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THE COHEN-MACAULAY MODULES OVER SIMPLE HYPERSURFACE SINGULARITIES

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Inv. Nr.: 2016
Stand-Nr.:

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MPI/SFB 84 - 51

Introduction:

The simple hypersurface singularities of dimension n over an algebraicly closed field k of characteristic zero are the singularities described by the following equations (cf.[1]):

$$A_\ell : x^{\ell+1} + z_2^2 + \ldots + z_{n+1}^2 = 0$$

$$D_\ell : x^2y + y^{\ell-1} + z_3^2 + \ldots + z_{n+1}^2 = 0$$

$$E_6 : x^3 + y^4 + z_3^2 + \ldots + z_{n+1}^2 = 0$$

$$E_7 : x^3 + xy^3 + z_3^2 + \ldots + z_{n+1}^2 = 0$$

$$E_8 : x^3 + y^5 + z_3^2 + \ldots + z_{n+1}^2 = 0$$

In this paper we want to show that there are up to isomorphism only finitely many indecomposable Cohen-Macaulay modules over the local ring[(1)] of such a simple singularity, and we compute their Auslander-Reiten quivers[(2)].

For the case of two-dimensional simple singularities this finiteness result is well known ([11], [3], [7]), and the Auslander-Reiten quiver is described in [3]. The one-dimensional simple hypersurface singularities have been discussed in [10], [5]. Therefore we study in general the relation between the Cohen-Macaulay modules over the local ring of an isolated hypersurface singularity $(Y,0)$ in $(k^n,0)$ given by an equation $f(x_1,\ldots,x_n)=0$

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(1) in the formal category
(2) for a discussion of the theory of Auslander-Reiten quivers see e.g. [8]

and of the singularity $(X,0)$ in $(k^{n+1},0)$ given by $f(x_1,\dots,x_n) + z^2 = 0$, using the fact that $(X,0)$ is a double ramified cover of $(k^n,0)$ branched over $(Y,0)$. In particular we show that the Auslander-Reiten quivers of indecomposable Cohen-Macaulay modules over $(Y,0)$ and over the singularity in $(k^{n+2},0)$ given by $f(x_1,\dots,x_n) + z_1^2 + z_2^2 = 0$ are isomorphic (theorem 2.1). Using this method we also obtain a description of the Cohen-Macaulay modules over simple plane curve singularities in terms of the representation theory of finite reflection groups in $GL(2,k)$ and qive a conceptual proof of the fact that their Auslander-Reiten quivers(3) coincide with the graphs described in [9] 3.7 (theorem 3.3).

This isomorphism of graphs had been observed by J.-L. Verdier; and it was the starting point for this work. I also want to thank M. Auslander, H. Esnault, E. Viehweg and in particular A. Wiedemann for many interesting and stimulating discussions.

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(3) as computed in [5]

1. Double ramified covers:

Let $f(x_1,\dots,x_n) = 0$ be the equation of an isolated hypersurface singularity in $(k^n,0)$, $n > 2$. Then the equation $f(x_1,\dots,x_n)+z^2=0$ again describes an isolated hypersurface singularity $(X,0) \subset (k^{n+1},0)$. The projection $pr : (X,0) \to (k^n,0)$, $(x_1,\dots,x_n,z) \mapsto (x_1,\dots,x_n)$ is a double ramified cover; its ramification locus $(Y,0) \subset (X,0)$ is mapped by pr isomorphically to the space in $(k^n,0)$ described by the equation $f(x_1,\dots,x_n) = 0$. The covering transformation of pr is the involution $\sigma : X \to X$, $(x_1,\dots,x_n,z) \mapsto (x_1,\dots,x_n,-z)$.

Let R be the local ring $\mathcal{O}_{X,0}$. The group $\mathbb{Z}/2\mathbb{Z}$ acts on R via σ ; we denote the twisted group ring by $R[\sigma]$ (i.e. $R[\sigma] = R \oplus R\cdot\sigma$ with the multiplication $(r_1+r_2\sigma)\cdot(r_1'+r_2'\sigma) = r_1r_1' + r_2\sigma(r_2') + (r_1r_2' + r_2\sigma(r_1'))\sigma$. Modules over $R[\sigma]$ correspond to R - modules M with an action of σ such that $\sigma(r\cdot m) = \sigma(r)\cdot\sigma(m)$ for all $r \in R$, $m \in M$. We call an $R[\sigma]$-module Cohen-Macaulay(4) if it is CM(4) as module over R .

R itself admits two σ - actions, namely $\varphi \mapsto \varphi\circ\sigma$ (the action induced by the action of σ on X), and $\varphi \mapsto -\varphi\circ\sigma$, We denote the corresponding $R[\sigma]$ - modules by R_+ (or sometimes just R) and R_- . Then $R[\sigma] \cong R_+ \oplus R_-$, and R_+ and R_- are the only indecomposable projective CM modules over $R[\sigma]$. If M,N are $R[\sigma]$ - modules then $\mathrm{Hom}_R(M,N)$ is again an $R[\sigma]$ - module, the action of σ being given by $\varphi \mapsto \sigma\circ\varphi\circ\sigma$. We put

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(4) here and in the sequel we abbreviate Cohen-Macaulay by CM .

$M^{\vee} := \mathrm{Hom}_R(M,R_+)$; if M is CM then $M \cong M^{\vee\vee}$. Furthermore we denote for an $R[\sigma]$ - modules M by M^{σ} (resp. M^{a}) the set of all σ - invariant (resp. σ - antiinvariant) elements of M . If M is CM of rank r over R then M^{σ} and M^{a} are CM over the regular ring $R^{\sigma} \cong \mathcal{O}_{k^n,0}$, thus they are free R^{σ} - modules of rank r .

For a CM module over $R[\sigma]$ of rank r we denote by $M' \subset M$ the submodule generated by M^{σ} . Since M^{σ} is free over R^{σ} we see that $M' = R \otimes_{R^{\sigma}} M^{\sigma} \cong R_+^r$. Obviously multiplication with the element $z \in R$ maps M into M' , so

$$F(M) := M/M'$$

is a module over $\hat{R} := R/zR = \mathcal{O}_{Y,0}$. If $\varphi : M \to N$ is a morphism of CM modules over $R[\sigma]$ then $\varphi(M') \subset N'$, so φ induces a morphism $F(\varphi) : F(M) \to F(N)$. Thus F is a functor from the category of CM $R[\sigma]$ - modules to the category of $\hat{R}$ - modules. One easily sees that $F(R_+) = 0$, $F(R_-) \cong \hat{R}$.

