Abstracts of the Conference "Partial Differential Equations"

Potsdam, September 6-10, 1993

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MPI / 93-90

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Preface

The present volume presents the abstracts of the lectures held in the international conference

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at the department of mathematics, University of Potsdam. The conference was supported by the Max-Planck-Gesellschaft, the Deutsche Forschungsgemeinschaft (Sonderforschungsbereich 288 "Geometrie und Quantenphysik"), and the Land Brandenburg.

The conference 1993 continued a series of earlier meetings, (Ludwigsfelde 1976, Reinhardsbrunn 1985, Holzhau 1988, Breitenbrunn 1990, Lambrecht 1991, Potsdam 1992, cf. MPI-Preprint 93-7). The general idea of the series is to bring together specialists in analysis, mathematical physics and geometry and to point out interactions and common aspects in the recent development of these fields.

Acknowledgement: The editors are indepted to Frau M. Bernhard, Frau Ch. Gottschalkson, and to the colleagues of the Max-Planck-group S. Behm, Ch. Dorschfeldt, T. Hirschmann, I. McGillivray, E. Ouhabaz, E. Schrohe for their effort in organizing the conference and carrying out the technicalities of this volume.

Potsdam, 30. November 1993

M. Demuth B.-W. Schulze

List of abstracts

Name	Title
Balslev, E.	Perturbation of embedded eigenvalues of Laplacians on hyperbolic manifolds
Bennish, J.	Mixed boundary value problems for elliptic and hyperbolic operators
Brzeźniak, Z.	The trace formula for the Schrödinger group, Gutzwiller trace formula and classical periodic orbits
Coulhon, T.	Liouville property and roughly isometric manifolds
Demuth, M.	Perturbation of analytically continued Dirichlet resolvents
Fedosov, B.V.	Symplectic reduction in deformation quantizations
Georgescu, V.	N-body Hamiltonians with hard-core interactions
Gramsch, B., Kaballo, W.	Multiplicative decompositions of holomorphic Fredholm functions for pseudo-differential operators and Ψ^*-algebras
Grigor'yan, A.	Derivatives of the heat kernel on a Riemannian manifold
Hoffmann-Ostenhof, M.	Regularity properties of the zero-set of solutions of Schrödinger equations
Höppner, W.	A variational formulation of ferromagnetism - on the regularity of the extremals
Holden, H.	On some stochastic partial differential equations
Jacob, N.	On the Dirichlet problem for pseudo-differential operators generating Feller semigroups
Khimshiashvili, G.	Some topological aspects of elliptic boundary problems
Komech, A.I., Merzon, A.E.	Stationary diffraction problems on the wedges with general boundary value conditions
Kondratiev, V.	On asymptotics of solutions of nonlinear elliptic equations in a neighbourhood of a conic point of the boundary
Konstantinov, A.Yu.	Eigenfunction expansion of multiparameter spectral problems
Kounchev, O.	Interior boundary value problems with singular interior boundary
Kutev, N., Lions, P.-L.	Nonlinear second order elliptic equations with jump discontinuous coefficients
Landis, E.M.	On non-divergent semi-linear elliptic equations with discontinuous coefficients
Lange, H., Kavian, O., Zweifel, P.F.	Periodic and stationary solutions to the Schrödinger-Poisson and Wigner-Poisson systems

Name	Title
Leichtnam, E.	Geometric remarks on an inversion formula for the heat equations
Lumer, G.	Diffusion phenomena with shocks: Results on pure and applied problems
Malamud, M.M.	Estimates for a system of differential operators
McGillivray, I.	A Feller property for some degenerate elliptic operators
Mikhailets, V.	Spectrum of the elliptic operator and boundary conditions
Neidhardt, H., Zagrebnov, V.	Singular perturbations and extension theory
Ouhabaz, E.	On spectra of elliptic and Schrödinger operators
Paneah, B.	Degenerate elliptic boundary value problems for weak coupled systems, solvability and maximum principle
Robinson, D.W.	Asymptotics of heat flow
Roitberg, I.Ya., Roitberg, Ya.A.	Green's formula for general parabolic problems and some of its applications
Roitberg, Ya.A.	Local increasing of smoothness of generalized solutions of elliptic boundary value problems in non-smooth domains
Saito, Y., Pladdy, Ch., Umeda, T.	Radiation condition for Dirac operators
Schrohe, E.	Boundary value problems in Boutet de Monvel's algebra for manifolds with conical singularities
Schulze, B.-W.	The wedge Sobolev spaces with branching discrete asymptotics
Seiler, R.	On the index of pair of projectors
Semenov, Yu.A.	Two-sided bounds on the heat kernel for the Schrödinger operator
Sheftel, Z.	Approximation by solutions of non-local elliptic problems
Shushkov, A.	Superoperators and existence of the wave operator
Siedentop, H.	Bound on the density at the nucleus
Sternin, B., Shatalov, V.	On a notion of resurgent function of several variables
Stollmann, P.	Absence of absolutely continuous spectra for high barriers
Sturm, K.-Th.	Analysis on local Dirichlet spaces
Tarkhanov, N., Fischer, B.	Structure of singularities of solutions of elliptic equations
Vogelsang, V.	On the time-asymptotic behaviour of the solutions of the Dirac equation for long range potentials
Weis, L.	A perturbation theorem for unbounded semi-groups and Schrödinger operators
Yagdjian, K.	Operators with multiple characteristics and related pseudodifferential operators

PERTURBATION OF EMBEDDED EIGENVALUES OF LAPLACIANS ON HYPERBOLIC MANIFOLDS.

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Let Δ be the Lapace-Beltrami operator on a non-compact finite-area Riemann surface $\Gamma\backslash h$, where Γ is a discrete subgroup of $P-SL_2(R)$ and h is the Poincaré halfplane. Selberg [Se] proved that for a congruence subgroup Γ_p of $P-SL_2(R)$, Δ has infinitely many eigenvalues embedded in the continuous spectrum $[\frac{1}{4},\infty]$. This conjecture has been disproved by Phillips and Sarnak [PS] under certain assumptions such as a generalized Lindelöf hypothesis. Their method consists in proving that sufficiently many cusp forms become resonances under a perturbation in the Teichmüller space T_Γ of the group Γ. This in turn is based on an explicit formula for Ima_2, where in the case of a simple eigenvalue $\kappa(\epsilon)=\kappa+a_1\epsilon+a_2\epsilon^2+O(\epsilon^2)$ is the perturbation expansion of the eigenvalue $\kappa(\epsilon)$ of $\Delta(\epsilon)$ for small $\epsilon\neq 0$. The method of proof of Phillips and Sarnak utilizes the Lax-Phillips scattering theory for the automorphic wave equation.

This formula for Ima_2 has been known in the physics literature of Schrödinger operators for a long time under the name of Fermi's Golden rule. It says that if $\Delta(\epsilon)=\Delta+\epsilon L+O(\epsilon^2)$ ($\epsilon\in(-\epsilon_0,\epsilon_0)$) is a real-analytic perturbation of Δ and $\kappa(\epsilon)=\kappa+a_1\epsilon+a_2\epsilon^2+O(\epsilon^3)$ is a Taylor expansion of a simple eigenvalue, then

$$Ima_2=-\frac{\pi}{4r}\sum_{l=1}^{m}\mid(Lu,E_l(\frac{1}{2}+ir,\cdot))\mid^2$$

where $\kappa=\frac{1}{4}+r^2$ and $E_l(s,\cdot)$, $l=1,...,m$ are the Eisenstein series associated with the m cusps (Note that $Re\,a_1=0$). It was proved by B. Simon [Si] utilizing the dilation-analytic theory of [BC], which in that case provided the basis for the application of analytic perturbation theory.

The identity of this formula in the Euclidean and hyperbolic cases suggests the possibility of proving it by the same method in both cases. The basic problem is the separation of the embedded eigenvalues from the continuous spectrum. In the Euclidean case the operator $-\Delta+V$ is transformed by a family of unitary operators induced by dilations of independent variables. Analytic continuation in the scaling parameter leads to a rotation of the continuous spectrum away from the eigenvalues; successively turning resonances into discrete eigenvalues. The analogous transformations in the hyperbolic case are dilations in the hyperbolic distance or, equivalently,

power transformations $U(t)$ of the independent variables. Since the continuous spectrum of Δ is entirely controlled by the 0'th Fourier mode and since the exponentially decreasing cusp forms explode under complex power transformations, these operators should be restricted to the y−coordinates in each cusp in the 0'th Fourier mode. The 0'th Fourier coefficient of the Eisenstein series $E(z,s)$ is transformed by $U(t)$ into a function $a_0(z,s,t)$ which is for large $y = Im\ z$ equal to $y^{\frac{1}{2}+(s-\frac{1}{2})t} + C(s)y^{\frac{1}{2}-(s-\frac{1}{2})t}$ in each cusp. This is why the continuous spectrum of $\Delta(t) = U(t)\Delta U(t^{-1})$ in the s-plane is given by $Arg\ i^{-1}(s-\frac{1}{2}) = -Arg\ t$. This is proved by a calculation of $\Delta(t)$.
The embedded eigenvalues are unchanged, since cusp forms are unchanged by $U(t)$. The 0'th Fourier coefficient of the resonance function with resonance ρ is for large y transformed into $Res_\rho C(s)y^{\frac{1}{2}-(\rho-\frac{1}{2})t}$ which becomes a square-integrable eigenfunction of $\Delta(t)$ for $Arg\, t < -Arg\, i^{-1}\left(\rho-\frac{1}{2}\right)$. Thus, the resonance ρ of Δ becomes an eigenvalue of $\Delta(t)$, when the continuous spectrum of $\Delta(t)$ crosses the resonance. As a consequence of the separation of the continuous spectrum from the eigenvalues Fermi's Golden Rule is now proved by the same proof as the one given by Simon in the case of Schrödinger operators.

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MIXED BOUNDARY VALUE PROBLEMS FOR ELLIPTIC AND HYPERBOLIC OPERATORS.

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The factorization method was used by Eskin to compute the asymptotics for elliptic boundary value problems for a scalar pseudo-differential equation. Other methods were developed by Schulze to treat the systems case as well as other types of problems on manifolds with singularities. In my talk I presented the result that the factorization method extends to elliptic boundary value problems for pseudo-differential systems. In this result the asymptotics are expressed as singular integrals.
The second part of my talk was on mixed initial boundary value problems for second-order hyperbolic equations. The main results are conormal regularity (that is, tangential regularity and regularity in weighted function spaces) and asymptotics for both the mixed Dirichlet-Neumann-Cauchy problem and problems satisfying the Shapiro-Lopatinski condition. The manuscript concerning the hyperbolic results is in preparation.

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THE TRACE FORMULA FOR THE SCHRÖDINGER GROUP, GUTZWILLER TRACE FORMULA AND CLASSICAL PERIODIC ORBITS

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The eigenvalues of the quantum harmonic oscillator $H_0 = -\frac{1}{2}\Delta + \frac{1}{2}\sum_{j=1}^d \omega_j^2 x_j^2$ acting in $L^2(I\!R^d)$ are well known, $\lambda_\alpha = \frac{1}{2}|\omega| + \alpha\cdot\omega$, where $\alpha \in I\!N^d$ and $\alpha\cdot\omega = \sum \alpha_j\omega_j$, $|\omega| = \sum \omega_j$. If we define $\operatorname{tr} e^{-itH_0} := \sum_\alpha e^{-it\lambda_\alpha}$ as a tempered distribution (in $t \in I\!R$) then one can easily calculate that $\operatorname{tr} e^{-itH_0} = e^{\frac{-it|\omega|}{2}} \prod_{j=1}^d (1 - e^{-it\omega_j})$, still in distributional sense. An easy consequence of the last formula is

$$\text{sing supp} \operatorname{tr} e^{-itH_0} = \{t \in I\!R : e^{-it\omega_j} \text{ for some } j \in \{1, \dots, d\}\}. \tag{1}$$

A natural question then arises as to what extent is the above property characteristic of the harmonic oscillator ?

Related problems have been studied (but for first order differential (or pseudo differential) operators on compact manifolds) by Y. C. de Verdier, J. Chazarin, Duistermaat and Guillemin, Melrose. For the Schrödinger equation case, see: Albeverio, Blanchard, Høegh–Krohn, Comm. Math. Phys. (1982), Boutet de Monvel–Berthier A., Boutet de Monvel L., Lebeau G., J. d'Anal. Math. (1993), Albeverio, Boutet de Monvel-Berthier A., Brzeźniak, The trace formula for Schrödinger operators from infinite dimensional oscillatory integrals, preprint (1992), Albeverio, Brzeźniak, Acta Math. Appl. (1994). In this talk we mainly follow the third paper cited above. We present two results from this work.

Theorem 1 *Let for a strictly positive symmetric matrix Ω^2 and for $h > 0$, $H_0(h) = -\frac{h}{2}\Delta + \frac{1}{2h} < \Omega^2 x, x >$ and $H(h) = -\frac{h}{2}\Delta + \frac{1}{2h} < \Omega^2 x, x > + \frac{1}{h}V_0(x)$ be respectively the free and the perturbed quantum harmonic oscillators (which are self-adjoint operators in $L^2(I\!R^d)$). Here we assume that $V_0(x) = \int e^{i <x,y>} d\mu_0(y)$ for some complex measure μ on $I\!R^d$ that has all moments finite.*

Then $\text{sing supp} \operatorname{tr} e^{-itH(h)}$ can be defined in a distributional sense as before and the following holds

$$\text{sing supp} \operatorname{tr} e^{-itH(h)} \subset \text{sing supp} \operatorname{tr} e^{-itH_0(h)}. \tag{2}$$

Since the RHS of (2) is h independent one may ask for the small h asymptotics of $\text{sing supp} \operatorname{tr} e^{-itH(h)}$. For this we also need some preliminary notation. Denote the total potential by $V_1(x) = \frac{1}{2} < \Omega^2 x, x > + V_0(x)$, $x \in I\!R^d$. We make the following assumptions on V_1. The set $\{x : \; V_1'(x) = 0\}$ is finite (and its elements will be denoted by $c_1, \ldots, c_{s_0}$, with $s_0 \in I\!N \setminus \{0\}$) and $\det \; V_1''(c_j) \neq 0$, $\det \cos \sqrt{t \; V_1''(c_j)} \neq 0$ for $j = 1, \ldots, s_0$. $t > 0$ is fixed such that any t–periodic, non-constant solution to the classical Hamiltonian system

$$\ddot{\gamma}(s) + V_1'(\gamma(s)) = 0, \quad 0 < s < t \tag{3}$$

is a nondegenerate periodic solution, see I. Ekeland, Convexity methods in Hamiltonian mechanics. One can prove that there exist a finite number of pairwise non-congruent t–periodic[1] solutions $\gamma_1, \ldots, \gamma_{s_1}$ to (3) such that any t–periodic solution γ to (3) is either a constant solution, $\gamma(s) = c_j$, $s \in [0, t]$ for some $j = 1, \ldots, s_0$ or $\gamma = (\gamma_j)_\tau$ for some $j \in \{1, \ldots, s_1\}$, $\tau \in [0, t]$. We also need the notion of oscillatory integral over a (infinite dimensional) Hilbert space, which we do not recall here. Then we have

Theorem 2 *If* $\mathcal{H}_{p,t} = \{\gamma \in H^1(0, t; I\!R^d) : \gamma(0) = \gamma(t)\}$ *is a Hilbert space with norm given by* $\|\gamma\|^2 = \int_0^t \{|\dot{\gamma}(s)|^2 + |\gamma(s)|^2\} \, ds$ *then*

$$\operatorname{tr} e^{-itH(h)} = k_t \tilde{\int_{\mathcal{H}_{p,t}}} e^{\frac{i}{2h}\{\|\gamma\|^2 - < B\gamma, \gamma> - <C\gamma, \gamma>\} - \frac{i}{h} V(\gamma)} \, d\gamma, \tag{4}$$

where the integral (with˜) on the RHS is the oscillatory integral mentioned before, B and C are trace class linear operators in $\mathcal{H}_{p,t}$ defined by $< B\gamma, \gamma > = \int_0^t < \Omega^2 \gamma(s), \gamma(s) > \, ds$, $< C\gamma, \gamma > = \int_0^t |\gamma(s)|^2 \, ds$, *while* $V(\gamma) = \int_0^t V_0(\gamma(s)) \, ds$. *Moreover, with some additional assumptions on the small potential V_0, and assuming that* $\det \sin(\frac{t}{2}\Omega) \neq 0$ *the following asymptotic formula as $h \searrow 0$ holds*

$$\begin{aligned} \operatorname{tr} e^{-itH(h)} &= \sum_{j=1}^{s_0} e^{\frac{i}{h} t V_1(c_j)} I_j^*(h) \\ &+ (2\pi i h)^{-\frac{1}{2}} \sum_{j=1}^{s_1} e^{\frac{i}{h} \int_0^t \left(\frac{1}{2}|\dot{\gamma}_j(s)|^2 - V_1(\gamma_j(s))\right) ds} I_j^{**}(h) + h^{-\frac{1}{2}} O(h), \end{aligned} \tag{5}$$

where I_j^* and I_j^{**} are C^∞ functions on $I\!R$ such that $I_j^*(0) = \left\{\det \cos \sqrt{t V_1''(c_j)}\right\}^{-\frac{1}{2}}$ *and* $I_j^{**}(0) = t(d_j)^{-\frac{1}{2}} \left\{\int_0^t \{|\dot{\gamma}_j(s)|^2 + |V_1'(\gamma_j(s))|\} \, ds\right\}^{\frac{1}{2}}$ *and d_j is the determinant of the linearization of the Poincare map corresponding to γ_j.*

[1] Two periodic solutions to (3) are called congruent iff each of them can be obtained from the other by means of time translation. If γ is such a solution, then by γ_τ we denote the translation of γ by time τ, i.e. $\gamma_\tau(s) = \gamma(s + \tau)$, if $s + \tau \leq t$ or $\gamma_\tau(s) = \gamma(s + \tau - t)$ otherwise.

Let us remark that for studying the limit $h \searrow 0$ we use results from two other papers: Albeverio, Brzeźniak, J. Funct. Anal. (1993) and Rezende, Comm. M. Phys. (1984).

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LIOUVILLE PROPERTY AND ROUGHLY ISOMETRIC MANIFOLDS.

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This is a report on a joint work with Laurent Saloff-Coste ([3]). We consider the equivalence relation of rough isometry between measured metric spaces, as introduced by Chavel and Feldmann [1], following Kanai [4]. Under very weak local geometry assumptions, we associate with a riemannian manifold M a weighted graph X which is roughly isometric to M. Several analytic features, such as the volumegrowth, the Sobolev type inequalities, transience or recurrence, can be transfered from M to its discretisation X or back from X to M; moreover, it can be checked that they are preserved under rough isometries between graphs. Therefore, all these properties are preserved under rough isometries between manifolds. This is nothing but a systematization of the work of Kanai (see [4], references herein and also [2]). In addition, it can be shown that the scaled Poincaré inequalities on balls of large radius are also preserved. Since Saloff-Coste has shown in [6] that the parabolic Harnack inequality is invariant under rough isometry between manifolds satisfying mild local geometry assumptions. As a corollary, one has that a manifold with Ricci curvature bounded from below, which is roughly isometric either to a manifold with non-negative Ricci curvature, or to a polynomial growth Lie group, has the strong Liouville property, i.e. there exist no non-trivial polynomial positive harmonic functions.
Let us recall in contrast the result of Lyons [5] that the Liouville property is not invariant under quasi-isometry.

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PERTURBATION OF ANALYTICALLY CONTINUED DIRICHLET RESOLVENTS.

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Let $(H_0)_\Sigma$ be the Dirichlet Laplacian in $L^2(\Sigma)$ where Σ is some open region in $\mathbb{R}^d$ such that $\mathbb{R}^d - \Sigma$ is bounded. Let $H_\beta = H_0 + \beta 1_{\mathbb{R}^d - \Sigma}$, with $H_0 = -\Delta$, $\beta > 0$. Assume that the sandwiched resolvent $e^{-|x|}((H_0)_\Sigma + z)^{-1} e^{-|x|}$ has an analytic continuation into a part G_∞ of the second sheet in the lower half-plane. A quantitative analyticity condition is given which ensures that $e^{-|x|}(H_\beta + z)^{-1} e^{-|x|}$ is analytic in a set $G_\beta \subset G_\infty$. Moreover, for large β the distance between the boundaries of G_β and G_∞ is estimated from below in terms of β.

SYMPLECTIC REDUCTION IN DEFORMATION QUANTIZATION

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Let $W(M)$ be the Weyl algebra bundle over a symplectic manifold (M, ω). Let D denote an Abelian connection on $W(M)$ with the curvature

$$\Omega_D = -\frac{i}{h}\omega,$$

and let $W_D(M)$ be the algebra of flat sections of $W(M)$ with respect to D. This algebra is called a deformation quantum algebra.

Let $H = H_0 + H_1 + \ldots$ be an element of $W_D(M)$ with a real function $H_0(x)$ as a leading term. Suppose the following conditions are fulfilled:

(i) $H_0(x)$ generates a 2π-periodic Hamiltonian flow, that is, a symplectic action of the group $U(1)$ on M,

(ii) $M_0 = \{H_0 = 0\}$ is a noncritical level manifold, which is compact and connected,

(iii) the orbit space $B = M_0/U(1)$ is a smooth manifold,

(iv) any solution of the Heisenberg equation

$$\dot{a} = \frac{i}{h}[H, a]$$

in $W_D(M)$ is 2π-periodic, that is the group $U(1)$ acts on $W_D(M)$ by automorphisms.

Under these conditions we prove a reduction theorem for the deformation quantum algebra, similar to the classical reduction theorem of Marsden-Weinstein.

Define the reduced quantum algebra as

$$R = A_H/J_H,$$

where A_H is the subalgebra of $W_D(M)$ consisting of elements commuting with H and J_H is an ideal in A_H generated by H. Let ω_B denote the Marsden-Weinstein symplectic form on B. There exists an Abelian connection $\tilde{D}$ on the Weyl algebra bundle $W(B)$ with the curvature

$$\Omega_{\tilde{D}} = -\frac{i}{h}\omega_B + \omega_0 + h\omega_1 + \dots,$$

where $\omega_0, \omega_1 \dots$ are closed two-forms on B, such that the reduced algebra R is isomorphic to the algebra $W_{\tilde{D}}(B)$ of flat sections of $W(B)$ with respect to the connection $\tilde{D}$.

This theorem allows us to construct an eigenstate functional on $W_D(M)$, that is, a functional $\langle a \rangle$ with values in $h^{-(n-1)}\mathbf{C}[[h]]$ having the property $\langle Ha \rangle = \langle aH \rangle = 0$ for any $a \in W_D(M)$. Combining this construction with the index theorem for quantum algebras, we obtain necessary conditions for existence of an asymptotic operator representation of the algebra $W_D(M)$ such that the operator $\hat{H}$ corresponding to H has a cluster of eigenvalues near zero. These conditions yield the Bohr–Sommerfeld quantization conditions and multiplicity of clusters.

N-BODY HAMILTONIANS WITH HARD-CORE INTERACTIONS.

