VECTOR BUNDLES OVER FINITE SETS AND HYPERELEMENTARY INDUCTION

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To Peter Hilton with congratulations and best wishes

The aim of this paper is to present an elementary and self-contained proof of a theorem of R.Brauer that every complex representation of a finite group G can be written as a Z-linear sum of representations induced up from hyperelementary subgroups of G. The key tools in this proof are the theorem of Solomon on permutation representations (theorem 2) and the Mackey Formula relating pullbacks and transfers of G-vector bundles (theorem 4).

The paper is organized as follows: §1 introduces the basic notions of G-sets and complex representations of G, namely the Burnside ring A(G), the complex representation ring R(G), the permutation homomorphism $C : A(G) \longrightarrow R(G)$ and gives a proof of the theorem of Solomon: there exist hyperelementary sets X and Y such that $1 = CX - CY$, where 1 in R(G) is the trivial representation of G; §2 develops the language of G-vector bundles over finite G-sets B, introduces the group $K_G(B)$, and for a G-map $f : B \longrightarrow C$ discusses the pullback homomorphism $f^! : K_G(C) \longrightarrow K_G(B)$ and the transfer $f_! : K_G(B) \longrightarrow K_G(C)$. The key result here is the Mackey formula. In the final section §3 we reinterpret Frobenius induction as the transfer along the collapsing map $c : G/H \longrightarrow G/G$, so the Brauer induction theorem says: given a finite G-set B there exists a G-map $f : B' \longrightarrow B$ such that B' is a hyperelementary set and the transfer $f_! : K_G(B') \longrightarrow K_G(B)$ is onto. The proof of the theorem is now very easy. We hope that the reader agrees that the language of vector bundles is convenient in thinking about representations of finite groups.

The language of K-theory has been introduced by M.F.Atiyah, F.Hirzebruch, and G.B.Segal [1], [2], [3], [12]. The Burnside ring has been introduced in the study of induction theorems by A.Dress [7], [8] and has been shown to be very useful in studying actions of compact Lie groups by T.tom Dieck [6]. Brauer's theorem (in the sharper form of elementary induction)

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was first proved in [4] , and a new proof was presented by Brauer and Tate in [5]. Our proof is of course inspired by D.M.Goldschmidt, I.M.Isaacs and L.Solomon [9], [10]. The same approach as used here proves the sharper elementar induction theorem, but one more notion (Frobenius induction on bundles) is needed - see [11] for details.

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1. <u>The Burnside ring and the representation ring of</u> G. Let G be a group, B and C finite G-sets. A function $f : B \rightarrow$ is said to be a G-map if $f(g.b) = g.f(b)$ for all g in G and b in B. If $f_1 : B_1 \longrightarrow C$ and $f_2 : B_2 \longrightarrow C$ are G-maps, we let

$$D = \{ (b_1, b_2) \in B_1 \times B_2 \mid f_1(b_1) = f_2(b_2) \}$$

and define the action of G on D by $g.(b_1, b_2) = (g.b_1, g.b_2)$. We have the projection maps $p_i : D \longrightarrow B_i$ defined by $p_i(b_1, b_2) = b_i$, and the diagram of G-maps

$$\begin{array}{ccc} D & \xrightarrow{p_2} & B_2 \\ \downarrow{\scriptstyle p_1} & & \downarrow{\scriptstyle f_2} \\ B_1 & \xrightarrow{f_1} & C \end{array}$$

is called the pullback diagram defined by f_1 and f_2.

The standard example of a G-set is a coset space G/H with action $g.xH = gxH$. If B is a G-set, then a map of G-sets $f : G/H \longrightarrow B$ is completely determined by $f(eH)$, which is an element of $B^H = \{ b \in B \mid h.b=b \text{ for all } h \in H \}$, the fixed point set of H in B. In particular,

$$\mathrm{Map}_G(G/H, G/K) = \{ gK \mid H \subset gKg^{-1} \}.$$

This means that two standard orbits G/H and G/K are G-equivalent if and only if H and K are conjugate subgroups of G. To ring yet one more change on this theme: G-equivalences of G/H are determined by the elements of N/H, where N is the normalizer of H in G. We call a G-equivalence class of standard orbits an orbit type of G. If $b \in B$, then the orbit

G.b of b is G-equivalent to G/G_b, where $G_b = \{ g \in G \mid g.b = b\}$ is the isotropy subgroup at b.

