcal{D}_X, \mathbb{Q}) = \{((M,F), K; W)\}$ where (M,F) are filtered $\mathcal{D}_X$-Modules such that M are regular holonomic and $Gr^F M$ are coherent over $Gr^F\mathcal{D}_X$, K are $\mathbb{Q}$-perverse sheaves with an isomorphism $\alpha : DR(M) \simeq \mathbb{C} \otimes K$ and W are the filtrations of (M, K), i.e. $DR(W_i M) \overset{\alpha}{\simeq} \mathbb{C} \otimes W_i K$.

v) Assume X is smooth and $K = L[\dim X]$ for a local system L on X. Then $((M,F), K; W)$ belongs to $\mathrm{Ob}\, MHM(X)$ iff it is an admissible variation of mixed Hodge structures in the sense of Kashiwara [K].

vi) For a principal divisor $Y = \{g = 0\}_{red}$ of X, we have an equivalence of categories:

$$MHM(X) \xrightarrow{\sim} MHM(U, Y)_{glu}$$

by assigning $(\underline{M}|_U, \varphi_{g,1}\underline{M}; \mathrm{can}, \mathrm{Var})$ to $\underline{M}$, where $j : U (= X \setminus Y) \to X$ and $MHM(U, Y)_{glu}$ is the category whose objects are $\{\underline{M}', \underline{M}''; u, v\}$ where $\underline{M}' \in MHM(U)$, $\underline{M}'' \in MHM(Y)$, $u : \psi_{g,1} j_* \underline{M}' \to \underline{M}''$, $v : \underline{M}'' \to \psi_{g,1} j_* \underline{M}'(-1)$ such that $vu = N$ (= the logarithm of the unipotent part of the monodromy, tensored by $(2\pi i)^{-1}$)

vii) For a closed immersion $i : X \to Y$, i_* is exact and we have an equivalence of categories:

$$i_* : MHM(X) \xrightarrow{\sim} MHM_X(Y)$$

where $MHM_X(Y)$ is the full subcategory of $MHM(Y)$ whose objects are supported in X.

Note that an object of $MHM(X)$ can be (locally) constructed by induction on the dimension of the support using v) vi) etc. If X is not smooth, we use an open covering $\{U_i\}$ of X with $U_i \to V_i$ closed immersions into smooth varieties. Here the $\mathbb{Q}$-part (K, W) is globally well-defined and it is enough to compare F for two embeddings $U_i \to V_i$ and $U_j \to V_j$. But it can be done using $U_i \cap U_j \to U_i \times U_j \to V_i \times V_j$ and the projections $V_i \times V_j \to V_i, V_j$.

We define $\mathbb{Q}_X^H = a_X^* \mathbb{Q}_{pt}^H$, where $a_X : X \to pt = \operatorname{Spec} \mathbb{C}$ and $\mathbb{Q}_{pt}^H = ((\mathbb{C}, F), \mathbb{Q}; W)$ with $Gr_i^F = Gr_i^W = 0$ for $i \neq 0$. If X is smooth, $\mathbb{Q}_X^H = ((\mathcal{O}_X, F), \mathbb{Q}_X[\dim X]; W)[-\dim X]$ with $Gr_i^F \mathcal{O}_X = 0$ $(i \neq 0)$, $Gr_i^W = 0$ $(i \neq \dim X)$. By ii) and v) $a_{X*} \mathbb{Q}_X^H$ is a bounded complex of graded polarizable mixed Hodge structures and it gives a natural mixed Hodge structure on $H^*(X, \mathbb{Q})$. We can also define the Hodge cycle class in (a generalization of) the $\mathbb{Q}$-Deligne cohomology for an irreducible closed subvariety of X.

