"ABELIAN GROUP ACTIONS ON ALGEBRAIC VARIETIES WITH ONE FIXED POINT"

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Introduction.. Since early stages of its developments, the theory of transformation groups has relied on comparison of linear representations with general group actions on manifolds from algebraic as well as geometric points of view. If G is a finite group and W is an orthogonal linear representation space of G , then it is an elementary fact that the unit sphere S(W) with the induced linear G–action has more than one G–fixed point if $S(W)^G \neq \phi$. The generalization of this fact to arbitrary smooth G–manifolds is neither elementary nor obvious. The first result in this direction is due to Conner–Floyd ([5] § 31) who proved that for $G = (\mathbb{Z}/2)^n$ acting smoothly on a closed manifold X , the fixed point set X^G cannot consist of one point. Conner and Floyd conjectured ([5] § 45) that the cyclic group $\mathbb{Z}/q^n$, where q is an odd prime, cannot act on a closed orientable manifold with only one fixed point. The example of Conner and Floyd for a smooth $\mathbb{Z}/4$ action on $\mathbb{R}P^2$ shows that this fixed–point property does not hold in general. The Conner–Floyd conjecture was proved by Atiyah and Bott ([3] Theorem 7.1) using their version of the Lefschetz fixed point formula for elliptic complexes (nowadays known as Atiyah–Bott–Lefschetz formula). Conner and Floyd also established their conjecture using their work on the cobordism of odd order periodic maps ([6] Theorem 8.3). They also construced a smooth G–action on a Riemann surface with exactly one fixed point for G a cyclic odd order group with at least two distinct primes dividing $|G|$.

A more general form of the Conner–Floyd conjecture for smooth odd order abelian p–group actions is due to W. Browder ([4]) based on his fixed–point Theorem and K–theoretic considerations. Browder ([4]) as well as Ewing and Stong ([8]) showed that the abelian hypothesis is necessary. In fact, based on the Atiyah–Bott and Conner–Floyd Theorem, Ewing and Stong ([8]) proved that if $G \neq (\mathbb{Z}/2)^n$ is a compact Lie group, then G can act smoothly on a closed (possibly non–orientable) manifold with one fixed point and in the orientable case, only abelian groups of odd prime power order cannot act with only one fixed point. The generalization and interpretation of the Conner–Floyd conjecture in an algebraic context was taken up by Assadi in [2] and [3]. Assadi's generalization to chain complexes and G–spaces with Poincaré duality provided an algebraic proof of the Conner–Floyd conjecture for $G = (\mathbb{Z}/p)^n$. For infinite dimensional Poincaré G–spaces and kG–Chain complexes satisfying Poincaré duality, the Conner–Floyd conjecture may be formulated in terms of the associated varieties [3]. Recently, W. Browder has extended his results in [5] to abelian p–group actions on finite dimensional simplicial complexes which are $(\mathbb{Z}/p)$ – homology manifolds [6], thus giving a further generalization of the Conner–Floyd conjecture.

In this paper, we prove that the Conner–Floyd conjecture generalizes to actions on complete non–singular algebraic varieties over an arbitrary algebraically closed field k. In particular:

<u>Theorem 2.1</u>. Let X be a complete algebraic variety over an algebraically closed field of characteristic $p \geq 0$, and let G be a finite ablian group of order q^r, where q is a prime different from p, acting on X via automorphisms. If the fixed point set consists of one point $x \in X$, then X is singular at the point x.

In fact, the following scheme–theoretic generalization is proved:

<u>Corollary 3.2</u>. Let G be a finite group of prime power order acting on a complete non–singular algebraic variety V defined over an algebraically closed

field of arbitrary characteristic. Then V^G cannot consist of an isolated point, i.e. $V^G \neq \mathrm{Spec}(k)$.

For varieties over $\mathbb{C}$, the underlying topological space of X in the analytic topology is triangulable according to Hironaka ([10]). Combining the above theorem and the results of Browder [6] or Assadi [2] [3] one concludes that in this case such an X does not satisfy Poincaré duality (even with $\mathbb{F}_p$–coefficients).