We give a second description of the functor F which will be useful for the discussion of some properties of F . For a CM module M over $R[\sigma]$ we put $\bar{M} := M^{\vee/\vee}$ and denote by i_M the canonical inclusion $M = M^{\vee\vee} \hookrightarrow M^{\vee/\vee} = \bar{M}$. This construction is functorial i.e. if $\varphi : M \to N$ is a morphism of CM $R[\sigma]$ - modules then there is a unique morphism $\bar{\varphi} : \bar{M} \to \bar{N}$ such that the diagram

$$\begin{array}{ccc} \overline{M} & \xrightarrow{\overline{\varphi}} & \overline{N} \\ \big\uparrow j_M & & \big\uparrow j_N \\ M & \xrightarrow{\varphi} & N \end{array}$$

commutes. Also one checks that $z \cdot \overline{M} \subset j_M(M)$, so $M \mapsto F'(M) := \overline{M}/_{j_M(M)}$ defines a functor F' from the category of CM $R[\sigma]$ - modules to the category of $\hat{R}$ - modules.

<u>Lemma 1.1</u>:

The functors F and F' are equivalent.

<u>Proof</u>: Let M be a CM module over $R[\sigma]$. Then

$$(1.2) \qquad F(M) = M/M' \cong M^a/z \cdot M^\sigma$$

as module over $\hat{R} \cong R^\sigma/z^2 \cdot R^\sigma$. If we identify M with $j_M(M)$ then

$$F'(M) = \overline{M}/M \cong \overline{M}^\sigma/ M^\sigma$$

since $\overline{M}^a = z \cdot \overline{M}^\sigma \subset M$ ($\overline{M}$ is isomorphic to R_+^r where $r := \mathrm{rank}_R M$). We define $\Psi_M : F'(M) \cong \overline{M}^\sigma/M^\sigma \to M^a/z \cdot M^\sigma \cong F(M)$ by $x \mapsto z \cdot x$. Obviously Ψ_M is injective; and it is surjective since $M^a = \overline{M}^a = z \cdot \overline{M}^\sigma$. Now it is easy to check that Ψ defines an equivalence of functors.

Remark 1.3: Let M,N be CM modules over $R[\sigma]$ and $\varphi' : M^a \to N^a$ a morphism of R^σ - modules such that $\varphi'(z \cdot M^\sigma) \subset z \cdot N^\sigma$. Then there is a unique morphism $\varphi : M \to N$ of $R[\sigma]$ - modules such that $\varphi/_{M^a} = \varphi'$. It is given by

$$\varphi(x) := \begin{cases} \varphi'(x) & \text{for } x \in M^a \\ \frac{1}{2}\ \varphi'(zx) & \text{for } x \in M \end{cases}$$

Theorem 1.4:

(i) For each CM module over $R[\sigma]$ the module $F(M)$ is CM over $\hat{R}$.

(ii) For each CM module W over $\hat{R}$ there is a CM module over $R[\sigma]$ such that $W \cong F(M)$.

(iii) If M,N are CM modules over $R[\sigma]$ with $F(M) \cong F(N)$ and if $r :=$ rank M - rank $N > 0$ then $M \cong N \oplus R_+^r$.

(iv) $F(M^\vee) = \mathrm{Hom}_{\hat{R}}(F(M),\hat{R})$ for every CM module M over $R[\sigma]$.

(v) For each CM module M over $R[\sigma]$ one has $M/z \cdot M \cong F(M) \oplus F(M \otimes R_-)$.

(vi) The functor F is exact.

(vii) If M,N are CM modules over $R[\sigma]$ and $\hat{\varphi} : F(M) \to F(N)$ is a morphism of $\hat{R}$ - modules then there is a morphism $\varphi : M \to N$ of $R[\sigma]$ - modules such that $\hat{\varphi} = F(\varphi)$.

(viii) If $\varphi : M \to N$ is a morphism between indecomposable CM modules over $R[\sigma]$ such that $F(\varphi)$ is an isomorphism then φ is an isomorphism.

(ix) If $\varphi : M \to N$ is a morphism between CM modules over $R[\sigma]$ then $F(\varphi) = 0$ if and only if there are morphisms $\varphi_1 : M \to R_+^r$, $\varphi_2 : R_+^r \to N$ for some $r > 0$ such that $\varphi = \varphi_2 \circ \varphi_1$.

Proof: (i) Applying the functor $\mathrm{Hom}_R(k,-)$ to the exact sequence

$$0 \to M' \to M \to F(M) \to 0$$

we see that $\mathrm{Ext}_R^i(k,F(M)) = \mathrm{Ext}_R^i(k,M) = 0$ for $0 \leq i < n-1$. So the depth of $F(M)$ as R - module is at least $n-1$. Since $\mathrm{depth}_R F(M) = \mathrm{depth}_{\bar{R}} F(M)$ this shows that $F(M)$ is Cohen-Macaulay.
(ii) By choosing a system of generators for W we obtain a surjection $R^r \twoheadrightarrow W$; let M be the kernel of this map. We endow R^r with the canonical σ - action, then $M \subset R_+^r$ inherits the structure of an $R[\sigma]$ - module. As above we apply the functor $\mathrm{Hom}_R(k,-)$ to the sequence

$$0 \to M \to R^r \to W \to 0$$

to see that M is Cohen-Macaulay.
The canonical inclusion $j : M \hookrightarrow R_+^r$ induces a σ - equivariant morphism $\bar{j} : \bar{M} \to \overline{R_+^r} = \bar{R}_+^r$ such that the diagram

$$\begin{array}{ccc} & & \bar{M} \\ & \overset{j_M}{\nearrow} & \\ M & & \downarrow \bar{j} \\ & \underset{j}{\searrow} & \\ & & R^r_+ \end{array}$$