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This is a resume of a work with Anne Boutet de Monvel-Berthier and Amy Soffer. Let X be an euclidean space, L a finite lattice and for each $a \in L$ let X^a be a subspace of X such that $a < b$ iff $X^a \subset X^b$ strictly, $X^c = X^a + X^b$ if $c = sup(a,b)$, $X^0 = \{0\}$ if $0 = inf L$, $X^1 = X$ if $1 = sup L$. We consider potentials $V^a : X^a \to R$ which have a short-range and a long-range component verifying rather standard conditions, and "hard-cores" $K^a \subset X^a$ which are compact, star-shaped with respect to the origin and with boundary of class C^1. Denote Π_a the orthogonal projection of X onto X^a and $V(a) = V_0^a \Pi_a$, $\psi(a)$ the characteristic function of the cylinder $\Pi_a^{-1}(K)$ with base K^a. For any number $\alpha > 0$, we have a selfadjoint operator $H_\alpha = \Delta + \sum_{a \in L}[V(a) + \alpha\psi(a)]$ in $\mathcal{H} = L^2(X)$, with form-domain $\mathcal{H}^1$ ($\mathcal{H}^s$ are the usual Sobolev spaces on X and $\mathcal{H}^s_t$ are the weighted Sobolev spaces defined by the norms $\left\|(1+p^2)^{\frac{s}{2}}(1+Q^2)^{\frac{t}{2}}u\right\|$, $p = -i\,\partial_x$, Q = multiplication by x). For each complex non-real z, $(z - H_\alpha)^{-1} \to R(z)$ as $\alpha \to +\infty$, strongly in $B(\mathcal{H}^{-1}, \mathcal{H}^{+1})$. The family $\{R(z) | z \in C, Im\, z \neq 0\}$ is a selfadjoint pseudo-resolvent and defines the selfadjoint *non-densely defined* hard-core Hamiltonian H in $\mathcal{H}$. We show that a generalized form of the conjugate operator method combined with graded C^*-algebra techniques can be used to prove an optimal version of the limiting absorption principle for H (as usual in N-body theory, the conjugate operator is the generator of the dilation group in X). There is a closed, countable set $\kappa(H) \subset R$ (the critical set composed of thresholds and eigenvalues) such that for real $\lambda \notin \kappa(H)$ $\lim_{\mu \to \pm 0} R(\lambda + i\mu) = R(\lambda \pm i0)$ exists in $B(\mathcal{H}^{-1}_t, \mathcal{H}^{+1}_{-t})$. For $t > \frac{1}{2}$ a more precise result holds in fact. In particular, H has no singular continuous spectrum and local decay holds at non-critical energies.

An important technical point of the proof is a regularity result for the Dirichlet problem in a non-smooth domain Ω for the Laplace operator Δ: if Ω has the uniform interior cone property and the uniform exterior ball property, then $u \in \mathcal{H}^1_0(\Omega)$ and $\Delta u \in L^2(\Omega)$ imply $u \in \mathcal{H}^2(\Omega)$.

Multiplicative decompositions of holomorphic Fredholm functions for pseudo–differential operators and Ψ^*–algebras.

B. Gramsch and W. Kaballo

Let Ψ denote the submultiplicative Fréchet algebra of Hörmander classes $\Psi^\circ_{\rho,\delta}$, $0 \leq \delta \leq \rho \leq 1$, $\delta < 1$, embedded into $\mathbb{L}(H)$, $H = L^2(\mathbb{R}^n)$. Let $\alpha_k(B)$, $k = 0, 1, 2, \ldots$, be the approximation numbers of $B \in \mathbb{L}(H)$ and let $\beta_k \geq \beta_{k+1} > 0$ be a null sequence. Define $\mathcal{J}_{<\beta>} := \{B \in \mathbb{L}(H) : \sup_k \beta_k^{-1} \alpha_k(B) < \infty\}$. Furthermore let $\Omega \subset \mathbb{C}^N$ be a holomorphy region and $\Phi(\Psi)$ the set of Fredholm operators $\Phi(H) \cap (\Psi)$.

<u>Theorem.</u> Let $T : \Omega \to \Phi(\Psi)$ be holomorphic and homotopic in $\mathcal{C}(\Omega, \Phi(\Psi))$ to an element of $\mathcal{C}(\Omega, \Psi^{-1})$. Then there exists a holomorphic function $A : \Omega \to \Psi^{-1}$ (group of invertible elements) and $S : \Omega \to \Psi^{-\infty} \cap \mathcal{J}_{<\beta>}$ such that

$$T(z) = A(z)(I + S(z)) \quad , z \in \Omega.$$

<u>Remarks:</u> (1) S is holomorphic with values in a locally convex Fréchet left ideal $\mathcal{I}_T \subset \Psi^{-\infty} \cap \mathcal{J}_{<\beta>}$.

(2) For $N = 1$ no homotopy assumption is needed, in this case there exists a rather sharp result of Leiterer (1978).

(3) Additive decompositions (related to multiplicative decompositions) have been considered e.g. by Krein, Trofimov 1969, Gramsch 1973, Gramsch, Kaballo 1978, 1989.

(4) The theorem above can be proved with slight changes for arbitrary submultiplicative Ψ^*–algebras; it seems to be new also for C^*–algebras of singular integral operators.

DERIVATIVES OF THE HEAT KERNEL

ON A RIEMANNIAN MANIFOLD

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Let M be a smooth connected non-compact geodesically complete Riemannian manifold, Δ be the Laplace operator associated with the Riemannian metric, $n \geqslant 2$ be the dimension of M. We are concerned with the heat kernel $p(x, y, t)$ (where $x, y \in M, t > 0$) being by definition the smallest positive fundamental solution to the heat equation

$$u_t - \Delta u = 0 \tag{0.1}$$

and which is known to exist on any manifold.
A question to be discussed here is estimations of derivatives of the heat kernel. They are based upon upper bounds of the heat kernel itself which were investigated in detail in [2] Let us introduce the notation

$$E_m(x,t) = \int_M \left|\nabla_y^m p\right|^2 (x,y,t) \exp\left(\frac{r^2}{Dt}\right) dy$$

where $r = \operatorname{dist}(x, y)$, $D > 2$ is a given constant and $m = 0, 1, 2, ..$. Let us specify that ∇^m means $\Delta^{m/2}$ if m is even and $\nabla \Delta^{\frac{m-1}{2}}$ if m is odd. In particular, E_0 contains no derivatives of the heat kernel:

$$E_0(x,t) = \int_M p^2(x,y,t) \exp\left(\frac{r^2}{Dt}\right) dy$$

Theorem 1 *If $D > 2$ then for any integer $m \geqslant 0$ the quantity E_m is finite. Moreover, for any $x \in M$ $E_m(x,t)$ is a continuous decreasing function of t. Besides, if E_0 is known to satisfy for some x and for $t \in (0, T)$ the inequality*

$$E_0(x,t) \leqslant \frac{1}{f(t)} \tag{0.2}$$

with a positive continuous function $f(t)$ then E_m is estimated as follows:

$$E_m(x,t) \leqslant \frac{\text{const}_{D,m}}{f_m(t)}, \ m = 1, 2, 3... \tag{0.3}$$

where f_m is the m-th integral of $f(t)$ i.e. is defined by induction:

$$f_0 = f, \ f_{k+1}(t) = \int_0^t f_k(\tau) d\tau, \ k = 0, 1, 2...$$

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Of course, one need the initial estimate (0.2) of E_0 in order to be able to apply this theorem and to obtain the inequality (0.3) . The necessary estimates of E_0 can be found in [2] . We only note here that the function $f(t)$ from (0.2) is expressed through some isoperimetric properties of the manifold.

The second result to be presented here is pointwise estimates of the time derivatives of the heat kernel. They can be obtained from the integral estimates (0.3) due to the fact that $\frac{\partial}{\partial t}p = \Delta p$ and upon application of the semigroup property.

Theorem 2 *Suppose that for two points $x, y \in M$ it is known that for all $t \in (0,T)$*

$$E_0(x,t) \leqslant \frac{\mathrm{const}}{f(t)},\ E_0(y,t) \leqslant \frac{\mathrm{const}}{g(t)}, \tag{0.4}$$

where $C_{1,2}$ are constants and f, g are continuous, increasing functions on $(0,T)$ such that the functions $\log f(t)$ and $\log g(t)$ are concave , then for any integer $m \geqslant 0$ and $t \in (0,2T)$

$$\left|\frac{\partial^m}{\partial t^m}p\right|(x,y,t) \leqslant \frac{\mathrm{const}}{\min\left(\sqrt{f(\frac{t}{2})g_{2m}(\frac{t}{2})}, \sqrt{g(\frac{t}{2})f_{2m}(\frac{t}{2})}\right)} \exp\left(-\frac{r^2}{2Dt}\right) \tag{0.5}$$

where $r = \mathrm{dist}(x,y)$.

For example, let

$$f(t) = \mathrm{const}\begin{cases} t^{\nu} & , t \leqslant 1 \\ t^{\mu} & , t > 1 \end{cases} \tag{0.6}$$

where $\nu, \mu > 0$ and suppose for the sake of simplicity that the estimate (0.2) holds for all x and for all $t > 0$. Then by Theorem 1 we have the estimate (0.3) which implies by Theorem 2 the pointwise upper bound (0.5) which acquires the form

$$\left|\frac{\partial^m}{\partial t^m}p\right|(x,y,t) \leqslant \frac{\mathrm{const}}{t^m f(t)} \exp\left(-\frac{r^2}{2Dt}\right) . \tag{0.7}$$

Let us note that the estimate (0.2) with the function (0.6) can be deduced from the pointwise estimate of the heat kernel:

$$p(x,x,t) \leqslant \frac{\mathrm{const}}{f(t)} \tag{0.8}$$

supposed to be true for all $x \in M$ and $t > 0$. Hence, (0.8) implies (0.7) . This fact was known before (see [1]) but the theorems 1,2 with the results of [2] enable us to do the same for a more general function $f(t)$ rather than (0.6) .

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REGULARITY PROPERTIES OF THE ZERO-SET OF SOLUTIONS OF SCHRÖDINGER EQUATIONS.

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Joint work with T.Hoffmann-Ostenhof and N.Nadirashvili.

Let $u \not\equiv 0$ be a real-valued distributional solution of the Schrödinger equation

$$(-\Delta + V)u = 0 \text{ in } \Omega,$$

Ω a domain in $I\!R^n$ and $V \in L^1_{loc}(\Omega)$, V real-valued. For $V \in K^{n,\delta}(\Omega)$ for some $\delta \in (0,1)$, $u \in C^{0,\delta}(\Omega)$. Let B_R denote the ball centred at the origin with radius R and $B_R \subset \Omega$ and let $N_u^{(1)}$ denote the set of points in B_R where u vanishes in first order. Based on a recent result [M.H.-O. and T.H.-O. 1992] we show that there is a constant $C < \infty$ such that $\forall x_0 \in N_u^{(1)} \cap B_{R/2}$ and $\forall \delta' < \delta$

$$| u(x) - \nabla u(x_0)(x - x_0) | \leq C \mid x - x_0 \mid^{1+\delta'}, \ \forall x \in B_R,$$

where $C = C(V, \delta, \delta', n, R, \sup\{| \ u(x) \ |: x \in B_R\})$. Using this estimate we then prove that $N_u^{(1)}$ is locally a $(n-1)-$dimensional hypersurface which is the graph of a $C^{1,\delta'}-$function, and hence "more regular" than u itself.

A VARIATIONAL FORMULATION OF FERROMAGNETISM – ON THE REGULARITY OF THE EXTREMALS.

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We consider a ferromagnetic body Ω under the presence of an applied magnetic field $\vec{H}_a$. If we neglect boundary anisotropy and magnetoelastic effects we may assume that the free energy U of Ω is given by the formula

$$U(\vec{M}) = \frac{c}{2}\int_\Omega \sum_{i,j}\left(\frac{\partial M_i}{\partial x_j}\right)^2 dx - \frac{1}{2}\int_\Omega \vec{H}_d(\vec{M})\vec{M}\,dx - \int_\Omega \vec{H}_a\vec{M}\,dx + \int_\Omega \varphi(\vec{M})\,dx$$

Here we have denoted by $\vec{M}$ the magnetization of Ω and by $\vec{H}_d(\vec{M})$ the demagnetizing field. The function φ is a polynomial. The integrals represent the exchange energy, the energy of the magnetic fields and the energy due to crystal anisotropy. If the temperature of Ω is constant the magnetization $\vec{M}$ has constant modulus, $|\vec{M}(x)| = M_s$, $x \in \Omega$ (cf.[3] and, for the mathematical setting, [4]).
Thus we consider the variational problem

$$U(\vec{M}) = \text{Min!} \tag{1}$$

on the set

$$W_2^1(\Omega, S^2) = \left\{\vec{M} \in W_2^1(\Omega)^3 \,\middle|\, |\vec{M}(x)| = M_s \text{ a.e}\right\} \tag{2}$$

of Sobolev mappings.

Theorem 1 *Let $\vec{M}$ be an absolute minimum of (1),(2). Then there exists an open set $\Omega_1 \subset \Omega$ such that $\vec{M}$ is smooth in Ω_1 and the one-dimensional Hausdorff measure of $\Omega - \Omega_1$ vanishes.*

Corresponding results are known for harmonic mappings. Our proof is based on a theorem of Giaquinta [2] on partial Hölder continuity and on a modification of the methods used by Evans [1] for studying the regularity of harmonic mappings.

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ON SOME STOCHASTIC PARTIAL DIFFERENTIAL EQUATIONS.

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In my talk I discussed the two related stochastic partial differential equations

$$\begin{gathered} u_t + f(u)_x = h(x,t,u) + g(u)W(t) \\ u(x,0) = u_0(x), \; x \in I\!R \end{gathered} \tag{1}$$

and

$$\begin{gathered} u_t + \lambda(u,\nabla)u = v\Delta u + w \\ u(x_1,...,x_n,0) = u_0(x_1,...,x_n) \end{gathered} \tag{2}$$

In (2), $u = (u_1(x_1,...,x_n,t)),...,u_n(x_1,...,x_n,t)$ and $(u,\nabla) = \sum_{j=1}^n u_j \frac{\partial}{\partial x_j}$. In the equations above, W and w both represent white noise. The initial data in (1) and (2) are deterministic.

In our analysis of (1), jointly with N.H.Rinebro (Oslo) [1], we interpret the equation in the weak sense, i.e., u is a solution of (1) iff

$$\begin{aligned} &\int_0^\infty dt \int_I\!R dx\, [\varphi_t u + f(u)\varphi_x] + \int_I\!R dx u_0(x)\varphi(x,\cdot) \\ &\quad = -\int_I\!R dx \int dB(t)\varphi g + \int_I\!R dx \int_0^\infty dt\, h\,\varphi \end{aligned} \tag{3}$$

for all $\varphi \in C_0^1(I\!R \times [0,\infty))$. The stochastic integral is interpreted in the Ito sense.

By an operator splitting technique, also called the fractional steps method, where we iterate beween the conservation law $u_t + f(u)_x = h$ and the stochastic differential equation $u_t = gW(t)$ interpreted as $du = g(u)\, dB(t)$, we prove existence of a solution of (1). In addition we provide a numerical method and illustrate the result with an example from flow in porous media.

The methods we use to analyze (2) are based on the so-called white noise analysis, and the results are joint with T.Lindstrøm, B.Øksendal (Oslo), J.Ubøe, T.-S.Zhang

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(Hangesund) [2]. Let $\mathcal{S}$ and $\mathcal{S}'$ denote the Schwartz space of rapidly decreasing functions on $I\!R$ and its dual respectively. Applying the Bochner-Minlos theorem we obtain the probability space $(\mathcal{S}', \mu)$ where μ is determined by

$$\int_{\mathcal{S}'} e^{i<w,\varphi>}\, d\mu(\omega) = e^{-\frac{1}{2}\|\varphi\|^2},\ \varphi \in \mathcal{S} \tag{4}$$

where $\|\cdot\|$ is the L^2-norm. We recover white noise as

$$\begin{aligned} W : \mathcal{S}' \times \mathcal{S} &\longrightarrow I\!R \\ W(\omega, \varphi) &= \langle \omega, \varphi \rangle \end{aligned} \tag{5}$$

Brownian motion can be determined by $B(x) = \langle \omega, X_{[0,x]} \rangle$. Let $h_n(x)$ denote the $n-$th Hermite polynomial, and $\xi_n(x) = c_n e^{-\frac{x^2}{2}} h_n(\sqrt{2}x)$. Then $\{\xi_n\}$ is an orthonormal basis for $L^2(I\!R)$. The Wiener-Ito chaos theorem says that $\left\{H_\alpha = \Pi_{j=1}^m h_{\alpha_j}\left(\langle w, \xi_j \rangle\right)\right\}$ with $\alpha = (\alpha_1, ..., \alpha_m)$, constitute an orthogonal basis for $L^2(\mathcal{S}', \mu)$.

Thus, if $X \in L^2(\mathcal{S}', \mu)$, we can write $X = \sum_\alpha c_\alpha H_\alpha$ uniquely. With this we can define the Wick product as

$$X \diamond Y = \sum_{\alpha,\beta} c_\alpha d_\beta H_{\alpha+\beta} \tag{6}$$

for $Y = \sum_\beta d_\beta H_\beta \in L^2(\mathcal{S}', \mu)$. One can prove that

$$\int Y_t\, dB_t = \int Y_t \diamond W_t\, dt \tag{7}$$

where we have on the left side a classical Ito integral of the adapted process Y_t. With this we interpret equation (2) as

$$u_t + \lambda u \diamond \frac{\partial u}{\partial x} = \nu \nabla u + W \tag{8}$$

For simplicity of notation we consider the scalar case, i.e., $n = 1$. Assume that $u = -\frac{\partial X}{\partial x}$ and $W = -\frac{\partial N}{\partial x}$. Then we obtain the KPZ-equation

$$\frac{\partial X}{\partial t} = \frac{\lambda}{2} \left(\frac{\partial X}{\partial x}^2 \right) + \nu \Delta X + N. \tag{9}$$

If we furthermore write $Y = Exp\left(\frac{\lambda}{2\nu} X\right)$ we obtain the heat equation with a stochastic potential, viz.

$$\begin{aligned} \frac{\partial Y}{\partial t} &= v \Delta Y + \frac{\lambda}{2\nu} Y \diamond N \\ Y(x, 0) &= f(x) \end{aligned} \tag{10}$$

We can solve this equation explicitly as

$$Y(t,x) = \hat{E}^x \left[f(b_{\alpha t}) Exp\left(\frac{\lambda}{2\nu} \int_0^t N(s, b_{\alpha s})\, ds \right) \right] \tag{11}$$

with $\alpha = \sqrt{2\nu}$ and $\hat{E}$ denotes expectation with respect to Brownian motion $(b_t, \hat{P}^x)$. For more precise statements of results and assumptions, as well as references to relevant literature, we refer to our papers listed below.

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On the Dirichlet problem for pseudo differential operators generating Feller semigroups

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In [2] we proved that there exists a large class of pseudo differential operators p(x,D) generating a Feller semigroup and satisfying the following estimates

(1) $$\|p(x,D)u\|_{a^2,s} \leq c\|u\|_{a^2,s+1},$$

(2) $$\|u\|_{a^2,s+1} \leq c(\|p(x,D)u\|_{a^2,s} + \|u\|_0),$$

(3) $$|B(u,v)| \leq c\|u\|_{a^2,1/2} \cdot \|v\|_{a^2,1/2},$$

(4) $$B(u,u) \geq c_0\|u\|^2_{a^2,1/2} - \|u\|^2_0,$$

where

(5) $$B(u,v) = \int_{\mathbb{R}^n} p(x,D)u(x) \cdot v(x)\, dx,$$

and for a fixed continuous negative definite function $a^2 : \mathbb{R}^n \to \mathbb{R}$, $a^2(\xi) \geq c|\xi|^r$ for $r > 0$ and $|\xi|$ large, the norm $\|.\|_{a^2,s}$ is defined by

(6) $$\|u\|_{a^2,s} = \int_{\mathbb{R}^n} (1 + a^2(\xi))^{2s} \cdot |\hat{u}(\xi)|^2\, d\xi.$$

In case that $p(x,D)$ is symmetric, $(B, H^{a^2,1/2}(\mathbb{R}^n))$ is a Dirichlet space. We are interested in solving the Dirichlet problem

(7) $$\begin{aligned} p(x,D)u &= f \text{ in } \Omega, \ \Omega \subset\subset \mathbb{R}^n, \\ u &= g \text{ in } \Omega^c. \end{aligned}$$

For $g = 0$ and $f \in L^2(\mathbb{R}^n)$ we have the following result for weak solutions

Theorem ([3]) For the representation problem $B(u_0,\varphi) = (f,\varphi)_0$ for all $\varphi \in C_0^\infty(\Omega)$ Fredholm's alternative theorem holds in the space $H_0^{a^2,1/2}(\Omega) := \overline{C_0^\infty(\Omega)}^{\|\cdot\|_{a^2,1/2}}$. Moreover, for any $\psi \in C_0^\infty(\Omega)$ it follows that $\psi u_0 \in H^{a^2,1}(\mathbb{R}^n)$. If in addition $f \in L^2(\mathbb{R}^n) \cap L^p(\mathbb{R}^n)$, $p > \frac{n}{r} \vee 2$, then $u_0 \in L^\infty(\mathbb{R}^n)$.

On the other hand, for $f = 0$ and $g \in H^{a^2,1/2}(\mathbb{R}^n)$ a generalized solution of (7) is given by $u_{pr}(x) = E^x(g(X_{\sigma_\Omega}))$, and $u_{pr} \in H^{a^2,1/2}(\mathbb{R}^n)$, where $(X_t)_{t\geq 0}$ denotes the Feller process generated by $p(x,D)$, and σ_Ω is the stopping time $\sigma_\Omega := \inf\{ t > 0 , X_t \notin \Omega \}$. By a result of M.Fukushima we have $B(u_{pr},\varphi) = 0$ for all $\varphi \in C_0^\infty(\Omega)$. From this and the theorem stated above, it follows for all $\psi \in C_0^\infty(\Omega)$ that $\psi u_{pr} \in H^{a^2,1}(\mathbb{R}^n)$.

The function $u := u_0 + u_{pr} \in H^{a^2,1/2}(\mathbb{R}^n)$ should be regarded as a generalized solution of (7). But in general it is open, whether

(8) $$\lim_{\Omega \ni x \to y \in \partial\Omega} u_{pr}(x) = g(y)$$

and

(9) $$\lim_{\Omega \ni x \to y \in \partial\Omega} u_0(x) = 0$$

hold. In some cases, for example for $p(x,D) = (-\Delta)^{t/2}$, $0 < t \leq 1$, such results do hold, see R.Song [4].

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SOME TOPOLOGICAL ASPECTS OF ELLIPTIC BOUNDARY PROBLEMS.

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In this informal report we describe a couple of topological observations on the L.Boutet de Monvel algebra (as presented in [1]) which emerge naturally (and perhaps may become useful) in the context of analytic K-homology [2].

Let X be a smooth manifold with (possibly empty) boundary Y and E, J be complex vector bundles over X, Y, respectively. Our main concern is the Boutet de Monvel Algebra $B = B(X, Y, E, E, J, J)$ (in the notation of [1]).

In fact, at the present moment we are able to obtain certain preliminary results only for trivial bundles but our basic considerations make sense also in the general case.

The strategy is to use recent developments within the (at least) three related approaches, namely, we would like to:

1) compute (at least some of) homological functors such as Hochschild homology $HH_*(B)$, cyclic (co)homology $CH_*^{(*)}(B)$ and Kasparov bifunctor $KK_*(B)$ [3];

2) compute its non-stable K-theory in the sense of M.Rieffel (cf., e.g., [4], [5]), that is the homotopy groups $\pi_*(GB)$, where GB, as usual, denotes the groups of units (invertible elements) of B or the groups $\pi_*(FB)$, where FB denotes the subset of elliptic (Fredholm) elements of B;

3) compute K-homology classes of extensions occuring in the CCR-tower of B in the sense of Dynin [6].

For scalar pseudodifferential operators (ΨDOs) on a manifold without boundary a substantial part of this program is already realized. In particular Brylinski-Getzler and also Wodzicki have computed Hochschild and cyclic homologies for the algebra of pseudodifferential symbols $B^\infty/B^{-\infty}$ [7] (there are also some related papers of Wodzicki yet inaccessible for the author), $\pi_*(FB)$ in certain cases were computed by the author [8] and the singular integral extension is realized as the fundamental class in K-homology [2].

