NONCOMMUTATIVE RIESZ-MARKOV-KAKUTANI THEOREM AND WEAK TWISTED BURNSIDE THEOREM

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Abstract. The well-known Riesz(-Markov-Kakutani) theorem identifies the space of linear functionals on algebra $A = C(X)$ and the space of regular measures on X. The first result of the present paper is a generalization of this theorem to the case of non-commutative C^*-algebra A via Dauns-Hofmann sectional representation theorem. The corresponding measures on Glimm spectrum are functional-valued.

In extreme situation this theorem is tautological, but for group C^*-algebras of discrete groups in many cases one obtains some new tool for counting twisted conjugacy classes. More precisely, let G be a countable discrete group and ϕ its automorphism. Let $g[a] = L_g a L_{\phi(g^{-1})}$ be the twisted action, $R(\phi)$ the number of twisted (or ϕ-) conjugacy classes, $R_*(\phi)$ the dimension of twisted invariant elements of Fourier-Stieltijes algebra, $S(\phi)$ the number of fixed points of the dual action on Jacobson spectrum of G, $S_*(\phi)$ the number of generalized fixed points on Glimm spectrum.

The second result of the paper (based on the first one) is the following weak form of twisted Burnside theorem: $R_*(\phi) = S_*(\phi)$. This result allows to obtain the strong form of twisted Burnside theorem $R(\phi) = S(\phi)$ in a number of cases. These reductions will be the subject of a forthcoming paper with A. Fel'shtyn.

Contents

1. Algebras of operator fields

Let us first remind the theory of operator fields following [2].

Definition 1.1. *A continuity structure for T and the $\{A_t\}$* is a linear space F of vector fields on T, with values in the $\{A_t\}$, satisfying

(1) If $x \in F$, the real-valued function $t \mapsto \|x(t)\|$ is continuous on T;
(2) for each $t \in T$, $\{x(t) \,|\, x \in F\}$ is dense in A_t;
(3) F is closed under pointwise multiplication and involution.

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Definition 1.2. A vector field x is *continuous* with respect to F at t_0, if for each $\varepsilon > 0$, there is an element $y \in F$ and a neighborhood U of t_0 such that $\|x(t) - y(t)\| < \varepsilon$ for all $t \in U$. The field x is *continuous on* S if it is continuous at all points of S

Definition 1.3. A *full algebra of operator fields* is a family A of operator fields on T satisfying:

(1) A is a *-algebra, i.e., it is closed under all the pointwise algebraic operations;
(2) for each $x \in A$, the function $t \mapsto \|x(t)\|$ is continuous on T and vanishes at infinity;
(3) for each t, $\{x(t) \mid x \in A\}$ is dense in A_t;
(4) A is complete in the norm $\|x\| = \sup_t \|x(t)\|$.

A full algebra of operator fields is evidently a continuity structure. If F is any continuity structure, let us define $C_0(F)$ to be the family of all vector fields x which are continuous on T with respect to F, and for which $t \mapsto \|x\|$ vanishes at infinity. In view of the preceding lemmas, $C_0(F)$ is a full algebra of operator fields — indeed, a maximal one.

Lemma 1.4. *For any full algebra A of operator fields on T, the following three conditions are equivalent:*

(1) *A is a maximal full algebra of operator fields;*
(2) *$A = C_0(F)$ for some continuity structure F;*
(3) *$A = C_0(A)$.*

Such a maximal full algebra A of operator fields may sometimes be called a *continuous direct sum* of the $\{A_t\}$. It is clearly *separating,* in the sense that , if $s, t \in T$, $s \neq t$, $\alpha \in A_s$, $\beta \in A_t$, there is an $x \in A$ such that $x(s) = \alpha$, $x(t) = \beta$. We will denote by $\widehat{a}$ the section corresponding to an element $a \in A$.