commutes. Since $z \cdot R^r \subset M$, $z \cdot \bar{M} \subset M$ and since all modules involved are reflexive of rank r , $\bar{j}$ is injective. Because $\bar{j}$ is σ - equivariant there are morphisms $\bar{j}_1 : \bar{M}^\sigma \to (R^\sigma)^r$, $\bar{j}_2 : \bar{M}^a \to (R^a)^r$ of R^σ - modules such that $j = j_1 \oplus j_2$ $\bar{j} = \bar{j}_1 \oplus \bar{j}_2 : \bar{M} = \bar{M}^\sigma \oplus \bar{M}^a \to (R^\sigma)^r \oplus (R^a)^r = R^r$ and such that the diagram

$$\begin{array}{ccc} \bar{M}^\sigma & \xrightarrow{\ z\ } & \bar{M}^a \\ \downarrow \bar{j}_1 & & \downarrow \bar{j}_2 \\ (R^\sigma)^r & \xrightarrow[\cong]{\cdot z} & (R^a)^r \end{array}$$

commutes. As $(R^a)^r = z \cdot (R^\sigma)^r \subset z \cdot R^r \subset M \subset \operatorname{im} \bar{j}$, $\bar{j}_2$ is surjective. Therefore $\bar{j}_1$ and $\bar{j}$ are also surjective, hence $\bar{j}$ is isomorphism. This shows that $W \cong \bar{M}/j_M(M) = F'(M)$.

(iii) We may suppose that the rank of N is minimal among all CM $R[\sigma]$ - modules $\tilde{N}$ with $F(\tilde{N}) \cong F(M)$. Then the isomorphism $F'(N) \xrightarrow{\cong} F'(M)$ can be lifted to a σ - equivariant injection $i : \bar{N} \to \bar{M}$ which maps $\bar{N}$ to a direct summand of $\bar{M}$. Decompose

$\overline{M}$ in the form

$$\overline{M} = i(\overline{N}) \oplus L$$

with $L \cong R_+^r$. As i induces an isomorphism between $\overline{N}/j_N(N)$ and $\overline{M}/j_M(M)$ we see that $L \quad j_M(M)$, $j_M(M) \cap i(\overline{N}) = i(j_N(N))$, hence $M \cong N \oplus L$.

(iv) We have

$$F(M) \cong M^a/z \cdot M^\sigma \quad , \; F(M^\vee) \cong M^{\vee a}/z \cdot M^{\vee\sigma}$$

If $\varphi \in M^{\vee a}$ then $\varphi(M^\sigma) \subset z \cdot R^\sigma$, hence φ induces a morphism $\hat{\varphi} : M^a/z \;\; M^\sigma \to R^\sigma/z^2 \cdot R^\sigma \cong \hat{R}$ of $\hat{R}$ - modules. For $\varphi \in z \cdot M^{\vee\sigma}$ we have $\varphi(M^a) \subset z \cdot R^a = z^2 \cdot R^\sigma$. So $\varphi \mapsto \hat{\varphi}$ defines a homomorphism $\psi : M^{\vee a}/z \cdot M^{\vee\sigma} \to \operatorname{Hom}_{\hat{R}}(M^a/z \cdot M^\sigma, R^\sigma/z^2 \cdot R^\sigma)$. If $\varphi \in M^{\vee a}$ with $\hat{\varphi} = 0$ (i.e. with $\varphi(M^a) \subset z^2 \cdot R^\sigma$) , we put

$$\varphi'(x) := \begin{cases} \frac{1}{2}\varphi(x) & \text{if} \quad x \in M^a \\ \\ \frac{1}{2}z \;\varphi(zx) & \text{if} \quad x \in M^\sigma \end{cases}$$

Then $\varphi' \in M^{\vee\sigma}$ and $\varphi = z \cdot \varphi' \in z\, M^{\vee\sigma}$. This shows that ψ is injective. It remains to show that ψ is surjective. So take $\hat{\varphi} \in \operatorname{Hom}_{\hat{R}}(M^a/z \cdot M^\sigma, R^\sigma/z^2R^\sigma)$. We lift $\hat{\varphi}$ to a morphism $\tilde{\varphi} : M^a \to R^\sigma$ of free R^σ - modules. Then $\varphi(z \cdot M^\sigma) \subset z^2\, R^\sigma = z \cdot R^a$. Define $\varphi : M \to R$ by

$$\varphi(x) := \begin{cases} \tilde{\varphi}(x) & \text{if} \quad x \in M^a \\ \frac{1}{z}\,\tilde{\varphi}(zx) & \text{if} \quad x \in M^\sigma \end{cases}$$

Obviously $\varphi \in M^{\vee a}$ and $\psi(\varphi) = \hat{\varphi}$.

(v) Let π_1, π_2 be the canonical projections from M to $F(M) = M/M'$, $F(M \otimes R_-) = M/R \cdot M^a$. By (1.2) they induce surjections $M^a \twoheadrightarrow M^a/zM^\sigma = F(M)$ resp. $M^\sigma \twoheadrightarrow M^\sigma/zM^a = F(M \otimes R_-)$. So the map

$$(\pi_1, \pi_2) : M \to F(M) \oplus F(M \otimes R_-)$$

is surjective and has kernel $z \cdot M = z \cdot M^a + z \cdot M^\sigma$.

(vi) If $0 \to M_1 \to M_2 \to M_3 \to 0$ is an exact sequence of CM modules, over $R[\sigma]$, then the induced sequences $0 \to M_1^\sigma \to M_2^\sigma \to M_3^\sigma \to 0$ and $0 \to M_1' \to M_2' \to M_3' \to 0$ are also exact (remember that $M_i' = M_i^\sigma \otimes_{R^\sigma} R$!) . So we also get an exact sequence $0 \to F(M_1) \to F(M_2) \to F(M_3) \to 0$.

(vii) is obvious for the functor F' .

(viii) As $\overline{M} \to F'(M)$, $\overline{N} \to F'(N)$ represent minimal systems of generators for $F'(M)$, $F'(N)$ and $F'(\varphi)$ is an isomorphism the determinant of the map $\overline{\varphi} : \overline{M} \to \overline{N}$ between free R - modules represents a non-zero element in $R/\mathfrak{m}_R = k$. So $\overline{\varphi}$ and hence φ is invertible.

(ix) If φ factors through R_+^r then obviously $F(\varphi)$ is zero. Conversely, if $F(\varphi) = 0$ then φ factors $M \to N' \hookrightarrow N$, and $N' \cong R_+^r$ for some r .