In the presence of boundary the results of such type are absent and we tried to do first steps in this direction. In particular, recent results of A.Wassermann on the cyclic homology of function algebras on Chevalley orbifolds [9] suggest the following computation which seemed to us somewhat related with B.

Represent $V := X$ as the simplest orbifold obtained from its double W with the evident action of $Z_2 = Z/2Z$ (reflection w. r. t. boundary). This Z-action lifts to all reasonable functional spaces, in particular, to the space of classical symbols of ΨDOs on W: $S = \Psi^{\infty}(W)/\Psi^{-\infty}(W)$.

Following Brylinski and Wodzicky one can construct now spectral sequences converging to $HH_*(T)$ and $CH_*(T)$. A formal analysis of these spectral sequences shows that they satisfy the conditions permitting to apply the "invariance principle" formulated by A.Wassermann ([9], main theorem of the fourth sequel) so that one can introduce the invariant subalgebra T of S and obtain certain reductions to usual (co)homology.

Theorem 1

$$HH_*(T) \cong H^{2n-*}(Z \times S^1, \mathbb{C}),$$

$$CH^*(T) \cong (\ker(d*^t) \cap \mathcal{D}_*) \oplus H_{*-2}(Z, \mathbb{C}) \oplus H_{*-4}(Z, \mathbb{C}) \oplus ...$$

where Z denotes the pair (S^*X, S^*Y), S^* standing for spherical cotangent bundle, $\mathcal{D}_*$ are spaces of the De Rham currents and d_*^t is the dual De Rham operator.

We would like to emphasize that the relation (if any) of these formulas to genuine algebra of boundary problems remains unclear. Initially, it seemed to the author that this may be applied for inner symbols with the transmission property. I was unable to explicate this idea and some remarks of Prof. E. Schrohe in the course of discussions during this conference have convinced me that it should not be possible. Besides I was neither able to find somewhere proofs of Wassermann's results nor to prove them myself. So that these formulas are to be considered as some conditional results (modulo some technicalities in the "invariance principle") and rather as an invitation to a collaboration with experts in boundary problems, a collaboration in course of which topology should benefit from differential equations as it has already happened many times.

In conclusion, we would like to outline another perspective concerning $\pi_*(FB)$. The point is that the well-known construction of boundary symbols going back to Vishik and Eskin assigns to a boundary problem $p \in B$ a family $s_Y(P)$ of Wiener-Hopf operators (WHOs) parametrized by points of S^*Y, and P is elliptic iff this family consists of invertible operators. One obtains thus a mapping $s_Y : FB \to C(S^*Y, GW)$, where W denotes now the algebra of matrix Wiener-Hopf operators in suitable function spaces on $\mathbb{R}$. The key observation is that it is possible to describe the set of connected components of the target space of mappings because the initial homotopy groups $\pi_*(GW)$ are already known [10] (in fact, we dealt there with singular integral operators but the reasoning applies also to WHOs) and the rest is a standard problem of algebraic topology which admits an algorithmic solution using Sullivan's theory of minimal models. As a trivial example, when X is a simply connected planar domain the boundary symbols for scalar problems define simply two classes in the fundamental group $\pi_1(GW) \cong Z$, that is two integers, and is not difficult to relate these integers with the index of the corresponding elliptic problem, which includes, in particular, the model result of I.Vekua on the oblique derivative problem. Similar connections are, of course, available also in higher dimensions.

Our last remark is that there are some recent computations of K-groups for solvable algebras of length two in the sense of Dynin and this is just the case for Boutet de Monvel algebras.

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STATIONARY DIFFRACTION PROBLEMS ON THE WEDGES WITH GENERAL BOUNDARY VALUE CONDITIONS

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We consider the boundary value problem for the Helmholtz equation in the plane angle Q of the arbitrary magnitude φ, $0 < \varphi < \pi$:

$$\mathcal{H}u(x) \equiv (\Delta + \omega^2)u(x) = 0, \quad x \in Q \qquad (1)$$

$$B_l u|_{\Gamma_l} = f_l(x), \quad l = 1,2 \qquad (2)$$

Here $\omega \in \mathbf{C}$, $B_{1,2}$ are arbitrary linear differential operators with constant complex coefficients. We denote by Γ_l the sides of angle Q , f_l are temperated distributions on Γ_l . We seek a solution u in the space $S'(Q)$.

It means that $u(x) \equiv u_0(x)|_Q$, where $u_0 \in S'(\mathbf{R}^2)$ and *supp* $u_0 \subset \bar{Q}$. Our result is the deriving of the explicit formulas of all solutions (1), (2). The boundary value problems for the Helmholtz equation with different boundary conditions were considered in [1-6, 8, 9]. Our results may be applied to the deriving of trapping modes in open wave guides, to the verification of the limit amplitude principle, to the finding of a scattering amplitudes, to the analysis of high order approximations of boundary value conditions in scattering problems etc.

1. Let $\varphi < \pi$. Then we apply the Paley - Wiener theory, the division theorem and the Cauchy - Kowalewski method to reduce (1), (2) to the SAE (system of algebraic equations) on the Riemannian surface [3, 8].If, for example, $\varphi = \frac{\pi}{2}$ then $Q \cong Q_{++} \equiv \{x \in \mathbf{R}^2 : x_1 > 0, x_2 > 0\}$ and we get SAE on the riemannian surface

$$V_{++} \equiv V \cap \{z \in \mathbf{C}^2 : Im\ z_l > 0, l = 1,2\}.$$

Here we denote by V the riemannian surface of complex characteristics of the Helmholtz operator $\mathcal{H}$:

$$V \equiv \{z \in \mathbf{C}^2 : \mathcal{H}(z) \equiv -z_1^2 - z_2^2 + \omega^2 = 0\}.$$

The SAE is the following:

$$d_+(z) \equiv \tilde{v}_1^1(z_1) - iz_2\tilde{v}_1^0(z_1) + \tilde{v}_2^1(z_2) - iz_1\tilde{v}_2^0(z_2) = 0, \quad z \in V_{++} \qquad (1')$$

$$B_l^0(-iz_l)\tilde{v}_l^0(z_l) + B_l^1(-iz_l)\tilde{v}_l^1(z_l) = \tilde{f}_l^0(z_l) + \sum c_l^k(-iz_l)^k, \quad l = 1,2, \quad Im\ z_l > 0. \qquad (2')$$

Here the functions $\tilde{v}_l^\beta(z_l)$ are analitic for $Im\ z_l > 0$, $F^{-1}_{z_l \to x_l}(\tilde{v}_l^\beta)|_{R^+}$ are equal to Cauchy data of the solution $u(x)$ of the problem (1), (2). The sums in (2') are finite. The solution may be represented by $\tilde{v}_l^\beta$ in the form

$$u(x) = F^{-1}_{z \to x}[\tilde{d}_+(z)/\mathcal{H}(z)], \quad x \in Q_{++}. \qquad (3)$$

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*) With the support by Grant of RAS.

We call (1') a connection equation. It is the relation between Cauchy data of solution $u(x)$ on the complex characteristics of Helmholtz operator $\mathcal{H}$.

We reduce the system (1'), (2') to RHP (Riemann - Hilbert problem) by V.A.Malyshev automorphic functions method as in [3, 8]. In the case $Im\ \omega \neq 0$ this RHP is well posed and may be solved explicitly by the traditional Riemann-Hilbert procedure as in [3].

In case when $Im\,\omega = 0$ the RHP is ill posed. This additional difficulty was overcome in [8], by introducing of the "retarded" solutions to the problem (1), (2). For the solutions the corresponding RHP is well posed.

2. Let now $\varphi > \pi$ and $Im\ \omega \neq 0$. In the case we cannot apply the Paley - Wiener theorem. Meanwhile in the case the connection equation similar to (1') exists too. For cxample, if $\varphi = \frac{3}{2}\pi$, and $Q = \mathbf{R}^2 \setminus \bar{Q}_{++}$ then roughly speaking, SAE (1'), (2') holds for $z \in V \setminus \bar{V}_{++}$.

Further we reduce corresponding SAE of the type (1'), (2') to RHP by the method of [4]. In the case, when $Im\ \omega \neq 0$ the solution $u(x)$ of (1), (2) may be expressed by (3).

3. Finally, let $\varphi > \pi$ and $Im\ \omega = 0$. In the case, (3) loses the sense. Then we derive for $u(x)$ the Sommerfeld type representation instead of (3). This representation we derive for "retarded" solutions to the problem (1), (2), mentioned above.

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ON ASYMPTOTICS OF SOLUTIONS OF NONLINEAR ELLIPTIC EQUATIONS IN A NEIGHBOURHOOD OF A CONIC POINT OF THE BOUNDARY.

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Consider equations of the form

$$\sum_{i,j=1}^{n} \frac{\partial}{\partial x_i}\left(a_{ij}(x)\frac{\partial u}{\partial x_j}\right) - a_0(x)|u|^{p-1}u = 0 \tag{1}$$

and

$$\sum_{i,j=1}^{n} a_{ij}(x)\frac{\partial^2 u}{\partial x_i \partial x_j} - a_0(x)|u|^{p-1}u = 0 \tag{2}$$

where $x = (x_1, ..., x_n)$, $m_1|\xi|^2 \leq \sum_{i,j=1}^{n} a_{ij}(x)\xi_i\xi_j \leq m_2|\xi|^2$, $\xi \in R^n$, $x \in \Omega$, $m_1, m_2 =$ const > 0, $a_0(x) \geq a_0 =$ const > 0. We assume that $a_{ij}(x)$ are measurable and $a_{ij}(x) \equiv a_{ji}(x)$.

A set $K \in R^n$ is called a cone if for any $x \in K$ and any constant $\lambda > 0$ the point λx belongs to K. We set $K^1 = K \bigcap \{x : |x| = 1\}$, $K_b = \{x : x \in K, |x| < b\}$

We consider solutions of equations (1), (2) on $\Omega = K_b$ with the boundary condition

$$\left.\frac{\partial u}{\partial v}\right|_{\partial K} = 0 \qquad if\,|x| < b \tag{3}$$

where $\frac{\partial}{\partial v}$ is the derivative of u in the conormal direction. Assume that $\partial K'$ satisfies a Lipschitz condition.

Theorem 1 *Let u be any weak solution of (1), (3) on K_b and*

$$p \geq \frac{n}{n-2}$$

Then

$$|u(x) - u(0)| \leq C|x|^s$$

for some $s > 0$. Here C, s depend on m_1, m_2, a_0, n, p.

Remark. *Assume* $1 \leq p < \frac{n}{n-2}$. *Then there exists a weak solution of (1), (3) such that*

$$\lim_{x \to 0} u(x)|x|^{n-2} = +\infty$$

Consider the solution of the problem (2), (3) on K_b such that $u(x) \in C^2(K_b \backslash 0) \cap C^1(\bar{K}_b \backslash 0)$. Assume that $\partial K' \in C^2$ and $p > 1$.

Theorem 2 *There exists a constant* $C_1 > 0$ *such that* $|u(x)| \leq C_1 |x|^{\frac{2}{1-p}}$.

Remark 1. *If* $|a_{ij}(x) - \delta_{ij}| \leq C|x|^\gamma$, $\gamma > 0$, $p \geq \frac{n}{n-2}$ *then* $|u(x) - u(0)| \leq C|x|^s$, *where* $u(x)$ *is any classical solution of (2), (3), and* $s > 0$.

Remark 2. *For any* p, $p > 1$ *there exists* $a_{ij}(x) \in C^\infty(K)$, *such that problem (2), (3) has unbounded solutions. This means that Theorem 1 is not valid for problem (2), (3).*

EIGENFUNCTION EXPANSION OF MULTIPARAMETER SPECTRAL PROBLEMS.

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We study multiparameter eigenvalue problems

$$A_j u_j = \sum_{k=1}^{n} \lambda_k B_{jk} u_j, \quad 0 \neq u_j \in H_j, \ 1 < j \leq n,$$

where A_j, B_{jk} are symmetric operators in separable Hilbert spaces H_j and B_{jk} are bounded. Here $\lambda = (\lambda_1, \ldots, \lambda_n) \in \mathbf{C}^n$ is the spectral parameter.
We discuss some recent expansion results for such problems. For the case of multiparameter problems with elliptic operators A_j and multiplication operators B_{jk} we prove theorems about smoothness of corresponding eigenfunctions.
Also we give an abstract approximation criterion for the existence of commuting self-adjoint extensions of a family of symmetric operators.

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INTERIOR BOUNDARY VALUE PROBLEMS WITH SINGULAR INTERIOR BOUNDARY.

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In analogy with the univariate spline-theory, we consider the problem

$$\int_{\Omega} (\Delta u(x))^2 \, dx \to \inf$$

where $n(x) = f(x)$, for $x \in \Gamma$, where Γ is a piecewise smooth variety of codim 1 in Ω, and $\Gamma \supset \partial\Omega$.

We prove theorems for existence and smoothness of the solution to the above problem. The solution u is combined of pieces u_j of biharmonic functions in every connected subdomain Ω_j of $\Omega \backslash \Gamma$.
The functions u_j and u_k satisfy some matching conditions on the joint part of the boundaries $\partial\Omega_j \cap \partial\Omega_k$.

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NONLINEAR SECOND-ORDER ELLIPTIC EQUATIONS WITH JUMP DISCONTINUOUS COEFFICIENTS.

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We investigate the Dirichlet problem for quasilinear second order, uniformly elliptic equations

$$a^{k,ij}(x,u,Du)u_{x_ix_j} + a^k(x,u,Du) = 0 \text{ in } \Omega_k,\ k = 1,2;\ u = g \text{ on } \partial\Omega \tag{1}$$

$$f^k(x,u,Du,D^2u) = 0 \text{ in } \Omega_k,\ k = 1,2;\ u = g \text{ on } \partial\Omega \tag{2}$$

in a bounded domain $\Omega \subset I\!R^n$, $n \geq 2$ with a smooth boundary $\partial\Omega$. We assume that $\Omega = \Omega_1 \cup \Omega_2 \cup S$ is divided into two subdomains Ω_1 and Ω_2 by a smooth surface S without self-crossing points. The coefficients of (1), (2) are supposed to be smooth on each side of S on $\bar{\Omega}_1$, $\bar{\Omega}_2$ respectively, and, when considered as functions on Ω, present some pure jump discontinuity on S.

We wish to show that under general and natural conditions on the coefficients, including uniform ellipticity, there exist unique solutions of (1) and (2) which belong at least to $C^1(\Omega) \cap C(\overline{\Omega})$.

In the case of (2), for general nonlinearities f^1, f^2, the equation will be understood in a viscosity sense. Viscosity solutions, as introduced by M.G. Crandall and P.-L. Lions and M.G. Crandall, L.C. Evans and P.L. Lions, have provided a general and efficient tool for studying the existence, uniqueness and regularity questions for fully nonlinear elliptic equations. The C^1 character of solutions is clearly fundamental for the uniqueness of solutions, since we can always prescribe u as we wish on S, solve (1) and (2) on each side, i.e. in Ω_1 and Ω_2, and obtain in this way a continuous (even Lipschitz contiuous) solution of (1) or (2) in $\overline{\Omega}$; which is therefore highly nonunique.

Our motivations for studying these problems: first, it is a natural first step towards the study of nonlinear elliptic equations with discontinuous coefficients. Next, when

f^1, f^2 are convex say in (Du, D^2u) so that (2) corresponds to the so-called Hamilton-Jacobi-Bellman equations of optimal stochastic control, problem (2) has a very natural interpretation in terms of optimal stochastic control since it only means that different sets of control can be used in Ω_1 and Ω_2. This has many applications to problems where some controls are forbidden in certain regions.

Theorem 1 *Suppose that $f^k \in C^1(\Omega_k \times I\!R \times I\!R^n \times S^n)$ and the uniform elliptic conditions hold i.e.*

$$\lambda|\xi|^2 \leq f^k_{r_{ij}}(x,z,p,r)\xi^i\xi^j \leq \Lambda|\xi|^2, \; k = 1,2$$

for $(x,z,p,r) \in \overline{\Omega_k} \times I\!R \times I\!R^n \times S^n$, $\xi \in I\!R^n$ and some positive constants λ, Λ. Let Ω be a bounded domain in $I\!R^n$ and S satisfy the exterior and interior sphere conditions. If $f^k(x,z,p,r)$ is a nonincreasing function of z then (1) has at most one solution $u \in C^2(\Omega_1 \cap \Omega_2) \cap C^1(\Omega) \cap C(\bar{\Omega})$.

Theorem 2 *Let Ω be a bounded $C^{2,\mu}$ smooth domain in $I\!R^n$, $n \geq 2$, $0 < \mu < 1$ and $a^{k,ij}$, $a^k \in C^1$, $S \in C^3$, $g \in C^{2,\mu}$. Suppose that the uniform ellipticity conditions*

$$\lambda|\xi|^2 \leq a^{k,ij}(x,z,p)\xi^i\xi^j \leq \Lambda|\xi|^2$$

hold as well as the natural structure conditions

$$\begin{aligned} |a^k| &\leq C(|z|)\left(1+|p|^2\right) \\ (1+|p|)\left|a^{k,ij}_p\right|, \; \left|a^{k,ij}_z\right|, \; \left|a^{k,ij}_x\right| &\leq c(|z|) \\ (1+|p|)\left|a^k_p\right|, \; \left|a^k_z\right|, \; \left|a^k_x\right| &\leq C(|z|)\left(1+|p|^2\right) \end{aligned}$$

If

$$za^k(x,z,0) \leq 0 \text{ for } x \in \overline{\Omega}, \; |z| \geq M, \; k = 1,2$$

for some nonnegative constant M, then problem (1) has at least one solution

$$u \in C^2(\overline{\Omega}_k \backslash \partial S) \cap C^{1,1}(\Omega) \cap C^{0,1}(\overline{\Omega}), \; k = 1,2$$

Next, a word on the method of proof. Uniqueness follows from some simple considerations or modifications of the maximum principle and/or of the uniqueness theory for viscosity solutions. Concerning existence, our strategy of proof is the following. We first regularize equations (1)-(2) by appropriately smoothing out the discontinuity of the coefficients across S. To find bounds on derivatives directly (without diff.) seems hopeless in view of the result of Safonov. This is why we first prove some estimates on tangential (to S) derivatives, namely, bounds and Hölder continuity; this follows from a modification of Bernstein's method, where we consider only the tangential part of the gradient, differentiating the approximated equation tangentially. Then we deduce similar estimates for the normal derivative to S.

ON NON-DIVERGENT SEMI-LINEAR ELLIPTIC EQUATIONS WITH DISCONTINUOUS COEFFICIENTS.

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Consider the Dirichlet problem for the equation

$$Lu \equiv a^{ij}(x)u_{x_ix_j} + F(x, Du) + \Phi(x, u) = 0; \tag{1}$$

$$Lu = 0,\ u|_{\partial G} = \varphi \tag{2}$$

in a bounded domain $G \subset \mathbb{R}^n$. We assume that $a^{ij}(x)$, $F(x,\xi)$, $\Phi(x,\xi)$ are measurable in all variables $a^{ij}(x) = a^{ji}(x)$, and

$$0 \leq d(x)|\eta|^2 \leq a^{ij}\eta_i\eta_j < M|\eta|^2,\ x \in G,\ \eta \in \mathbb{R}^n. \tag{3}$$

We also assume that $\Phi(x, 0) = 0$ and $\Phi(x, \xi)$ monotonically decreases in ξ.
1. The maximum principle is true: for $u, v \in C^2(G) \cap C(\overline{G})$ from the conditions $Lu \geq 0$, $Lv \leq 0$, and $u|_{\partial G} \leq v|_{\partial G}$ it follows $u \leq v$ in G.
2. Let $F(x,\eta) = \sum b^i(x)|\eta_i|^{1+\beta} sgn\ \eta_i$, $\|b\| > 1$, $1 < \beta < 2$. A function $v \in C^2(G) \cap C(\overline{G})$ is called a super (sub) function if $v|_{\partial G} \geq \varphi$ ($v|_{\partial G} \leq \varphi$) and $Lv \leq 0$ ($Lv \geq 0$) in G. The function $u^+ = \inf v$ ($u^- = \sup v$) where the infimum (supremum) is taken over all super (sub) functions is called a super (sub) solution. From the maximum principle it follows that $u^- \leq u^+$.

Theorem 1 *The functions u^+ and u^- satisfy Hölder's condition in any strictly interior subdomain of G with the exponent $(\beta - 1)/\beta$.*

Theorem 2 *If in (3), $\alpha(x) > const > 0$ in a neighborhood of ∂G, φ in (2) satisfies a Hölder condition in G and $u^+|_{\partial G} = u^-|_{\partial G} = \varphi$.*

3. Let $F \equiv 0$ and $|\Phi| \geq C|u|^q$, $0 < q < 1$. Define u^+ and u^- as before.

Theorem 3 *The functions u^+ and u^- satisfy Hölder's condition in any strictly interior subdomain of G. If ∂G satisfies a Lipschitz condition and φ satisfies Hölder's condition, then $u^+|_{\partial G} = u^-|_{\partial G} = \varphi$.*

4. Let in (3), $\alpha(x) > \alpha > 0$ everywhere in G. Denote by $\lambda(x)$ the maximal eigenvalue of the matrix $a(x) = \|a^{ij}(x)\|$. Define by $a_h^{ij}(x)$ the convolution of $a^{ij}(x)$ with a positive function with support in a ball of radius h. Assume that $|F(x,\xi)| \leq \eta^{1+\beta}$, $0 < \beta < 1$, F is odd in η and $|\Phi(x,\xi)|$ is decreasing in ξ. Assume for simplicity that F and Φ are infinitely differentiable. Consider a solution of the problem

$$a_h^{ij}(x)\, u^h_{x_i x_j} + F(x, Du^h) + \Phi(x, u^h) = 0;$$

$$u^h|_{\partial G} = \varphi.$$

If ∂G satisfies a Lipschitz condition and φ satisfies a Hölder condition then the family $\{u^h\}$ is compact in $C(\overline{G})$. Choose $u^{h_k} \rightrightarrows u_0$.

Theorem 4 *(i)* $u^- \leq u_0 \leq u^+$ *and*

(ii) for almost every point $x_0 \in G$ *there exists a second order polynomial* $\mathcal{P}_{x_0}(x)$ *such that* $L\left(\mathcal{P}_{x_0}(x)\right)\big|_{x=x_0} = 0$ *and* $u_0(x) - \mathcal{P}_{x_0}(x) = o\left(|x - x_0|^2\right)$.

Theorem 5 *Let* $\bar{e} = \sup_{x\in G}(tr\,\|a_{ij}\|/\,\lambda_{max}(x))$ *and* $\bar{e} > 2$. *Let* $N \subset G$ *be a closed subset in* G *and* $cap_{\bar{e}-2}N = 0$. *Assume that for any point* $x_0 \in G\backslash N$,

$$\underset{O(x_0)}{osc}\, a^{ij}(x) < \lambda(x)/n$$

in some neighbourhood $U(x_0)$ *of* x_0. *Then*

$$u_0 \equiv u^+ \equiv u^-$$

PERIODIC AND STATIONARY SOLUTIONS TO THE SCHRÖDINGER–POISSON AND WIGNER–POISSON SYSTEMS

O. Kavian, H. Lange, P.F. Zweifel

We are concerned with the study of the Wigner–Poisson (WP) and Schrödinger–Poisson (SP) systems with space periodic boundary conditions on the unit cube $Q = [0,1]^3$ in $\mathbb{R}^3$. Both (WP) and (SP) are quantum transport models, they describe the quantum mechanical motion of a large ensemble of electrons in a vacuum under the action of repulsive or attractive Coulomb forces generated by the charge of the electrons. Thus, both models play an important role e.g. in semiconductor theory. We refer to a couple of recent paper's on (WP) and (SP) for an introduction to the subject; see BREZZI–MARKOWICH [1], ILLNER–LANGE–ZWEIFEL [2], LANGE–ZWEIFEL [3], [4], BOHUN–ILLNER–ZWEIFEL [5], ZWEIFEL [6], ILLNER [7], ARNOLD–NIER [8], ARNOLD–MARKOWICH [9]; a general introduction may be found in TATARSKII [10].