2. Functionals and measures

Definition 2.1. A (*bounded additive*) *algebra of operator field measure* (BA AOFM) related to a maximal full algebra of operator fields $A = \Gamma(\mathcal{A})$ is a set function $\mu : S \to \Gamma(\mathcal{A})^* = A^*$, where $S \in \Sigma$, some algebra of sets,

- being additive: $\mu(\sqcup S_i)(a) = \sum_i \mu(S_i)(a)$
- $\mu(S)(a) = 0$ if $\operatorname{supp} \widehat{a} \cap S = \emptyset$.
- bounded: sup over partitions $\{S_i\}$ of T of $\sum_i \|\mu(S_i)\|$ is finite and denoted by $\|\mu\|$

Definition 2.2. It is *-*weak regular* (RBA AOFM) if for each $E \in \Sigma$, $a \in A$, and $\varepsilon > 0$ there is a set $F \in \Sigma$ whose closure is contained in E and a set $G \in \Sigma$ whose interior contains E such that $\|\mu(C)a\| < \varepsilon$ for every $C \in \Sigma$ with $C \subset G \setminus F$.

We will use as Σ all sets and the algebra generated by closed sets in T.

Definition 2.3. Let AOFM λ be defined on a field Σ of sets in T for which $\lambda(\emptyset) = 0$. A set $E \in \Sigma$ is called λ-*set* if for any $M \in \Sigma$

$$\lambda(M) = \lambda(M \cap E) + \lambda(M \cap (T \setminus E)).$$

Lemma 2.4. *Let λ be an* AOFM *defined on a field Σ of sets in T with $\lambda(\emptyset)=0$. The family of λ-sets is a subfield of Σ on which λ is additive. Furthemore, if E is the union of a finite sequence $\{E_n\}$ of disjoint λ-sets and $M\in\Sigma$, then $\lambda(M\cap E)=\sum_n \lambda(M\cap E_n)$.*

Proof. It is clear, that the void set, the whole space, and the complement of any λ-set are λ-sets. Now let X and Y be λ-sets, and $M\in\Sigma$. Then, since X is a λ-set,

$$\lambda(M\cap Y)=\lambda(M\cap Y\cap X)+\lambda(M\cap Y\cap(T\setminus X)), \tag{1}$$

and since Y is a λ-set,

$$\lambda(M)=\lambda(M\cap Y)+\lambda(M\cap(T\setminus Y)), \tag{2}$$

$$\lambda(M\cap(T\setminus(X\cap Y)))=\lambda(M\cap(T\setminus(X\cap Y))\cap Y)+\lambda(M\cap(T\setminus(X\cap Y))\cap(T\setminus Y)),$$

hence,

$$\lambda(M\cap(T\setminus(X\cap Y)))=\lambda(M\cap(T\setminus X)\cap Y)+\lambda(M\cap(T\setminus Y)). \tag{3}$$

From (1) and (2) it follows that

$$\lambda(M)=\lambda(M\cap Y\cap X)+\lambda(M\cap Y\cap(T\setminus X))+\lambda(M\cap(T\setminus Y)),$$

and from (3) that

$$\lambda(M)=\lambda(M\cap Y\cap X)+\lambda(M\cap(T\setminus(X\cap Y))).$$

Thus $X\cap Y$ is a λ-set. Since $\cup X_n=T\setminus\cap(T\setminus X_n)$, we conclude that the λ-sets form a field. Now if E_1, and E_2 are disjoint λ-sets, it follows, by replacing M by $M\cap(E_1\cup E_2)$ in Definition 2.3, that

$$\lambda(M\cap(E_1\cup E_2))=\lambda(M\cap E_1)+\lambda(M\cap E_2).$$

The final conclusion of the lemma follows from this by induction. □

It is clear that AOFM can be decomposed in a canonical way into a linear combination of *positive* AOFM, i.e., such that

$$\mu(E)[a^*a]\ge 0$$

for any $E\in\Sigma$. Such a set function is non-decreasing.