Next we study the relation between CM modules over R and over R[O] . If M is a CM module over R we put

$$(1.5) \qquad \tilde{M} := M \oplus \sigma^*(M)$$

and endow it with the action $(x,y) \mapsto (y,x)$ of σ . In this way $\tilde{M}$ is a module over $R[\sigma]$. Furthermore $\tilde{M} \cong \tilde{M} \otimes R_-$, an isomorphism between the two $R[\sigma]$ - modules is given by $(x,y) \mapsto (x,-y)$.

If M itself already had the structure of an $R[\sigma]$ - module (denote the $\mathbb{Z}/2\mathbb{Z}$ action on M by σ') then $\tilde{M}$ is isomorphic to $M \oplus (M \otimes R_-)$ as $R[\sigma]$ - module, the isomorphism being given by

$$(1.6) \qquad \begin{array}{ccc} M \oplus (M \otimes R_-) & \to & M \oplus \sigma^*(M) = \tilde{M} \\ (x,y) & \mapsto & (x+y, \sigma'(x)-\sigma'(y)) \end{array}$$

<u>Proposition 1.7</u>:

(i) Let M be a CM module over R . Then M admits the structure of an $R[\sigma]$ - module if and only if $M \cong \sigma^*(M)$.

(ii) Let M_1, M_2 be indecomposable CM modules over $R[\sigma]$ such that M_1 and M_2 are isomorphic as R - modules. Then as $R[\sigma]$ - modules $M_1 \cong M_2$ or $M_1 \cong M_2 \otimes R_-$.

(iii) Let M be an indecomposable CM module over $R[\sigma]$. Then either M is indecomposable as R - module or there is an indecomposable R-module N with $N \not\cong \sigma^*(N)$ such that $M \cong \tilde{N} = N \oplus \sigma^*(N)$.

Proof: (i) Let $\psi : M \to \sigma^*(M)$ be an isomorphism. ψ induces a morphism $\psi^* : \sigma^*(M) \to \sigma^*\sigma^*(M) = M$; and ψ defines the structure of an $R[\sigma]$ - module on M if and only if $\psi^* \circ \psi = id \in E := End_R(M)$.

Let $I \subset E$ be the ideal $I := \{\varphi \in End_R(M) / \varphi(M) \subset \mathfrak{m}_R \cdot M\}$.

Then $E/I \cong k$ (Proof: Let $0 \to K \to R^r \to M \to 0$ represent a minimal system of generators for M . Then E/I is canonically embedded in $End(R^r/\mathfrak{m}_R R^r) \cong GL(r,k)$; and invertible elements in E are represented by invertible elements in $End_R(R^r)$. Conversely, if $\bar{\varphi} : R^r \to R^r$ is a morphism with $\varphi(K) \subset K$ that induces an isomorphism $R^r/\mathfrak{m}_R \cdot R^r \to R^r/\mathfrak{m}_R \cdot R^r$ then the induced map $\varphi : M \to M$ is surjective. As M is free over R^σ the map φ is also injective, hence invertible. By [13]2.19 the matrix algebra E/I does not contain any idempotents apart from $\pm id$, so $E/I \cong k$).

So we may assume that

$$\psi^2 = id + \rho \qquad \text{with} \quad \rho \in I$$

We now define a sequence of morphisms

$\psi_i : M \to \sigma^* M$ by

$$\psi_1 := \psi$$
$$\psi_{i+1} := \frac{3}{2}(\psi_i - \frac{1}{3}\psi_i^3)$$

Then one easily sees by induction that

$$\psi_i^2 = id + \rho_i \qquad \text{with} \quad \rho_i(M) \subset \mathfrak{m}_R^i \cdot M .$$

$\sigma^*(M)$ is equal to M as R^σ - module. As $\mathfrak{m}_R^{2i} \cdot M \subset \mathfrak{m}_{R^\sigma}^i$ each ψ_{2i} induces an idempotent map

$$\bar{\psi}_{2i} : M/\mathfrak{m}_{R^\sigma}^i \cdot M \to M/\mathfrak{m}_{R^\sigma}^i \cdot M \quad .$$

From the construction it follows that for each $j > 2i$ the map induced by ψ_j on $M/\mathfrak{m}_{R^\sigma}^i \cdot M$ equals to $\bar{\psi}_{2i}$. So the sequence of the ψ_i defines a morphism of R - modules $\psi_\infty : M \to \sigma^*(M)$ with $\psi_\infty^2 = id$.

(ii) By theorem 1.4.v we have

$F(M_1) \oplus F(M_1 \otimes R_-) \cong F(M_2) \oplus F(M_2 \otimes R_-)$ as $\hat{R}$ - modules. Then the Krull-Schmidt-theorem [13] 2.22 implies that

$F(M_1) \cong F(M_2)$ or $F(M_1) \cong F(M_2 \otimes R_-)$. The claim now follows from part (iii) of theorem 1.4.

(iii) If $M \cong R_+, R_-$ the statement is trivial. In the other cases $M/z \cdot M$ has precisely two summands by theorem 1.4. So M as an R - modules has at most two summands. Suppose that N is a summand of M as R - module, $N \neq M$. Since $N/z \cdot N$ is a summand both of $M/z \cdot M$ and $F(\tilde{N}) = F(\tilde{N} \otimes R_-)$; it follows from 1.4.v that $M \cong \tilde{N}$. Furthermore N does not admit the structure of an $R[\sigma]$ - module, for otherwise $N/z \cdot N \subsetneqq M/z \cdot M$ would already have two summands.

<u>Corollary 1.8</u>

There are finitely many isomorphism classes of indecomposable CM modules over R if and only if there are finitely many isomorphism classes of indecomposable CM - modules over $\hat{R}$.

Proof:

If there are only finitely many indecomposable CM modules over R then by proposition 1.7 the same is true for $R[\sigma]$. Conversely suppose that there are infinitely many isomorphism classes of indecomposable CM modules over R . For each such module M choose an indecomposable summand N_M of the $R[\sigma]$ - module $M = M \oplus \sigma^* M$. If M_1, M_2 are two indecomposable CM R - modules such that $N_{M_1} \cong N_{M_2}$ then by (1.7.iii) and the Krull-Schmidt-theorem $M_1 \cong M_2$ or $M_1 \cong \sigma^*(M_2)$. So there are also infinitely many isomorphism classes of indecomposable CM $R[\sigma]$ - modules.