When studying (WP) and (SP) for periodic boundary conditions some slight changes in the equations have to be taken into account. The physical model should be a plasma of electrons moving in a background of fixed positive charge density (say $C(x)$) whereas the overall plasma is charge neutral. Thus, the Poisson equation takes a different form than that of most of the references, namely it should read as

$$\text{(P)} \qquad \Delta V(x,t) = C(x) - n(x,t)$$

where $n(x,t)$ is the density of negative charge which (together with $C(x)$) is normalized to

$$\int_Q n(x,t)dx = \int_Q C(x)dx = 1 \ ;$$

for simplicity in this note we consider only the case $C(x) \equiv l$. Furthermore, the momentum is quantized to $v_k = 2\pi k$, $k \in Z^3$. Following the discussions in [5], [6], [7] the (WP) and (SP) systems in our case can be shown to take the form

$$\text{(WP)} \qquad \partial_t w_k + v_k \cdot \nabla_x w_k + \Theta(V) w_k(x,t) = 0 \quad , \quad w_k(x,t) = w_k^0(x) \ ;$$

here $k \in Z^3$, $w_k(x,t) = w(x, v_k, t)$ (where $w(x,v,t)$ is the original Wigner function), and $\Theta(V)$ is the pseudo–differential operator given by (for quantum number $\hbar = 1$)

$$\Theta(V)w(x,t) = -i \sum_{k' \in Z^3} \int_{2Q} [V(\frac{x+z}{2}, t) - V(\frac{x-z}{2}, t)] w_{k'}(x,t) e^{2\pi i(k-k')z} dz \quad ;$$

here $V(x,t)$ is a solution of Poisson's equation (P) and $n(x,t) = \sum_{k \in Z^3} w_k(x,t)$; the (SP) system reads as

$$\begin{aligned} i\partial_t \psi_m &= -\frac{1}{2}\Delta\psi_m + V(x,t)\psi_m \ , \\ \Delta V &= 1 - n(x,t) \ , \quad \psi_k(x,0) = \varphi_k(x) \ , \quad n(x,t) = \sum_{k \in Z^3} \lambda_m \mid \psi_m(x,t) \mid^2 \ , \end{aligned}$$

both (WP) and (SP) are subject to 1–periodic boundary conditions on Q; furthermore in (SP) one should assume that $\|\varphi_m\|_{L^2(Q)} = 1$, $(\varphi_m, \varphi_l) = \delta_{ml}$; whereas the (λ_m) are the occupation probabilities of the initial pure states (φ_m) which build up the initial mixed state. The link between solutions of (SP) and (WP) is given by the WIGNER–transform

$$w_k(x,t) = \sum_{m=1} \lambda_m \int_{2Q} \psi_m(\frac{x-z}{2}, t) \psi_m^*(\frac{x+z}{2}, t) e^{2\pi i k z} dz \quad , \tag{1}$$

whereas in terms of $\Psi = (\psi_m)$ $\Theta(V)$ is given by

$$\begin{aligned} & \Theta(V)w(x,t) \\ = & -i \sum_{m \in N} \lambda_m \int_{2Q} [V(\frac{x+z}{2}, t) - V(\frac{x-z}{2}, t)] \psi_m(\frac{x-z}{2}, t) \psi_m^*(\frac{x+z}{2}, t) e^{2\pi i k z} dz \ . \end{aligned}$$

We formulate results on

a.) Existence and uniqueness of global–in–time strong 1–periodic solutions of (SP);

b.) Existence of countably many 1–periodic stationary states of (SP), i.e. solutions of type

$$\Psi(x,t) = e^{i\omega t}\Phi(x)$$

with some $\omega \in I\!R$ and a real 1–periodic function Φ.

Theorem 1 *Let $\Phi = (\varphi_m) \in X^2$ with L^2–norm equal to 1; then for any $T > 0$ there is a unique global strong 1–periodic solution (Ψ, V, n) of (SP) which satisfies the conservation laws*

$$\|\Psi(\cdot,t)\|^2_{L^2(Q)} = 1 \quad , \quad \|\nabla\Psi(\cdot,t)\|^2_{L^2(Q)} + \|\nabla V(\cdot,t)\|^2_{L^2(Q)} = const. \qquad (\forall t \in S_T). \tag{2}$$

Theorem 2 *There exists a countably infinite number of different stationary states* $\Psi_j(x,t) = e^{i\omega_j t}\Phi_j(x)$ *with* 1*-periodic real functions* $\Phi_j \in H^2 \cap C^\infty$ *such that* $\omega_j \to \infty$ *for* $j \to \infty$.

Remark: All results stated for (SP) transfer by using the WIGNER transform (1) to similar results for (WP) which we do not formulate here.

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GEOMETRIC REMARKS ON AN INVERSION FORMULA FOR THE HEAT EQUATION.

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This report is a brief summary of two forthcoming papers: one in collaboration with François Golse the other in collaboration with François Golse and Matthew Stenzel.

To motivate our work let us recall Lebeau's ([2]) inversion formula for the heat operator $\partial_t + \Delta$ (Δ is nonnegative) of (the flat euclidean space) $\mathbb{R}^n$. For any $u \in L^2(\mathbb{R}^n)$

$$e^{-t\Delta}u(x) = \frac{1}{(4\pi t)^{\frac{n}{2}}} \int_{\mathbb{R}^n} e^{-\frac{(x-y)^2}{4t}}\, u(y)\, dy$$

defines for all $t > 0$ a holomorphic function on $\mathbb{C}^n$ denoted by $Tu(t,\cdot)$. Lebeau's formula is stated as follows: for all $y \in \mathbb{R}^n$ and $\varepsilon > 0$, one has:

$$u(y) = c(\varepsilon,\eta) \int_0^\infty e^{-\frac{\varepsilon^2}{4t}} \frac{dt}{t^{n+1}} \int_{|\omega|=\varepsilon} \left(1 - 2t\sqrt{-1}\varepsilon^{-2}\omega \cdot \mathrm{grad}_x\right) Tu\left(t, y - \sqrt{-1}\omega\right) dt\, d\omega$$

for all $u \in C_0^\infty(\mathbb{R}^n)$.

We will state an analogous formula when $\mathbb{R}^n$ is replaced by any real analytic Riemannian orientable compact n-dimensional manifold (X,g). The volume form defining the orientation is denoted by μ, $\Delta = -\mathrm{div}(\mathrm{grad}\cdot)$ denotes the Laplacian of (X,g), $p(x,\xi)$ is the principal symbol of Δ and $d(x,y)$ denotes the geodesic distance on X.

Let us recall the following theorem of Guillemin and Stenzel (see [3]).

Theorem 1 *There is an open neighborhood U of X in T^*X and unique complex structure on U such that:*

(i) $(x,\zeta) \mapsto \sigma(x,\zeta) = (x,-\zeta)$ *defines an antiholomorphic involution on* $U = \sigma(U)$.

(ii) On U the standard 1-form $\zeta \cdot dx$ is equal to $+Im\,\bar{\partial}\varphi$.

In this work we shall work with such a complexification U of X.

Theorem 2 *For $\varepsilon > 0$ small enough the complexification of the exponential map is given by:*

$$T_\varepsilon X = \{(x,\xi) \in T_x X; \|\xi\|_x < \varepsilon\} \longrightarrow U\,(\subset T^*X)$$
$$(x,\xi) \mapsto \mathrm{Exp}_x(i\xi)$$

$\zeta : h \mapsto < h, \xi >_x$ *where* $<,>$ *is the scalar product of* $T_x X$ *and* $\|\xi\|_x^2 = < \xi, \xi >_x$.

Thus $\mathrm{Exp}_x(i\xi) \mapsto x$ defines the usual cotangent fibration of X.

Let us fix $x \in X$ and write $Y = \{\mathrm{Exp}_x(i\xi)/\|\xi\|_x < \varepsilon\}$ (for ε small enough). We orientate Y and ∂Y in a compatible way.

Roughly speaking if we consider the holomorphic extension of the metric g (of X) to U and restrict it to Y we obtain a non degenerate $\mathbb{C}$-quadratic form g^Y on Y. In the same way we construct (from μ on X) an n-form μ^Y on Y which never vanishes. We can define on Y a notion of grad^Y.

Definition 3 *(i) For any complex C^∞ vector field $v \in \Gamma(Y, T^{\mathbb{C}}Y)$ let us denote $\mathrm{div}^Y v$ the scalar function defined by $d(i_v^Y \mu^Y) = \mathrm{div}^Y v\, \mu^Y$ where i_v^Y is the interior product and d the exterior derivative on Y.*

(ii) $\Delta^Y = -\mathrm{div}^Y \mathrm{grad}^Y(\cdot)$

We can now state our inversion formula for the heat equation (recall that $Tf(t,m) = (e^{-t\Delta} f)(m)$ for $(t,m) \in \mathbb{R}^{+*} \times U$). We still denote $d^2(\cdot,\cdot)$the holomorphic extension of $d^2(\cdot,\cdot)$ to a neighborhood of the diagonal of $X \times X$ in $U \times U$.

Theorem 4 *Let us assume that $\sqrt{-Re\, d^2(\cdot,\cdot)}$ defines a distance in the fiber Y (it is non negative for $\varepsilon > 0$ small enough).*

(i) There is a function $K(t,m) \in C^\infty(\mathbb{R}^{+*} \times Y)$ such that $(\partial_t - \Delta^Y)K \equiv 0$ and $K(t,m) \rightharpoonup \delta(x)$ (the dirac mass) when $t \to 0^+$.

(ii) For every $f \in C^\infty(X)$ satisfying $\int_X f\, d\mu = 0$ we have:

$$f(x) = \int_0^\infty dt \int_{\partial Y} \left[Tf\, i^Y_{\mathrm{grad}^Y K} \mu^Y - K\, i^Y_{\mathrm{grad}^Y Tf} \right]$$

When X is a symmetric space of the compact type then Y is an open subset of the "dual" (see [4]) of X and the preceeding geometric objects of Y have remarkable properties. For instance, $-g^Y$ defines a *real* definite positive quadratic form on Y and $\sqrt{-d^2(\cdot,\cdot)}$ defines the geodesic distance of $-g^Y$ (so the hypothesis of the theorem is satisfied).

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DIFFUSION PHENOMENA WITH SHOCKS: RESULTS ON PURE AND APPLIED PROBLEMS.

Günter Lumer

We develop an operator-theoretic approach in the setting of a Banach space X, and in the classical PDE context with $X = C(\overline{\Omega})$ appropriate for a realistic treatment of diffusion shocks (abrupt changes in boundary values), or heat shocks, with specific applications. The evolution of such systems between shocks at $t = t_0$ and $t = t_1 > t_0$ is described by an initial value-boundary value (Banach space X) problem:

$$\frac{du}{dt} = \hat{A}u + F(t)$$

$$u(t_0) = f \text{ (mild g.s.)} \tag{1}$$

$$Bu(t) = \varphi, \qquad t_0 < t \leq t_1 \qquad (\varphi \in H), \tag{2}$$

where $\hat{A}$ is of the form: $D(\hat{A}) = D(A) \oplus H$, A is the generator of an irregular bounded analytic semigroup $Q(t)$ on X, H the space of "A-harmonic" elements, $\hat{A}\hat{f} = Af$ for $\hat{f} = f + h$ (with $f \in D(A)$, $h \in H$). Details of much of what is mentioned here will appear in [1], [2].

Theorem 1 $\forall f \in X$, $\varphi \in H$, $F \in L^1_{loc}([0, +\infty[, X)$, *(1) has a unique optimal regular solution* $u :]t_0, t_1] \to D(\hat{A})$ *given explicitly by* $(u(t, f, F) = u(t))$

$$u(t) = \varphi + Q(t - t_o)(f - \varphi) + \int_{t_0}^{t} Q(t - s) F(s)\, ds. \tag{3}$$

We treat periodic shock problems with period $2r > 0$, boundary values $\varphi, 0, \varphi, 0, \ldots$, $F = 0$.

Theorem 2 *For the above-mentioned periodic shocks problem there is a unique* $2r$-*periodic asymptotic attractor* u^*, *where* $u^*(t) = u^*(t, w_r)$ *and*

$$w_r = (1 + Q(r))^{-1} \varphi \tag{4}$$

The above formulas can be, and actually are, used for computer analysis of particular situations and concrete applications.

Among other things, one studies the steady "shock pattern", $\delta_{-,r}(t) = u^*(t_0) - u^*(t) = w_r - Q(t)w_r$ for small $t > 0$. One has:

Theorem 3 $\forall r_1, r_2 > 0$, $\|\delta_{-,r_1}(t) - \delta-, r_2(t)\| \to 0$ *as* $t \to 0_+$.

More precise facts will appear in [2].

These results can be applied in particular with $X = C(\overline{\Omega})$, Ω a bounded smooth domain in $\mathbb{R}^N$, A essentially an elliptic second order operator $-A(x, D)$ with appropriate regularity conditions on the variable coefficient,

$$\begin{aligned} &D(A) = \left\{f \in C_0(\Omega) \cap W^{2,p}(\Omega) : A(x,D)f \in C(\overline{\Omega})\right\}, \\ &Af = A(x,D)\, f \text{ in } \Omega \qquad (p > N) \end{aligned} \tag{5}$$

The theory, and computer analysis based on the theoretical formulas, has technological implications (see [2]) and has been used in connection with solar cells on "spinning communication satellites" (and analogous systems). For instance, it gives useful information on (the delicate matter of) "accelerate testing" of the mentioned satellites and analogous systems (as far as comparing the results of testing with actual functioning). Finally, ramified (transmission) shock problems are studied, bringing up special difficulties concerning the analyticity of $Q(t)$ in a $C(\overline{\Omega})$ setup where $\omega = \bigcup \Omega_i$, with transmission. We have results on this in quite particular cases, but sufficient to cover some interesting technological applications (for instance: the qualitative behavior in solar cells on spinning satellites is not essentially modified by the presence of protective glasses – mathematically this follows from the study of a heat shocks problem with transmission on three adjacent domains).

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ESTIMATES FOR A SYSTEM OF DIFFERENTIAL OPERATORS.

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1. We consider the problem of describing certain linear spaces $L(P_1, ...P_N)$ (depending, in general, on the domain $\Omega\,(\subset I\!R^n)$ and on $p \in [1, \infty]$) of differential polynomials $Q(D)$ satisfying the estimate

$$\|Q(D)f\|_{L_P(\Omega)} \leq C \left[\sum_{1\leq j\leq N} \|P_j(D)f\|_{L_P(\Omega)} + \|f\|_{L_P(\Omega)} \right] \qquad \forall f \in C^\infty(\overline{\Omega}) \tag{1}$$

with some constant C being independent of $f \in C^\infty(\overline{\Omega})$.

Theorem 1 *Let Ω be a bounded region in $I\!R^n$, $\{P_j(D)\}_1^N$ be differential polynomials whose symbols $\{P_j(\xi)\}_1^N$ are algebraically independent, and let the generic fiber of the mapping $P = (P_1, ..., P_N) : \mathbb{C}^n \to \mathbb{C}^N$ be irreducible (i.e. the variety*

$$V_\alpha = V(P_1 - \alpha_1, ..., P_N - \alpha_N) = \{\xi \in \mathbb{C}^n : P_1(\xi) - \alpha_1 = ... = P_N(\xi) - \alpha_N = 0\}$$

is irreducible for almost all $\alpha \in \mathbb{C}^N$). Then estimate (1) is equivalent to the equality

$$Q(\xi) = \sum_{1\leq j\leq N} \lambda_j P_j(\xi) + \lambda_0 \tag{2}$$

for some $\lambda_j \in \mathbb{C}$ $(0 \leq j \leq N)$, i.e. to the equality $\dim L(P_1, ..., P_N) = N + 1$.

Remark 1. *The condition of algebraic independence of the polynomials $\{P_j(\xi)\}_1^N$ is equivalent to their functional independence. So it can be reformulated in terms of the Jacobian matrix $(\partial P_j/\partial \xi_k)$ as follows: there exist $\xi^0 = (\xi_1^0, ..., \xi_n^0) \in \mathbb{C}^n$ such that $rank\,(\partial P_j(\xi_1, ..., \xi_n)/\partial \xi_k)_{\xi=\xi^0} = N$.*

Corollary 2 *Let $l = (l_1, ..., l_n) \in (Z_+\backslash 0)^n$, let Ω be a bounded region in $I\!R^n$ and let*

$$P_j(D) = \sum_{1\leq k\leq n} a_{jk} D_k^{l_k}, \qquad (1 \leq j \leq n-1)$$

be linearly independent differential operators whose linear span contains no differential monomial $D_k^{l_k}$, $1 \leq k \leq n$. Then $\dim L(P_1, ..., P_{n-1}) = n$.

2. Let us consider the consequences of estimate (1), assuming that the transcendence degree of the field $\mathbb{C}\,(P_1(\xi), ..., P_N(\xi))$ (over $\mathbb{C}$) is equal to one. The latter condition is equivalent to the representability of each of the polynomials $P_j(\xi)$ in the form $P_j(\xi) = \tau_j(w(\xi))$, where $\tau_j(t) \in \mathbb{C}[t]$ and $w(\xi) \in \mathbb{C}\,[\xi_1, ..., \xi_n]$ is irreducible.

Theorem 3 *Let Ω be a bounded region in $\mathbb{R}^n$ and let $P_j(\xi) = \tau(w(\xi)) \in \mathbb{C}\,[\xi_1, ..., \xi_n]$ where $\tau_j(t)$ is a polynomial in one variable and $w(\xi_1, ..., \xi_n)$ is irreducible. If for some $p \in [1, \infty)$ estimate (1) holds for all $f \in C^\infty(\overline{\Omega})$ then either $Q(\xi)$ has the form (2) for some $\lambda_j \in \mathbb{C}$, $0 \leq j \leq N$ or $w(\xi) = \langle \xi, x^0 \rangle = \xi_1 x_1^0 + ... + \xi_n x_n^0$, where $x^0 = (x_1^0, ..., x_n^0) \in \mathbb{R}^n$ and $Q(\xi) = q\,(\langle \xi, x^0 \rangle)$, $q(t) \in \mathbb{C}[t]$ with*

$$deg\, q(t) \leq \max_{1 \leq j \leq n} deg\, \tau_j(t)$$

3. The well-known result of Hörmander now follows easily from Theorems 1 and 2 combined with the following.

Proposition 4 *Let $P(\xi_1, ..., \xi_n)$ be a polynomial. If $P(\xi) - \alpha$ is reducible for all $\alpha \in \mathbb{C}$ then $P(\xi) = \tau(w(\xi))$, where $\tau(t) \in \mathbb{C}[t]$ and $w(\xi)$ ($\in \mathbb{C}[\xi_1, ..., \xi_n]$) is irreducible.*

Theorem 5 (Hörmander) *If Ω is a bounded region in $\mathbb{R}^n$ and the differential polynomials $P(D)$ and $Q(D)$ satisfy the condition (1) then either $Q(\xi) = aP(\xi) + b$ ($a, b \in \mathbb{C}$) or $P(\xi) = \tau\,(\langle \xi, x^0 \rangle)$ and $Q(\xi) = q\,(\langle \xi, x^0 \rangle)$ where $x^0 \in \mathbb{R}^n$ and $\tau(t)$, $q(t)$ in $\mathbb{C}[t]$, $deg\, q(t) \leq deg\, \tau(t)$.*

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A FELLER PROPERTY FOR SOME DEGENERATE ELLIPTIC OPERATORS.

I. McGillivray

In the theory of Dirichlet forms, statements are valid in general only up to sets of zero capacity. It is desirable to specify circumstances in which "q.e." valid statements are in fact valid everywhere.

The notion of (r,p)-capacity for general submarkovian semigroups T_t, a generalisation of the Bessel capacity, was introduced by Fukushima and Kaneko. We give a condition to guarantee that each point in the state-space has (r,p)-capacity uniformly bounded away from zero for r,p sufficiently large. Our condition is verified, for example, whenever we have a Sobolev inequality. We have found the following abstract regularity condition: if the L^p–generator of the semi-group T_t, or its square-root, contains a core of continuous functions, and the above capacitory condition is valid, then

$$T_t f \text{ admits a continuous version for all } f \in L^1 \cap L^\infty$$

We apply this result to degenerate elliptic operators. Suppose we have a gradient Dirichlet form with weight function belonging to the Muckenhoupt class of order 2. Combining methods of Gilbarg and Trudinger, and Fabes, Kenig and Serapioni, we show there exists an operator core of continuous functions and hence the above weak version of the Feller property holds.

SPECTRUM OF THE ELLIPTIC OPERATOR AND BOUNDARY CONDITIONS.

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Let $\Omega \subset I\!R^n$ be a bounded domain, $\mathcal{A}(x,D)$ be a selfadjoint elliptic differential operator of $2m-th$ order on Ω with smooth coefficients, A be the selfadjoint realization of $\mathcal{A}(x,D)$ in the Hilbert space $L_2(\Omega)$. The spectral properties of A are analyzed. We assume that the minimal operator of $\mathcal{A}(x,D)$ is positive.

Theorem 1 *If*

$$D(A) \subseteq H^s(\Omega),\ s \in (0,2m] \tag{1}$$

then the functions

$$N_{\pm}(\lambda;A) = \#\left\{k : \pm\lambda_k \in (0,\lambda]\right\},\ \lambda > 0$$

satisfy the asymptotic formulas

$$\begin{aligned} &N_-(\lambda;A) = O\left(\lambda^{\frac{n-1}{s}}\right), \\ &N_+(\lambda;A) = \begin{cases} O\left(\lambda^{\frac{n-1}{s}}\right), & s \in (0,s_0] \\ w\lambda^{\frac{n}{2m}} + O\left(\lambda^{\frac{n-1}{s}}\right), & s \in (s_0,2m] \end{cases} \end{aligned} \tag{2}$$

where $\lambda \to +\infty$, $w = w(\Omega,\mathcal{A}) > 0$, $s_0 = 2m\frac{n-1}{n}$.

The case of a positive realization A and $s = 2m$ has been studied in many papers (see [1], [2]).

The formulas (2) are precise for the class of selfadjoint elliptic differential operators under condition (1).

Let A_M be a "soft" selfadoint extension of a minimal operator. A question concerning the asymptotic behaviour of nonzero eigenvalues of the operator A_M have been raised in [3]. G. Grubb in [4] proved that the number $N_0(\lambda;A_M)$ of eigenvalues in $(0,\lambda]$ satisfies the asymptotic formula

$$N_0(\lambda;A) = w\lambda^{\frac{n}{2m}} + O\left(\lambda^{\frac{n-\theta}{2m}}\right),\ \lambda \to \infty \tag{3}$$

where

$$\theta = \max\left\{\frac{1}{2} - \varepsilon, \frac{2m}{2m+n-1}\right\},\ \varepsilon > 0$$

Theorem 2 *The asymptotic formula (3) is valid for any* $\theta \in (0, 1)$.

It should be expected that formula (3) is valid also for $\theta = 1$.

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SINGULAR PERTURBATIONS AND EXTENSION THEORY.