Let $MH(X,n)^p$ be the full subcategory of $MHM(X)$ whose objects $\underline{M}$ satisfy $Gr_i^W \underline{M} = 0$ for $i \neq n$. An object of $MH(X,n)^p$ is called a polarizable Hodge Module of weight n. We say $\underline{M} \in \operatorname{Ob} MH(X,n)^p$ has strict support Z if $\operatorname{rat}(\underline{M})$ is an intersection complex with support Z. Let $MH_Z(X,n)^p$ the full subcategory of such objects. Then we have $MH(X,n)^p = \bigoplus_Z MH_Z(X,n)^p$ and an equivalence of categories:

$$(**) \qquad MH_Z(X,n)^p = VHS_{gen}(Z, n - \dim Z)^p$$

where the right hand side is the category of polarizable variation of Hodge structures of weight $n - \dim Z$ defined on some smooth open dense subset of Z. The equivalence

(**) holds also in the analytic case, and in this case we have the cohomological functors $\underline{H}^i f_* : MH(X, n)^p \to MH(Y, n+i)^p$ with the hard Lefschetz and the polarization on the primitive part, if f is projective. If f is proper Kähler, we have the functors $\underline{H}^i f_* : MH(X, n)^p \to MH(Y, n+i)$ (but not the hard Lefschetz nor the (global) polarizability for the moment). Note that it implies Kollár's torsion freeness in the proper Kähler case. As to the Ohsawa-Kollár vanishing we can deduce it from the Kodaira vanishing:

Let Z be a projective variety embedded in $X = \mathbb{P}^N$ by L^m where L is an ample line bundle on Z. Let $\underline{M}$ be an object of $MHM(Z)$ represented by $((M, F), K; W) \in MHM(X)$. Then $Gr_p^F DR_X(M, F) \in D^b(\mathcal{O}_Z)$ (and is independent of the embedding) and

$$H^i(Z, Gr_p^F DR_X(M,F) \otimes_{\mathcal{O}_Z} L) = 0 \quad \text{for } i > 0$$
$$H^i(Z, Gr_p^F DR_X(M, F) \otimes_{\mathcal{O}_Z} L^{-1}) = 0 \quad \text{for } i < 0.$$

This implies also the vanishing of Guillen-Navarro-Puerta because $\mathbb{C}_Z[\dim Z]$ is semi-perverse, i.e. ${}^p\underline{H}^i(\mathbb{C}_Z[\dim Z]) = 0$ if $i > 0$.

As a corollary of the above arguments, we get a natural Hodge structure on $IH^q(X, L)$ (resp. mixed Hodge structure on $H^*(U, L)$) if X is compact and of class C and L is a local system on a Zaris open dense smooth subset U of X, which underlies a polarizable variation of Hodge structures (resp. admissible variation of mixed Hodge structures in the sense of Kashiwara [K]). Here we use [K-K]. (In the projective case, [Z] is enough.)

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L functions and the Oscillator Representation

The local and global theory of L functions for classical groups as developed in [2] and [3] is outlined. The use of the Rankin Selberg method and the doubling construction for classical groups yield Eulerian integral representations of such L functions. Such constructions can also be applied to yield information about specific values of such L functions (i.e. poles, etc.) Namely it is possible to express the Petersson norm (on the orthogonal group O(Q)) of the Θ correspondant of an automorphic cusp form of a symplectic group Sp in terms of the special values of such L functions (here O(Q) × Sp is a dual reductive pair). This gives a criterion to detect the nonvanishing of Θ correspondants by means of (i) global data, i.e. nonvanishing of the L function at specific values (ii) local data, i.e the occurrence of a certain irreducible representation in a local oscillator representation (see [4], [5]). The key tool for this is the extension of the Siegel formula as set forth by Weil to a new range of cases where the adelic Weil Siegel Θ integral is absolutely convergent ([1]).

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Titel: LOWER CURVATURE BOUNDS AND TOPOLOGICAL FINITENESS

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The purpose of this talk is to report on some very recent developments ([GP], [SY], [AG] concerning finiteness problems for the topology of complete riemannian spaces with bounded geometry. In a general setting, one has to consider restrictions on the most basic geometric data: curvature, diameter, and volume. We will discuss both sectional and Ricci curvature. In light of Toponogov's Triangle Theorem and the Bishop–Gromov comparison for volumes of metric balls (cf. [G] for many facts and additional references), lower curvature bounds play by far the more fundamental role than upper bounds, in particular in the case of Ricci curvature.

