MATHEMATISCHE ARBEITSTAGUNG 1977

UNIVERSITÄT BONN

Sonderforschungsbereich 40

Theoretische Mathematik

Wegeler Str. 10

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Inhalt

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Mathematisches Institut
der Universität Bonn

Programm der Mathematischen Arbeitstagung 1977 (I)

Dienstag, den 21.6.:

17.15 - 18.15 Uhr:	M.F. Atiyah: The Classical Geometry of Yang-Mills Fields

Mittwoch, den 22.6.:

10.00 - 11.00 Uhr:	J.-P. Serre: Function field analogue of $SL_2(\mathbb{Z})$
12.00 - 13.00 Uhr:	H.W. Lenstra: Euclidean number fields
17.00 - 18.00 Uhr:	T. Tromba: Recent progress in Plateau's problem

Donnerstag, den 23.6.:

10.00 - 10.15 Uhr:	Festlegung der nächsten Vorträge
10.15 - 11.15 Uhr:	W. Feit: Projective modules of some finite classical groups
12.30 - ca. 20.00 Uhr:	Dampferfahrt auf dem Rhein nach Leutesdorf; Abfahrt am "Alten Zoll" mit Motorschiff "Carmen Silva"

Die Vorträge finden alle im "Großen Hörsaal" (Wegelerstraße 10) statt.

Erfrischungspausen mit Tee: Mittwoch vormittags von 11.15-12.00 Uhr vor dem Großen Hörsaal, nachmittags ab 15.30 Uhr im Diskussionsraum Beringstraße 1.

Die Post liegt während der Vormittags-Teepausen aus.

Tischtennis im Keller des Hauses Beringstraße 4.

Den Tagungsbeitrag bitte an Frau Gerber (SFB-Büro Beringstr. 4) bezahlen.

Alle Teilnehmer mögen sich bitte in die Teilnehmerliste eintragen.

Alle Informationen und die Teilnehmerlisten liegen vor dem Diskussionsraum Beringstraße 1 aus.

Mathematisches Institut
der Universität Bonn

Programm der Mathematischen Arbeitstagung 1977 (III)

Sonntag, den 26.6.:

10.00 - 11.00 Uhr: Ch. Thomas: Space form problems

12.00 - 13.00 Uhr: R. Finn: Surface tension phenomena and geometry

17.00 - 18.00 Uhr: J.-P. Bourguignon and C.L. Terng: Solution of the Calabi conjecture

Montag, den 27.6.:

10.00 - 11.00 Uhr: C. Procesi: Ideals of determinants and Young diagrams

12.00 - 13.00 Uhr: F. Sakai: Kodaira dimension of open complex manifolds

17.00 - 18.00 Uhr: A. Andreotti: Domain of regularity of solutions of partial differential equations

Die Vorträge finden alle im "Großen Hörsaal" (Wegelerstr. 10) statt.

Erfrischungspausen mit Tee: Sonntag und Montag vormittags ab 11.15 Uhr vor dem großen Hörsaal, nachmittags ab 15.30 Uhr im Diskussionsraum der Beringstraße 1.

Die Post liegt während der Vormittags-Teepause aus.

Tischtennis im Keller des Hauses Beringstraße 4.

! Die Referenten werden nochmals gebeten, ihre Kurzfassungen möglichst bald !
! bei Herrn Kraft abzugeben, da wir den Tagungsbericht allen Teilnehmern !
! noch vor ihrer Abreise aushändigen möchten. !

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Title: CLASSICAL GEOMETRY OF YANG MILLS FIELDS I, II

Name of author: M. F. ATIYAH

Address: MATHEMATICAL INSTITUTE 24 ST. GILES OXFORD ENGLAND

Bibliography:

M. F. Atiyah, N J Hitchin and I. M. Singer, Deformations of Instantons, Proc Nat. Acad. Sci USA 1977.

M. F. Atiyah and R. Ward, Instantons and Algebraic Geometry, Comm. Math. Phys. 1977 (to appear)

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S. Coleman, Classical lumps and their quantum descendants, Erice Summer School in Physics 1975

R. Jackiw, Quantum meaning of classical solutions Phys Rev (to appear)

§1 The Yang-Mills Equations

Modern elementary particle physics uses gauge theories as mathematical models which will hopefully provide the right quantum field theory. In mathematical terms gauge theories are fibre bundles with connections (and other data such as sections etc.). The classical (i.e. pre-quantized) model deals with certain non-linear partial differential equations in space-time which arise from a suitable Lagrangian. For certain purposes one sometimes replaces time by imaginary time, so that Minkowski space becomes Euclidean space. Moreover conditions at infinity are imposed which lead one to compactify R^4 to the 4-sphere S^4. This is natural in the important case of conformally invariant equations

The simplest and most basic equations
of this type are the Yang-Mills
equations, which in mathematical terms
are the following. Let $P \to M$ be
a differentiable fibre bundle with group
G (a compact Lie group) and M a
compact Riemannian manifold (usually S^4).
Let A be a connection on P, $F(A)$ its
curve and $\|F\|^2$ its natural L^2-norm
using the metric on M and a bi-invariant
metric on G. This is our Lagrangian
and we look for its critical points. The
Euler equations can be written as $D * F = 0$
where $*$ is the duality operator on forms in M
and D is the covariant derivative skew-
symmetrized.

