Mapping class groups

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Every finite symmetric set G of generators for a finitely generated group Γ defines a *word norm* $\| \,\|_G$ on Γ and hence a distance function $d_G(h, h') = \|h^{-1}h\|_G$.

The *extended mapping class group* $\mathcal{M}^0(S)$ of a closed surface S of genus $g \geq 2$ is the *outer automorphism group* $\mathrm{Out}(\pi_1(S))$ of the fundamental group $\pi_1(S)$ of S. If $g \neq 2$ it is also precisely the isometry group of the *complex of curves* $\mathcal{C}(S)$ for S. The one-skeleton of this complex of curves is the (locally infinite) graph whose vertices are the free homotopy classes of simple closed curves on S and where two such vertices are connected by an edge if and only if they can be realized disjointly.

Since $\pi_1(S)$ is center-free, there is an exact sequence

$$0 \to \pi_1(S) \to \mathrm{Aut}(\pi_1(S)) \xrightarrow{\Pi} \mathcal{M}(S) \to 0.$$

The pre-image under Π of every finitely generated subgroup Γ of $\mathcal{M}^0(S)$ is then a finitely generated group. We characterize those subgroups Γ of $\mathcal{M}(S)$ for which $\Pi^{-1}(\Gamma)$ is word hyperbolic by their action on the complex of curves: Namely, for an arbitrary $\alpha \in \mathcal{C}(S)$ the orbit map $g \in \Gamma \to g\alpha \in \mathcal{C}(S)$ is a quasi-isometric embedding of Γ into $\mathcal{C}(S)$ if and only if the group $\Pi^{-1}(\Gamma)$ is word hyperbolic.

Moreover, the extended mapping class group $\mathcal{M}^0(S)$ acts properly discontinuously and cocompactly on a directed metric graph whose vertices are the complete *train tracks* on S. This is used to show that $\mathcal{M}^0(S)$ admits a biautomatic structure.

References:

U. Hamenstädt, *Word hyperbolic extensions of surface groups*,

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