Conformal Field Theory and Topological Quantum Field Theory

Kenji Ueno
Seki Kowa Institute of Matheamtics
Yokkaichi University

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Hirzebruch

Topological Methods of Algebraic Geometry
Japanese translation 「代数幾何学における位相的方法」

1972 February IMU Lectures in Tokyo

1996 Seki Kowa Prize
Mathematical Society of Japan

1996 Order of the Sacred Treasure, Gold and Silver Star
勲二等瑞宝章

Hirzebruch

On behalf of all Japanese and German mathematicians who once stayed in Bonn and collaborated together, secretaries of SFB40 and the Max Planck Institute for Mathematics who helped their activities, and Deutsche Forschungsgemeinschaft and Max Planck Gesellschaft supporting them financially.

Modular Functor

MF1. *Disjoint union axiom*: The operation of disjoint union of labeled marked surfaces is taken to the operation of tensor product, i.e. for any pair of labeled marked surfaces there is an isomorphism

$$V((\mathbf{\Sigma}_1, \lambda_1) \sqcup (\mathbf{\Sigma}_2, \lambda_2))) \cong V(\mathbf{\Sigma}_1, \lambda_1) \otimes V(\mathbf{\Sigma}_2, \lambda_2).$$

The identification is associative.

MF2. *Glueing axiom*: Let $\mathbf{\Sigma}$ and $\mathbf{\Sigma}_c$ be marked surfaces such that $\mathbf{\Sigma}_c$ is obtained from $\mathbf{\Sigma}$ by glueing at an ordered pair of points and projective tangent vectors with respect to a glueing map c. Then there is an isomorphism

$$V(\mathbf{\Sigma}_c, \lambda) \cong \bigoplus_{\mu \in \Lambda} V(\mathbf{\Sigma}, \mu, \hat{\mu}, \lambda),$$

Modular Functor

MF3. *Empty surface axiom*: Let $\emptyset$ denote the empty labeled marked surface. Then

$$\text{Dim } V(\emptyset) = 1.$$

MF4. *Once punctured sphere axiom*: Let $\mathbf{\Sigma} = (S^2, \{p\}, \{v\}, 0)$ be a marked sphere with one marked point. Then

$$\text{Dim } V(\mathbf{\Sigma}, \lambda) = \begin{cases} 1, & \lambda = 0 \\ 0, & \lambda \neq 0. \end{cases}$$

Modular Functor

MF5. *Twice punctured sphere axiom*: Let $\mathbf{\Sigma} = (S^2, \{p_1, p_2\}, \{v_1, v_2\}, \{0\})$ be a marked sphere with two marked points. Then

$$\mathrm{Dim}\, V(\mathbf{\Sigma}, (\lambda, \mu)) = \begin{cases} 1, & \lambda = \hat{\mu} \\ 0, & \lambda \neq \hat{\mu}. \end{cases}$$

MF

There is an isomorphism of modular functors

$$I_{N,K} : \mathcal{V}_K^{SU(N)} \to \mathcal{V}_{N,K}^{\dagger},$$

i.e. for each extended labeled marked surface $(\mathbf{\Sigma}, \lambda)$ we have an isomorphism of complex vector spaces

$$I_{N,K}(\mathbf{\Sigma}, \lambda) : \mathcal{V}_K^{SU(N)}(\mathbf{\Sigma}, \lambda) \to \mathcal{V}_{N,K}^{\dagger}(\mathbf{\Sigma}, \lambda),$$

which is compatible with all the structures of a modular functor.

Hirzebruch

N. Reshetikhin, V. Turaev, Invariants of 3-manifolds via link polynomials and quantum groups, Inventiones Mathematicae 103 (1991) 547–598.

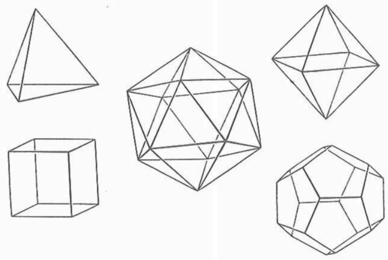
Hirzebruch

第二回日本数学会関孝和賞受賞講演
‘正多面体とサッカーボール’

フリードリッヒ・ヒルツェブルッフ
(80 年から 95 年まで，マックス・プランク数学研究所所長)

1996 年 11 月 9 日東京大学における受賞講演

正多面体はユークリッドの本（BC 300 年頃）の中で論じられています．まず，正多面体をお見せしましょう．



正多面体は自然界や芸術作品にも登場します．正 12 面体の例として次のようなものがあります．