We let fo(G) be the set of finite orbit types of G. If X is a finite G-set, then it determines a counting function $(X) : fo(G) \longrightarrow Z$ defined by (X)(type G/H) = number of orbits in X of type G/H. The function (X) is finitely non-zero and takes values in the set of natural numbers.

THEOREM 1 (Structure of finite G-sets). If X and Y are finite G-sets, then X is G-equivalent to Y if and only if (X) = (Y).

The proof is immediate: if $f : X \longrightarrow Y$ is a G-equivalence, then of course it sets up a type-preserving one-to-one correspondence between the orbits in X and the orbits in Y, so we have (X) = (Y). Conversely, if (X) = (Y), then since this means that we can set up a type-preserving one-to-one correspondence between the orbits in X and Y, it follows that there exists a G-equivalence $f : X \longrightarrow Y$. This is because it is enough to define f orbit by orbit, and if $h : G.x \longrightarrow G/H$ and $k : G.y \longrightarrow G/H$ are G-equivalences, then of course $k^{-1}h : G.x \longrightarrow G.y$ is a G-equivalence.

An example: let G = (Z,+), the additive group of the integers. The orbit types are given by Z/(n) with n a positive integer (and Z/(n) is known as an n-cycle). The structure theorem gives as an immediate consequence the structure of the conjugacy classes of elements in the symmetric groups S_k.

We let A(G) be the set of all finitely non-zero functions $f : fo(G) \longrightarrow Z$. Notice that each f in A(G) can be written as f = (X) - (Y) , that is f is the difference of counting functions of finite G-sets (of course we define the addition in A(G) by (f+f')(G/H) = f(G/H) + f'(G/H)). Indeed, A(G) is a free abelian group with the counting functions (G/H) as a basis (where we choose exactly one H from its conjugacy class. This addition operation corresponds to disjoint sum of G-sets: $(X \sqcup Y) = (X) + (Y)$. We define the multiplication in A(G) to correspond to Cartesian product of G-sets: (X)x(Y) = (XxY), where G acts diagonally on XxY : g.(x,y) = (g.x,g.y). Notice that evaluation of counting functions is not in general a homomorphism of rings from A(G) to Z.

We can sum up the construction of the Burnside ring A(G) as follows. Let Set-G denote the set of G-equivalence classes of finite G-sets. Disjoint sum defines addition and Cartesian product defines multiplication in Set-G. The Burnside ring A(G) is the completion of Set-G to a ring. Indeed, the construction of the Burnside ring is just an extension of the familiar process of constructing the integers Z from the natural numbers N. More precisely, if E is the group consisting of the identity element alone, then Set-E = N with the usual addition and multiplication, and A(E) = Z.

Here is a second instance of this process of completion to a ring: the representation ring R(G) of complex representations of G. A complex representation of G is a C-linear action $G \times V \longrightarrow V$ on a finite dimensional vector space V over C. That is, for each g in G the map $g.:V \longrightarrow V$ is a C-linear map. Equivalently, a representation is a homomorphism $G \longrightarrow GL(V)$ into the full linear group of V. If V and W are representations of G, then a C-linear map $T:V \longrightarrow W$ is said to be a map of representations (or a CG-map) if T(g.v) = g.T(v) for all g in G and v in V. That is, a CG-map is both a C-map and a G-map (equivalently, it is a map of CG-modules over the group ring CG of G over C). We let Rep-G be the set of CG-isomorphism classes of representations of G over the complex numbers. Direct sum of vector spaces defines addition in Rep-G and tensor product of vector spaces over C defines multiplication in Rep-G. We let R(G) be the completion of Rep-G to a ring. Elements of R(G) (often known as virtual representations of G) are formal differences V - W of CG-isomorphism types of finite dimensional representations of G. Just as in the case of the Burnside ring A(G), the representation ring R(G) turns out to be a free abelian group with irreducible complex representations forming a free basis (surprisingly enough, this will not be used in the proof of Brauer's hyperelementary induction theorem). Given an element g of G, it defines a function $\text{Trace } g : \text{Rep-}G \longrightarrow C$ by setting $\text{Trace } g\,(V) = \text{Trace } g.:V \longrightarrow V$. Since Trace g preserves addition and multiplication, it induces a ring homomorphism $\text{Trace } g : R(G) \longrightarrow C$. We will use the fact (see Serre [13] for an easy and elegant proof) that the functions Trace g separate elements in R(G). Indeed, we will use the even weaker consequence: a necessary and sufficient condition that an element u of R(G) is the unit element 1 = C , that is the class of the trivial representation g.=1 : C $\rightarrow$ C is that for each g in G we have Trace g (u) = 1 .