H. Neidhardt and V.Zagrebnov

Let A and V be two nonnegative self-adjoint operators on the separable Hilbert space $\mathcal{H}$. Further, let $\mathcal{D} \subseteq dom(A) \cap dom(V)$ be a dense subset of $\mathcal{H}$ such that

$$(Vf,f) \leq a(Af,f) + b\|f\|^2, \qquad f \in \mathcal{D}, \qquad 0 < a < 1. \tag{1}$$

We introduce the abstract operator $\dot{H}$

$$\dot{H}f = Af - Vf, \qquad f \in \mathrm{dom}(\dot{H}) = \mathcal{D}. \tag{2}$$

The operator $\dot{H}$ is symmetric, closable and semibounded with lower bound $-b$. However, the operator $\dot{H}$ is in general not essentially self-adjoint. Let us assume that $\dot{H}$ is not essentially self-adjoint. Since $\dot{H}$ is semibounded the Friedrichs extension $\hat{H}$ exists. Moreover, denoting by $\hat{A}$ the Friedrichs extension of $\dot{A} = A|\mathcal{D}$ it is not hard to see that $\hat{H}$ coincides with the form sum of $\hat{A}$ and $-V$. Next let us introduce a regularizing sequence for the singular perturbation.

Definition 1 A sequence $\{V_n\}_{n\geq 1}$ of bounded non-negative self-adjoint operators is called a regularizing sequence of V if

(i) $V_1 \leq V_2 \leq \ldots \leq V_n \leq \ldots \leq V$

(ii) $lim_{n\to\infty}(V_n f, f) = (Vf,f)$, $f \in \mathcal{D} \subseteq dom(V)$.

Let $\tilde{A}$ any semibounded self-adjoint extension of $\dot{A}$. With the regularizing sequence $\{V_n\}_{n=1}^{\infty}$ we associate the following sequence of self-adjoint operators $\tilde{H}_n$,

$$\tilde{H}_n = \tilde{A} - V_n, \qquad n = 1, 2, \ldots. \tag{3}$$

The problem is now to find conditions which guarantee that the sequence $\{\tilde{H}_n\}_{n=1}^{\infty}$ tends to the Friedrichs extension $\hat{H}$, i.e.,

$$s - \lim_{n\to\infty}(\tilde{H}_n - z)^{-1} = (\hat{H} - z)^{-1}, \qquad \Im(z) \neq 0 \tag{4}$$

for every semibounded self-adjoint extension $\tilde{A}$ of $\dot{A}$. In general we cannot expect that the sequence $\tilde{H}_n$ tends to $\hat{H}$ assuming only that $\{V_n\}_{n\geq 1}$ is a regularizing sequence. Actually we need a little bit more.

Proposition 2 *Let $\{V_n\}_{n\geq 1}$ be a regularizing sequence of V. If for every self-adjoint extension $\tilde{A}$ of $\dot{A} = A|\mathcal{D}$ obeying $\tilde{A} \geq \eta$, $\eta < 0$, the convergence (4) takes place, then*

$$\sup_{n\geq 1}(V_n h, h) = +\infty \tag{5}$$

for every nontrivial h of $\mathcal{N}_\eta = ker(\dot{A}^* - \eta)$.

By this proposition it seems to be natural to introduce the following notation.

Definition 3 Let $\{V_n\}_{n\geq 1}$ be a regularizing sequence of V. The sequence is called admissible with respect to $\dot{A} = A|\mathcal{D}$ if there is a $\eta < 0$ such that for every nontrivial $h \in \mathcal{N}_\eta = ker(\dot{A}^* - \eta)$ the condition (5) is satisfied.

To solve our problem an optimal result would be to show that the inverse of Proposition 2 is true, i.e., if $\{V_n\}_{n\geq 1}$ is an admissible regularizing sequence of V with respect to $\dot{A} = A|\mathcal{D}$, then for every semibounded self-adjoint extension $\tilde{A}$ of $\dot{A}$ we have that the convergence (4) is valid. Till now we cannot prove this conjecture in full generality. However, if we restrict the set of semibounded self-adjoint extensions $\tilde{A}$ of $\dot{A}$, then we can do it. To describe these restrictions we use a description of all semibounded self-adjoint extensions which goes back to [1]. Let $\tilde{A}$ be any semibounded self-adjoint extension of $\dot{A} = A|\mathcal{D}$ with lower bound greater than $\eta < 0$, i.e. $\tilde{A} \geq \eta$. By $\tilde{\nu} \geq \eta$ we denote the closed quadratic form which corresponds to $\tilde{A}$, i.e.

$$\tilde{\nu}(f,f) = ((\tilde{A} - \eta)^{1/2} f, (\tilde{A} - \eta)^{1/2} f) + \eta(f,f), \quad f \in \mathrm{dom}(\tilde{\nu}) = \mathrm{dom}((\tilde{A} - \eta)^{1/2}). \tag{6}$$

In particular, by $\hat{\nu} \geq 0$ we denote the closed quadratic form which corresponds to the Friedrichs extension $\hat{A}$ of $\dot{A}$. In accordance with [1] we have now a one-to-one correspondence between the set of all semibounded self-adjoint extensions $\tilde{A}$ of $\dot{A}$ obeying $\tilde{A} \geq \eta$ and all non-negative closed quadratic forms $\tilde{q}$ on the deficiency subspace $\mathcal{N}_\eta = ker(\dot{A}^* - \eta)$, where the form $\tilde{q}$ is not necessarily densely defined on $\mathcal{N}_\eta$. The correspondence is given by the formulas

$$\mathrm{dom}(\tilde{\nu}) = \mathrm{dom}(\hat{\nu}) \dot{+} \mathrm{dom}(\tilde{q}), \tag{7}$$

where $\dot{+}$ means $\mathrm{dom}(\hat{\nu}) \cap \mathrm{dom}(\tilde{q}) = \{0\}$, and

$$\tilde{\nu}(g+h, g+h) = \hat{\nu}(g,g) + \tilde{q}(h,h) + 2\,\mathrm{Re}(g,h) + \eta(h,h), \tag{8}$$

$g \in \mathrm{dom}(\hat{\nu}), h \in \mathrm{dom}(\tilde{q}) \subseteq \mathcal{N}_\eta$. This means, having an extension $\tilde{A}$ which obeys $\tilde{A} \geq \eta$ we can find a unique non-negative closed quadratic form $\tilde{q}$ on $\mathcal{N}_\eta$ such that (7) and (8) hold. Conversely, if we have a non-negative closed quadratic from $\tilde{q}$ on $\mathcal{N}_\eta$, then we can define by (7) and (8) a semibounded extension $\tilde{A}$ of $\dot{A}$ obeying $\tilde{A} \geq \eta$. The domain of $\tilde{q}$ may be a closed subspace of $\mathcal{N}_\eta$ or not. The Friedrichs extension $\hat{A}$ corresponds to the trivial form $\hat{q}$, i.e., dom$(\hat{q}) = \{0\}$. Very often this is expressed by $\hat{q} = +\infty$. The Krein extension $\check{A}$ is given by a form $\check{q}$ which is zero on the whole deficiency subspace $\mathcal{N}_\eta$, i.e., $\check{q} = 0$. All other forms $\tilde{\nu}$ are between $\check{\nu}$ and $\hat{\nu}$ and which yields $\check{A} \leq \tilde{A} \leq \hat{A}$. Using this description our main theorem goes now as follows.

Theorem 4 *Let $\{V_n\}_{n\geq 1}$ be an admissible regularizing sequence of V with respect to $\dot{A}$ and let $\tilde{A} \geq \eta$ be a self-adjoint extension of $\dot{A}$ obeying $\tilde{A} \geq \eta$ for some $\eta < 0$. If $\tilde{A}$ corresponds to a closed quadratic form $\tilde{q}$ on $\mathcal{N}_\eta = ker(\dot{A}^*)$ whose domain $dom(\tilde{q})$ is a closed subspace of $\mathcal{N}_\eta$, then we have*

$$s - \lim_{n\to\infty} (\tilde{H}_n - z)^{-1} = (\hat{H} - z)^{-1}, \qquad \Im(z) \neq 0, \tag{9}$$

where $\hat{H}$ is the Friedrichs extension of $\dot{H} = (A - V)|\mathcal{D}$.
In particular, if $\check{A}$ denotes the Krein extension of $\dot{A}$ with respect to the lower bound $\eta < 0$, then we have

$$s - \lim_{n\to\infty} (\check{H}_n - z)^{-1} = (\hat{H}_n - z)^{-1}, \qquad \Im(z) \neq 0. \tag{10}$$

Corollary 5 *If the deficiency indices of $\dot{A}$ are finite, then for every self-adjoint extension $\tilde{A}$ of $\dot{A}$ we have (9).*

Corollary 5 strengthens the results of Section 3 of [2].

Corollary 6 *If $\tilde{A}$ is a semibounded self-adjoint extension of $\dot{A}$ such that*

$$dim(dom(\tilde{\nu}) \setminus dom(\hat{\nu})) < +\infty, \tag{11}$$

then (9) is valid.

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ON SPECTRA OF ELLIPTIC AND SCHRÖDINGER OPERATORS.

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Given an elliptic operator A on $L^2(\mathbb{R}^N)$ defined formally by

$$A_2 = -\sum_{k,j=1}^{N} D_j\left(a_{kj} D_k\right) + \sum_{k=1}^{N} a_k D_k + D_k(b_k \cdot) + c,$$

we ask when the essential spectrum $\sigma_{ess}(A) = [0, \infty)$.
Assume that $a_k \in W^{1,\infty}(\mathbb{R}^N)$ and a_{kj} are real $1 \leq k, j \leq N$. We show that if one of the following conditions holds

1. $a_k - \nu_k \delta_{kj}$, a_k, b_k, $c \in L_2(R^N)$ and $a_{kj} = a_{jk}$

2. $a_{kj} - \nu_k \delta_{kj}$, a_k,b_k $c \in L_0^2 := \overline{L^2 \cap L^\infty}^{L^\infty}$

for some constants $\nu_k > 0$, then $\sigma_{ess}(A) = [0, \infty)$.

The method consists in showing that the resolvent difference $(\lambda + A)^{-1} - (\lambda + H_0)^{-1}$ is a compact operator in $L^2(\mathbb{R}^N)$. Here $H_0 = -\sum_{k=1}^{N} \nu_k D_k^2$.
We use the same method to get similar results for the general Schrödinger operator

$$A(\vec{b}) = -\sum_{k,j=1}^{N} (D_j - ib_j)\left(a_{kj}(D_k - ib_k)\right) + \sum_{k,j=1}^{N} a_k\left((D_k - ib_k)\right) + \sum_{k=1}^{N} (D_k - ib_k)(c_k \cdot) + c$$

where $\vec{b} = (b_1, ..., b_N)$ is a magnetic potential.

DEGENERATE ELLIPTIC BOUNDARY VALUE PROBLEMS FOR WEAK COUPLED SYSTEMS. SOLVABILITY AND MAXIMUM PRINCIPLE

Boris Paneah

Let Ω be a bounded domain in $\mathbb{R}^n$, $n \geq 3$, with a smooth boundary $\partial\Omega$. We denote by τ a smooth nonsingular vector field on $\partial\Omega$ and by ν a unit vector field of interior normals to $\partial\Omega$. In the first part of the talk we consider the following boundary value problem

$$\begin{aligned} Lu + Hu &= F \quad \text{in } \Omega \\ \frac{\partial}{\partial\tau}u + A\frac{\partial}{\partial\nu}u + Bu &= f \quad \text{on } \partial\Omega \,. \end{aligned} \tag{1}$$

Here $u = (u_1, u_2, ..., u_N)$ in an unknown vector–function: $\Omega \to \mathbb{R}^N$. L is a scalar second order elliptic differential operator on Ω, H is an arbitrary first order differential operator, A and B are smooth $N \times N$ matrices on $\partial\Omega$. So, the problem (1) is weak coupled in Ω but not on the boundary $\partial\Omega$.

It is well known that the ellipticity of the problem (1) is equivalent to the condition $\det A \neq 0$ on $\partial\Omega$. For this reason we suppose further that the set $\mu = \{q \in \partial\Omega \mid \det A(q) = 0\}$ is not empty and, moreover, μ is a submanifold of codim 1. Let us assume also that the vector field τ is transversal to μ.

Consider a sufficiently small tubular neighborhood $\mathcal{U}$ of the submanifold μ with the normal coordinates (t, x) on $\mathcal{U}$. Here $x = (x_1, x_2, ..., x_{n-2})$ are local coordinates on μ.

Condition 1. The zero eigensubspace $\mathcal{R}$ of the matrix $A(O, x)$ is p-dimensional, $p \geq 1$, and does not depend on $x \in \mu$.

This means that in some basis in $\mathbb{R}^N$ the matrix $A(t, x)$ has a form $((A_{ij}))_{i=1,j=1}^{2,2}$, where A_{11} and A_{22} are square matrices of orders p and $N - p$ respectively; $A_{i1}(t, x) = t^k A'_{i1}(t, x)$ for some integer $k \geq 1$ and smooth matrices A'_{i1}, $i = 1, 2$.

Condition 2. There exists a smooth function $a(x) \neq 0$ on μ such that $A'_{11}(O, x) = a(x)\Gamma$ where Γ is a constant $p \times p$ matrix without eigenvalues on the imaginary axis.

Denote by πu the orthogonal projection of the vector u on the subspace $\mathcal{R}$. Let $\pi_{\pm}u$ be the projections of πu on the spectral subspaces $\mathcal{R}^{\pm}$ corresponding to positive (resp. negative) eigenvalues of Γ.

Theorem 1 *If the number k is even then the problem (1) is Fredholm (between suitable spaces).*

Theorem 2 *If k is odd then the problem (1) is not Fredholm. But the modified problem*

$$\begin{aligned} Lu + Hu &= F \quad \text{in } \Omega \\ \frac{\partial}{\partial \tau}u + A\frac{\partial}{\partial \nu}u + Bu &= f \quad \text{on } \partial\Omega \setminus \mu \\ \pi_- u &= g \quad \text{on } \mu \end{aligned} \tag{2}$$

is Fredholm (between suitable spaces). Moreover, for sufficiently smooth F, f, g the restriction $u \mid \partial\Omega$ of the solution u belongs to Sobolev space $H^1(\partial\Omega\backslash\mu)$ and $[u]_\mu = [\pi_+ u]_\mu$ where $[w]_\mu$ denotes the jump of w on μ.

The second part of the talk is devoted to the strong maximum principle for boundary value problems of the type (1) (and (2) as well). We consider the following problem

$$\begin{aligned} \mathcal{L}_k u_k + \sum_{j=1}^{N} h_{kj} u_j = F_k \quad &\text{in} \quad \Omega \\ \frac{\partial}{\partial \tau_k} u_k + a_k \frac{\partial}{\partial \nu} u_k + \sum_{j=1}^{N} b_{kj} u_j = f_k \quad &\text{on} \quad \partial\Omega, \ \ 1 \le k \le N. \end{aligned} \tag{3}$$

Here all of $\mathcal{L}_k$ are elliptic second order operators on Ω with positive principle parts, τ_k are some nonsingular vector fields on $\partial\Omega$ and h_{kj}, a_k, b_{kj} are smooth functions. It is obvious that this is weak coupled boundary value problem. If all the functions a_k do not vanish on $\partial\Omega$ then (3) is an elliptic problem. Let us introduce the sets

$$\gamma_k = \{q \in \partial\Omega \mid a_k(q) = 0, \ k = 1, 2, ..., N\}$$

and let us assume for short that all γ_k are not empty. We only need the following condition on γ_k. The set γ_k, $k = 1, 2, ..., N$, does not contain any maximal trajectory of the field τ_k. (If γ_k is a submanifold this condition is fulfilled automatically). To describe the class of matrices $((h_{jk}))$ and $((b_{jk}))$ under consideration, let us introduce two definitions. We say that a matrix $((c_{ij}))_1^N$ satisfies the (rd)-condition if for all $j, k = 1, 2, ..., N$

(rd) $$c_{jj} c_{jk} \le 0, \quad k \ne j; \quad c_{jj} \sum_k c_{jk} \ge 0 \ .$$

We say that a matrix-valued function $((c_{jk}(x)))_1^N$ satisfies (zs)-condition if for $j < k$ and for every x

(zs) $$c_{jk}(x) = 0 \Rightarrow c_{kj}(x) = 0 \ .$$

Theorem 3 *(Strong maximum principle). Let $u = (u_1, u_2, ..., u_N)$ be a solution of the problem (3). Assume that*

1° *each function a_k does not change sign on $\partial\Omega$;*

2° *$a_j b_{jj} \leq 0$ on $\partial\Omega$; $h_{jj} \leq 0$ in Ω for all $j = 1, 2, ..., N$.*

3° *The matrices $((h_{jk}))$ and $((b_{jk}))$ satisfy (rd)- and (zs)-conditions.*

If $F_k \geq 0$ in Ω and $a_k f_k \geq 0$ on $\partial\Omega$ for all $k = 1, 2, ..., N$ then each function u_k either > 0 in Ω or $\equiv$ const.

Remark. This result can be generalized to the case when some coefficients a_k change sign on $\partial\Omega$. Then the strong maximum principle is formulated for the modified problem (2).

Asymptotics of Heat Flow

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The diffusion of heat in a system with periodic conductivity is governed by two scales of length. The small time diffusion is described by the geodesic distance but the large time behaviour is dictated by the distance associated with an homogenized system obtained by a suitable nonlinear averaging process.

Let H be the positive self-adjoint operator

$$H = -\sum_{i,j=1}^{d} \partial_i c_{ij} \partial_j$$

on $L_2(R^d\,; dx)$ with real-valued coefficients $c_{ij} = c_{ji} \in L_\infty(R^d\,; dx)$ satisfying

$$C = (c_{ij}) \geq \mu I > 0 \quad ,$$

in the sense of $d \times d$-matrices, uniformly on R^d. The semigroup S generated by H describes the heat flow governed by the coefficients of conductivity c_{ij}. The action of S is determined by a positive kernel K,

$$(S_t\varphi)(x) = (K_t * \varphi)(x) = \int_{R^d} dy\, K_t(x\,; y)\varphi(y) \quad ,$$

which is known to satisfy Gaussian upper and lower bounds and the behaviour of the heat flow has largely been analyzed by successive improvements of such bounds.

First, recall that if the coefficients c_{ij} are constant then

$$K_t(x\,; y) = (4\pi t)^{-d/2} |\det C|^{-1/2} \exp(-d_C(x\,; y)^2/4t)$$

with $d_C(x\,; y)^2 = ((x-y), C^{-1}(x-y))$. Thus the asymptotic behaviour is a dissipation governed by the distance d_C, the geodesic distance associated with the Riemannian metric C^{-1}.

Secondly, as a result of the work of many authors, it has been established that for variable coefficients

$$K_t(x\,; y) \sim t^{-d/2} e^{-d_C(x;y)^2/4t}$$

for all small $t > 0$ with d_C the appropriate geodesic distance. It has also been a common belief that a similar result should be true for large t. But Davies established that this is not always the case for one-dimensional systems. In particular he showed that for large t the kernel resembles a Gaussian distribution with a distance which is generally larger than the geodesic distance. In fact for periodic multi-dimensional systems Batty, Bratteli, Jørgensen and Robinson (J. Geom. Anal., to appear) have recently demonstrated that the asymptotic behaviour can be exactly identified.

Theorem *Assume the coefficients c_{ij} are periodic. Then there is a constant coefficient $d \times d$-matrix $\widehat{C}$, satisfying $\widehat{C} \geq \mu I$, with corresponding semigroup $\widehat{S}$ and kernel $\widehat{K}$, such that one has uniform convergence*

$$\lim_{t\to\infty} \|S_t - \widehat{S}_t\|_{p\to p} = 0$$

on each of the L_p-spaces over R^d.

Moreover,

$$\lim_{t\to\infty} \sup_x t^{d/2p} \left(\int_{R^d} dy\, |K_t(x\,;y) - \widehat{K}_t(x\,;y)|^q \right)^{1/q} = 0$$

with $p^{-1} + q^{-1} = 1$.

The matrix $\widehat{C}$ corresponds to the homogenization, in the sense of Bensoussan, Lions and Papanicolau, of the conductivity matrix C. It is a type of average of C and the result reflects the physical fact that the inhomogeneities of the system are averaged out with time.

It should be emphasized that the map $C \to \widehat{C}$ is highly nonlinear, although it is order preserving. If M denotes the usual mean value over R^d one has

$$M(C^{-1})^{-1} \le \widehat{C} \le M(C)$$

and $\widehat{C} = M(C)$ if and only if the coefficients are constant. These statements follow most readily from two variational principles given by Norris (Bull. Lond. Math. Soc., to appear) who extended many of Davies' estimates to the multi-dimensional case. In addition, in one-dimension one has $\widehat{C} = M(C^{-1})^{-1}$ but necessary and sufficient conditions for this identification are not known.

Finally, although the theorem identifies the asymptotic form of S its proof does not give any indication about the rate of convergence to the limit, nor does it adapt to the addition of first-order terms. It would be of interest to understand the situation with drift terms and to find methods of dealing with more general non-periodic systems.

GREEN'S FORMULA FOR GENERAL PARABOLIC PROBLEMS AND SOME OF ITS APPLICATIONS.

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Let $G \subset I\!R^n$ be a bounded domain with a boundary $\partial G \in C^\infty$, $\Omega = G \times (0,T)$, $0 < T < \infty$, $\Omega' = \partial\Omega \times (0,T)$. We denote by $(\,,\,)_G$, $(\,,\,)_\Omega$, $<\,,\,>_{\partial G}$, $<\,,\,>_{\Omega'}$ the scalar products (or their extension in $L_2(G)$, $L_2(\Omega)$, $L_2(\partial G)$, $L_2(\Omega')$ respectively.

In Ω we consider a general parabolic boundary value problem

$$Lu \equiv L(x,t,D_x,\partial_t) = f(x,t) \qquad ((x,t) \in \Omega);$$

$$B_j u \equiv B_j(x,t,D_x,\partial_t)u(x,t)|_{x=x'} = \varphi_j(x',t) \qquad (j = 1,...,m;\ x' \in \partial G); \tag{1}$$

$$\partial_t^{k-1} u|_{t=0}\varphi_{ok}\ (k = 1,...,\kappa).$$

Here $D_x = (D_1,...,D_n)$, $D_j = i\partial/\partial x_j$, $\partial_t = \partial/\partial t$, $ord\, L = 2m$, $ord\, B_j = m_j$. The order of the operator is defined as its terms highest order; the order of the $D_x^\alpha \partial_t^\beta = D_1^{\alpha_1}...D_n^{\alpha_n}\partial_t^\beta$ is $|\alpha| + 2b\beta = \alpha_1 + ... + \alpha_n + 2b\beta$; b is the divisor of the number m. The number $2b$ is called the parabolic weight of the problem, $\kappa = \frac{m}{b}$. The m_j $(j = 1,...,m)$ are arbitrary nonnegative numbers. Let r_j be an order of the expression B_j with respect to derivatives $D_\nu = i\partial/\partial\nu$ (ν is the normal to Ω') and

$$r = \max\{2m, r_1 + 1, ..., r_m + 1\} \tag{2}$$

Theorem 1 *Let the problem (1) be parabolic. Then expressions $C_j(x,t,D_x,\partial_t)$, $B_j'(x,t,D_x,\partial_t)$, $C_k'(x,t,D_x,\partial_t)$ (of corresponding orders l_j, m_j', l_k' $(j = 1,...,m;\ k = 1,...,r-m)$) exist such that Green's formula*

$$(Lu,v)_\Omega + \sum_{j=1}^{m}\langle B_j u, C_j' v\rangle + \sum_{j:1\le j\le r-2m}\langle D_\nu^{j-1}Lu, C_{m+j}'v\rangle_\Omega + \sum_{k=0}^{\kappa-1}((\partial_t^k u)(x,0), T_k v(x,0))_G$$

$$= (u, L^+ v)_\Omega + \sum_{j=1}^{m}\langle C_j u, B_j' v\rangle_\Omega + \sum_{k=0}^{\kappa-1}((T_k' u)(x,T),\, \partial_t^k v(x,T))_G \quad (u,v \in C^\infty(\bar\Omega)) \tag{3}$$

is valid.

Here T_k, T'_k are $(2m-(k+1)b)$-order differential expressions,

$$m_j + l'_j = l_j + m'_j = 2m - 1 \quad (j = 1, ...m); l'_{m+j} = -j \quad (j : 1 \le j \le r - 2m) \qquad (4)$$

(if $r = 2m$ then the third term of the left part of (3) is absent).