In dimension 4 (for oriented M) we
have $*^2 = 1$ and we can decompose
$F = F^+ \oplus F^-$ into eigenspaces of $*$.
Taking $G = SU(2)$ as the interesting case

one finds $\|F\|^2 = \|F^+\|^2 + \|F^-\|^2$

while $8\pi^2 k = \|F^+\|^2 - \|F^-\|^2$

where k is an integer topological invariant of P (essentially the Pontryagin number). It follows that (if $k > 0$) the absolute minimum of $\|F\|^2$ occurs when $F^- = 0$, i.e. when $*F = F$. These are the self-dual Yang-Mills equations and solutions of these have been called "instantons".

§2. Algebraic Geometry

Using the fibration $P_3(C) \to S^4$ one finds that the equations $*F = F$ on S^4 correspond to certain algebraic vector bundles on $P_3(C)$ of rank 2. These are the stable bundles extensively studied by algebraic geometers and in this way the instantons can be constructed in principle from algebraic geometry. The results are briefly as follows:

1) All solutions F are given by rational functions whose poles (in the complexification) are of order k (in the conformal or Plücker coordinates).

2) The space of solutions $M(k)$ is a real manifold of dimension $8k-3$.

3) For $k=1$, $M(k)$ is the interior of S^4, i.e. the hyperbolic 5-space, which is an orbit of the conformal group $SO(5,1)$

4) For $k=2$, $M(k)$ is connected but not simply-connected (and is explicitly known).

5) For $k \leq 2$ all solutions are given by the Ansatz of t' Hooft, but this is false for $k \geq 3$.

6) Solutions can be constructed from certain special algebraic curves in P_3. $k+1$ disjoint lines give the t' Hooft solution. The next class uses elliptic curves and gives all solutions for $k = 3$ or 4

Title: Function field analogue of $SL_2(\mathbf{Z})$

Name of author: Jean-Pierre SERRE

Address: 6 avenue de Montespan 75116 PARIS France

Bibliography: Arbres, Amalgames, SL_2, to appear (Oct. 1977) in Asterisque, S.M.F.

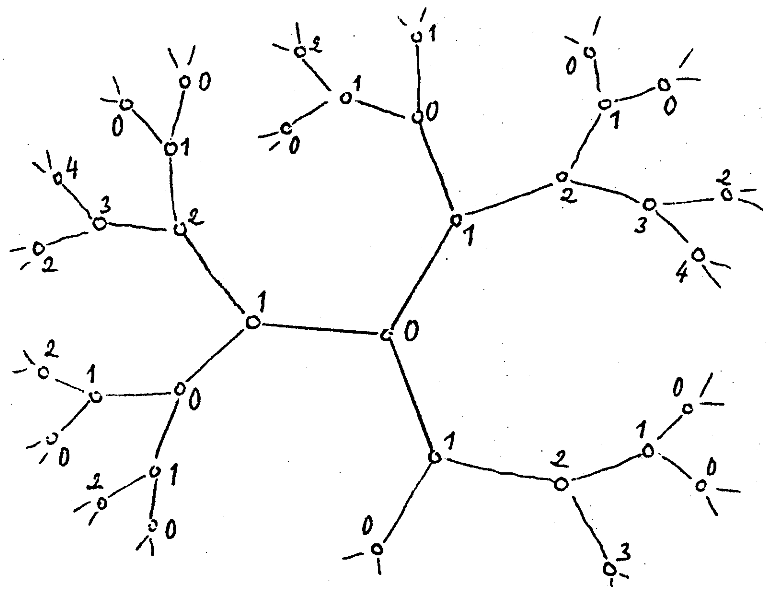
The " function field analogue " of $\mathbf{Z}$ (resp. $\mathbf{Q}$) is the affine ring A (resp. the field K of fractions of A) of an affine curve $C^{aff} = C - \{P\}$ having just one point at infinity P, defined over some field k_o. To study the group $\Gamma = GL_2(A)$, we make it act on the Bruhat-Tits tree X of $SL_2(K)$ relative to the valuation v of K defined by P. The vertices of the quotient graph $\Gamma\backslash X$ can be interpreted as classes of 2-dimensional vector bundles over C whose restriction to C^{aff} is trivial (two bundles E_1 and E_2 are put in the same class if there is $n \in \mathbf{Z}$ such that $E_1 \simeq E_2 \otimes I_P^{\otimes n}$, where I_P is the line bundle defined by P). Known results on vector bundles allow one to determine completely $\Gamma\backslash X$ in some simple cases. For instance :

a) If C^{aff} is the affine line, we have $A = k_o[t]$, and the quotient graph $\Gamma\backslash X$ looks like :

