ε-factors in the de Rham setting

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The classical theory of ε-factors (Tate, Langlands, Deligne, Laumon..) assigns to every represntation ρ_v of the Galois group of a local field F_v a function $\psi_v \mapsto \varepsilon_v(\rho_v, \psi_v)$ on the set of non-trivial additive characters of F_v. The ε-factors satisfy the key product formula: Let ρ be a Galois representation of a global field F and $\varepsilon(\rho)$ the constant in the functional equation for the L-function of ρ; then for every character $\psi = (\psi_v)$ of A_F/F the product of $\varepsilon_v(\rho_v, \psi_v)$ (v runs the set of places of F) equals $\varepsilon(\rho)$.

One may hope that in case F = field of rational functions on a curve over a field k there exists a geometric version of the theory of ε-factors which would assigne to every ρ_v as above a 1-dimensional local system $\varepsilon(\rho_v)$ on the moduli of ψ_v's so that in the global setting the tensor product of $\varepsilon(\rho_v)$ equals the determinant of the global cohomology with coefficients in ρ. The classical theory arises when k is a finite field by considering the Frobenius action on the ε-lines.

In the talk (which is an exposition of a joint work with S.Bloch and H.Esnault) the desired geometric theory is constructed in the de Rham setting. The construction is closely related to Tate's linear algebra construction of residues: the ε-lines arise as "matrix coefficients" of certain operators in appropriate Clifford algebra.