Lemma 2.5. *The sets F and G in the* Definition 2.2 *can be chosen in such a way that $\|\mu(C)(fa)\|<\varepsilon$ for any continuous function $f:T\to[0,1]$.*

Proof. Let us take the decompositions $\mu=\sum_{i=1}^4 x_i\mu_i$, $a=\sum_{j=1}^4 y_ja_j$, where μ_i and a_j are positive, x_i, y_j are complex numbers of norm ≤ 1. Let us choose the sets F and G, as in Definition 2.2, for $\varepsilon/16$ and for all pairs μ_i, a_j simultaneously. Then

$$0\le\mu_i(C)(f\cdot a_j)=\mu_i(C)((a_j)^{1/2}f(a_j)^{1/2})\le\mu_i(C)(a_j)\le\frac{\varepsilon}{16},$$

and

$$\|\mu_i(C)(f\cdot a)\|\le\sum_{i,j=1}^4|x_iy_j|\cdot|\mu_i(C)(a_j)|\le 16\cdot\frac{\varepsilon}{16}=\varepsilon.$$

□

Theorem 2.6. *The functionals on $\Gamma(\mathcal{A})$ can be identified with* RBA AOFM.

Proof. Obviously RBA AOFM form a linear normed space with respect to $\|.\|$.

First, we want to prove that the natural linear map $\mu \mapsto \mu(T)$ is an isometry of RBA AOFM into A^*. Since $\|\mu(T)\| \le \|\mu\|$, it is of norm ≤ 1. Let now take an arbitrary small $\varepsilon > 0$. Let $E_1, \dots, E_n$ be a partition of T such that

$$\sum_{i=1}^{n} \|\mu(E_i)\| \ge \|\mu\| - \varepsilon.$$

Let $a_i \in A$ be such elements of norm 1, that $\mu(E_i)(a_i) \ge \|\mu(E_i)\| - \varepsilon/n$.

By *-weak regularity of μ and normality of T one can take closed sets C_i, disjoint open sets G_i, and continuous functions $f_i : T \to [0,1]$ such that $C_i \subset E_i$, $\|\mu(E_i \setminus C_i)(a_j)\| \le \varepsilon/n^2$, $C_i \subset G_i$, $\|\mu(G_i \setminus C_i)(a_j)\| \le \varepsilon/n^2$, (and estimations hold for multiplication by positive functions as well, as in Lemma 2.5) $f_i(s) = 0$ if $s \notin G_i$, $f_i(s) = 1$ if $s \in C_i$, $i, j = 1, \dots, n$.

Consider the element $a := \sum_i f_i a_i \in \Gamma(\mathcal{A}) = A$. Then $\|a\| \le 1$ and

$$| \, \mu(S)(a) - \|\mu\| \, | \le \sum_{i=1}^{n} |\mu(E_i)(a) - \mu(E_i)| + \varepsilon$$

$$\le \sum_{i=1}^{n} |\mu(E_i \setminus C_i)(a) + \mu(C_i)(a) - \mu(E_i)(a_i)| + 2\varepsilon$$

$$= \sum_{i=1}^{n} \left| \sum_{j=1}^{n} \mu(E_i \setminus C_i)(f_j a_j) + \mu(C_i)(a_i) - \mu(E_i)(a_i) \right| + 2\varepsilon$$

$$\le \sum_{i,j=1}^{n} |\mu(E_i \setminus C_i)(f_j a_j)| + \sum_{i=1}^{n} |\mu(E_i \setminus C_i)(a_i)| + 2\varepsilon \le n^2 \, \frac{\varepsilon}{n^2} + n \, \frac{\varepsilon}{n^2} + 2\varepsilon \le 4\varepsilon.$$

Since ε is arbitrary small, $\|\mu\| = \|\mu(S)\|$.

It remains to represent the general functional φ by RBA AOFM. This functional on $\Gamma(\mathcal{A})$ can be extended by Hahn-Banach theorem to a continuous functional ψ on $B(\mathcal{A}) = \prod_{t \in T} A_t$ (the algebra of possibly discontinuous cross-sections of $\mathcal{A}$ with sup-norm). This functional can be decomposed $\psi = \sum_{i=1}^{4} \alpha_i \psi_i$, where ψ_i are positive functionals, $\alpha_i \in \mathbb{C}$, $|\alpha_i| = 1$, $\|\psi_i\| \le \|\psi\|$. Let us define

$$\lambda(E)(a) := \psi(\chi_E a), \qquad \lambda_i(a) := \psi_i(\chi_E a), \quad i = 1, \dots, 4.$$