On the other hand theorem 1.4 implies that there are finitely many indecomposable CM modules over $\hat{R}$ if and only this is true for $R[\sigma]$.

Finally we study the relation between the Auslander-Reiten-quivers of indecomposable CM modules over $\hat{R}$, $R[\sigma]$ and R . By [2] sect.8 almost-split sequences exist in all these categories.

Lemma 1.9:

(i) Let $0 \to M_1 \overset{\alpha}{\to} M_2 \overset{\beta}{\to} M_3 \to 0$ be an exact sequence of CM modules over $R[\sigma]$ with M_1, M_3 indecomposable. This sequence is almost-split if and only if the induces sequence $0 \to F(M_1) \overset{F(\alpha)}{\longrightarrow} F(M_2) \overset{F(\beta)}{\longrightarrow} F(M_3) \to 0$ is almost-split.

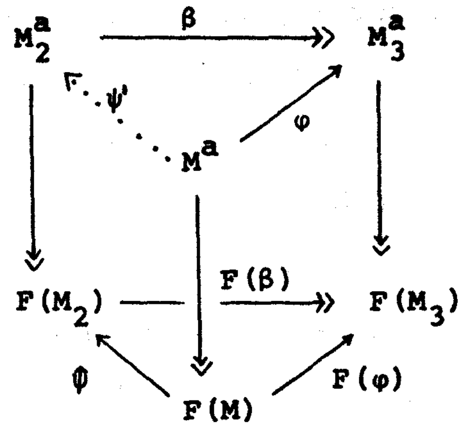
(ii) Let M, N be indecomposable CM modules over $R[\sigma]$ not isomorphic to R_+ . There is an irreducible morphism of $R[\sigma]$- modules $M \to N$ if and only if there is an irreducible morphism $F(M) \to F(N)$.

<u>Proof</u>: (i) Suppose first that the original sequence is almost-split. Then it is clear that $0 \to F(M_1) \xrightarrow{F(\alpha)} F(M_2) \xrightarrow{F(\beta)} F(M_3) \to 0$ is either split or almost-split. Assume it splits. Then there is a direct summand N of M_3 such that $F(\beta)$ induces an isomorphism between $F(N)$ and $F(M_3)$. By part (viii) of theorem 1.4 the map β induces an isomorphism between N and M_3 , so the original sequence splits.

Conversely suppose that $0 \to F(M_1) \xrightarrow{F(\alpha)} F(M_2) \xrightarrow{F(\beta)} F(M_3) \to 0$ is almost-split. Let $\varphi : M \to M_3$ let be a morphism from an indecomposable CM $R[\sigma]$ - module M to M_3 . Then by (1.4.viii) the map $F(\varphi)$ is not an isomorphism, so there is a morphism $\hat{\psi} : F(M) \to F(M_2)$ such that the diagram

$$\begin{array}{ccccc} 0 \to F(M_1) & \xrightarrow{F(\alpha)} & F(M_2) & \xrightarrow{F(\beta)} & F(M_3) \to 0 \\ & & {\scriptstyle \hat{\psi}}\nwarrow & & \nearrow{\scriptstyle F(\varphi)} \\ & & & F(M) & \end{array}$$

commutes. Then there is a homomorphism $\psi' : M^a \to M_2^a$ such that the diagram



where the vertical arrows are given by (1.2), commutes. Obviously $\psi'(z \cdot M^{\sigma}) \subset z \cdot M_2^{\sigma}$, so by (1.3) there is a morphism $\psi : M \to M_2$ of $R[\sigma]$ modules such that the diagram

$$\begin{array}{ccccccccc} 0 & \to & M_1 & \xrightarrow{\alpha} & M_2 & \xrightarrow{\beta} & M_3 & \to & 0 \\ & & & & {\scriptstyle\psi}\nwarrow & & \nearrow{\scriptstyle\varphi} & & \\ & & & & & M & & & \end{array}$$

commutes.

(ii) Since almost-split sequences exist in the categories of CM modules over $\hat{R}$ and $R[\sigma]$, this follows directly from (i), the way irreducible morphism are obtained from almost-split sequences (cf. [8] 6.1) and the Krull-Schmidt theorem.

<u>Lemma 1.10</u>:

Consider an almost-split sequence

$$(1.11) \qquad 0 \to M_1 \xrightarrow{\alpha} M_2 \xrightarrow{\beta} M_3 \to 0$$

of CM modules over $R[\sigma]$.

(i) If $M_3 \cong N_3 \oplus \sigma^*(N_3)$ with an indecomposable CM module N_3 over R as in (1.7), (1.9), let $0 \to N_1 \overset{\alpha'}{\to} N_2 \overset{\beta'}{\to} N_3 \to 0$ be the almost-split sequence of CM R - modules ending at N_3 . Then the sequence (1.11) is isomorphic to

$$(1.12) \quad 0 \to N_1 \oplus \sigma^* N_1 \overset{\alpha' \oplus \sigma^* \alpha'}{\to} N_2 \oplus \sigma^* N_2 \overset{\beta' \oplus \sigma^* \beta'}{\to} N_3 \oplus \sigma^* N_3 \to 0$$

(ii) If M_3 is irreducible as R - module then (1.11) is an almost-split sequence of R - modules.

<u>Proof</u>: (i) One easily checks that (1.12) has the universal property of [8] , 1.4, so it suffices to show that $N_1 \oplus \sigma^*(N_1)$ does not split as $R[\sigma]$ - module. Otherwise N_1 would admit the structure of an $R[\sigma]$ - module, so $\sigma^*(N_1) \cong N_1$. But the almost-split sequence starting with $\sigma^*(N_1)$ is

$$0 \to \sigma^*(N_1) \overset{\sigma^*(\alpha')}{\to} \sigma^*(N_2) \overset{\sigma^*(\beta')}{\to} \sigma^*(N_3) \to 0 .$$

As the almost-split sequence is uniquely determined by its initial term we would have $\sigma^*(N_3) \cong N_3$. So by Prop. 1.7 the $R[\sigma]$ - module $M_3 \cong N_3 \oplus \sigma^*(N_3)$ were reducible.

(ii) is proven in the same way.