From the point of view of varieties, the condition $p \neq q$ in Theorem 2.1 is necessary, since the action of $\mathbb{Z}/p$ on $\mathbb{P}^1(\overline{\mathbb{F}}_p)$ given by $(x_0 ; x_1) \longrightarrow (x_0 ; x_0 + x_1)$ has precisely one fixed point. On the other hand, it is easily seen that this fixed point has multiplicity two, so that Corollary 3.2 applies to this case. Finally, one may ask to what extent the non–singularity of X plays a role for the truth of the Conner–Floyd conjecture. By means of examples (Section 3) one can see that there exists one–fixed point actions on projectively normal subvarieties of $\mathbb{P}^N$ which have only a normal singularity at the fixed point. Moreover, for $k = \mathbb{C}$, the link of the singularity at X^G could be quite complicated (see Corollary 3.3).

In the next section we discuss some preliminary notions from algebraic geometry which may not be well–known to researchers in topological transformation groups. In Section 2 we give the proof of the main theorem. In Section 3 we discuss some examples including the case of p–groups actions in characteristic p.

Section One. Preliminaries.

In the sequel, k denotes an algebraically closed field of characteristic $p \geq 0$, and G will be a finite group. If $(|G|, p) = 1$ ($|G|$ = order of G), then the element $\frac{1}{|G|} \sum_{g \in G} g$ is an idempotent in kG. Consequently, all kG–modules are kG–projective and the group algebra kG is semisimple, which shows that all kG–modules are completely reducible (i.e. kG–isomorphic to a direct sum of irreducible kG–submodules). This result (known as Maschke's Theorem, cf. Curtis–Reiner [7]), refines further when G is abelian. Namely, any n–dimensional kG–module W is G–isomorphic to a direct sum of one–dimensional (over k) kG–submodules : $W \cong \bigoplus_{i=1}^{n} L_i$, $\dim_k L_i = 1$. Further, the representation of G on L_i factors through $G \twoheadrightarrow \mathbb{Z}/\ell$ where $\mathbb{Z}/\ell$ acts on L_i via an appropriate ℓ–th root of unity. Similarly for infinite dimensional representation the above idempotent may be used.

The standard reference for notation and definitions from algebraic geometry is Hartshorne [9] and the reader will find the details in [9] as appropriatly referred to them. In particular, the term variety refers to an irreducible variety. Basic properties of projective varieties are adequately covered in [9] Chapter I where the reader may replace "complete" by "projective". Let G act effectively on a projective k–variety X by automorphisms. Then the geometric orbit space X/G exists and it is a projective variety as well. In the case $k = \mathbb{C}$, this coincides with the orbit space under the usual (Euclidean) topology. We will need this fact only for curves in the positive characteristic where complete and projective are equivalent (indeed for non–singular curves only). This case is handled by the following elementary considerations. The G–action on X induces a G–action on the field of rational functions $K(X) \overset{\text{def}}{\equiv} K$ leaving the subfield k fixed. In particular, the fixed field $L \overset{\text{def}}{\equiv} K^G$ is a finitely generated field extension of k of transcendence degree one, and the extension $L \subset K$ is Galois. It is well–known that there is a unique (up to isomorphism) projective non–singular curve X' whose function field is isomorphic to L. Moreover, there is a k–morphism $\pi : X \longrightarrow X'$ inducing the inclusion $L \subset K$ (cf. [9] Ch. I § 6).

In the classical case, i.e. $k = \mathbb{C}$, the map π is a ramified (i.e. branched) covering, and ramification occurs over the orbits whose isotropy subgroups are non–trivial. Let g and g' be the genera of X and X' respectively. Then, the Riemann–Hurwitz formula relates g and g' when X and X' are non–singular:

$$\text{(GRH)} \qquad 2g - 2 = |G|(2g' - 2) + \deg R .$$

Here R is the ramification divisor ([9] Chapter IV, § 2).

The proof of this theorem for $k = \mathbb{C}$ is an elementary Euler–Poincaré characteristic count, and simplifies to the following:

$$\text{(RH)} \qquad 2g - 2 = |G|(2g' - 2) + \sum_{x \in X} (|G| - |G(x)|)$$

where $|G(x)|$ is the number of points in the orbit of $x \in X$ which is equal to $|G|$ except for finitely many x.