There is an important relation between the Burnside ring A(G) and the representation ring R(G), namely the permutation representation homomorphism. Given a finite G-set X, we consider the vector space CX with X as basis over C. We notice that g.:X $\longrightarrow$ X gives a C-linear map g. : CX $\rightarrow$ CX, so CX is a representation of G. Since C(X $\sqcup$ Y) = CX + CY and C(X x Y) = CX $\otimes$ CY , this means that the function C : Set-G $\longrightarrow$ Rep-G induces a homomorphism of rings

$$C : A(G) \longrightarrow R(G) .$$

The theorem of Solomon will say that there is an element of very special form in A(G) whose image under C is 1.

If G = E , the identity group, then A(E) = Z = R(E), and C : A(E) $\longrightarrow$ R(E) is the identity map. In general C has both a nontrivial kernel (for example $G=S_3$, the symmetric group on three letters) and a nontrivial cokernel (for example, G = Z/(3)).

We introduce the notion of a family of G-sets. A subset $\mathcal{F}$ of Set-G is said to be a family if for each X in $\mathcal{F}$ and each G-map f : X' $\longrightarrow$ X we also have X' in $\mathcal{F}$. For example, if we have a pullback diagram

$$\begin{array}{ccc} D & \xrightarrow{p_2} & B_2 \\ \downarrow{\scriptstyle p_1} & & \downarrow{\scriptstyle f_2} \\ B_1 & \xrightarrow{f_1} & C \end{array}$$

and B_1 is in $\mathcal{F}$, then D is also in $\mathcal{F}$. In particular, $B_1 \times B_2$ is in $\mathcal{F}$ (take C to be a single point * = G/G). If we let $A_{\mathcal{F}}(G)$ to be the subset of A(G) consisting of elements of the form (X) - (Y) with X and Y in $\mathcal{F}$, then $A_{\mathcal{F}}(G)$ is an ideal in the ring A(G): not is it only closed under addition, but it is closed under multiplication by all elements in A(G).

Let $\mathcal{C}$ be the collection of all G-sets B such that for each b in B the isotropy subgroup $G_b = \{g \in G \mid g.b = b\}$ is a cyc subgroup of G. We claim: $\mathcal{C}$ is a family, for if $f : B' \to B$ is a G-map, then for b' in B' the isotropy subgroup $G_{b'}$ is a subgroup of $G_{f(b)}$, so if this group is cyclic, $G_{b'}$ is also cyclic.

If p is a prime, let $\mathcal{C}'_p$ be the collection of all G-set B such that G_b is cyclic of order prime to p for each b in B It is also immediate that $\mathcal{C}'_p$ is a family for each prime p.

For our next family we need the notion of a covering. A G-map $q : X \longrightarrow Y$ is said to be a covering if q is onto and the inverse image of an orbit in Y is a single orbit in (this is really a connectedness hypothesis). A covering tra formation is a G-map $c : X \longrightarrow X$ which makes the followin diagram commute:

$$\begin{array}{ccc} X & \xrightarrow{c} & X \\ \downarrow q & & \downarrow q \\ Y & \xrightarrow{1} & Y \end{array},$$