2. Periodicity:

We now suppose that the function f has the form

$$f(x_1,\dots,x_n) = g(x_1,\dots,x_{n-1})+x_n^2 \quad , \quad n > 3$$

and write y for x_n . $\tau : (x_1,\dots,x_{n-1},y,z) \mapsto (x_1,\dots,x_{n-1},-y,z)$ induces an action of $\mathbb{Z}/2\mathbb{Z}$ on R and $\hat{R}$. As above we denote by $\hat{R}[\tau]$ the twisted group ring, and by $\hat{R}_+,\hat{R}_-$ the $\hat{R}[\tau]$ - modules $\hat{R}$ defined by the τ - action $\varphi \to \varphi \circ \tau$ resp. $\varphi \to -\varphi \circ \tau$. By ch.1 the Auslander-Reiten-quiver of indecomposable CM - modules over $k[[x_1,\dots,x_{n-1}]]/(g)$ is isomorphic to the Auslander-Reiten-quiver of indecomposable CM $\hat{R}[\tau]$ - modules, with the vertex corresponding to $\hat{R}_+$ deleted.

Theorem 2.1:

There is a bijection G between the set of indecomposable CM - modules over $\hat{R}[\tau]$ which are not isomorphic to $\hat{R}_+$ and the set of indecomposable CM - modules over R such that

(i) If W is an indecomposable CM $\hat{R}[\tau]$ - module not isomorphic to $\hat{R}_+$, $M = G(W)$ and $\tilde{M} = M \oplus \sigma^*(M)$ as in (1.5) then

$$W \cong F(\tilde{M}) \qquad \text{as} \qquad \hat{R}\text{ - module}$$

(ii) There is an irreducible morphism $W \to W'$ of CM $\hat{R}[\tau]$ - modules if and only if there is an irreducible morphism of R - modules $G(W) \to G(W')$.

The rest of this chapter is devoted to the proof of theorem 2.1. We put

$$\eta := y + \sqrt{-1} \cdot z \quad , \quad \zeta := y - \sqrt{-1} \cdot z \ ,$$

then $\eta \cdot \zeta = g(x_1, \ldots, x_{n-1})$ in R .

Now let W be an indecomposable CM -module over $\hat{R}[\tau]$ which is not ismorphic to $\hat{R}_+$. We choose an $\hat{R}^\tau$ - basis $e_1, \ldots, e_r$ of the τ - invariant part of W and an $\hat{R}^\tau$ - basis $f_1, \ldots, f_r$ of the τ - antiinvariant part of W . As in ch.1 this system of generators for W defines an exact sequence

$$0 \to N \to R_+^r \oplus R_+^r \to W \to 0 \quad .$$

N is an $R[\sigma]$ - module with $F(N) = W$. If we let τ act on $R^r \oplus R^r$ by $\tau' : (u,v) \mapsto (\tau(u), -\tau(v))$ then $\tau'(N) = N$. One easily sees that

$$N^a = z \cdot (R^\sigma)^r \oplus z \cdot (R^\sigma)^r$$

and that N is generated over R by N^a and elements in N^σ of the form

$$(0, \ldots, 0, \underset{\substack{\uparrow \\ i}}{y}, 0, \ldots, 0 \ ; \ \varphi_{1i}(x), \ldots\ldots, \varphi_{ri}(x))$$

$$(\varphi'_{1i}(x), \ldots, \varphi'_{ri}(x); \ 0, \ldots, 0, \underset{\substack{\uparrow \\ i}}{y}, 0, \ldots\ldots, 0 \ )$$

with $\varphi_{ij}(x), \varphi'_{ij}(x) \in R$ invariant under σ and τ , $i,j=1,\ldots,r$.
We put for $i=1,\ldots,r$

$$a_i := (0,\ldots,0,\eta,0,\ldots 0;\ \varphi_{1i}(x),\ldots\ldots,\varphi_{ri}(x))$$

$$b_i := (0,\ldots,0,\underset{\substack{\uparrow\\ i}}{\zeta},0,\ldots 0;\ \varphi_{1i}(x),\ldots\ldots,\varphi_{ri}(x))$$

$$a_{r+i} := (\varphi'_{1i}(x),\ldots,\varphi'_{ri}(x);\ \ 0,\ldots,0,\zeta,0,\ldots\ldots,0\ )$$

$$b_{r+i} := (\varphi'_{1i}(x),\ldots,\varphi'_{ri}(x);\ \ 0,\ldots,0,\underset{\substack{\uparrow\\ i}}{\eta},0,\ldots\ldots,0\ )$$

and $N_1 := R\cdot a_1 + \ldots + R\cdot a_{2r}$, $N_2 := R\cdot b_1 + \ldots + R\cdot b_{2r}$. Then

$$N = N_1 + N_2$$

and

$$(2.2) \qquad \sigma(N_1) = \tau(N_1) = N_2$$

Since $\zeta\cdot a_i \in \sum_{i=r+1}^{2r} R\cdot a_i$ we see that $\operatorname{rank}_R N_1 < r$ [5]. Similarly $\operatorname{rank}_R N_2 < r$. As $\operatorname{rank}_R N = 2r$ this implies that $N_1 \cap N_2 = \{0\}$, so

$$(2.3) \qquad N = N_1 \oplus N_2 .$$

---

(5) As $\dim X > 2$, X is irreducible. So it makes sense to speak of the rank of an R - module.

Lemma 2.4:

The R - module N_1 is irreducible.

Proof:

Suppose that N_1 has a decomposition into irreducible summands

$$N_1 = U_1 \oplus \ldots \oplus U_l$$

with $l > 2$. Then $N/z \cdot N$ has at least $2l$ summands, so by (1.4.v) and prop. 1.7 the module W is reducible over $\hat{R}$ and $l=2$. We may suppose that $W = W' \oplus \tau(W')$ for some indecomposable $\hat{R}$ - module W' . Without loss of generality we can then assume that $e_1+f_1,\ldots,e_r+f_r$ is a system of generators for $W' \oplus \{0\} \subset W' \oplus \tau^*(W')$. Then the $R[\sigma]$ - linear involution

$$(2.5) \qquad \kappa : R^r \oplus R^r \to R^r \oplus R^r$$
$$(u,v) \mapsto (v,u)$$

maps N into itself; and

$$\kappa(N_1) = N_2 .$$

As $N_2 = \sigma(U_1) \oplus \sigma(U_2)$ we have

$$U_1 \cong \sigma(U_1) \quad \text{or} \quad U_1 \cong \sigma(U_2)$$

In the first case by prop. 1.7 and (1.6) $W = F(N) = F(U \oplus \sigma(U) \oplus V \oplus \sigma(V))$

has at least four summands as $\hat{R}$ - module, in contradiction to (1.7.iii).