Expressions C_j, B'_j, C'_j $(j = 1, ..., m)$ are differential with respect to derivatives D_ν and they are pseudodifferential with respect to tangential derivatives along the Ω'.

Theorem 2 *The problem*

$$L^+ v = g \quad (in\,\Omega);\; B'_j v|_{\Omega'} = \varphi_j \quad (j = 1, ..., m); \quad \partial_t^{k-1} v|_{t=T} = \phi_{0k} \quad (k = 1, ..., \kappa)$$

(with the changed time axis direction) that is adjoint to problem (1) with respect to Green's formula (3) is parabolic if and only if the problem (1) is parabolic.

Let us note a few applications and generalizations.

A) Both for problem (1) and for problem (5) the theorem on a complete collection on isomorphics holds. By means of passage to the limit (3) may be established for the corresponding space of distributions.

B) It follows from theorems 1,2 that if we change arbitrarily $f(x,t)$ in $G_0 \times (0,T)$ or $\varphi_j(x,t)$ $(j = 1, ..., m)$ in $\gamma_0 \times (0,T)$, ($G_0 \subset G$, $\gamma_0 \subset \partial G$ are open subsets; the diameters of G_0 and γ_0 are arbitrarily small) then any vectorfunction defined on the manifold $\gamma \times (0,T)$ may be approximated by solutions of the problem (1) and their derivatives (here γ is an open piece of the (n-1)-dimensional manifold $\gamma_1 \subset G$).

C) We study L_2-theory in this work. But L_p-theory $(1 < p < \infty)$ can also be treated here.

LOCAL INCREASING OF SMOOTHNESS OF GENERALIZED SOLUTIONS OF ELLIPTIC BOUNDARY VALUE PROBLEMS IN NON-SMOOTH DOMAINS.

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This work consists of two parts. The first part was obtained together with B. Ya. Roitberg; the second part was obtained together with A. V. Sklyarets.
Let $G \subset I\!R^n$ be a bounded domain. The boundary ∂G contains the conical points, edges, etc.; let $M \subset \partial G$ be a singular pointset, $\partial G \backslash M \in C^\infty$.

In G we consider the elliptic boundary value problem

$$L(x, D)u = f,\ B_j u|_{\partial G\backslash M} = \Phi_j\ (j = 1, ..., m;\ ord\, L = 2m,\ ord\, B_j = m_j). \tag{1}$$

Let

$$r = \max\{2m, m_1 + 1, ...m_m + 1\}. \tag{2}$$

By means of integration by parts we find

$$(Lu, v) = (u, L^+v) + \sum_{1\le j\le 2m} \left\langle D_\nu^{j-1}u, \Lambda_{2m-j+1}v\right\rangle\ (u \in C^\infty(\bar{G}\backslash M), v \in C_M^\infty(\bar{G})); \tag{3}$$

here, if $v \in C_M^\infty(\overline{G})$ vanishes in some neighborhood in $\overline{G}$ of the set M then $v \in C_M^\infty(\overline{G})$, $(\ ,\ )$, $<\ ,\ >$ is the scalar product in $L_2(G_1)$, $L_2(\partial G_1)$ respectively; $D_\nu = i\partial/\partial\nu$; $\nu-$ is a normal to ∂G. Therefore the equation $Lu = f$ is equivalent to the equality

$$(u_0, L^+v) + \sum_{1\le j\le 2m} \langle u_j, \Lambda_{2m-j+1}v\rangle = (f_0, v)\ \left(v \in C_M^\infty(\overline{G})\right). \tag{4}$$

Here $u_0 = u|_{\overline{G}\backslash M}$, $u_j = D_\nu^{j-1}u|_{\partial G\backslash M}$, $f_0 = f|_{\overline{G}\backslash M}$.

Similarly, if $B_j(x, D) = \sum_{1\le k\le m_j+1} B_{jk}(x, D')D_\nu^{k-1}$ ($B_{jk}(x, D')$ is the tangential operator) then

$$B_j u|_{\partial G\backslash M} \equiv \sum_{1\le k\le m_j+1} B_{jk}(x, D')u_k = \phi_j\ (j = 1, ..., m). \tag{5}$$

We identify $u \in C^\infty(\overline{G}\backslash M)$ with the vector $(u_0, ..., u_r)$, and f with the vector $(f_0, ..., f_{r-2m})$, $f_0 = f|_{\overline{G}\backslash M}$, $f_k = D_\nu^{k-1} f|_{\partial G\backslash M}$ $(k = 1, ..., r-2m)$. Then $u = (u_0, ..., u_r)$ is the solution of the problem (1) if and only if equalities (4), (5) and

$$\left(D_\nu^{j-1} L\right)\big|_{\partial G\backslash M} = f_j \quad (j = 1, ..., r-2m) \tag{6}$$

are valid. Here the left-hand part of (6) is expressed in terms of $u_1, ..., u_r$ by formulas as (5).

Definition 1 *Let u_0, f_0 be (generalized) functions in G; u_j, f_k be (generalized) functions on $\partial G\backslash M$. Then a vector is called a generalized solution of the problem (1) if (4)-(6) are valid.*

The problem (1) (or equalities (4)-(6)) defines a mapping $A : U = (u_0, ..., u_r) \mapsto F = (f, \varphi) = (f_0, ..., f_{r-2m}, \varphi_1, ..., \varphi_m)$. To study it we must introduce some functional spaces.

For any $s \geq 0$ and $p \in (1, \infty)$ we denote by $H^{s,p}(G)$ the Bessel potential space, and let $H^{-s,p}(G) = \left(H^{s,p'}(G)\right)^*$, $1/p + 1/p' = 1$; $\| \|_{s,p}$ is the norm in $H^{s,p}(G)$ $(s \in R$, $1 < p < \infty)$. By $B^{s,p}(\partial G\backslash M)$ $(s \in R$, $1 < p < \infty)$ we denote the Besov space, $<< >>_{s,p}$ is the norm in $B^{s,p}(\partial G\backslash M)$.

Let $r > 0$ be a fixed integer, $s, p \in I\!R$, $1 < p < \infty$, $s \neq k + 1/p$ $(k = 0, 1, ..., r-1)$. By $\tilde{H}^{s,p,(r)}(\overline{G}\backslash M)$ we denote the completion of $C^\infty(\overline{G})$ with respect to the norm

$$|||u, \overline{G}\backslash M|||_{s,p,(r)} \equiv \left(\|u\|_{s,p}^p + \sum_{1 \leq j \leq r} \langle\langle D_\nu^{j-1} u, \partial G\backslash M \rangle\rangle_{s-j+1-\frac{1}{p},p}^p \right)^{\frac{1}{p}} \tag{7}$$

The closure S of the mapping $u \mapsto \left(u|_{\overline{G}\backslash M}, u|_{\partial G\backslash M}, ..., D_\nu^{r-1} u|_{\partial G\backslash M}\right)$ $(u \in C^\infty(\overline{G}))$ is the isometry between $\tilde{H}^{s,p,(r)}(\overline{G}\backslash M)$ and the subspace of direct product $\mathcal{F}^{s,p} := H^{s,p}(G) \times \prod_{1 \leq j \leq r} B^{s-j+1-\frac{1}{p},p}(\partial G\backslash M)$.
Therefore we can identify $u \in \tilde{H}^{s,p,(r)}(\overline{G}\backslash M)$ with the element $Su \in \mathcal{F}^{s,p}$. We shall write $u = (u_0, ..., u_r)$ $(\forall u \in \tilde{H}^{s,p,(r)}(\overline{G}\backslash M))$. Hence, the space $\tilde{H}^{s,p,(r)}(\overline{G}\backslash M)$ consists of vectors $u = (u_0, ..., u_r)$. This is the space of the solutions of the problem (1) (or (4)-(6)). The mapping introduced above $A : u \mapsto F = (f, \varphi)$ acts continuously from $\tilde{H}^{s,p,(r)}(\overline{G}\backslash M)$ to $\tilde{K}^{s,p} := \tilde{H}^{s-2m,p,(r-2m)}(G\backslash m) \times \Pi_{j=1}^m B^{s-m_j-\frac{1}{p},p}(\partial G\backslash M)$.

By means of the complex interpolation method we define the space $\tilde{H}^{s,p,(r)}(\overline{G}\backslash M)$ and the norm (7) for $s = \frac{k+1}{p}$ $(k = 0, ..., r-1)$.

Theorem 2 *Let $u \in \tilde{H}^{s,p,(r)}(\overline{G}\backslash M)$ $(s \in \mathbb{R},\ p \in (1,\infty))$ be a generalized solution of the problem (1) with $F \in \tilde{K}^{s_1,p_1}$. Let $x_0 \in \partial G\backslash M$, and F belong to $\tilde{K}^{s_1,p_1}$ $(s_1 \geq s, p_1 \geq p)$ locally up to the boundary in some neighborhood in $\overline{G}\backslash M$. Then the solution u belongs to $\tilde{H}^{s_1,p_1,(r)}$ locally up to the boundary in this neighborhood. A similar theorem is valid also for the elliptic problem for the Douglis-Nirenberg system.*

This theorem gives an answer to a question formulated by professor V.A.Kondratjev in 1992 at the conference in Rostock.

In the second part Sobolev's problem is studied. In this problem the boundary of the domain consists of smooth manifolds with different dimensions. A theorem on the complete collection of isomorphisms for this problem is obtained.

RADIATION CONDITION FOR DIRAC OPERATORS

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In the papers [1] and [2], results from the theory of pseudodifferential operators and spectral analysis of Schrödinger operators were combined to discuss the asymptotic properties of the Dirac operator

$$H = -i\sum_{j=1}^{3} \alpha_j \frac{\partial}{\partial x_j} + \beta + Q(x). \tag{1}$$

Here $i = \sqrt{-1}$, $x = (x_1, x_2, x_3) \in \mathbf{R}^3$ and α_j, β are the Dirac matrices, i.e., 4×4 Hermitian matrices satisfying the anticommutation relation

$$\alpha_j\alpha_k + \alpha_k\alpha_j = 2\delta_{jk}\, I, \qquad (j, k = 1, 2, 3, 4) \tag{2}$$

with the convention $\alpha_4 = \beta$, δ_{jk} being Kronecker's delta and I being the 4×4 identity matrix. The potential $Q(x) = (q_{jk}(x))$ is a 4×4 Hermitian matrix-valued function. Here we assume that $Q(x)$ is short-range in the sense that each element q_{jk} satisfies

$$\sup_{x\in\mathbf{R}^3} \left[(1 + |x|)^{1+\epsilon}|q_{jk}(x)|\right] < \infty \qquad (x \in \mathbf{R}^3,\ j, k = 1, 2, 3, 4), \tag{3}$$

where ϵ is a positive constant. The free Dirac operator H_0 is defined by

$$H_0 = -i\sum_{j=1}^{3} \alpha_j \frac{\partial}{\partial x_j} + \beta. \tag{4}$$

The aim of this talk is to show how the Dirac operator and the Schrödinger operator are related each other and how some properties of the Dirac operator and the solutions of the Dirac equation can be obtained from the corresponding properties of the Schrödinger operator. Since we have from the anticommutation relation (2)

$$(H_0)^2 = (-\Delta + 1)I, \tag{5}$$

we can anticipate a close relationship between these two operators. We also want to show that some results from the theory of pseudodifferential operators, which were used in [1] and [2], are useful in discussing our problems.

Our strategy is to combine a representation formula for the resolvent $R_0(z)$, which was originated in Yamada [3] and used in [1] and [2] with some known results on Schrödinger operators to study some new properties of the extended resolvent $R^{\pm}(\lambda)$ of the Dirac operator H with a short-range potential Q. Let

$$R^{\pm}(\lambda)f(x) = {}^t(v_1^{\pm}(x), v_2^{\pm}(x), v_3^{\pm}(x), v_4^{\pm}(x)), \tag{6}$$

where ${}^tA$ is the transposed matrix (or vector) of A, and

$$f \in L_2(\mathbf{R}^3, (1 + |x|^2)^{-\delta}\, dx) \tag{7}$$

with a fixed constant δ satisfying $\delta > 1/2$. In order to simplify the description, here we assume that $\lambda > 1$. Then, after giving another proof of the limiting absorption principle for the Dirac operator (1), we are going to prove the following:

(1) Each element $v_j^{\pm}(x)$, $j = 1, 2, 3, 4$, satisfies the radiation condition

$$\begin{cases} v_j^{\pm} \in L_2(\mathbf{R}^3, (1 + |x|^2)^{-\delta}\, dx), \\ (\partial_\ell \mp i\sqrt{\lambda^2 - 1}\tilde{x}_\ell)v_j^{\pm} \in L_2(\mathbf{R}^3, (1 + |x|^2)^{\delta - 1}\, dx), \end{cases} \tag{8}$$

where $\ell = 1, 2, 3$, $\partial_\ell = \partial/\partial x_\ell$, and $\tilde{x}_\ell = x_\ell/|x|$.

(2) $v = R^{\pm}(\lambda)f$ is characterized as a unique solution of the equation $(H - \lambda)v = f$ with the radiation condition (8).

(3) Each element $v_j^{\pm}(x)$, $j = 1, 2, 3, 4$ satisfies the asymptotic behavior

$$v_j^{\pm}(r\cdot) \sim c(\lambda, f)r^{-1}e^{\pm i\sqrt{\lambda^2 - 1}r} \quad \text{in } L_2(S^2) \tag{9}$$

as $r = |x| \to \infty$, where S^2 is the unite sphere in $\mathbf{R}^3$, and $c(\lambda, f) \in L_2(S^2)$ is determined by λ and f.

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BOUNDARY VALUE PROBLEMS IN BOUTET DE MONVEL'S ALGEBRA FOR MANIFOLDS WITH CONICAL SINGULARITIES

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In joint work with B.-W. Schulze a pseudodifferential calculus for boundary value problems on manifolds with finitely many conical singularities is constructed [4].
The idea is to combine Boutet de Monvel's concept for smooth manifolds with boundary [1] with the calculus of B.-W. Schulze [5, 6] for singular manifolds without boundary. On the smooth part of the manifold, the operators we are considering are standard elements in Boutet de Monvel's algebra. Near one of the singularities we assume that the manifold looks like the cone $X \times \overline{\mathbf{R}}_+/X \times \{0\}$, where X is a smooth compact manifold with boundary.
All the analysis is then performed on the cylinder $X \times \mathbf{R}_+$. Choosing coordinates (x, t) in $X \times \mathbf{R}_+$ we introduce Mellin symbols with values in Boutet de Monvel's algebra: the action is of Mellin type with respect to the $t-$direction, while it is pseudodifferential (in the sense of Boutet de Monvel) on the cross-section X.
The operators correspondingly act on Sobolev spaces involving the Mellin transform. Similarly as before, these spaces coincide with the standard L^2-Sobolev spaces outside the singularities. Close to $\{t = 0\}$ we additionally use weight functions $\sim t^\gamma, \gamma \in \mathbf{R}$ and the Mellin action with respect to t combined with the pseudodifferential action in x.
The construction of both, the operators and these Sobolev spaces requires the introduction of a parameter-dependent version of Boutet de Monvel's algebra.
In this calculus, the parameter plays the role of an additional covariable. Instead of relying on the theory proposed in [2], we present a new approach to Boutet de Monvel's calculus based on operator-valued pseudodifferential symbols on spaces with an $\mathbf{R}_+$ group action, cf. [6], Section 3.2.
This point of view allows a considerably faster access. Moreover, it makes some of the constructions in Boutet de Monvel's algebra more transparent and brings the concept of ('singular') Green, potential, and trace operators closer to the usual pseudodifferential theory, cf. also [3].

In order to be able to handle the asymptotics of solutions near the singularites, discrete asyptotics types play an important role in the definitions of the operators and the spaces they are acting on.

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THE WEDGE SOBOLEV SPACES WITH BRANCHING DISCRETE ASYMPTOTICS.

B.-W. Schulze

Pseudo-differential operators on manifolds with edges are a generalization of boundary value problems. They form an algebra analogously to Boutet de Monvel's algebra, [2], [5], [9], but the analogue of the transmission properties is typically violated. The "edge theory" is in a sense a combination of the theory of pseudo-differential boundary value problems and that for conical singularities. A question is, in particular, the nature of the elliptic regularity in the weighted "wedge Sobolev spaces" $W^{s,\gamma}$, $s \in \mathbb{R}$ being the smoothness, $\gamma \in \mathbb{R}$ the weight. Consider those spaces over the infinite open stretched wedge

$$\mathbb{R}_+ \times X \times \mathbb{R}^q \ni (t, x, y),$$

where X is a closed compact C^∞ manifold, $n = dim\, X$, the base of the (stretched) model cone $\mathbb{R}_+ \times X =: X^\wedge$ of the wedge, and $\mathbb{R}^q \ni y$ is the edge. If $K^{s,\gamma}(X^\wedge)$ denotes the weighted Sobolev space on $X^\wedge$ (defined by means of the Mellin transform in t near $t = 0$) and equal to $H^s(\mathbb{R} \times X)|_{(\varepsilon,\infty)\times X}$ for every $\varepsilon > 0$, there is an $\mathbb{R}_+$ action κ_λ on $K^{s,\gamma}(X^\wedge)$ defined by $(\kappa_\lambda u)(t,x) = \lambda^{\frac{(n+1)}{2}} u(\lambda t, x)$, $\lambda \in \mathbb{R}_+$. Set $< \eta >= (1 + |\eta|^2)^{\frac{1}{2}}$ and $\kappa(\eta) := \kappa_{<\eta>}$. Then $\mathcal{W}^{s,\gamma}(X^\wedge \times \mathbb{R}^q)$ is defined as the closure of $\mathcal{S}(\mathbb{R}^q, C_0^\infty(X^\wedge))$ with respect to the norm

$$\left\{ \int < \eta >^{2s} \left\| \kappa^{-1}(\eta) \left(F_{y\to\eta} u \right)(\eta) \right\|^2_{\mathcal{K}^{s,\gamma}(X^\wedge)} d\eta \right\}^{\frac{1}{2}}. \tag{1}$$

These spaces have been introduced in [5] in a more general set up for an arbitrary Banach space E instead of $\mathcal{K}^{s,\gamma}(X^\wedge)$, with a corresponding group $\{\kappa_\lambda\}_{\lambda\in\mathbb{R}_+}$ of isomorphisms. In [2] there were obtained the asymptotics of solutions for elliptic wedge problems in the case of constant exponents along the edge.
The singular terms are of the form

$$F^{-1}_{\eta\to y} \left\{ < \eta >^{\frac{1}{2}} c_{jk}(x, \hat{\eta})(t < \eta >)^{-p_j} log^k(t < \eta >) \omega(t < \eta >) \right\} \tag{2}$$

with $c_{jk}(x, y') \in C^\infty\left(X, H^s\left(\mathbb{R}^q_{y'}\right)\right)$, and hat indicating the image under the Fourier transform $F_{y'\to\eta}$. Here $p_j \in \mathbb{C}$, $Re\, p_j \to -\infty$ as $j \to \infty$. For the case of variable (along

$\mathbb{R}^q \ni y)$ and branching $p_j = p_j(y)$ the singular functions were first obtained in [6], [7]. They have the form (modulo "easier" singular terms for $s = \infty$)

$$F^{-1}_{\eta \to y} \left\{ < \eta >^{\frac{1}{2}} \left\langle \zeta(y;\hat{\eta}), (t < \eta >)^{-z} \right\rangle \omega(t < \eta >) \right\} \tag{3}$$

Here $\zeta(y;y')$ is an element of

$$C^\infty \left(\Omega_y, \mathcal{A}' \left(K, C^\infty \left(X, H^s \left(\mathbb{R}^q_{y'} \right) \right) \right) \right) \tag{4}$$

$\zeta(y,\hat{\eta}) = (F_{\eta \to y}\zeta)(y;y')$ K, V $\subset$ $\mathbb{C}_z$ compact, where $\mathcal{A}'(K,V)$ is the space of V-valued analytic functionals, carried by K. The element $\zeta \in (4)$ involved in(3) is pointwise (i.e. for every y) discrete in the sense of (2). The functions (2), (3) belong to $\mathcal{W}^{s,\gamma}(X^\wedge \times \mathbb{R}^q)$. The formula (2) shows how the smoothness in y of the coefficients of the edge asymptotics depends on $Re\, p_j$. The corresponding jumping and branching smoothness in the general case (3) is adequately formulated in the form (4). This fits to the shape of smoothing Green and smoothing Mellin operators in the wedge algebra with branching discrete asymptotics. Details for $dim\, X = 0$ are contained in [7]. The generalization to arbitrary $dim\, X$ is of analogous structure.

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ON THE INDEX OF PAIR OF PROJECTORS

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We study the relative index of two orthogonal infinite dimensional projections which, in the finite dimensional case, is the difference in their dimensions. We relate the relative index to the Fredholm index of appropriate operators, discuss its basic properties, and obtain various formulas for it. We apply the relative index to counting the change in the number of electrons below the Fermi energy of certain quantum systems and interpret it as the charge deficiency. Wy study the relation of the charge deficiency with the notion of adiabatic charge transport that arises from the consideration of the adiabatic curvature. It is shown that, under a certain covariancc, (homogeneity), condition the two are related. The relative index is related to Bellissard's theory of the Integer Hall effect. For Landau Hamiltonians the relative index is computed explicitly for all Landau levels.

TWO-SIDED BOUNDS ON THE HEAT KERNEL FOR THE SCHRÖDINGER OPERATOR

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Let us consider the operator $H = -\Delta + V$ acting in $L^p = L^p(\mathbb{R}^d)$, $d \geq 2$, for some $p \in [1, \infty[$. In this report we are interested in assumptions on V, guaranteeing the following estimates

$$c_L g_t(|\, x - y \,|) \leq e^{-tH}(x,y) \leq c_V g_t(|\, x - y \,|) \ , \quad t > 0, \ x, y \in \mathbb{R}^d \tag{1}$$

where c_L and c_V are positive numbers, $g_t(a) = (4\pi t)^{-\frac{d}{2}} e^{-\frac{a^2}{4t}}$ and $e^{-tH}(x,y)$ is the integral kernel of e^{-tH}.
Simple arguments show that the condition $\|(-\Delta)^{-1} V\|_\infty \leq e_V - 1$ (for $V \leq 0$) is necessary for pointwise (a.e.) inequality $e^{-tH}(x,y) \leq e_V g(|\, x - y \,|)$ to hold. On the other hand, the condition

$$\inf_{\lambda > 0} \|(\lambda - \Delta)^{-1} \,|\, V \,|\, \|_\infty \equiv a_1(|\, V \,|) < 1$$

is sufficient for the correct definition of H and $e^{-tH}(x,y)$ and for the following estimates to hold

$$c_1 e^{-\lambda_1 t} g_{ta_1}(|\, x - y \,|) \leq e^{-tH}(x,y) \leq c_2 e^{\lambda_1 t} g_{ta_2}(|\, x - y \,|)$$

where λ_1, $c_{1,2}$, $a_{1,2}$ are positive numbers, $a_1 < 1$, $a_2 > 1$ (see [6], [3], [4]).
The main goal of this report is to prove (1) under some assumptions on V. In particular, I will show the implication

$$\|(-\Delta)^{-1} \,|\, V \,|\, \|_\infty < 1 \Rightarrow (1) \ , \qquad d \leq 3 \tag{2}$$

and related results for other $d's$.
The material is, partly, a joint work with V.A. Liskevich.
The method of proving (1), presented here, is based on some ideas of J. Nash [5] and E.B. Davies [2].
We always assume that $V \in L^1_{loc}(\mathbb{R}^d)$, $V = V^+ - V^-$, $V^\pm \geq 0$ and V^- is a $-\Delta$-form bounded potential, that is $V^- \leq \beta(-\Delta) + c(\beta)$ for some numbers $\beta < 1$ and $c(\beta) \in \mathbb{R}^1$.
Let H be the form sum $-\Delta + V$.
Let $\mathcal{S}(\alpha) f(x) = e^{\alpha \cdot x} f(x)$, $\alpha \in \mathbb{R}^d$.