We first consider the class $\mathcal{M}_{k\,d\,v}^{K\,D\,V}$ of all compact riemannian manifolds, in a fixed dimension n, for which K, D, V and k, d, v are upper and lower bounds for their sectional curvature, diameter, and volume, respectively. The following subclasses are most interesting:

$$\mathcal{M}_{k\ \ v}^{K\,D} \subset \mathcal{M}_{k\ \ v}^{\ \ D} \subset \mathcal{M}_{k}^{\ \ D}.$$

(Blanks indicate that no corresponding bounds are imposed. Obviously, one bound can always be normalized by rescaling the metric.)

There are now finiteness results for all these three topologically distinct categories: In 1970, J. Cheeger showed that $\mathcal{M}_k^{K\,D}{}_v$ contains only finitely many diffeomorphism types of manifolds; A. Weinstein had obtained a special case somewhat earlier. A key step was to estimate the injectivity radius, which gives rise to coverings by a bounded number of convex sets. The whole argument has been simplified by work of E. Heintze, H. Karcher, and S. Peters. In 1981, M. Gromov proved that the Betti numbers (for arbitrary coefficients in a field) are a priori bounded for spaces in the class $\mathcal{M}_k^D$. Note in particular that in case $k>0$, D is automatically bounded in terms of k, by Myers' Theorem. However, $\mathcal{M}_k^D$ contains infinitely many homotopy types for higher dimensions. The beautiful and intricate arguments were based just on angle comparison and a powerful new idea of Grove, which allows to use distances like smooth Morse functions and characterize non-critical points on the cut locus very geometrically. A considerable improvement of the (very large) explicit estimates for the Betti numbers, as well as an extension of this result to non-compact manifolds which are asymptotically non-negatively curved, was given by U. Abresch [A].

The missing link between the above two major results is a new theorem of K. Grove and P. Petersen [GP]: The class $\mathcal{M}_k^D{}_v$ contains only finitely many homotopy types.

The crucial idea here is to show that although there are no lower bounds for critical points of distance functions from points in $M \in \mathcal{M}_{k,v}^{D}$, there does exist an a priori estimate $r > 0$ for the radius of metric tubes about the diagonal $\Delta \subset M \times M$ in which the distance $d(x,y)$ has no critical points outside Δ. This yields a strong deformation retraction (with bounded dilatation) of such tubes onto Δ, which in turn shows that sufficiently close maps into M are always homotopic. From there it is not too difficult and more standard to obtain bounds for the number of homotopy types in terms of 1-skeletons of the nerves of coverings by balls associated with minimal ε-nets.

In the case of Ricci curvature, one is first of all interested in the class $\mathcal{R}(k,D)$ of compact n-manifolds with a lower curvature bound $(n-1)k$ and an upper diameter bound D. This set is precompact in the Hausdorff topology by a theorem of M. Gromov. However, it is now known that there is no general finiteness result at the level of Betti numbers, even for $k \geq 0$. J.-P. Sha and D.-G. Yang have constructed very important examples of positively Ricci curved manifolds, in all dimensions ≥ 7 (and also in dimension 5), with arbitrarily large topology, [SY]. In particular, connected sums of any number of copies of $S^m \times S^n$, $3(n-1) \geq m \geq n+1 \geq 3$, admit metrics with Ric > 0. Their construction is only semilocal. Thus finiteness results for Ricci curvature, if they exist, will require further restrictions.

We want to point out that the Cha-Yang examples also show that the class of manifolds with positive Ricci curvature is topologically always considerably larger than the class of manifolds with nonnegative sectional curvature, in higher dimensions. This was, of course, expected, but had been established only in the non-compact case recently by examples of D. Gromoll, W.T. Meyer, and L. Bérard-Bergery.