In the general case, the above formula (GRH) is valid, and the only delicate point is the computation of $\deg R$. However, when $(|G_x|, p) = 1$ for all points $x \in X$ with non–trivial isotropy groups, the ramification is called tame, and $\deg R = \sum_{x \in X} (|G| - |G(x)|)$ so that (RH) holds in the following discussion (cf. [9] Ch. IV, § 2).

We will also need to consider desingularization of curves and an equivariant analogue. Suppose X is a possibly singular projective curve on which G acts by automorphisms. Then the set of singular points of X is a finite G–invariant set, i.e. a G–set. Also, G acts on the function field $K(X) \equiv K$ by k–automorphisms as above. Let Y be the unique non–singular projective curve such that $K(Y) \overset{\text{def}}{\equiv} K$. Then the G–action on K induces a G–action on the set of discrete valuation rings of K (which is the underlying set of Y). Thus G acts on Y by isomorphisms. Every local ring $\mathcal{O}_{X,x}$, $x \in X$, when regarded as a subring of K, is dominated by a discrete valuation ring $\mathcal{O}_{Y,y}$ ([9] Ch. I, § 6). The inclusions $\mathcal{O}_{X,x} \subset \mathcal{O}_{Y,y}$ for various $x \in X$ and $y \in Y$ give rise to a map $f : Y \longrightarrow X$ which turns out to be a morphism. Clearly, f will be equivariant with respect to the given G–actions. Further, Y is the normalization of X, and in the complement of the singular set, say $X_{reg} = X - X_{sing}$, $f : f^{-1}(X_{reg}) \longrightarrow X_{reg}$ is an isomorphism, and $f : f^{-1}(X_{sing}) \longrightarrow X_{sing}$ is a G–map of G–sets.

Remark. Although we will not need the following, it is interesting to notice that Y/G is the normalization of X/G, and the suitable generalization of the Hurwitz formula for singular curves should agree with the standard one for $Y \longrightarrow Y/G$.

Section Two.

We will keep the hypotheses and notation of the previous section.

2.1. Theorem. Let X be a complete algebraic variety over an algebraically closed field of characteristic $p \geq 0$, and let G be a finite abelian group of order q^r, where q is a prime different from p. Suppose that G acts on X via automorphisms, and X^G consists of one point. Then X^G is a singular point of X.

As a corollary of this theorem we have the following analogue of the Conner–Floyd conjecture and its generalitzation by Browder [5] [6].

2.2. Corollary. Let X be a complete non–singular variety over an algebraically closed field of characteristic $p \geq 0$. Suppose G is an abelian group of order q^r, where q is a prime different from p, and G acts on X via automorphisms. Then X^G cannot consist of one point.

Proof.: To get a contradiction, assume that $X^G = \{x_0\}$, X is smooth at x_0, and the action on $X - \{x_0\}$ is fixed–point free, i.e. for all $x \neq x_0$, the isotropy subgroup $G_x \neq G$. Notice that $T_{x_0}X$ is a G–representation. Choose a basis $e_1, \ldots, e_n$ of eigenvectors of $T_{x_0}X$ and let $x_1, \ldots, x_n$ be the corresponding coordinate functions. Let $\mathfrak{m}$ be the maximal ideal of the local ring $\mathcal{O}_{X,x_0}$ at x_0. Then we have a projection $\mathfrak{m} \longrightarrow \mathfrak{m}/\mathfrak{m}^2 \cong (T_{x_0}X)^*$. Choose a G–equivariant splitting (using semisimplicity) $\varphi : (T_{x_0}X)^* \longrightarrow \mathfrak{m}$ and let