Proposition 1 *Assume that*

$$\|\mathcal{S}(\alpha)e^{-tH}\mathcal{S}(-\alpha)f\|_1 \leq ce^{\alpha^2 t+\lambda_0 t}\|f\|_1 \quad , \qquad f \in L^1 \cap L^2$$

for some numbers c, $\lambda_0 \geq c(\beta)$, *all* $\alpha \in \mathbb{R}^d$ *and* $t > 0$. *Then for pointwise a.e.* (x, y)

$$e^{-tH}(x,y) \leq c^2 \left(\frac{2\pi d c_N(d)}{1-\beta}\right)^{\frac{d}{2}} \cdot e^{\lambda_0 t} g_t(|\, x-y\,|) \quad ,$$

where $c_N(d)$ *is the best constant in Nash's inequality*

$$\|\varphi\|_2^{2+\frac{4}{d}} \leq \kappa \|\nabla\varphi\|_2^2 \cdot \|\varphi\|_1^{\frac{4}{d}} \quad .$$

Proof: One can show that $T^t(\alpha) =: \mathcal{S}(\alpha)e^{-t(H+\alpha^2)}\mathcal{S}(-\alpha)$ is a holomorphic semigroup on L^2 and $-\frac{d}{dt}T^t(\alpha) = H(\alpha)T^t(\alpha)$, where $H(\alpha) =: H + 2\alpha \cdot \nabla$. Since $\langle \alpha \cdot \nabla f, f\rangle = 0$, $\forall f \in H^1(\mathbb{R}^d)$, now one can follow [5] to obtain

$$e^{\alpha\cdot x}e^{-tH}(x,y)e^{-\alpha\cdot y} \leq \left(\frac{2\pi d c_N(d)}{1-\beta}\right)^{\frac{d}{2}} \cdot c^2 e^{\lambda_0 t+\alpha^2 t}(4\pi t)^{-\frac{d}{2}}$$

and then put $\alpha = \frac{x-y}{2t}$.

<u>Results</u>

First of all notice that the operator $\Delta - 2\alpha \cdot \nabla$ defined on $(1-\Delta)^{-1}L^p$ generates a holomorphic semigroup on L^p, $1 \leq p < \infty$, $e^{t(\Delta-2\alpha\cdot\nabla)} = \mathcal{S}(\alpha)e^{t(\Delta-\alpha^2)}\mathcal{S}(-\alpha)$ and $(-\Delta+2\alpha\cdot\nabla)^{-1}$ has the integral kernel $(-\Delta+2\alpha\cdot\nabla)^{-1}(x,y) = e^{\alpha\cdot(x-y)}(\alpha^2-\Delta)^{-1}(x,y)$. Assume for a moment that $\|V^-(-\Delta+2\alpha\cdot\nabla)^{-1}\|_{1,1} = \delta < 1$. Then according to J. Voigt's theorem [8] $\Delta - 2\alpha\cdot\nabla + V^-$ defined on $(1-\Delta)^{-1}L^1$ generates a bounded C_0–semigroup on L^1 and $\|e^{t(\Delta-2\alpha\cdot\nabla+V^-)}\|_{1,1} \leq \frac{1}{1-\delta}$.

Proposition 2 *Let* $d = 3$. *Assume that* $\|(-\Delta)^{-1}V^-\|_\infty = \delta < 1$. *Then*

$$e^{-tH}(x,y) \leq c_0 g_t(|\, x-y\,|)\ , \quad c_0 \leq (1-\delta^{-\frac{d+4}{2}} \cdot (2\pi d c_N(d))^{\frac{d}{2}} \quad .$$

Proof:

$$\begin{aligned}(-\Delta+2\alpha\cdot\nabla)^{-1}(x,y) &= (4\pi\,|\,x-y\,|)^{-1}e^{-|\alpha||x-y|+\alpha\cdot(x-y)} \\ &\leq (4\pi\,|\,x-y\,|)^{-1} = (-\Delta)^{-1}(x,y)\end{aligned}$$

Therefore

$$\begin{aligned}\|Y^-(-\Delta+2\alpha\cdot\nabla)^{-1}\|_{1,1} &= \|(-\Delta-2\alpha\cdot\nabla)^{-1}V^-\|_\infty \\ &\leq \|(-\Delta)^{-1}V^-\|_\infty = \delta < 1\end{aligned}$$

and

$$
\begin{aligned}
\|\mathcal{S}(\alpha)e^{-tH}\mathcal{S}(-\alpha)f\|_1 &= \|e^{-t(H+2\alpha\cdot\nabla-\alpha^2)}f\|_1 \\
&\leq e^{\alpha^2 t}\|e^{t(\Delta-2\alpha\cdot\nabla+V^-)} \mid f \mid \|_1 \\
&\leq \frac{e^{\alpha^2 t}}{1-\delta}\|f\|_1 \ , \quad f\in L^1\cap L^2 \ .
\end{aligned}
$$

Appealing to Prop. 1 one can get the desired result.

Let us now consider the case $d=5$. We have ($z=x-y$)

$$
\begin{aligned}
(-\Delta+2\alpha\cdot\nabla)^{-1}(x,y) &= \frac{1}{8\pi^2}\frac{1+\mid\alpha\mid\mid z\mid}{\mid z\mid^3}e^{-|\alpha||z|+\alpha\cdot z} \\
&\leq (-\Delta)^{-1}(x,y)+\frac{1}{8\pi^2}\frac{\mid\alpha\mid}{\mid z\mid^2}e^{-|\alpha||z|+\alpha\cdot z} \quad , \\
\|(-\Delta+2\alpha\cdot\nabla)^{-1}V^-\|_\infty &\leq \|(-\Delta)^{-1}V^-\|_\infty \\
&+ \frac{1}{8\pi^2}\mid\alpha\mid\sup_{x\in\mathbb{R}^d}\int_{\mathbb{R}^d}\mid z\mid^{-2} e^{-|\alpha||z|+\alpha\cdot z}V(z+x)dz \ .
\end{aligned}
$$

Since

$$
\begin{aligned}
\int_{\mathbb{R}^5}\mid z\mid^{-2} e^{-|\alpha||z|+\alpha\cdot z}V^-(z+x)dz &\leq \|V^-\|_{\frac{d}{2}}\left(\int_{\mathbb{R}^5}[\mid x\mid^{-2} e^{-|\alpha||x|+\alpha\cdot x}]^{\frac{d}{d-2}}\right)^{\frac{d-2}{d}} dx \\
&= \|V^-\|_{\frac{d}{2}}\cdot\mid\alpha\mid^{-1}\cdot c(d) \quad \text{with explicit } c(d),
\end{aligned}
$$

then $\|(-\Delta+2\alpha\cdot\nabla)^{-1}V^-\|_\infty\leq\|(-\Delta)^{-1}V^-\|_\infty+\frac{1}{8\pi^2}c(d)\|V^-\|_{\frac{d}{2}}$. Thus we get

Proposition 3 *Let $d=5$. Assume that*

$$\|(-\Delta)^{-1}V^-\|_\infty+\frac{c(d)}{8\pi^2}\|V^-\|_{\frac{d}{2}}=\delta<1 \ .$$

Then $e^{-tH}(x,y)\leq c_0 g_t(\mid x-y\mid)$, $c_0\leq(1-\delta)^{-\frac{d+4}{2}}(2\pi d c_N(d))^{\frac{d}{2}}$.

Remark. (2) is a simple consequence of Prop. 2. Surely, the proof of Prop. 3 works also in other dimensions. Related to Prop. 3 the bound $e^{-tH}(x,y)\leq c_V e^{\lambda_0 t}g_t(\mid x-y\mid)$, $\lambda_0>0$, has been discussed in [1] for $V\in L^q(\mathbb{R}^d)$, $q>\frac{d}{2}$, and in [7] under the assumptions $a_1(V^-)=0$ and $V^-\in L^{\frac{d}{2}}(\mathbb{R}^d)$.

Concluding I give a variant of Prop 3.

Proposition 4 *Let $d\geq 4$. Assume that $\|(-\Delta)^{-1}V^-\|_\infty=\delta<1$ and $V^-\leq\varepsilon(-\Delta)+c_1 e^{\frac{d}{\varepsilon}}$ ($\forall\varepsilon>0$) for a suitably small constant c_1. Then there exist $c_V<\infty$ depending only on d,δ, and c_1 such that $e^{-tH}(x,y)\leq c_V g_t(\mid x-y\mid)$.*

Proof: Choose $d = 5$ for simplicity. Let $\psi(y) = \mid y \mid^{-1} e^{-\frac{|\alpha||y|+\alpha\cdot y}{2}}$ $\psi_x(\cdot) = \psi(\cdot - x)$. Then $\varepsilon\|\nabla\psi\|_2^2 + c_1 e^{\frac{1}{\varepsilon}}\|\psi\|_2^2 \geq \langle\psi_x, V^-\psi_\alpha\rangle$. A straightforward calculation gives $\|\nabla\psi\|_2^2 \leq a \mid \alpha \mid^{-1} \lg \mid \alpha \mid$, $\|\psi\|_2^2 \leq b \mid \alpha \mid^{-2}$. The statement of Prop. 4 now easily follows.

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APPROXIMATION BY SOLUTIONS OF NON-LOCAL ELLIPTIC PROBLEMS.

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Let $G \subset I\!R^n$ be a bounded domain with the boundary $\Gamma \in C^\infty$, G_1 is a subdomain of G with the boundary $\gamma \in C^\infty$, $\Gamma \cap \gamma = \emptyset$, $G_2 = G \backslash G_1$; $\alpha : \Gamma \to \gamma$ is a diffeomorphism; for any function $u(y)$ $(y \in \gamma)$ we set $(Ju)(x) := u(\alpha x)$ $x \in \Gamma$. We consider the non-local elliptic problem [1]

$$L_i u_i(x) = f_i(x) \ (x \in G_i;\ ord\, L_i = 2m_i;\ i = 1, 2),$$
$$B_j u := J\left(B_{j1}u_1(y) + B_{j2}u_2(y)\right)(x) + B_{j3}u_2(x) = \varphi_j(x)$$
$$(x \in \Gamma;\ y = \alpha x \in \gamma;\ j = 1, ..., l;\ l = m_1 + 2m_2)$$

The system of boundary expressions B_{ji} is assumed to be normal [1]. In this case one can introduce the adjoint with respect to Green's formula non-local problem for the formally adjoint expressions L_i^+; corresponding boundary expressions we denote $B'_{ji}(j = 1, ..., l; i = 1, 2, 3)$ The adjoint problem is also elliptic.

Let Λ_1 be a smooth (n-1)-dimensional manifold without border, situated in G_1 or in G_2, Λ is an open subset of Λ_1 having sufficiently smooth boundary. For $u = (u_1, u_2)$, where u_1, u_2 are sufficiently smooth functions, we set $\nu_r u := (u|_\Lambda, ..., d_\nu^{r-1}u|_\Lambda)$, $D_\nu = \frac{i\partial}{\partial\nu}$, ν is the normal to Λ. Let G_0 be a domain having arbitrarily small diameter situated in G_1 or G_2. We put

$$M(G_0) := \left\{u = (u_1, u_2) \in C^\infty(G_1) \times C^\infty(G_2) : \operatorname{supp} Lu \subset G_0; B_j u = 0,\ j = 1, ..., l\right\},$$

$$\nu_r M(G_0) := \{\nu_r u : u \in M(G_0)\}$$

Theorem 1 *Let a) $G_0 \subset G_2$, $\Lambda \subset G_1$, $G_1 \backslash \bar{\Lambda}$ connected;*
b) the expression L_2^+ has in G_2 the property of uniqueness for Cauchy problem: if $L_2^+ \nu = 0$ in a domain $G' \subset G_2$ and $\nu = 0$ in $G'' \subset G'$, then $\nu = 0$ in G';
c) the problem $L_1^+ \nu_1 = 0$ in $G_1 \backslash \bar{\Lambda}$, $B'_{j1}\nu = 0$ on γ $(j = 1, ..., l)$ has only zero solution. Then $\nu_{2m_1} M(G_0)$ is dense in $\prod_{j=1}^{2m_1} B^{s_j, p}(\Lambda)$ for any $s_j \geq 0$, $1 < p < \infty$.

Theorem 2 *Let be*
a) $G_0 \subset G_2$, $\Lambda \subset G_1$, $\bar{\Lambda}$ the boundary of a subdomain $G' \subset G_1$

b) L_2^+ has in G_2 the property of uniqueness for Cauchy problem;
c) the Dirichlet problem for $L_1^+\nu_1 = 0$ in G' has no more than one solution;
d) the problem $L_1^+\nu_1 = 0$ in $G_1\backslash\bar{G}'$, $B_{j1} = 0$ on γ has only zero solution.

Then $\nu_{m_1}M(G_0)$ is dense in $\prod_{r=1}^{m_1} B^{m_1-r+1-\frac{1}{p},p}(\Lambda)$.

Let now Γ_0 be an open subset of Γ; the diameter of Γ_0 may be arbitrarily small. We put $M(\Gamma_0) := \{u \in C^\infty(G_1) \times C^\infty(G_2) : Lu = 0; \operatorname{supp} B_j u \subset \Gamma_0, j = 1, ..., l\}$, $\nu_r M(\Gamma_0) := \{\nu_r u : u \in M(\Gamma_0)\}$.

Theorem 3 *Let be $\Lambda \subset G_2$ $G_2\backslash\bar{\Lambda}$ is connected and let the expression L_2^+ has in G_2 the property of uniqueness for Cauchy problem. Then $\nu_{2m_2}M(\Gamma_0)$ is dense in $\prod_{j=1}^{2m_2} B^{s_j,p}(\Lambda)$ for any $s_j \geq 0$ and $1 < p < \infty$.*

Some more assertions of such are proved. Similar questions for usual boundary value probems were studied since 1960 by many authors (see [2], [3] and its references).

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SUPEROPERATORS AND EXISTENCE OF THE WAVE OPERATORS.

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We study a problem of existence of the wave operators using Liouville superoperators terms

$$[A,B]T = AT - TB,\ W_\pm(T) = s-\lim_{t\to\mp\infty} e^{i[A,B]t}T.$$

We consider the model problem. Let be $\mathcal{L} = [x,x]$ where x is the operator of multiplication by x in $L_2(R)$.

The following results are obtained.

Theorem 1. *Let $P_\pm$ be orthogonal projections on Hardy spaces $H_2^\pm$ respectively, T be a bounded operator such that $\mathcal{L}T \in S_2$. If $P_\pm \mathcal{L}T \in S_1$ Then $w-\lim_{t\to\mp\infty} e^{ixt}Te^{-ixt}$ exists.*

Theorem 2. a) *Let T be a bounded operator such that $\mathcal{L}T$ has L_2^{loc}-kernel $k(x,y)$. Suppose that*

$$\frac{\partial k}{\partial x} - \frac{\partial k}{\partial y} \in L_2^{loc}.$$

Then $w-\lim_{t\to\mp\infty} e^{ixt}Te^{-ixt}$ exist.

b) *Let be $\hat{\mathcal{L}} = \left[\frac{1}{i}\frac{\partial}{\partial x}, \frac{1}{i}\frac{\partial}{\partial x}\right]$ T be a bounded operator such that $\mathcal{L}T \in S_2$, $\hat{\mathcal{L}}\mathcal{L}T \in S_2$, $\hat{\mathcal{L}}T \in B$. Then $W_\pm(T) = w-\lim_{t\to\mp\infty} e^{ixt}Te^{-ixt}$ exists. Moreover*
$\lim_{t\to\mp\infty} \|e^{ixt}(T - W_\pm(T)e^{-ixt}f\|$ exists for all $f \in L_2$.
In particular, if T is unitary and $W_\pm(T)$ is unitary then

$$W_\pm(T) = s-\lim_{t\to\mp\infty} e^{ixt}Te^{-ixt}.$$

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Bound on the Density at the Nucleus

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The Hamiltonian of an atom of N electrons with q spin states each and a fixed nucleus of charge Z located at the origin is given by

$$H_{N,Z} = \sum_{\nu=1}^{N}\left(-\Delta_\nu - \frac{Z}{|\mathfrak{r}_\nu|}\right) + \sum_{\substack{\mu,\nu=1\\ \mu<\nu}}^{N} \frac{1}{|\mathfrak{r}_\mu - \mathfrak{r}_\nu|} \tag{1}$$

self-adjointly realized in $\bigwedge_{\nu=1}^{N}(L^2(\mathbb{R}^3)\otimes\mathbb{C}^q)$. Furthermore we write ψ for an eigenfunction that belongs to the bottom of the spectrum of $H_{N,Z}$, i.e., a ground state eigenfunction and

$$\rho_\psi(\mathfrak{r}) = \sum_{\sigma_1,\ldots,\sigma_N=1}^{q}\sum_{\nu=1}^{N}\int_{\mathbb{R}^{3(N-1)}} |\psi(\mathfrak{r}_1,\sigma_1;\ldots;\mathfrak{r}_{\nu-1},\sigma_{\nu-1};\mathfrak{r},\sigma_\nu;\mathfrak{r}_{\nu+1},\sigma_{\nu+1};\ldots;\mathfrak{r}_N,\sigma_N)|^2 \, d\mathfrak{r}_1\ldots d\mathfrak{r}_{\nu-1}d\mathfrak{r}_{\nu+1}\ldots d\mathfrak{r}_N \tag{2}$$

for the corresponding density.

A quantity of particular interest is the ground state density $\rho_\psi(0)$ at the nucleus. Recently Narnhofer [3] argued that one might expect for an atom $\rho_\psi(0) = O(Z^3)$. In this talk we will outline a proof of this conjecture. In fact we can prove

Theorem 1 *Let ρ_ψ be a ground state density of $H_{N,Z}$ and $N = Z$, then*

$$\rho_\psi(0) \leq \frac{\pi}{48} q Z^3 + const\ Z^{161/54}.$$

This value is not in disagreement with Lieb's Strong Scott Conjecture [2] which is actually older and stronger than Narnhofer's: according to Lieb the scaled atomic density $\rho_\psi(\mathfrak{r}/Z)/Z^3$ should converge to the corresponding quantity of the bare Schrödinger operator $H^0_{N,Z}$ which equals $H_{N,Z}$ except for the omission of the second sum, the electron-electron interaction. If one assumes this convergence to be pointwise this would predict $\rho_\psi(0) = \frac{\zeta(3)}{8\pi} q Z^3 + o(Z^3)$ (Lieb [2], (7.35)). We are now explicitly able to see that our theorem does support this conjecture, since $\frac{\pi}{48}$, which is about 0.065, is bigger than $\frac{\zeta(3)}{8\pi}$, which is about 0.048. On the other hand it shows that our estimate is rather good – we loose only using an inequality of Hoffmann-Ostenhof et al. [1] right at the beginning of our proof while all other estimates are asymptotically correct using phase space localizations developed in [4] and [5] – but presumably not yet sharp.

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ON A NOTION OF RESURGENT FUNCTION OF SEVERAL VARIABLES.

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The aim of this report is the definition of a resurgent function of several independent variables. The resurgent function theory introduced by Jean Ecalle (see [1]) has up to the moment wide applications to different mathematical and physical problems. For example, such problems are: investigation of the thin structure of the spectrum for Schrödinger operators, Dulac's problem on finiteness of the number of limit cycles and others.

However, some mathematical and physical problems require the notion of a resurgent function of *several* variables. These are: the investigation of solutions to differential equations at infinity, investigation of wave diagrams in electromagnetic theory and so on.

The notion of a resurgent function of several variables can be introduced as follows.

Let $\mathbf{C}^n$ be the complex space with coordinates $x = (x^1, \dots, x^n)$.

Definition 1 The function $f(x^1, \dots, x^n)$ is called *a resurgent function* if it admits a representation of the form

$$f(x) = \ell[\tilde{f}(\zeta, x)] = \int_{\Gamma} e^{-\zeta} \tilde{f}(\zeta, x) d\zeta, \tag{1}$$

the function $\tilde{f}(\zeta, x)$ being an infinitely continuable homogeneous (hyper)function of (ζ, x) of degree -1. Here the contour Γ goes to infinity along the direction of the real axis of the plane $\mathbf{C}_\zeta$. The set of singular points of $\tilde{f}$ surrounded by Γ is called *a support* of the resurgent function f.

Let $\tilde{\mathcal{R}}$ be a convolution algebra of functions $\tilde{f}(\zeta, x)$ of the type described above and let $\mathcal{R}$ be a set of resurgent functions. The following two assertions describe the main properties of the operator (1).

Theorem 1 *The set $\mathcal{R}$ is an algebra with respect to the standard multiplication. The operator ℓ defined by formula (1) is a homomorphism of algebras*

$$\ell : \tilde{\mathcal{R}} \to \mathcal{R}.$$

Theorem 2 *The following commutation formulas*

$$\frac{\partial}{\partial x^i} \ell(\tilde{f}) = \ell\left(\left(\frac{\partial}{\partial \zeta}\right)^{-1} \left(\frac{\partial}{\partial x^i}\right) \tilde{f}\right), \; i = 1, \dots, n$$

hold.

We present also the investigation of asymptotic expansions of resurgent functions at infinity. It happens that an analogue of the elementary resurgent symbol [1,2] (which is essentially an asymptotic expansion of the resurgent function at infinity) for several variables is an expansion of the type

$$f(x) = e^{-S(x)} \sum_{k=0}^{\infty} A_k(x), \tag{2}$$

where $S(x)$ is a homogeneous function of degree 1 and $A_k(x)$ are homogeneous functions of degree $-k$.

The corresponding asymptotic expansion of the function $\tilde{f}(\zeta, x)$ with respect to smoothness has the form

$$\tilde{f}(\zeta, x) = \frac{A_0(x)}{\zeta - S(x)} + \ln(\zeta - S(x)) \sum_{k=0}^{\infty} \frac{(\zeta - S(x))^k}{k!} A_{k+1}(x). \tag{3}$$

Functions having singularities of the type (3) are called *resurgent functions with simple singularities* (for one variable see, for example, [2]).

However, in general the function $S(x)$ can have ramification at some points of $\mathbf{C}^n \setminus \{0\}$ where the function $f(x)$ is regular (the so-called *focal* points). At such points the expansions (2) and (3) do not work and, hence, the notion of resurgent functions with simple singularities is to be modified. Such a modification, done in the report, uses the so-called $\partial/\partial\zeta$*-transformation* introduced by the authors (see [4]):

$$F^{\partial/\partial\zeta}[f(\zeta, x)] = \left(\frac{i}{2\pi}\right)^{n/2} \left(\frac{\partial}{\partial\zeta}\right)^{n/2} \int\limits_{h(\zeta,p)} e^{-xp\frac{\partial}{\partial\zeta}} f(\zeta, x) dx.$$

With the help of this transformation we write down modifications of (2) and (3) at focal points being invariant along solutions of partial differential equations.

The introduced notions can be applied to certain mathematical and physical problems (such as obtaining asymptotic expansions at infinity of solutions to partial differential equations, investigation of wave diagrams in electromagnetic theory and others).

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ABSENCE OF ABSOLUTELY CONTINUOUS SPECTRA FOR HIGH BARRIERS.

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The result presented in this talk was obtained jointly with I.McGillivray and G. Stolz. It concerns the nature of the spectrum of $-\frac{1}{2}\Delta+V$ if the potential V has high barriers. We say that V has barriers of form $S_1 \subset \mathbb{R}^d$ height $h_1 > 0$ and width $w_h > 0$ provided

(i) $\mathbb{R}^d \setminus \bigcup_n S_n$ has only bounded connected components.