In the second case N_1 and N_2 admit σ - actions with $N_i \cong N_i \otimes R_-$. As $N_1 \cong N_2$ as R - module we see by (1.6) that the two $\hat{R}$ - summands of W are isomorphic, i.e. $\tau^*(W') \cong W'$. This is again a contradiction to (1.7iii).

We put

$$G(W) := N_1 \ , \ G'(W) := N_2 \ ;$$

obviously $G(W)$, $G'(W)$ and the inclusion $G(W) \oplus G'(W) \hookrightarrow R^r \oplus R^r$ are - up to isomorphy - uniquely determined by W . By (2.2) and (2.3) we have $G'(W) \cong \sigma^*(G(W))$, $F(G(W) \oplus \sigma^*(G(W))) \cong F(N) = W$. So G is an injective map from the set of indecomposable CM modules over $R[\tau]$ that are not isomorphic to $\hat{R}_+$ to the set of indecomposable CM modules over R . One easily sees that

$$G'(W) = G(W \otimes \hat{R}_-) \qquad \text{for any indecomposable CM module } W \text{ over } \hat{R}[\tau] \text{ differt from } \hat{R}_+ .$$

This shows that G is also surjective, thus part (i) of theorem 2.1 is proven.

For the proof of part (ii) it suffices to show

<u>Lemma 2.6</u>:

Let $0 \to W_1 \overset{\alpha}{\to} W_2 \overset{\beta}{\to} W_3 \to 0$ be an almost-split sequence of CM mo-

dules over $\hat{R}[\tau]$. Then there is an almost-split sequence
$0 \to G(W_1) \to G(W_2) \to G(W_3) \to 0$ [6] .

<u>Proof</u>:

If $r_i := \text{rank}_R W_i$ then it is easy to see that the sequence of (2.6) can be embedded in a commutative diagram

$$\begin{array}{ccccccccc}
 & & 0 & \longrightarrow & 0 & \longrightarrow & 0 & & \\
 & & \uparrow & & \uparrow & & \uparrow & & \\
0 & \longrightarrow & W_1 & \xrightarrow{\alpha} & W_2 & \xrightarrow{\beta} & W_3 & \longrightarrow & 0 \\
 & & \uparrow & & \uparrow & & \uparrow & & \\
0 & \longrightarrow & R_+^{r_1} \oplus R_+^{r_1} & \xrightarrow{\bar{\alpha}} & R_+^{r_2} \oplus R_+^{r_2} & \xrightarrow{\bar{\beta}} & R_+^{r_3} \oplus R_+^{r_3} & \longrightarrow & 0 \\
 & & \uparrow & & \uparrow & & \uparrow & & \\
0 & \longrightarrow & G(W_1) \oplus G'(W_1) & \xrightarrow{\alpha'} & G(W_2) \oplus G'(W_2) & \xrightarrow{\beta'} & G(W_3) \oplus G'(W_3) & \longrightarrow & 0 \\
 & & \uparrow & & \uparrow & & \uparrow & & \\
 & & 0 & \longrightarrow & 0 & \longrightarrow & 0 & &
\end{array} \qquad (2.7)$$

where $\bar{\alpha},\bar{\beta},\alpha',\beta'$ represent the direct sum decomposition.

<u>Case 1</u>: W_3 is indecomposable as $\hat{R}$ - module:
By lemma 1.10 also W_1 is indecomposable over $\hat{R}$. Hence $G(W_1) \oplus G'(W_1)$ and $G(W_3) \oplus G'(W_3)$ are both indecomposable over $R[\sigma]$. By lemma 1.9 the bottom row of (2.7) is an almost-split sequence of $R[\sigma]$ - modules, so the claim follows from lemma 1.10.

(6) Here $G(W_2)$ denotes the direct sum of the $G(W')$, where W' runs through the direct summands of W_2 .

<u>Case 2</u>: W_3 is decomposable over $\hat{R}$.

Then we have a decomposition $W_i = W_i' \oplus \tau^*(W_i')$, $i=1,2,3$; and the top row of (2.7) splits into a direct sum of two almost-split sequences over $\hat{R}$:

$$0 \to W_1' \oplus \tau^* W_1' \to W_2' \oplus \tau^* W_2' \to W_3' \oplus \tau^* W_3' \to 0$$

If we adjust the systems of generators for the W_i as in the proof of lemma 2.4 we have involutions $\kappa_i : R^{r_i} \oplus R^{r_i} \to R^{r_i} \oplus R^{r_i}$ preserving the submodules $G(W_i) \oplus G'(W_i)$ as in (2.5) .

Put $N_i' := (1 + \kappa_i)(G(W_i))$, $N_i'' := (1 - \kappa)(G(W_i))$. Then N_i', N_i'' are preserved by the σ -actions on $R_+^{r_i} \oplus R_+^{r_i}$, $W_i = F(N_i')$.

Furthermore $N_i' \cong G(W_i)$ as R - module, and the diagramm (2.7) gives a diagram

$$\begin{array}{ccccccccc}
 & & 0 & & 0 & & 0 & & \\
 & & \uparrow & & \uparrow & & \uparrow & & \\
0 & \longrightarrow & W_1' \oplus \tau^* W_1' & \longrightarrow & W_2' \oplus \tau^* W_2' & \longrightarrow & W_3' \oplus \tau^* W_3' & \longrightarrow & 0 \\
 & & \uparrow & & \uparrow & & \uparrow & & \\
0 & \longrightarrow & R_+^{r_1} \oplus R_+^{r_2} & \longrightarrow & R_+^{r_2} \oplus R_+^{r_2} & \longrightarrow & R_+^{r_3} \oplus R_+^{r_3} & \longrightarrow & 0 \\
 & & \uparrow & & \uparrow & & \uparrow & & \\
0 & \longrightarrow & N_1' \oplus N_1'' & \longrightarrow & N_2' \oplus N_2'' & \longrightarrow & N_3' \oplus N_3'' & \longrightarrow & 0 \\
 & & \uparrow & & \uparrow & & \uparrow & & \\
 & & 0 & & 0 & & 0 & &
\end{array}$$

where all arrows preserve the direct sum decomposition. The claim now follows again from (1.9) and (1.10).