$f_i := \varphi(x_i) \in \mathcal{O}_{X,x_0}$. Let $V_0 \subseteq X$ be an open neighborhood of x_0 where all f_i are defined, and let $V := \bigcap_{g \in G} g V_0$ be a G–invariant open neighborhood. This induces a G–equivariant morphism $\psi : V \longrightarrow T_{x_0} X : \psi(v) = \Sigma f_i(v) e_i$. Since $x_i \equiv f_i \pmod{\mathfrak{m}^2}$, the differential $d\psi$ induces an isomorphism of the Zariski cotangent spaces, which implies that ψ is étale at x_0 . Now let $C := \{v \in V \mid f_2(v) = \ldots = f_n(v) = 0\}$. C is smooth at x_0 by the Jacobian criterion. Let $\Sigma_0 :=$ component of C passing through x_0 . Thus Σ_0 is a curve passing through x_0 and non–singular at x_0 . Moreover, Σ_0 is G–invariant and $\Sigma_0^G = \{x_0\}$. Let Σ_1 be the closure of Σ_0 in X , (i.e. add the finitely many possibly missing closed points to Σ_0 to get a complete, possibly singular curve Σ_1). It follows that Σ_1 is also non–singular at x_0 and the G–invariant finite set of singular points of Σ_1 lies in the fixed–point free part of Σ_1 . Now let $\pi_1 : \Sigma \longrightarrow \Sigma_1$ be the equivariant normalization of Σ_1 as described in Section One. Thus, π_1 is a finite proper morphsim which restricts to a G–isomorphism onto the open subset of regular points of Σ_1 , i.e. an open G–invariant neighborhood of x_o . Hence, Σ^G consists of one point, namely $\pi_1^{-1}(x_0)$.

To summerize, we have produced a nonsingular complete curve Σ on which g acts by automorphisms and Σ^G consists of one point, call it x_0 again. Let $\Sigma' = \Sigma/G$ and $\pi : \Sigma \longrightarrow \Sigma/G$ be the projection onto the orbit space (cf. Section One), and let $x_0' = \pi(x_0)$. Since $(|G|,p) = 1$, the ramifications of π are all tame, and we may apply the Riemann–Hurwitz formula (RH) of Section One. For each branch point $x' \neq x_0'$, $x' \in \Sigma'$, and each ramification point $x \in \pi^{-1}(x')$ lying above x' , the ramification index is $|G_x| \neq |G|$. Let $g = \text{genus}(\Sigma)$ and $g' = \text{genus}(\Sigma')$. Thus (RH) becomes

$$2g-2 = |G|(2g'-2) + \sum_{x \neq x_0} |G| \cdot \left[1 - \frac{1}{|G_x|}\right] + |G| \cdot \left[1 - \frac{1}{|G|}\right] .$$

Hence $2g-2 \equiv -1 (\mathrm{mod}\ q)$.

On the other hand, consider the space of differential one–forms $\Omega^1_{\Sigma/k}$ which is a g–dimensional k–vector space on which G acts linearly. From the above conclusion of (RH), we conclude that $g \neq 1$. For $g = 0$, $\Sigma \cong \mathbb{P}^1(k)$ and it is well–known that the automorphism group of $\mathbb{P}^1(k)$ is PGL(2,k) and as a result, up to G–isomorphism, the G–action on $\mathbb{P}^1(k)$ is linear. It follows that any such effective linear G–action on $\mathbb{P}^1(k)$ must have at least two fixed points. Hence $g \geq 2$, and $\Omega^1_{\Sigma/k} \neq 0$. According to Section One, $\Omega^1_{\Sigma/k} \cong \bigoplus_{i=1}^{g} L_i$, where $\dim_k L_i = 1$ and the G–action on L_i factors through a projection $G \xrightarrow{\theta_i} \mathbb{Z}/q^{s_i} \cong \langle \zeta_i \rangle$, where ζ_i is a primitive q^{s_i}–th root of unity. Thus, we have a basis $T = \{t_1, \ldots, t_g\}$ of G–invariant differential one–forms on which $(\zeta_j, t_j) \longrightarrow \zeta_j^{n_j} \cdot t_j$ where $n_j \not\equiv 0 (\mathrm{mod}\ q^{s_j})$ and $q^{s_i} > 1$ since the action of G on Σ is not trivial. At least one element of T , say t_1 , must not vanish at x_0 . Since the degree of the divisor (t_1) is $2g-2 > 0$, t_1 must vanish at some point $y \neq x_0$. Let $\varphi : \Sigma \longrightarrow \Sigma$ be the isomorphism which represents the generator of $\mathbb{Z}/q^{s_1} \cong \theta_1(G)$ so that $\varphi^* t_1 = \zeta^{n_1} \cdot t_1$. Let $y_1 = f^{-1}(y) \in G(y) \equiv$ orbit of y . Then $t_1(y_1) = t_1(f^{-1}(y)) \overset{\mathrm{def}}{\equiv} (f^* t_1)(y) = \zeta^{n_1} \cdot t_1(y) = 0$, and we conclude that the order of vanishing of t_1 at all points of the orbit of y are the same. Hence, the degree of the divisor (t_1) is divisible by $\gcd\left\{ |\frac{G}{G_y}| : t_1(y) = 0 \right\} \equiv 0 \ (\mathrm{mod}\ q)$. It follows that $2g-2 \equiv 0 \ (\mathrm{mod}\ q)$, contradicting the conclusion of the Hurwitz formula above. This contradiction proves the theorem. □