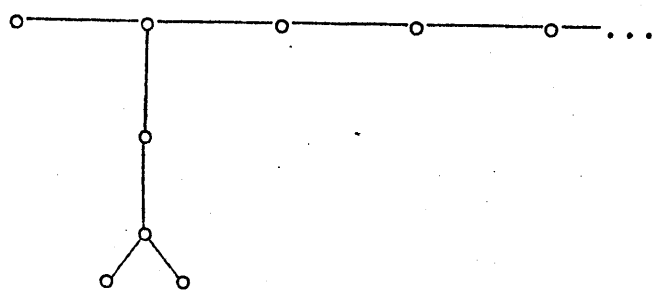
$$\underset{0}{\circ}\text{———}\underset{1}{\circ}\text{———}\underset{2}{\circ}\text{———}\underset{3}{\circ}\text{—}\ \cdots$$

where the vertex labelled n corresponds to the vector bundles $\underline{1} \oplus I_P^{\otimes n}$ (that there are no more vector bundles than those is Grothendieck's theorem on bundles over the projective line).

For the structure of X itself, see next page.



b) If $A = \mathbb{F}_2[u,v]/(u^2 + u + v^3 + v + 1)$, then C is an elliptic curve "without points" so that A is a principal domain. The graph $\Gamma \backslash X$ is then :



When k_o is finite , one finds that $\Gamma \backslash X$ is the union of a finite graph Y , and of finitely many " cusps " , each of them being isomorphic to :

These cusps correspond to the elements of Pic(A).

Using this information, it is not hard to see that Γ is an "amalgam" of finite groups (viz. the stabilizers of the vertices of a "fundamental domain"). For $A = k_o[t]$, for instance, this gives Nagao's theorem :

$$GL_2(k_o[t]) = GL_2(k_o) *_{B(k_o)} B(k_o[t]) ,$$

where B is the standard Borel subgroup of GL_2.

One also finds that $H_i(\Gamma, \mathbb{Q})$ is 0 for $i \geqslant 2$ and is finite dimensional for $i = 1$.

The <u>Euler-Poincare characteristic</u> of Γ is defined by :

$$\chi(\Gamma) = \sum_{x \in \mathrm{som}(\Gamma\backslash X)} 1/|\Gamma_x| - \sum_{y \in \mathrm{ar.geom}(\Gamma\backslash X)} 1/|\Gamma_y|$$

where Γ_x (resp. Γ_y) is the stabilizer of x (resp. y) in Γ. This definition extends to subgroups of finite index of Γ. For $\Gamma_1 = SL_2(A)$, one finds (using the fact that the Tamagawa number of SL_2 is 1) that

$$\chi(\Gamma_1) = \zeta_{C,P}(-1) ,$$

where $\zeta_{C,P}$ is the zeta function of C^{aff} (i.e. the zeta function of C with the P-factor removed).

When $\Gamma' \subset \Gamma$ has no ℓ-torsion for $\ell \neq \mathrm{charc}.k_0$, one finds that $\chi(\Gamma')$ <u>is a negative integer</u>, which has the following homological interpretation :

Call St the Steinberg module (over Z) of $GL_2(K)$, i.e. the reduced 0-th homology group of the Tits building of GL_2 over K. Then St <u>is a</u> $Z[\Gamma']$ -<u>projective module of finite type</u>, <u>which is stably free</u>, <u>of rank</u> $-\chi(\Gamma')$.

In other words, $\chi(\Gamma')$ can be interpreted as a <u>relative</u> Euler-Poincare characteristic.

<u>Note</u> - Most of the results stated in this lecture are special case of general results of Harder on S-arithmetic groups in the function field case ; see in particular his forthcoming paper in Invent.Math.

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Title: Euclidean number fields

Name of author: H.W. Lenstra, Jr.

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Bibliography: P. Samuel, About Euclidean rings, J. Algebra 19 (1971), 287-301; HWLJ, Lectures on Euclidean rings, Bielefeld 1974; id., Euclidean number fields of large degree, Invent. Math. 38 (1977), 237-254; id., On Artin's conjecture and Euclid's algorithm in global fields, to appear.

Let R be a commutative ring with 1 without zero-divisors. A map $\phi: R-\{0\} \to \{0,1,2,\ldots\}$ is called a <u>euclidean algorithm</u> on R if for all $a, b \in R$, $b \neq 0$, there exist $q, r \in R$ such that $a = qb + r$ and either $r = 0$ or $\phi(r) < \phi(b)$. If such a ϕ exists, then R is called <u>euclidean</u>. It is well-known that any euclidean R is a principal ideal domain.

