MPIM Spring 2024 Report

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Visit period: August 12, 2024 to September 11, 2024
Collaborator and host at MPIM: Dr. Pieter Moree

I visited Max Planck Institute for Mathematics in Bonn within the period from August 12, 2024 to September 11, 2024 to work with Dr. Pieter Moree on a monograph entitled "A computational history of prime numbers and Riemann zeros". This is a joint collaboration with Dr. Junghun Lee, Dr. Izabela Petrykiewicz and Prof. Dr. Alisa Sedunova. In this visit, we made huge changes in Chapter 3 and that is now close to finish. We removed the sections which are inactive and not very relevant to the main topic of the book. Currently inactive sections are put in the end of Section 5 which basically contains various expanded topics but is still a mess.

I attended the Number Theory talks on August 14, August 15, August 21, September 4, and September 11. I also had several discussions with MPIM postdoc Dr. Steve (Kai) Fan. We are working on a paper generalizing my work with Dr. Jordan Schettler and two students of San Jose State University, about the direct connection between the error in the asymptotic formula for average Goldbach problem and the zero-free region of the Riemann zeta function. In particular, we consider functions in a subclass of the Selberg class and consider their associated zero-free regions, and analogues to the prime number theorem and average Goldbach. We are trying to make the criteria of such a subclass as explicit as possible.

Finally, let me explain about the progress of our book project in a bit more detail. We finalized the structure of Chapter 3 regarding the distribution of prime numbers, by removing old sections which are incomplete and are currently inactive. The final structure of Section 3 is composed of 5 sections which focuses on methods to compute the number of prime numbers up to some given number and its asymptotic behavior (i.e. the Prime Number Theorem). Section 3.2 expands the discussion on prime computation to explicit prime number bounds, Section 3.3 gives the theoretical explanation to the methods used, mainly by connecting the distribution of prime numbers to zeros of the Riemann zeta function, and the rest of Chapter 3 expands that connection to the average number of Goldbach representations. In Section 3.3, we introduce zero-free regions of the Riemann zeta function and explain how they determine the error in the Prime Number Theorem. They in fact also determine the error in approximating the average number of Goldbach representations which we describe in Section 3.5. Section 3.4 is a preliminary to Section 3.5 where we introduce the Goldbach conjecture and how to count the number of corresponding representations. There is one issue, however, that unlike prime counting where we can easily switch between direct counting and the von Mangoldt function weighted counting, the case of counting Goldbach representations is not as straightforward. Unfortunately we have not yet found an easy explanation to these two different counting methods, and this leaves Section 3.4 unfinished, and Section 3.5 not perfectly complete. We aim to finish these incomplete parts of Chapter 3 and move on to Chapter 4 in the next visit.

We remark, however, that we have reached our minimum target number of pages and the book is still expanding, so we are moving closer to the completion of this book. It was very helpful to have Dr. Moree's around for instant discussions that we managed to polish the book to its current shape. Last but not least, we are deeply thankful to the comfortable and supportive research environment at Max Planck Institute for Mathematics, including the convenient library access and huge collection of papers, which helped us realize these improvements to our book project. We hope to resume working on the book project next summer.