Section Three. In this section we discuss some consequences of the main theorem as well as the case of p–group actions on varieties in characteristic p , where a scheme–theoretic version of Corollary 2.2 is valid. As before, let k be algebraically closed of characteristic p , and let X be an affine k–scheme with coordinate ring $k[X]$. The fixed–point scheme X^G is the closed subscheme defined by the ideal $I = \{f^g - f \mid g \in G, f \in k[X]\}$. Thus, the k–algebra $R := k[X]/I$ is the largest quotient of $k[X]$ on which G acts trivially, and $X^G = \mathrm{Spec}(R)$.

3.1. Theorem. Let X be an irreducible k–scheme of positive dimension on which a finite p–group G acts by k–isomorphisms. Then the fixed–point scheme X^G cannot consist of an isolated point, i.e. $X^G \neq \mathrm{Spec}(k)$.

3.2 Corollary. Let G be a finite group of prime power order acting on a complete non–singular algebraic variety V defined over an algebraically closed field of arbitrary characteristic. Then V^G cannot consist of an isolated point, i.e. $V^G \neq \mathrm{Spec}(k)$.

The above corollary follows from 3.1 and Corollary 2.2.

Proof of 3.1. We may assume $X^G(k) \neq \phi$. Let $x_0 \in X^G(k)$, and let U be an affine open neighborhood of x_0 . Then $U_0 := \bigcap_{g \in G} gU$ is a G–invariant affine open neighborhood of x_0 , and we may prove the theorem for U_0 . Therefore, we may assume that X is affine. Let m be the maximal ideal of the local ring $\mathcal{O}_{X,x_0}$, and consider the finite dimensional k–vector space m/m^2 on which G acts linearly. Since G is a p–group and $\mathrm{char}(k) = p$, there is a non–zero vector

$\alpha_0 \in (m/m^2)^*$ which is fixed under G . The pair consisting of the closed point x_0 and the non–zero tangent vector α_0 at x_0 which is fixed under G is equivalent to a surjective k–homomorphism $R \longrightarrow k[\varepsilon]/(\varepsilon^2)$, where $\mathrm{Spec}(R) = X^G$ as discussed above. Therefore $\mathrm{Spec}(R) \neq \mathrm{Spec}(k)$ as claimed. □

A version of 3.1 has been proved for unipotent actions in a different context by Meyer–Oberst [13].

As pointed out in the Introduction, the case $k = \mathbb{C}$ implies that if a complete variety X has a G–action with X^G = one point , then the link of the singularity at the fixed point X^G is not (mod q)–homology equivalent to a sphere, provided that $X - X^G$ is regular.

<u>3.3. Corollary</u>. Suppose $k = \mathbb{C}$ and X is a complete variety on which G (as in Theorem 2.1) acts with $X^G = \{x_0\}$. Suppose that X is non–singular in the complement of x_0 . Then the link of the singularity at x_0 is not (mod q)–homology equivalent to a sphere.