that is $qc = q$. Given x_0 and x_1 with $q(x_0) = q(x_1)$ there may not be any covering transformation c with $c(x_0) = x_1$ (or there may be many, if Y consists of more than one orbit). However, if there exists a covering transformation $c : X \to X$ with $c(x_0) = x_1$, then its value on the orbit of x_0 is compl tely determined by this condition. This is easy to see: if $K_0 = G_{x_0}$, $H_0 = G_{q(x_0)}$, then we may replace q by the canoni quotient map $p : G/K_0 \longrightarrow G/H_0$ induced by the inclusion of K_0 into H_0. By our normalization the point $x_0 = eK_0$, and there exists a G-map $c : G/K_0 \longrightarrow G/K_0$ with $c(eK_0) = x_1 = hK_0$ if and only if $K_0 \subset hK_0h^{-1}$ (notice that $h \in H_0$, sin $q(x_0) = q(x_1)$ by hypothesis). If we impose the condition tha X is a finite G-set, then covering transformations are G-equ valences. Our discussion above allows us to talk of the gro of covering transformations at x_0 in X (we will denote this group by Cov (q, x_0)). Notice that this group depends just on the orbit of x_0 under G, rather than on x_0 itself. We sha call the covering $q : X \longrightarrow Y$ a principal covering if for each x_0 in X the group of covering transformations at x_0 acts transitively on the fiber of q above $q(x_0)$. That is, $q : X \longrightarrow Y$ is a principal covering if and only if for any x_0 and x_1 in X with $q(x_0) = q(x_1)$ there exists a covering transformation $c : X \longrightarrow X$ with $c(x_0) = x_1$ (notice that c defines a unique element of $Cov(q,x_0)$).

If $f : Y' \longrightarrow Y$ is a G-map and $q : X \longrightarrow Y$ is a covering, then the pullback

$$\begin{array}{ccc} X' & \xrightarrow{f'} & X \\ \downarrow{\scriptstyle q'} & & \downarrow{\scriptstyle q} \\ Y' & \xrightarrow{f} & Y \end{array}$$

has the property that $q' : X' \longrightarrow Y'$ is a covering as well. If q is a principal covering, then so is q'. Indeed $Cov(q',x') = Cov(q,f'(x'))$.

We are now ready to define hyperelementary sets: if p is a prime, we shall say that Y is a p-hyperelementary set if there exists a principal covering $q : X \longrightarrow Y$ with X a $\mathcal{C}'_p$ -set such that for each x in X the group $Cov(q,x)$ is a p-group. Locally this means the following: we have a normal cyclic group C in H such that the order of C is prime to p and H/C is a p-group (such an H is known as a p-hyperelementary group), and $q: X \longrightarrow Y$ locally looks like the canonical projection $G/C \longrightarrow G/H$ with the group of covering transformations H/C. The remarks above on pullbacks show that p-hyperelementary sets form a family in Set-G . We shall denote the union of these families over all primes p by $\mathcal{H}$, and call elements of $\mathcal{H}$ hyperelementary sets. We can now state and prove the theorem of L.Solomon. G is now a finite group

THEOREM 2 (Solomon) . Let $\mathcal{H}$ be the family of hyperelementary sets, and $A_{\mathcal{H}}(G)$ the corresponding ideal of elements of the form $(X) - (X')$ with X,X' in $\mathcal{H}$. Then there exists an element $u \in A_{\mathcal{H}}(G)$ such that $Cu = 1$.

<u>Proof</u>. This is a desert island proof - each step forces the next. The argument is really the original one of Solomon.

First notice that it is enough to show that for each element g of G there is an element u_g in $A_{\mathcal{H}}(G)$ such that Trace g $(Cu_g) = 1$, for we then have

$$\prod_{g \in G} (1 - u_g) = 1 - u$$

with $u \in A_{\mathcal{H}}(G)$ and Trace g $(Cu) = 1$ for all g in G, so $Cu=1$.