where $a \in \Gamma(\mathcal{A})$, and χ_E is the characteristic function of E. Evidently, $\lambda(T)(a) = \psi(a)$ and λ is a BA AOFM. Indeed, the first two properties of Definition 2.1 are evident. The third one can be verified for each λ_i, $i = 1, \dots, 4$:

$$\sum_{j=1}^{N} |\lambda_i(E_j)| = \sum_{j=1}^{N} \lambda_i(E_j)(\mathbf{1}) = \sum_{j=1}^{N} \psi_i(\chi_{E_j} \mathbf{1}) = \psi_i(\mathbf{1}) \le \|\psi_i\|,$$

hence,

$$\sum_{j=1}^{N} |\lambda(E_j)| \le \sum_{i=1}^{4} \sum_{j=1}^{N} |\lambda_i(E_j)| \le \sum_{i=1}^{4} \|\psi_i\| \le 4 \cdot \|\psi\|.$$

Now we want to find an RBA AOFM μ such that $\mu(T)(a) = \lambda(T)(a)$. Without loss of generality it is sufficient to do this for a positive $\lambda = \lambda_i$.

Let F represent the general closed subset, G the general open subset, E the general subset of S. Define μ_1 and μ_2 by putting

$$\mu_1(F)(a^*a) = \inf_{G \supset F} \lambda(G)(a^*a), \qquad \mu_2(E)(a^*a) = \sup_{F \subset E} \mu_1(F)(a^*a),$$

and then by taking linear extension. More precisely, due to separability one can take a confinal sequence $\{G_i\}$. The unit ball in the dual space is weakly compact and one can take a weakly convergent sequence $\lambda(G_{i_k})$. Its limit ψ is a positive functional on A enjoing inf-property on positive elements. In particular, it is independent of the choice of $\{G_i\}$ and $\{G_{i_k}\}$. In a similar way for sup.

These set functions are non-negative and non-decreasing. Let G_1 be open and F_1 be closed. Then if $G \supset F_1 \setminus G_1$, it follows that $G_1 \cap G \supset F_1$ and $\lambda(G_1) \le \lambda(G_1) + \lambda(G)$ so that $\mu_1(F_1) \le \lambda(G_1) + \lambda(G)$. Since G is an arbitrary open set containing $F_1 \setminus G_1$ we have

$$\mu_1(F_1) \le \lambda(G_1) + \mu_1(F_1 \setminus G_1).$$

If F is a closed set it follows from this inequality, by allowing G_1 to range over all open sets containing $F \cap F_1$, that

$$\mu_1(F_1) \le \mu_1(F \cap F_1) + \mu_2(F_1 \setminus F).$$

If E is an arbitrary subset of T and F_1 ranges over the closed subsets of E it follows from the preceding inequality that

$$\mu_2(E) \le \mu_2(E \cap F) + \mu_2(E \setminus F). \tag{4}$$

It will next be shown that for an arbitrary set E in T and arbitrary closed set F in T we have

$$\mu_2(E) \ge \mu_2(E \cap F) + \mu_2(E \setminus F). \tag{5}$$

To see this let F_1 and F_2 be disjoint closed sets. Since T is normal there are disjoint neighborhoods G_1 and G_2 of F_1 and F_2 respectively. If G is an arbitrary neighborhood of $F_1 \cup F_2$ then $\lambda(G) \ge \lambda(G \cap G_1) + \lambda(G \cap G_2)$ so that

$$\mu_1(F_1 \cap F_2) \ge \mu_1(F_1) + \mu_2(F_2).$$

We now let E and F be arbitrary sets in T with F closed and let F_1 range over closed subsets of $E \cap F$ while F_2 ranges over the closed subsets of $E \setminus F$. The preceding inequality then proves (5). From (4) and (5) we have

$$\mu_2(E) = \mu_2(E \cap F) + \mu_2(E \cap (T \setminus F)) \tag{6}$$

for any E in T and F closed. The function μ_2 is defined on the field of all subsets of T and it follows from (6) that every closed set F is a μ_2-set. If μ is the restriction of μ_2 on the field determined by the closed sets it follows from Lemma 2.4 that μ is additive on this field. It is clear from the definition of μ_1 and μ_2 that $\mu_1(F) = \mu_2(F) = \mu(F)$ if F is closed and thus $\mu(E) = \sup_{F \subset E} \mu(F)$. This shows that μ is *-weak regular and since $\|\mu(T)\| < \infty$, we have μ is RBA AOFM.