<u>Proof</u>: According to Hironaka [10], X is triangulable, and we may choose x_0 to be a vertex of an underlying symplicial structure. Further, by triangulating the orbit space X/G , we may assume that G acts simplicialy on X . Passing to the second barycentric subdivision, results in a G–CW–structure for X . Therefore the cellular chain complex $C_*(X)$ becomes a finite–dimensional permutation G–chain complex (cf. Assadi [1] Ch. I). If the link of the singularity at x_0 is a (mod q)–homology sphere, then X becomes a (mod q)–homology manifold, and consequently, the permutation G–chain

complex $C_*(X) \otimes \mathbb{F}_q$ satisfies duality. According to the construction, in the G–sets providing permutation bases for $C_i(X)$, only x_0 has isotropy subgroup G. But this contradicts Browder's Theorem [6] (see the Introduction).

In the same direction, the combination of Theorem 2.1 above, Browder [6], and Hironaka's triangulation Theorem [10] yields the following:

3.4. Corollary. Suppose $k = \mathbb{C}$, and X is a complete variety and G acts on X as in Theorem 2.1 above with X^G = one point. Then the underlying topological space of X in the analytic (i.e. Euclidean) topology does not satisfy Poincarè duality with mod q coefficients (and hence $\mathbb{Z}$–coefficients). □

Note: (1) When $G = (\mathbb{Z}/q)^r$, then we can also apply Assadi [2] [3] in conjunction with Hironka's result [10] to obtain this special case of Corollaries 3.3 and 3.4 above. This was the original form of 3.3 and 3.4 in the first version of this paper. We would like to thank Bill Browder for communicating his results to us, as well as bringing to our attention the following result of G. Bredon [14]. Bredon has shown that if $G = \mathbb{Z}/p$ acts on a connected finite Poincaré complex X of positive formal dimension, then X^G cannot be mod p acyclic. Thus, for $G = \mathbb{Z}/p$, Brendon's Theorem also implies 3.3 and 3.4.

To point out a concrete example confirming the above results, we consider an example of a complex projective surface X on which the group $G = \mathbb{Z}/p$ acts with only one fixed point. The link of this point is a rational homology sphere, in fact the 3–dimensional classical Lens space $L^3(\mathbb{Z}/5)$, which is not a $(\mathbb{Z}/5)$ – homology sphere.

<u>3.5. Example</u>. Let X be the quintic hypersurface in $\mathbb{P}^3(\mathbb{C})$ given by the equation $x_1^5 + x_2^5 + x_3^5 + x_1x_2x_4^3 = 0$ where (x_1,x_2,x_3,x_4) are the homogeneous coordinates of $\mathbb{P}^3(\mathbb{C})$. The action of $\mathbb{Z}/5$ on $\mathbb{P}^3$ is given by $(\varepsilon,x_i) \longmapsto \varepsilon^i x_i$ where ε is a fifth root of unity generating $\mathbb{Z}/5$. As one computes easily, X is invariant under $\mathbb{Z}/5$ and of the four fixed points in $\mathbb{P}^3$, exactly one point lies in X , namely $P = (0,0,0,1)$. At P , X is analytically isomorphic to the affine hypersurface $xy = z^5$ near the origin, by the Morse Lemma. This is a rational double point of type A_4 and it is a quotient $\mathbb{C}^2/(\mathbb{Z}/5)$ where $\mathbb{Z}/5$ acts by $(\varepsilon,(s,t)) \longmapsto (\varepsilon s,\varepsilon^4 t)$. Here, $x = s^5$, $y = t^5$, $z = st$. On the other hand, X is nonsingular at any point different from P . Thus, the link of the singularity is the classical lens space $L^3(1,4)$ with fundamental group $\mathbb{Z}/5$ which is a rational homology sphere, but not a (mod 5) – homology sphere. In particular, X does not satisfy Poincaré duality with $(\mathbb{Z}/5)$ – coefficients, although it is a rational Poincaré complex.

Finally, it appears that the analogues of Corollary 2.3 and 2.4 for varieties defined over fields of positive characteristic remain true when we formulate them in terms of local cohomology.

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