The second step consists in showing that the homomorphis[m] of rings

$$\mathrm{Trace}\, g \circ C : A_{\mathcal{H}}(G) \longrightarrow Z$$

is onto Z. Of course, we have to remark that Trace $g(CX) = |\mathrm{Fix}(g,X)| = |X^g|$, the number of points in X fixed under the element g, so we indeed land in the integers. To show that $\mathrm{Trace}\, g \circ C : A_{\mathcal{H}}(G) \longrightarrow Z$ is onto, it is enough to show that 1 is in the image, or since $\mathrm{Trace}\, g \circ C(A_{\mathcal{H}}(G)) =$ [(n)] is an ideal, it is enough to show that for each prime p the natural number n is not divisible by p (that is, n=1 as was claimed). To recapitulate: given g and a prime p, we wish t[o] construct a hyperelementary set X with $|X^g| \neq 0 \mod (p)$. Since we only have g and p to work with, let's start with $\langle g \rangle$ the cyclic group generated by g in G. Write $\langle g \rangle = C \times D$ as a product of two cyclic groups C and D, with D a p-group and the order of C prime to p. Let $N = N_G(C)$ be the normali[zer] of C in G - the principle of maximum unhappiness leads us to expect that the quotient group N/C fails to be a p-group. St[ill] N/C has at least one p-Sylow subgroup P. We let H be the set of elements in N which represent the cosets of P. This means that G/H is a p-hyperelementary set. Wouldn't it be nice if $|(G/H)^g| \neq 0 \mod (p)$? Let's prove it: this will complete th[e] argument of the theorem. An element xH is fixed under g if and only if $x^{-1}gx \subset H$, so $x^{-1}\langle g \rangle x \subset H$, in particular $x^{-1}Cx$ [⊂ H]. But this means that $x^{-1}Cx \subset C = \mathrm{Ker}\; H \longrightarrow H/C = P$, since the target is a p-group and the order of $x^{-1}Cx$ is relatively prime to p. This means that $x^{-1}Cx = C$, or $x \in N$, that is $(G/H)^g = (N/H)^g = (N/H)^g = (N/H)^D$, the last since $C \subset N$, so acts trivially on N/H. Since D is a p-group, we have $|(N/H)^D| = |N/H| \mod (p)$, but $|N/H| = |(N/C)/(H/C)| \neq 0 \mod (p)$, for H/C has been chosen to be a p-Sylow subgroup of N/C. To sum up: Trace $g\,(C(G/H)) \neq 0 \mod (p)$ for the hyperelementary set G/H, hence Trace $g\,(C(A_{\mathcal{H}}(G))) = (1) = Z$, as we wanted to show.

2. Mackey's Formula for G-vector bundles. A G-vector bundle over a G-set B is a G-map $p: E(p) \longrightarrow B$ such that for each b in B the fiber $p_b = p^{-1}(b) = \{ y \in E(p) \mid p(y) = b \}$ has the structure of a complex vextor space and for each g in G the map $g. : p_b \longrightarrow p_{g.b}$ is a C-linear map. If $B = *$ is a single point, then a G-vector bundle over B is just a complex representation of G. More generally, a G-vector bundle over G/H is completely determined by its fiber over eH which is a representation of H. Conversely, given a representation W of H, we consider the G-set $Gx_H W$ which is a quotient of G x W under the relation $(gh,w) \sim (g,hw)$ for h in H, with G-action $g'.(g,w) = (g'g,w)$. The map $p : G \, x_H W \longrightarrow G/H$ induced by the projection $p(g,w) = gH$ is a G-vector bundle over G/H with $W = p_{eH}$. Let us note this down:

LEMMA3. Restriction to the fiber at eH gives a one-to-one correspondence $\mathrm{Vect}_G(G/H)$ with Rep-H.

If $f : B \longrightarrow C$ is a G-map of finite G-sets, then we define the pullback and transfer of G-vector bundles over f. The pullback

$$f^! : \mathrm{Vect}_G C \longrightarrow \mathrm{Vect}_G B$$

is defined by setting $(f^! p)_b = p_{f(b)}$. The transfer

$$f_! : \mathrm{Vect}_G B \longrightarrow \mathrm{Vect}_G C$$

is defined by setting

$$(f_! q)_c = \bigoplus_{f(b)=c} q_b \, .$$

Of course, it is much better to define $f^!$ and $f_!$ by their universal mapping properties, but these definitions have the virtue of giving explicit models for the pullback and transfer. By the way, notice that in our definition of G-vector bundles the fibers may have varying dimensions - for example, $(f_! q)_c = 0$ if c is not in the image of f. Of course the dimensions of fibers over a single orbit are the same, since $g. : p_b \longrightarrow p_{g.b}$ furnish C-isomorphisms.

We now exhibit a fundamental relation between pullback and transfer which is known as Mackey's Formula.