3. Simple hypersurface singularities:

The two-dimensional simple singularities are just the two-dimensional isolated hypersurface singularities which are quotient singularities (cf. [6]). So there are only finitely many isomorphism classes of indecomposable CM modules over their local rings ([11], [3], [7]). By iterated application of corollary 1.8 we get

Theorem 3.1:

Let (X,0) be an n-dimensional simple hypersurface singularity. Then up to isomorphism there are only finitely many indecomposable CM modules over the local ring $\mathcal{O}_{X,0}$.

Remark 3.2:

One would expect that this property characterizes the simple singularities among all isolated hypersurface singularities (and hence among all isolated Gorenstein singularities by [11], 1.2). This is true in dimension 2 (cf. [7], §1), hence by corollary 1.8 it also holds for curve singularities (see also [10],)

The Auslander-Reiten quivers of indecomposable CM modules over two-dimensional simple singularities were computed by M. Auslander [3] . His arguments are based on the fact, that these singularities are of the form k^2/Γ with a finite group $\Gamma \subset SL(2,k)$; and he used the Mc Kay correspondence [12] to identify the Auslander-Reiten quivers as Dynkin quivers of type A,D,E . We want to give a similar description of the Auslander-Reiten quivers of the simple plane curve singularities(7).

Let G be a finite subgroup of $GL(2,k)$ which is generated by reflections, and denote by $\varepsilon : G \to \{\pm 1\}$ the linear character $g \mapsto \det g$. The kernel Γ of ε is a subgroup of index 2 in G ; we denote a generator of the group $G/\Gamma \cong \mathbb{Z}/2\mathbb{Z}$ by σ . The singularity of $X := k^2/\Gamma$ at the origin is a simple surface singularity. σ acts as an involution on X , and by Chevalleys theorem [4] V.5.3 the quotient $X/\langle\sigma\rangle$ is smooth. The branch locus $(Y,0) \subset (X,0)$ of the projection $X \to X/\langle\sigma\rangle$ is isomorphic to a simple plane curve singularity - and every simple plane curve singularity can be obtained in this way.
Let S be the local ring $\mathcal{O}_{k^2,0}$. Then $R := \mathcal{O}_{X,0} = S^\Gamma$. To determine the Auslander-Reiten quiver of indecomposable CM modules over $\mathcal{O}_{Y,0}$ it suffices by theorem 1.4 and lemma 1.2 to compute the Auslander-Reiten quiver of indecomposable CM modules over $R[\sigma]$.
If $\rho : G \to GL(E)$ is a representation of G over k we put

$$M_\rho := (S \otimes E)^\Gamma .$$

This is in a natural way a CM module over $R[\sigma]$ (cf. [9], [3], [7]). $\rho \mapsto M_\rho$ is an additive functor from the category of $k[G]$ - modules to the category of CM modules over $R[\sigma]$. By c we de-

(7) They were computed explicitely in [5].

note the canonical representation $c : G \to GL(2,k)$ of G.

Theorem 3.3:

(i) For each CM module M over $R[\sigma]$ there is a unique representation ρ of G such that $M \cong M_\rho$.

(ii) Let ρ, ρ' be irreducible representations of G . Then there exists an irreducible morphism $M_\rho \to M_{\rho'}$ if and only if ρ is a direct summand of $\rho' \otimes c$.

Remark 3.4:

This shows that the Auslander-Reiten quiver of $\mathcal{O}_{Y,0}$ is isomorphic to the graph computed in [9], 3.7, with the vertex corresponding to the trivial representation deleted.

Proof of theorem 3.3:

(i) For a representation $\rho : G \to GL(E)$ we have $S \otimes E \cong (M_\rho \otimes_R S)^{\vee\vee}$, hence E as $k[G]$ - module can be recovered from M_ρ . So $\rho \mapsto M_\rho$ is an injection from the set of indecomposable $k[G]$ - modules to the set of indecomposable CM modules over $R[\sigma]$. To prove that it is surjective let M be an indecomposable CM module over $R[\sigma]$. By [7] there is a representation $\rho' : \Gamma \to GL(E')$ such that $M \cong (S \otimes E')^\Gamma$. as R-module. Let $\rho'' : \Gamma \to GL(E'')$ be the representation $\rho'' := \rho \circ \sigma$, where the generator σ of G/Γ acts on Γ by conjugation. Obviously $\sigma^*(M) \cong (S \otimes E'')^\Gamma$ as R-module. As $M \cong \sigma^*(M)$ it follows from [2] or [7] that the representations ρ' and $\rho'' = \rho \circ \sigma$ are isomorphic. Hence there is a representation ρ of G such that $\rho|\Gamma \cong \rho'$. By prop.1.7 we have $M_\rho = M$ or $M \cong M_{\varepsilon\rho}$.

$$(3.5) \qquad 0 \to R_- \to M_c \to \mathfrak{m}_{R_+} \to 0$$

representing the unique non-trivial extension of $\mathfrak{m}_{R_+}$ by R_-. (cf. [3], observe that $R_- \cong \omega_{X,0}$ as $R[\sigma]$ - module). By [3] one obtains for each non-trivial indecomposable CM modules M over R the almost-split sequence ending with M by tensoring M with the sequence (3.5) and taking reflexive hulls:

$$0 \to M \otimes R_- \to (M \otimes M_c)^{\vee\vee} \to M \to 0 \quad .$$

It follows from lemma 1.10 that this statement also holds in the category of CM modules over $R[\sigma]$. Now if $M = M_\rho$ for some irreducible representation ρ of G then one easily checks that $(M_c \otimes M_\rho)^{\vee\vee} = M_{\rho\otimes c}$ (cf. [3]). So the CM modules over $R[\sigma]$ admitting an irreducible morphism to M are just the modules $M_{\rho'}$, where ρ' is an irreducible summand of $\rho \otimes c$.

<u>Remark 3.6</u>:

Part (i) of theorem 3.3 can also be proven using (ii) and the completeness result [14] prop. 1 .

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