We finally mention new results concerning non-compact manifolds M^n with nonnegative Ricci curvature. Here the main problem is whether or not M^n has finite topological type, as in the situation of nonnegative sectional curvature. Again J.-P. Cha and D.-G. Yang were able to modify their construction and obtained manifolds with strictly positive Ricci curvature and unbounded topology, in all dimensions ≥ 7. The underlying space, for example, can be chosen to be $\mathbb{R}^4 \times S^3$ with infinitely many copies of $S^4 \times S^3$ attached to it by surgery. Thus additional assumptions are required for finiteness results. U. Abresch and D. Gromoll [AG] have shown that if M^n has diameter growth $o(r^{1/n})$, then M^n has the topological type of the interior of a compact manifold with boundary, provided the sectional curvature is bounded away from $-\infty$. Actually this result can be proved already

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for asymptotically nonnegatively Ricci curved manifolds. The Sha-Yang examples have diameter growth $o(r^{2/3})$, so do some examples of Bérard-Bergery; the Gromoll-Meyer examples have bounded diameter near infinity. All these examples have sectional curvature bounded away from $-\infty$.

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Titel: Moment map Problems in Infinite Dimensions

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The definition of Stability for vector bundles (in algebraic geometry) is berry developed to resemble the theory of stable orbits in geometric invariant theory. This is explained in a preprint of Donaldson [Do 2]. Moreover, while Donaldson's work concerns pure Yang-Mills equations, recent work of Hitchin, Corlette and Simpson on three different problems of stability, where coupled problems with "Higgs fields" emerge, indicates that there will be a very useful theory of such infinite dimensional problems. In each case, the group corresponds to the group of gauge transformations (or automorphisms).

We describe briefly the basic example of the uncoupled Yang-Mills equation. Let X be a complex Kähler manifold, $\begin{smallmatrix} E \\ \downarrow \\ X \end{smallmatrix}$ a topological vector bundle of rank N over X, and $\Omega^{1,1}$ the space of ~~conn~~ Hermitian connections with curvature of type (1,1) in E. $\Omega^{1,1}$ inherits a complex structure from the complex structure on X, and the Kähler form ω on induces a Kähler form Ω on $\mathcal{O}^{1,1}$

$$\Omega(A,B) = \int_X \operatorname{tr}(A\wedge B)\wedge\omega^{m-1}.$$

The unitary gauge group (which plays the role of maximal compact subgroup) acts in the usual way on connections. The complexified group acts by extension on D'' ($D = D' + D''$ according to the complex structure) and by duality on D'. There is a moment map for the unitary action

$$\mathcal{O}^{1,1} \longrightarrow \mathfrak{g}^* \overset{L^2}{\simeq} C^\infty(\mathfrak{g}_E)$$

given by

$$D \to (\omega, F_D).$$

A connection $D \in \mathcal{O}^{1,1}$ induces a holomorphic structure on E. The usual definition of stability is to assert that if $\mathcal{F} \subseteq E$ is any coherent subsheaf (possibly "singular" holomorphic sub-bundle), then E is stable if $\mu(\mathcal{F}) < \mu(E)$. Here

$$\mu(\mathcal{F}) = \frac{(c_1(\mathcal{F})\wedge[\omega]^{m-1})[X]}{\operatorname{rank}\mathcal{F}}.$$

<u>Theorem</u>: Donaldson, Uhlenbeck-Yau. Every stable vector-bundle has a unique (up to unitary equivalence) connection D for which $(\omega, F_D) = \mu I$.

Corlette's work is on flat bundles over Kähler manifolds, the first version being, of course, flat bundles over curves. Here the space $\mathcal{A}^{1,1}$ is replaced by $\mathcal{A}^f$, the space of flat, not nessarily unitary connections. The complex gauge groups acts by conjugation. Stability can be defined as the absence of sub-bundles which are carried into themselves by the connection. The following theorem may also have been proved by Donaldson.

<u>Theorem (Corlette)</u>: Every stable flat bundle (over a Riemann surface) has a metric in which

$$D = \begin{pmatrix} \partial^A + \phi \\ \bar\partial^A + \phi^* \end{pmatrix}$$

and for which the moment map $\bar\partial^A \phi = 0$. Note that for flat unitary connections $\phi = 0$. Corlette is able to oblain new results on representations this way.