Finally, by the definition, $\mu(S)(a) = \lambda(S)(a) = \psi(a) = \varphi(a)$ for $a \in \Gamma(\mathcal{A})$. $\square$

3. Twisted-invariant AOFM

The most part of the argument is valid for various representations of algebras by operator fields, but now we will pass to the case of group C^*-algebra of a discrete group and concentrate ourselves on the following important case due to Dauns and Hofmann [1, Corollary 8.13]:

Theorem 3.1. *Consider a C^*-algebra A (with or without an identity) and the set B of all its primitive ideals in the hull-kernel topology Q.*

(1) The complete regularization $\varphi : (B, Q) \to (M, T)$ can be taken to consist of closed ideals m of A satisfying $m = \cap\varphi^{-1}(m)$. The topology T of M contains the weak-star topology T^* induced on M by A.

Denote by $\mathcal{K}$ the family of all T-compact subsets of M of the form $\{m \in M | \|a+m\| \geq \varepsilon\}$, $a \in A$, $0 < \varepsilon$. Let $(\pi'' : E' \to (M, T)$ be the uniform field of C^*-algebras obtained by first forming the canonical field from A, M and then enlarging the weak-star topology T^* of M up to T. For each $a \in A$, $\hat{a}$ is the map $\hat{a} : M \to E'$, $\hat{a}(m) = a + m \in A/m$. The C^*-algebra of all sections vanishing at infinity with respect to the class $\mathcal{K}$ is denoted by $\Gamma_0(\pi'')$.

(2) Then the map $A \to \Gamma_0(\pi'')$, $a \mapsto \hat{a}$ is an isometric star-isomorphism. In particular, if M is T-compact, then $M \in \mathcal{K}$ and $A \cong \Gamma(\pi'')$.

Definition 3.2. Ideals m from the previous theorem, i.e., points of M, are called *Glimm ideals*.

We will use the notation T instead of M for Glimm spectrum to agree with the first section.

Now we consider a countable discrete group G and its automorphism ϕ. Also, $A = C^*(G)$. One has the twisted action of G on A:

$$g[a] = L_g a L_{\phi(g^{-1})},$$

where L_g is the left shift, and respectively on functionals.

Definition 3.3. The dimension $R_*(\phi)$ of the space of twisted invariant functionals on $C^*(G)$ is called *generalized Reidemeister number of ϕ.*

Definition 3.4. A (Glimm) ideal I is a *generalized fixed point of $\widehat{\phi}$* if the linear span of elements $b - g[b]$ is not dense in $A_I = A/I$, where $g[.]$ is the twisted action, i.e. its closure K_I does not coincide with A_I.

If we have a finite number of such fixed points, then the twisted invariant RBA AOFM's are concentrated in these points. Indeed, let us describe the action of G on RBA AOFM's in more detail. The action of G on measures is defined by the identification of the measures and functionals on A.

Lemma 3.5. *For any Borel $E \subset T$ the functional $\mu(E)$ is twisted invariant.*

Proof. This is an immediate consequence of *-weak regularity. Let $a \in A$, $g \in G$ and $\varepsilon > 0$ be an arbitrary small number, and U and F as in Lemma 2.5, for a and $g[a]$

simultaneously. Let us choose a continuous function $f : T \to [0,1]$, with $f|_F = 1$ and $f|_{T\setminus U} = 0$. Then

$$\begin{aligned}|\mu(E)(a-g[a])| = |\mu(E\setminus F)(a)+\mu(F)(a)-\mu(E\setminus F)(g[a])-\mu(F)(g[a])| \le \\ \le |\mu(F)(a)-\mu(F)(g[a])|+2\varepsilon = |\mu(F)(fa)-\mu(F)(g[fa])|+2\varepsilon = \\ = |\mu(U)(fa)-\mu(U\setminus F)(fa)-\mu(U)(g[fa])+\mu(U\setminus F)(g[fa])|+2\varepsilon \le \\ \le |\mu(U)(fa)-\mu(U)(g[fa])|+4\varepsilon = |\mu(T)(fa)-\mu(T)(g[fa])|+4\varepsilon = 4\varepsilon.\end{aligned}$$