So far we have been working only with the sets $Vect_G B$. They have the direct sum operation

$$+ : Vect_G B \times Vect_G B \longrightarrow Vect_G B$$

defined by $(p + p')_b = p_b \oplus p'_b$, the direct sum of fibers, and the product operation

$$\smile : Vect_G B \times Vect_G B \longrightarrow Vect_G B$$

defined by $(p \smile p')_b = p_b \otimes p'_b$, the tensor product of fiber We now introduce additive inverses and complete $Vect_G B$ to a ring called $K_G(B)$. If * denotes a point, then $K_G(*) = R(G)$, the representation ring of G. More generally, Lemma 3 gives us $K_G(G/H) = R(H)$, the isomorphism given by restriction to fibers over eH. If $f : B \longrightarrow C$ is a G-map, then pullback induces a ring homomorphism

$$f^! : K_G(C) \longrightarrow K_G(B) ,$$

so in particular we can think of $K_G(B)$ as a $K_G(C)$-module. The transfer along f induces a homomorphism of additive group

$$f_! : K_G(B) \longrightarrow K_G(C)$$

which in general is not a homomorphism of rings, but Corolla says that $f_!$ is a homomorphism of $K_G(C)$-modules, namely

$$f_! (f^! p \smile p') = p \smile f_!(p') .$$

In particular if we want to prove that $f_!$ is onto $K_G(C)$ it is enough to show that 1 is in the image of $f_!$, where 1 is the trivial line bundle over C.

3. <u>The Brauer induction theorem</u>. Before we state the Brauer induction theorem for the family of hyperelementary subgroups of the finite group G, we need to introduce the concept of Frobenius induction of representations. If H is a subgroup of G and W is a representation of H, we think of W as a CH-module, and define

$$\mathrm{Ind}_H^G W = CG \otimes_{CH} W ,$$

with the action of CG coming from multiplication in CG. If we let $G/H = \{g_1H, \ldots , g_rH\}$, then $\mathrm{Ind}_H^G W$ is the direct sum of $g_i \otimes W$ (each isomorphic to W as a vector space over the complex numbers C), with the action of G defined as follo if $gg_i = g_jh$, then $g.(g_i \otimes w) = g_j \times h.w$. That is, g. pe mutes the summands and twists by the original action of H on

LEMMA 6. Let $c : G/H \longrightarrow G/G=*$ be the collapsing map, then $c_! = \mathrm{Ind}_H^G : R(H) \longrightarrow R(G)$.

Proof. Suppose that $p \in \mathrm{Vect}_G G/H$ is a G-vector bundle with $p_{eH} = W$, that is $E(p) = G \times_H W$ with $p_{g_iH} = [g_i, W]$, the set of equivalence classes of (g_i, w). In particular, $g_i \cdot : W = p_{eH} \longrightarrow p_{g_iH}$ is an isomorphism. We thus have

$$c_! p = \bigoplus_i [g_i, W],$$

and we have to determine the action of G. Now if $gg_i = g_j h$ with h in H, then $g.[g_i, w] = [gg_i, w] = [g_j h, w] = [g_j, h.w]$, so

$$c_! p = \mathrm{Ind}_H^G W,$$

as claimed.

We can now state Brauer's hyperelementary induction theorem. Recall that a finite group H is called hyperelementary if there exists a prime p and a normal cyclic subgroup C of H of order prime to p such that H/C is a p-group.

THEOREM 7 (Brauer hyperelementary induction). Let G be a finite group, then

$$\sum_H \mathrm{Ind}_H^G : \bigoplus_{\substack{H \\ \text{hyperelementary}}} R(H) \longrightarrow R(G)$$

is onto.

We can restate this theorem in the language of vector bundles.

THEOREM 8 (Brauer hyperelementary induction). Let G be a finite group, B a finite G-set. There exists a G-map $f : B' \longrightarrow B$ with B a hyperelementary G-set such that

$$f_! : K_G(B') \longrightarrow K_G(B)$$

is onto.

Proof. The case of general B follows from the special case of B = * (and this is a faithful translation of Theorem 7).

According to Solomon's theorem there exists a $u \in A_{\mathfrak{X}}(G)$ such that $Cu = 1$ in $R(G) = K_G(*)$. Write $u = (X) - (Y)$ with X and Y hyperelementary sets. Let $1_X : X \times C \longrightarrow X$ be the trivial line bundle over X, then if $c : X \longrightarrow *$ is the collapsing map, we have $c_! 1_X = CX$. We let $B' = X \sqcup Y$ and define $w = (1_X, - 1_Y)$, where we have used the canonical isomorphism

$$K_G(X \sqcup Y) = K_G(X) \oplus K_G(Y) .$$

We notice that $c_! w = c_! 1_X - c_! 1_Y = CX - CY = Cu = 1 \in R(G)$, so $c_! : K_G(B') \longrightarrow K_G(*) = R(G)$ is onto, proving the hyperelementary induction theorem.

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