Simpson's work concerns the variation of Hodge Structure. I rephrase it here in the language of differential geometry. The data are a holomorphic vector bundle $\begin{matrix} E \\ \downarrow \\ X \end{matrix}$ (or a point $D \in \mathcal{O}\!\mathcal{L}^{1,1}$) and a Higg's field which he calls $\mathcal{K} : C^\infty(E) \to T^{1,0}(X) \otimes C^\infty(E)$ and which Hitchin calls ϕ. $D''\mathcal{K} = 0$ by assumption.

In Simpsons case, E is graded

$$E = \sum_{p+q=w} E^{p,q} \quad \text{and} \quad K : C^\infty(E^{p,q}) \to C^\infty(T^{1,0}(x) \otimes E^{p-1,q+1}).$$

The grading reduces the gauge group to the product of the diagonal $\times\, \mathcal{G}^{p,q} = \times C^\infty(\mathcal{G}_{E^{p,q}})$ but is probably not necessary for the analytic theorem. Stability is defined by requiring that if $\mathcal{F} \subseteq E$ is a sub-sheaf, $K : C^\infty(\mathcal{F}) \to C^\infty(T^{1,0}(x) \otimes \mathcal{F})$ then

$$\mu(\mathcal{F}) < \mu(E).$$

The following theorem has been proved by Hitchin for curves, and Simpson for the graded general case.

<u>Theorem</u>: If (D, K) is a stable pair, then there exists a unique metric for which the pair is Hermitian-~~[illegible]~~ Yang-Mills

$$(\omega, (F_D + [K, K^*])) = \mu I$$

The sign coming from grading, or "imposed" is important, as the reverse sign $(\omega, (F_D - [K, K^*])) = \mu I$ is related to harmonic maps into the unitary group rather than the negatively curved symmetric space, and all convexity is lost. Hitchin and Corlette obtain roughly the same moduli space but for different geometric problems.

Simpson combines the ideas of Donaldson and

Uhlenbeck-Yau to give the proof which is closest to a geometric invariant theory proof.

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Titel: Zeta-Funktionen von Shimura varietäten und Dieudonnémoduln

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Let S be a scheme of finite type over $\mathbb{Z}$. One defines

$$t \frac{d}{dt} Z_p(S,t) =\cdot \sum_{n\geq 1} \# S(\mathbb{F}_{p^n})\, t^n$$

If $S \times \mathbb{F}_p$ is smooth $Z_p(S,t)$ is a rational function, which may be expressed by the étale cohomology. The Zeta-function of S is

$$(1) \qquad \zeta_S(s) = \prod_{p \in \operatorname{Spec} \mathbb{Z}} Z_p(S, p^{-s}) \qquad s \in \mathbb{C} \quad \operatorname{Re} s \gg 0.$$

One can expect that ζ_S contains significiant information about the arithmetic of S.

If S is a Shimura curve ζ_S may be written as a product of L-series associated to cusp forms, which are eigenforms for Heckeoperators (Shimura).

For higher dimensional Shimura varieties Langlands has devellopped a program to express ζ_S as a product of L-series of automorphic representations. Equivalently one should have a formula of the following type:

$$(2) \qquad \# S(\mathbb{F}_{p^n}) = \sum \operatorname{Tr} \pi_N (f^N)$$

The sum runs over certain automorphic representations π_N of certain groups N.

The first step in this programm is a decomposition of $S(\overline{\mathbb{F}}_p)$ into orbit spaces:

$$(3) \qquad S(\overline{\mathbb{F}}_p) = \coprod_{\varphi} H(\mathbb{Q}) \backslash X_\varphi \times G(\mathbb{A}_f^p) / C^p$$

One needs also a description of the Frobenius Φ acting on the right hand side. $S(\mathbb{F}_{p^n})$ are the fixed points of Φ^n. Kottwitz has expressed the number of the fixed points in terms of orbital an twisted orbital integrals. By a suitable trace formula one hopes to derive from this (2). The program was carried out by Langlands in the case where S is the Shimura variety associated to a quaternion algebra over a totally real number field.