(ii) $V(x) \geq h_n$ for $dist(x, S_n) \leq \frac{w_n}{2}$.

Theorem *Let $V \in L^1_{loc} - K_d$ have barriers of form S_n, height h_n and w_n. If $h_n \to \infty$ and $\sum_n \sigma(S_n)\, exp\left(-\frac{1}{8+\varepsilon}\sqrt{h_n}w_n\right) < \infty$ for some $\varepsilon > 0$ then $\sigma_{ac}\left(-\frac{1}{2}\Delta + V\right) = \emptyset$.*

The above theorem is a multidimensional extension of results of Simon and Spencer [1]. The corresponding generalization of the Simon and Spencer technique for "wide well" potentials has been given in [3]; see also [4]. While the general scheme of the proof is a decoupling method as in [1], there is a difference: in the one-dimensional situation one can use resolvents and their explicit representation by solutions to obtain the necessary trace estimates. In the multidimensional case more involved techniques are necessary. Apart from a factorization technique developed in [2,3] the following propabilistic estimate is the key ingredient of the proof. Denoting Brownian motion by $(\Omega, \mathbf{P}^x, X_t)$ and the first hitting time of a set S by $\tau_s(w) = \inf\{s > 0; X_s(w) \in S\}$ we have:

Occupation time lemma *Let $t > 0$ and $S \subset \mathbb{R}^d$ compact, and $T_{S^\delta} := \lambda\{0 < s < t; dist(X_s(w), S) \leq \delta\}$. Then, for $0 < \alpha < t$,*

$$\mathbf{P}^x\left[\tau_s \leq t, T_{s^\delta} \leq \alpha\right] \leq c \cdot exp\left(-\frac{\delta^2}{(4+\varepsilon)\alpha}\right),$$

where c only depends on $\varepsilon > 0$ and the dimension d.

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Analysis on Local Dirichlet Spaces

Karl-Theodor Sturm, Erlangen

Every regular Dirichlet $(\mathcal{E}, \mathcal{D}(\mathcal{E}))$ form on a locally compact space X defines in an intrinsic way a metric ρ on X. This metric ρ is the key to prove various results in the context of local Dirichlet forms which are known either in differential geometry or in p.d.e.

For instance, we give sharp conditions for recurrence as well as for conservativeness and sharp spectral bounds. These conditions are in terms of the volume growth $v : r \mapsto m\,(B_r(x))$ of concentric balls $B_r(x) \subset X$ which are defined intrinsically by means of the metric ρ. For instance, if

$$\int_1^\infty \frac{r}{v(r)}\,dr = \infty$$

(e.g. if $v(r) \leq C \cdot r^2$ for large r) then $\mathcal{E}$ is **recurrent** and if

$$\int_1^\infty \frac{r}{\log v(r)}\,dr = \infty$$

(e.g. if $v(r) \leq \exp(C \cdot r^2)$ for large r) then $\mathcal{E}$ is **conservative**. This improves or generalizes results by CHENG/YAU, KARP, KARP/LI, GRIGOR'YAN and TAKEDA. We also derive L^p-growth conditions for nonnegative sub- or supersolutions on X. In particular, we obtain L^p-Liouville theorems which extend results by YAU and KARP. Finally, we prove a sharp integrated **heat kernel estimate** of the form

$$\int_A \int_B p(t,x,y)\,m(dy)\,m(dx) \leq \sqrt{m(A)}\sqrt{m(B)} \cdot \exp\left(-\frac{\rho^2(A,B)}{2t}\right) \cdot \exp\left(-\lambda \cdot t\right)$$

generalizing recent results by DAVIES and GRIGOR'YAN.

In order to get the sharp pointwise heat kernel estimates of LI/YAU, DAVIES, VAROPOULOS and SALOFF-COSTE we have to assume that the doubling property holds true for the intrinsic balls and that on these balls a uniform Poincare inequality is satisfied. Under these assumptions BIROLI/MOSCO derived a uniform elliptic Harnack inequality. We prove that the latter two properties already imply that a uniform **parabolic Harnack inequality** holds true. For instance, this in turn implies that all solutions of the parabolic equation $Lu = \frac{\partial}{\partial t}u$ are Hölder continuous (w.r.t. the intrinisic metric).

Actually, we prove also the converse: if a uniform parabolic Harnack inequality holds true then the doubling property and a uniform Poincare inequality must hold true. This extends recent results by SALOFF-COSTE and GRIGOR'YAN.

We emphasize that the scope of applications of these results is much broader than classical Riemannian geometry. The results also apply to uniformly elliptic operators on Riemannian manifolds (cf. SALOFF-COSTE) as well as to uniformly elliptic operators with weights (cf. FABES/KENIG/SERAPIONI and FABES/JERISON/KENIG), to Hörmander type operators and general subelliptic operators on $\mathbb{R}^N$ (cf. FEFFERMAN/PHONG, NAGEL/STEIN/WAINGER, FEFFERMAN/SANCHEZ-CALLE, JERISON, JERSISON/SANCHEZ-CALLE, BIROLI/MOSCO).

STRUCTURE OF SINGULARITIES OF SOLUTIONS OF ELLIPTIC EQUATIONS

B. Fischer and N. Tarkhanov

Let P be an elliptic differential operator with real analytic coefficients on an open set $X \subset \mathbb{R}^n$. Weak solutions of the equation $Pf = 0$ on an open set $U \subset X$ are known to be real analytic functions in U.

Given a closed set $S \subset X$ and a solution f of $Pf = 0$ in $X \backslash S$, the set S may be considered as the set of singularities of f in X.

Example 1. Denote by Φ a fundamental solution of P in X which exists because of Malgrange's theorem (1955). For a distribution $h \in \mathcal{E}'$ with support in S the potential $\Phi(h)$ satisfies the equation $P\Phi(h) = 0$ outside S. Such singularities are considered to be the simplest singularities on S.

The question arises whether an arbitrary singularity on S may be decomposed into the simplest singularities. If S is compact the question was answered by Tarkhanov (1989).

A measure m on S is said to be massive if every subset of S of zero measure m has empty interior.

Example 2. Choose a dense sequence $\{y_\nu\}$ of points of S and a sequence $\{m_\nu\}$ of positive numbers such that $\sum_\nu m_\nu < \infty$. Set $m(\sigma) = \sum_{y_\nu \in \sigma} m_\nu$ for a subset $\sigma \subset S$. Then m is a massive measure on S.

Thus a massive measure always exists on S and we fix such a measure, say, m.

Theorem 1. *Suppose that S is a locally connected compact subset of X. Then for each solution f of $Pf = 0$ in $X \backslash S$ there exist a unique solution f_e of $Pf_e = 0$ in X and a sequence $\{c_\alpha\} \subset L^2(m)$ satisfying $\|\alpha! c_\alpha\|^{1/|\alpha|}_{L^2(m)} \to 0$, such that*

$$f(x) = f_e(x) + \sum_\alpha \int_S D_y^\alpha \Phi(x,y) c_\alpha(y) dm(y) \quad \textit{for} \quad x \in X \backslash S.$$

Proof. See Tarkhanov [3]. ∎

For arbitrary closed subsets S of X this theorem fails. The obvious reason is that the derivatives $D_y^\alpha \Phi(x,y)$ may be not in $L^2(m)$ for a fixed $x \in X \backslash S$. However, there may be deeper obstacles also. At least, the technique of Tarkhanov [3] does not work in the case.

In the paper we investigate the singularities in the small, i.e., within a relatively compact open set U in X.

Moreover, we limit ourselves to the singularities laying on a smooth submanifold S of X. Then one has the natural choice of a massive measure m on S, namely, $m = ds$ where ds is the induced Lebesgue measure on S.

Theorem 2. *For each solution f of $Pf = 0$ in $U \backslash S$ there exist a solution f_e of $Pf_e = 0$ in U and a sequence $\{c_\alpha\} \subset L^q(S \cap U)$ $(q < \infty)$ satisfying $\|\alpha! c_\alpha\|^{1/|\alpha|}_{L^q(S\cap U)} \to 0$, such that*

$$f(x) = f_e(x) + \sum_\alpha \int\limits_{S\cap U} D^\alpha_y \Phi(x,y) c_\alpha(y) dm(y) \quad \text{for} \quad x \in U \backslash S.$$

Proof. The proof is given by combining some abstract theorems of functional analysis and very precise estimates for solutions of the transposed equation $P'g = 0$ near S using methods of complex analysis. ■

Mention some consequences of this theorem.

Corollary 1. *Let S be a locally connected compact subset of X and let $\{y_\nu\}$ be a dense sequence of points of S. Then for every solution f of $Pf = 0$ in $X \backslash S$ there are a solution f_e of $Pf_e = 0$ in X and a sequence $\{f_\nu\}$ of solutions of $Pf_\nu = 0$ in $X \backslash y_\nu$, such that $f = f_e + \sum_\nu f_\nu$ in the topologiy of $\mathcal{E}(X \backslash S)$.*

Therefore compact singularities of solutions of $Pf = 0$ may be separated into one-point singularities.

Corollary 2. *Let $\mathcal{O}$ be a relatively compact subdomain of X wih piecewise smooth boundary. Then for each solution f of $Pf = 0$ in $\mathcal{O}$ there exist a sequence $\{c_\alpha\} \subset L^q(\partial\mathcal{O})$ $(q < \infty)$ satisfying $\|\alpha! c_\alpha\|^{1/|\alpha|}_{L^q(\partial\mathcal{O})} \to 0$, such that*

$$f(x) = \sum_\alpha \int\limits_{\partial\mathcal{O}} D^\alpha_y \Phi(x,y) c_\alpha(y) ds(y) \quad \text{for} \quad x \in \mathcal{O}.$$

Thus we can represent by boundary integrals not only solutions smooth enough near the boundary but also quite arbitrary solutions without any boundary values on $\partial\mathcal{O}$.

Corollary 3. *Every hyperfunction on $S \cap U$ has a representative of the form*

$$\sum_\alpha \int\limits_{S\cap U} D^\alpha_y \Phi(x,y) c_\alpha(y) dm(y)$$

with $\{c_\alpha\} \subset L^q(S \cap U)$ $(q < \infty)$ satisfying $\|\alpha! c_\alpha\|^{1/|\alpha|}_{L^q(S\cap U)} \to 0$.

Finaly we formulate an open question.

Conjecture 1. *Theorem 2 holds even if S is a locally connected closed subset of X.*

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ON THE TIME-ASYMPTOTIC BEHAVIOUR OF THE SOLUTIONS OF THE DIRAC EQUATION FOR LONG RANGE POTENTIALS.

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The problem of determining the time asymptotic behaviour of the solution $u(t)$ of the instationary Dirac equation

$$u_t + i\,Au = e^{-ikt}f(x),\; u(t=0) = 0,\; k^2 > 1 \tag{1}$$

with the Dirac operator

$$A = A_0 + P(x),\; A_0 = -i\sum_{j=1}^{3}\alpha_j\partial_j + \alpha_4$$

essentially relies on the proof of the Hölder continuity of the resolvent boundary values $R_{\lambda+i0}$ on $\sigma_{ac}(A)$. Namely, the solution $u(t)$ possesses the representation

$$iu(t) = e^{-ikt}U + w(t),\; w(t) = p.v.\int e^{-it\lambda}\frac{(R_{\lambda+i0} - R_{\lambda-i0})f}{2\pi i(k-\lambda)}\,d\lambda \tag{2}$$

where U is satisfying the stationary Dirac equation

$$(A-k)U = f \tag{3}$$

and Sommerfeld's radiation condition

$$\left(\partial_\theta + i\sqrt{k^2-1}\right)U = o(|x|^{-1})\;(|x|\to\infty). \tag{4}$$

Because of the time harmonic perturbation on the r.h.s of (1) we can expect that the remainder $w(t)$ in (2) is vanishing for $t\to\infty$. In order to derive this limiting amplitude principle he assumption "f orthogonal to the point eigenspace" is necessary.

For the free Dirac operator A_0 our problem can be precisely solved by Fourier transform such that

$$w(t) = O\left(t^{-\frac{3}{2}}\right) \quad \text{in } L^2_{-\alpha} \mapsto L^2_\alpha,\; \alpha > 1,$$

with an optimal exponent $\frac{3}{2}$. For short range Potentials $P(x) = O\,(|x|\to\infty)$ there exists the scattering operator S,and the operators A and A_0 are unitary equivalent on

σ_{ac}. Therefore in such cases the problem can be reduced on the free Dirac operator A_0 with the same decay $O\left(t^{\frac{-3}{2}}\right)$ assuming that the thresholds $\lambda = \pm 1$ are no eigenvalues or resonances of A([K]). We remark that in the dilation-analytical situation the boundary values $R_{\lambda \pm iO}$ even are analytical across $\{\lambda^2 > 1\}$ ([B], [R]).
For long range potentials we exactly have the subsequent

Theorem. *Let $a_1, ..., a_N \in R^3$, $\alpha > 1$, $f \in L^2_\alpha$, $k^2 > 1$, k no embedded eigenvalue of A, f orthogonal to the point eigenspace of A and $P(x)$ be a 4×4 hermitian matrix valued potentials such that $|P(x)| \leq \mu |x - a_j|^{-1}$ in $U(a_j)\,(j = 1, ..., N)$, $\mu < 1$, $P(x) = O(|x|^{-\varepsilon})$, $\partial_r P(x) = O(|x|^{-1-\varepsilon})$ $(x \to \infty)$, $\varepsilon > 0$ and $P \in L^{3+\varepsilon}_{loc}$ else.*
Then with the notaions (1) - (4) for the spinor $\breve{u}(t)$ we have the decay

$$w(t) = o(1)\,(t \to +\infty)\,in\,L^2_{-\alpha},\ \alpha > 1;$$

and $w(x,t) = o(1)\,(t \to +\infty)$ locally uniformly in $x \neq a_j$ $(j = 1, ..., N)$, if $f \in D(A)$ additionally.

Remark *(i) By the additional assumption*

$$P(x) - p(|x|) = o\left(|x|^{-\frac{1}{2}}\right)\ (|x| \to \infty)$$

for some real functions $p(r)$ there are no embedded eigenvalues of A on $\{\lambda^2 > 1\}$ ([V1]).
(ii) If f do not be orthogonal to E_p of A then the point spectrum $\sigma_p(A)$ induces an additional oscillating part $\breve{u}_p(t)$ in (2) such that

$$\|u_p(t)\|_{L^2} \leq c\|f\|_{L_2}$$

and $u_p(x,t) = O(1), (t \to +\infty)$ locally uniformly in $x \neq a_j$ $(j = 1, ..., N)$, if $f \in D(A)$ again.

The proof of the theorem is essentially based on the resolvent estimate ([V2])

$$\|R_{\lambda \pm iO} f\|_{-\alpha} \leq c\|f\|_\alpha\,,\ \alpha > 1,$$

and the new radiation estimate

$$\left\|\left(\partial_r \pm i\sqrt{\lambda^2 - 1}\right) R_{\lambda \pm iO} f\right\|_{\delta - 1} \leq c\|f\|_\alpha$$

for some positive "Hölder exponent" δ following some idea of Eidus ([E1], [E2])

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A PERTURBATION THEOREM FOR UNBOUNDED SEMI-GROUPS AND SCHRRÖDINGER OPERATORS.

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In recent years one has studied "singular" perturbations of differential operators (e.g. by potentials defined in terms of measures) which are not covered by the classical semigroup perturbation theorems, using quadratic form methods or the Feyman-Kac formula.

In this talk we propose a functionalanalytic framework for "singular" perturbations: If A generates a c_0-semigroup of operators on a Banach space X we give a relative boundness condition on a perturbation B that insures that (an extention of) $A+B$ generates a semigroup of unbounded operators. Adding analyticity conditions or positivity assumptions we even get that (the extention of) $A+B$ generates a semigroup again.

We show that in the Hilbert space case this extention coincides with the one given by the $KLMN$-theorem. We apply our result to Schrödinger operators, in particular to perturbation by measures.

OPERATORS WITH MULTIPLE CHARACTERISTICS AND RELATED PSEUDODIFFERENTIAL OPERATORS

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In this talk, we shall describe two classes of pseudodifferential operators arrised in the construction of the parametrix for the Cauchy problem for hyperbolic operators with characteristics with variable multiplicity [1], [2].

1. We describe the first class by means of real-valued function $\lambda \in C^\infty([0,T])$, T>0 , such that $\lambda(0)=\lambda'(0)=0$, $\lambda'(t)>0$ when t>0. Here $\lambda' = d\lambda/dt$. For $\lambda(t)$ we define $\Lambda(t)=\int_0^t \lambda(r)dr$ and assume that $c|\lambda(t)/\Lambda(t)| \le |\lambda'(t)/\lambda(t)| \le c_0|\lambda(t)/\Lambda(t)|$, $|\lambda^{(k)}(t)| \le c_k|\lambda'(t)/\lambda(t)|^{k-1}|\lambda'(t)|$, for all k=1,2,..., and all t>0, with the positive constants c, c_0, c_k, where $c>(m-1)/m$ and $m \ge 2$. For positive numbers M, N let us denote

$$Z_h(M,N)=\{(t,x,\xi)\in[0,T]\times R_x^n\times R_\xi^n \mid \Lambda(t)\langle\xi\rangle \ge N\ln\langle\xi\rangle,\ \ \langle\xi\rangle \ge M\}$$
$$Z_{pd}(M,N)=\{(t,x,\xi)\in[0,T]\times R_x^n\times R_\xi^n \mid \Lambda(t)\langle\xi\rangle \le N\ln\langle\xi\rangle,\ \ \langle\xi\rangle \ge M\}$$

Here $\langle\xi\rangle=(e+|\xi|^2)^{1/2}$. We denote by $S^m_{\rho,\delta}$ Hörmander classes while by $C([0,T];S^m_{\rho,\delta})$ a continuous mapping of $[0,T]$ into $S^m_{\rho,\delta}$.

Definition 1. *Let m_1, m_2, m_3, ρ be a real numbers while M and N are positive. By $S_{\rho,\delta}\{m_1,m_2,m_3\}_{M,N}$ we denote the set of all functions $a(t,x,\xi)\in C^\infty([0,T]\times R_x^n\times R_\xi^n)$ such that $a\in C([0,T];S^m_{\rho',\delta})$ for some m,ρ' and such that for any k,α,β there exists a constant $C_{k,\alpha,\beta}$ such that*

$$\left|D_t^k D_\xi^\alpha D_x^\beta a(t,x,\xi)\right| \le C_{k,\alpha,\beta}\langle\xi\rangle^{m_1-\rho|\alpha|+\delta|\beta|}\lambda(t)^{m_2}\left|\frac{\lambda(t)}{\Lambda(t)}\right|^{m_3+k} \quad \textit{for all } (t,x,\xi)\in Z_h(M,N).$$

We also denote $\kappa_{\rho,\delta}\{m_1,m_2,m_3\}_{M,N}=\bigcap_{k=0}^{\infty} S_{\rho,\delta}\{m_1-k,m_2-k,m_3+k\}_{M,N}$.

Proposition 1. *Let $a_k(t,x,\xi)\in S_{\rho,\delta}\{m_1-k,m_2-k,m_3+k\}_{M,N}$, $k=0,1,\ldots$, and assume that $a_k(t,x,\xi)=0$ for all $(t,x,\xi)\in Z_{pd}(M,N)$ and all $k=0,1,\ldots$. Then there exists a symbol $a(t,x,\xi)\in S_{\rho,\delta}\{m_1,m_2,m_3\}_{M,N}$, $\operatorname{supp} a\subset Z_h(M,N)$ for which*

$$a \sim a_0+a_1+a_2+\ldots \mod \kappa_{\rho,\delta}\{m_1,m_2,m_3\}_{M,N}$$

in the sense that $a-a_0-a_1-\ldots-a_{k-1}\in S_{\rho,\delta}\{m_1-k,m_2-k,m_3+k\}_{M,N}$ for all k, and any symbols with the last property differ by the elements of $\kappa_{\rho,\delta}\{m_1,m_2,m_3\}_{M,N}$.

2. We describe the second class by means of real-valued function $\lambda\in C^\infty(R^n)$ whose derivatives of any order are bounded. We denote $Z=\{x\in R^n \mid \lambda(x)=0\}$, $NZ=R^n\backslash Z$ and assume that for every α there exists constant C_α such that $\left|D_x^\alpha \lambda(x)\right| \le C_\alpha K^{|\alpha|-1}(x)\left(|\lambda(x)|+|\nabla\lambda(x)|\right)$ for all $x \in NZ$. Here we use the notation $K(x) = 1+|\nabla\lambda(x)|/|\lambda(x)|$. We also assume that there is positive $\varepsilon<1/2$ such that $K(x) \le C|\lambda(x)|^{-\varepsilon}$ for all $x \in NZ$. Let M and N be positive constants. We define

$$Z_h(M,N) = \left\{(x,\xi) \in R_x^n \times R_\xi^n \;\middle|\; \lambda^2(x)\langle\xi\rangle^2 \ge N^2 \ln^2\langle\xi\rangle,\ \ \langle\xi\rangle \ge M\right\}$$

$$Z_{pd}(M,N) = \left\{(x,\xi) \in R_x^n \times R_\xi^n \;\middle|\; \lambda^2(x)\langle\xi\rangle^2 \le N^2 \ln^2\langle\xi\rangle,\ \ \langle\xi\rangle \ge M\right\}$$

Definition 2. *Let m_1, m_2, m_3, ρ be a real numbers while M and N are positive. By $S_{\rho,\delta}\{m_1,m_2,m_3\}_{M,N}$ we denote the set of all functions $a(x,\xi) \in C^\infty\left(R_x^n \times R_\xi^n\right)$ such that $a \in S^m_{\rho',\delta}$ for some m,ρ' and such that for any α,β there exists a constant $C_{\alpha,\beta}$ such that*

$$\left|D_\xi^\alpha D_x^\beta a(x,\xi)\right| \le C_{\alpha,\beta}\langle\xi\rangle^{m_1-\rho|\alpha|+\delta|\beta|}|\lambda(x)|^{m_2} K(x)^{m_3+|\beta|} \textit{ for all } (x,\xi) \in Z_h(M,N).$$

We also denote $\kappa_{\rho,\delta}\{m_1,m_2,m_3\}_{M,N} = \bigcap_{k=0}^{\infty} S_{\rho,\delta}\{m_1-k, m_2-k, m_3\}_{M,N}$.

Preposition 2. *Let $a_k(t,x,\xi) \in S_{\rho,\delta}\{m_1-k, m_2-k, m_3\}_{M,N}$, $k=0,1,\dots$, and assume that $a_k(x,\xi)=0$ for all $(x,\xi) \in Z_{pd}(M,N)$ and all $k=0,1,\dots$. Then there exists a symbol $a(x,\xi) \in S_{\rho,\delta}\{m_1,m_2,m_3\}_{M,N}$, supp $a \subset Z_h(M,N)$ for which*

$$a \sim a_0 + a_1 + a_2 + \dots \mod \kappa_{\rho,\delta}\{m_1,m_2,m_3\}_{M,N}$$

in the sense that $a - a_0 - a_1 - \dots - a_{k-1} \in S_{\rho,\delta}\{m_1-k, m_2-k, m_3\}_{M,N}$ for all k, and any symbols with the last property differ by the elements of $\kappa_{\rho,\delta}\{m_1,m_2,m_3\}_{M,N}$.

For the pseudodifferential operators with the symbols from both first and second classes the following theorem holds.

Theorem. *Let A and B be pseudodifferential operators with the symbols $a \in S_{\rho,\delta}\{m_1,m_2,m_3\}_{M,N}$ and $b \in S_{\rho,\delta}\left\{m_1', m_2', m_3'\right\}_{M,N}$, respectively. Then the product AB is a pseudodifferential operator with a symbol belonging to $S_{\rho,\delta}\left\{m_1+m_1', m_2+m_2', m_3+m_3'\right\}_{M,N}$.*

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