□

Lemma 3.6. *For any twisted-invariant functional φ on $C^*(G)$ the corresponding measure μ is concentrated in the set GFP of generalized fixed points.*

Proof. Let $\|\mu\| = 1$. Suppose opposite: there exists an element $a \in A$, $\|a\| = 1$, vanishing on generalized fixed points, such that $\varphi(a) \neq 0$. Let $\varepsilon := |\varphi(a)| > 0$. In each point $t \notin GFP$ we can find elements $b_t^i \in A$, $g_t^i \in G$, $i = 1,\dots k(t)$, such that

$$\|a(t) - \sum_{i=1}^{k(t)} (g_t^i[b_t^i](t) - b_t^i(t))\| < \varepsilon/4.$$

Then there exists a neighborhood U_t such that for $s \in U_t$ one has

$$\|a(s) - \sum_{i=1}^{k(t)} (g_t^i[b_t^i](s) - b_t^i(s))\| < \varepsilon/2.$$

Let us choose a finite subcovering $\{U_{t_j}\}$, $j = 1,\dots,n$, of $\{U_t\}$ and a Borel partition $E_1,\dots,E_n$ subordinated to this subcovering. Then

$$\varphi(a) = \sum_{j=1}^n \mu(E_j)(a) = \sum_{j=1}^n \mu(E_j)\left(a - \sum_{i=1}^{k(t_j)} (g_{t_j}^i[b_{t_j}^i] - b_{t_j}^i)\right) + \\ + \sum_{j=1}^n \sum_{i=1}^{k(t_j)} \mu(E_j)(g_{t_j}^i[b_{t_j}^i] - b_{t_j}^i).$$

By Lemma 3.5 each summand in the second term iz zero. The absolute value of the first term is less then $\sum_j \|\mu(E_j)\|\varepsilon/2 \le \|\mu\|\cdot\varepsilon/2 = \varepsilon/2$. A contradiction with $|\varphi(a)| = \varepsilon$. □

Since a functional φ on A_I is twisted-invariant if and only if $\operatorname{Ker}\varphi \supset K_I$, the dimension of the space of these functionals equals the dimension of the space of functionals on A_I/K_I and is finite dimensional if and only if the space A_I/K_I is finite dimensional. In this case the dimension of the space of twisted-invariant functionals on A_I equals $\dim(A_I/K_I)$.

Definition 3.7. *Generalized number $S_*(\phi)$ of fixed points* of $\widehat{\phi}$ on Glimm spectrum is

$$S_*(\phi) := \sum_{I\in GFP} \dim(A_I/K_I).$$

The argument above gives the following statement.

Theorem 3.8 (Weak generalized Burnside)**.**

$$R_*(\phi) = S_*(\phi)$$

if one of these numbers is finite.

In [4] (see also [3, 5]) it was proved for groups of type I that the Reidemeister number $R(\phi)$, i.e., the number of twisted (or ϕ-)conjugacy classes, coincides with the number of fixed points of the corresponding action $\widehat{\phi}$ on the dual space $\widehat{G}$. This was a generalization of Burnside theorem. The proof used identification of $R(\phi)$ and the dimension of the space of (L^∞-) twisted invariant functions on G, i.e. twisted invariant functionals on $L^1(G)$. Since only part of L^∞-functions defines functionals on $C^*(G)$ (namely, Fourier-Stieltijes functions), so, *a priori* one has $R_*(\phi) \le R(\phi)$. Nevertheless, the functions with some symmetry conditions very often are in Fourier-Stieltijes algebra, so one can conjecture that $R(\phi) = R_*(\phi)$ if $R(\phi) < \infty$. This is the case for all known examples.

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