We want to explain the decomposition (3) in the case, where S is associated to the symplectic group. Let V be a $\mathbb{Q}$-vector space of dimension $2g$ together with a symplectic form $\langle\, ,\, \rangle$. Let be $G/\mathbb{Q}$ the group of symplectic similitudes.

$$G = \{ g \in Gl(V) \mid \langle gv, gw \rangle = \mu(g) \langle v, w \rangle,\ \mu(g) \in \mathbb{Q} \}.$$

There exists up to conjugation a unique morphism $h : \mathbb{C}^\times \longrightarrow G_{\mathbb{R}}$ with the following properties:

a) h defines on $V_{\mathbb{R}}$ the structure of a complex space

b) $\langle v, h(i) w \rangle$ is a symmetric and positive definit form on $V_{\mathbb{R}}$.

We denote the conjugacy class of h by X_∞. An element $h \in X_\infty$ defines a polarized abelian variety (A, λ) over $\mathbb{C}$ together with a symplectic isomorphism $\eta_{\mathbb{Q}} : H_1(A, \mathbb{Q}) \simeq V$.

Since $\eta_{\mathbb{Q}}$ makes algebraically no sense, it is replaced by a symplectic similitude

$$(4) \quad \eta : H_1(A, \mathbb{A}_f) \longrightarrow V \otimes \mathbb{A}_f \mod C$$

Here $\mathbb{A}_f$ is the ring of finite adeles and $C \subset G(\mathbb{A}_f)$ is a congruence subgroup.

Let be (A, Λ, η) a triple, where $A/\mathbb{C}$ is an abelian variety up to isogeny, $\Lambda = \mathbb{Q}_+ \lambda$ is a polarization up to a factor and η is as under (4). One sees that those triples correspond bijectively to the set

$$S(\mathbb{C}) = G(\mathbb{Q}) \backslash X_\infty \times G(\mathbb{A}_f) / C .$$

In fact S becomes a scheme over $\mathbb{Q}$, since we have a moduli problem. This is the Shimura variety associated to G.

For a suitable model $\tilde{S}/\mathbb{Z}$ of S and a sufficiently general prime p the set $\tilde{S}(\overline{\mathbb{F}}_p)$ is described as the set of triples $(A, \Lambda\ \eta^p)$, s. th.

a) A is an abelian variety over $\overline{\mathbb{F}}_p$ up to isogeny of order prime to p.

b) Λ is $\mathbb{Q}_+\lambda$ where λ is a polarization of A

c) $\eta^p : H_1(A, \mathbb{A}_f^p) \xrightarrow{\sim} V \otimes \mathbb{A}_f^p \bmod C^p$ is a symplectic similitude.

Here $\mathbb{A}_f^p = (\prod_{\ell \neq p} \mathbb{Z}_\ell) \otimes \mathbb{Q}$ and $C^p \subset G(\mathbb{A}_f^p)$ is a congruence subgroup.

A has complex multiplication $K \xrightarrow{\iota} \operatorname{End} A \otimes \mathbb{Q}$, where K is commutative, $[K : \mathbb{Q}] = 2g$ and λ induces on K the conjugation. Assume that A, ι and Λ are defined over $\mathbb{F}_q$. Let be $\pi \in K$ the Frobenius over $\mathbb{F}_q$. We choose a diagram

$$(5) \quad \overline{\mathbb{Q}} \begin{array}{l} \nearrow \mathbb{C} \\ \searrow \hat{\overline{\mathbb{Q}}}_p \end{array}$$

In the sense of this diagram A is the reduction up to isogeny of an abelian variety $A_{\mathbb{C}}/\mathbb{C}$, s. th. the operation of K and Λ lift to $A_{\mathbb{C}}$. Choose an isomorphism

$$(6) \qquad H_1(A_{\mathbb{C}}, \mathbb{Q}) \simeq V$$

of symplectic modules. Let be $T = \{x \in K \mid x\bar{x} \in \mathbb{Q}\}$, where $x \mapsto \bar{x}$ is the conjugation. Since K acts on $H_1(A_{\mathbb{C}}\ \mathbb{Q})$ we get a morphism $T \longrightarrow G$. Hence we can view π as an element of T and G. The CM-type $\Phi \subset \operatorname{Hom}(K, \mathbb{C})$ of $A_{\mathbb{C}}$ defines a cocharacter $\mu : \mathbb{C}^\times \longrightarrow T_{\mathbb{C}}$, s. th. $(\mu, \varphi) = 1$ if $\varphi \in \Phi$ an zero otherwise.

We call to points (A, Λ, η) and (A', Λ', η') isogenous if (A, Λ) and (A', Λ') are isogenous. The set of points in such an isogeny class φ is described as an orbit space.

$$(7) \quad H \backslash X_\varphi \times G(\mathbb{A}_f^p)/C^p$$

Of course φ is entirely determined by the pair (μ, π). We want a description of (7) in terms of μ, π.

One has $H = \{ x \in \operatorname{End} A \otimes \mathbb{Q} \mid xx^* \in \mathbb{Q} \}$, where $x \mapsto x^*$ is the Rosati involution. The isomorphism $H_1(A, \mathbb{A}_f^p) \simeq H_1(A_{\mathbb{C}}, \mathbb{A}_f^p) \simeq V \otimes \mathbb{A}_f^p$ by ( ) gives us an embedding $H \longrightarrow G(\mathbb{A}_f^p)$.

On the other hand let be $M_{\mathbb{Q}}$ the covariant rational Dieudonné module of A. Let be B the completion of the maximal unramified extension of $\mathbb{Q}_p$. The $M_{\mathbb{Q}}$ is a free B-module. We have in fact an isomorphism (noncanonical)

$$(8) \qquad H_1(A_{\mathbb{C}}, \mathbb{Q}) \otimes B \simeq M_{\mathbb{Q}}$$

We choose it in such a way that the action of K and the symplectic structure on both sides is respected. The Frobenius $F: A \longrightarrow A^{(p)}$ induces an operator $V: M_{\mathbb{Q}} \longrightarrow M_{\mathbb{Q}}$ which is called the Verschiebung. X_p is the set of O_B-modules $N \subset M_{\mathbb{Q}}$, s. th. $N \otimes \mathbb{Q} = M_{\mathbb{Q}}$ and s. th.

a) N is invariant under V and pV^{-1},

b) $p^r\langle\ ,\ \rangle$ is perfect on N for some $r \in \mathbb{Z}$.

The Frobenius on $S(\bar{\mathbb{F}}_p)$ acts on the orbit space (7) via X_p:

$$N \longmapsto V^{-1} N$$

We denote by σ the Frobenius of B. The action of V on the left hand side of may be written:

$$(9) \quad Vx = \sigma^{-1}(bx), \quad x \in H_1(A_{\mathbb{C}}, B), \quad b \in K \otimes B$$

Let be $\mathcal{B}(T) = \{ \beta \in K \otimes B \mid \beta\bar{\beta} = p^a,\ a \in \mathbb{Z} \} / \beta \equiv \sigma^{-1}(c)\beta c^{-1}$.

Here c runs through the elements $c \in K \otimes B$, $c\bar{c} = 1$.

Let be L/B a finite normal extension, that contains K and $\sqrt{-p}$. Fix an element g, s. th. $g^2 = -p$. Let be $\pi \in L$ a prime element, s. th. $\mathrm{Nm}_{L/B[\sqrt{-p}]}\, \pi = g$. We define a map (independent of π)

$$k_g : X_*(T) \longrightarrow \mathcal{B}(T)$$
$$\nu \longmapsto Nm_{L|B}\, \nu(\pi)$$

The following theorem was proved by Harry Reimann and the author. An independent proof of a slightly weaker version (however sufficient for the purposes at hand) was given by Kottwitz.

<u>Theorem</u>: $b \equiv k_g(\mu)$, for $g = -\left(\left(\frac{p-1}{2}\right)!\right)^{-1}\gamma$, where

$$\gamma^2 = (-1)^{\frac{p-1}{2}}\, p \,, \quad \gamma \in \mathbb{R}_+ \cup \mathbb{R}_+ i \quad \text{(in the sense of diagram (5))}.$$

This theorem gives a full description of the orbit spaces in question.

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