Define subsets $R_n \subset R$ for $n = -1, 0, 1, 2, \ldots$ by $R_{-1} = \{0\}$, $R_n = \{x \in R$: the natural map $R_{n-1} \to R/Rx$ is surjective$\} \cup \{0\}$ $(n \geq 0)$. Then $R_{-1} \subset R_0 \subset R_1 \subset R_2 \subset \ldots$, and Motzkin (Bull. Amer. Math. Soc. 55 (1949), 1142-1146) proved that R is euclidean if and only if $R = \bigcup_n R_n$. Moreover, if this condition is satisfied, then the map $\theta: R-\{0\} \to \{0,1,2,\ldots\}$ defined by $\theta(x) = n$ if $x \in R_n - R_{n-1}$ is a euclidean algorithm on R; it is, in fact, the <u>smallest</u> one in the sense that $\theta(x) \leq \phi(x)$ for all $x \in R-\{0\}$ and any euclidean algorithm ϕ on R. Notice that $R_0 = U(R) \cup \{0\}$, where $U(R)$ denotes the group of units of R.

If $R = k[t]$, with k a field and t a polynomial variable, then one easily proves by induction that $R_n = \{f \in k[t]: \deg(f) \leq n\}$ for all n (with $\deg(0) = -1$), so that $\theta = \deg$. It turns out that θ has a similar description on some other well-known euclidean rings, as follows.

Suppose $x \in R - R_0$ is such that any $y \in R$ can be written as $y = \Sigma_{j=0}^{n} u_j x^j$, with n an integer ≥ 0 and $u_j \in R_0$ for $0 \leq j \leq n$; then we define $\psi_x(y)$ to be the smallest n for which such a representation exists.

It can be proved that $\psi_x = \theta$ whenever ψ_x is a euclidean algorithm on R. We now have:

<u>Theorem 1.</u> $\theta = \psi_t$ on $R = k[t]$,

$\theta = \psi_2$ on $R = \mathbb{Z}$,

$\theta = \psi_{1+i}$ on $R = \mathbb{Z}[i]$, $i^2 = -1$,

$\theta = \psi_{1-\rho}$ on $R = \mathbb{Z}[\rho]$, $\rho^2 = -\rho-1$.

The proofs of the last two cases of this theorem are fairly delicate. A better understanding of why the theorem is true would be desirable. It is probable, though unproved, that also $\theta = \psi_{1+i}$ on the quaternion ring $\mathbb{Z}[i,j,(1+i+j+ij)/2]$, $i^2 = j^2 = -1$, $ji = -ij$ (the non-commutativity of the ring doesn't hurt).

The behaviour of the rings mentioned in theorem 1 is not typical for number rings: if R is an algebra of finite type over $\mathbb{Z}$, or over a field, and R has a euclidean algorithm of the form ψ_x, with $x \in R-R_0$, then R is isomorphic to one of the rings mentioned in theorem 1.

In the remainder of this summary we let K be a number field, i.e. a finite field extension of the field $\mathbb{Q}$ of rational numbers, S denotes a finite set of places of K containing the set S_∞ of infinite places, and for R we take the ring of S-integers of K, i.e. $R = \{x \in K: |x|_{\underline{p}} \leq 1$ for all places $\underline{p}$ of K which are not in S$\}$. Thus, if $S = S_\infty$, then R is the ring of algebraic integers in K.

It is well-known that U(R) is finite if and only if $|S| = 1$, which implies that $R = \mathbb{Z}$ or that R is the ring of integers in an imaginary quadratic number field. If, in addition, R is a principal ideal domain, then R is one of

$$\mathbb{Z},\ \mathbb{Z}[\rho],\ \mathbb{Z}[i],\ \mathbb{Z}[(1+\sqrt{-7})/2],\ \mathbb{Z}[\sqrt{-2}],\ \mathbb{Z}[(1+\sqrt{-11})/2],$$
$$\mathbb{Z}[(1+\sqrt{-19})/2],\ \mathbb{Z}[(1+\sqrt{-43})/2],\ \mathbb{Z}[(1+\sqrt{-67})/2],\ \mathbb{Z}[(1+\sqrt{-163})/2].$$

The first six of these are euclidean (even norm-euclidean, see below), the other four are not.

In the case of infinite U(R) we have:

<u>Theorem 2.</u> Let, with the above notations, $|S| \geq 2$, and assume the truth of certain generalized Riemann hypotheses. Then R is euclidean if and only if R is a principal ideal domain. Moreover, if R is euclidean, then θ is determined by

$\theta(x) = 1$ if x is a prime element of R for which the natural map $U(R) \to U(R/Rx)$ is surjective,

$\theta(x) = 2$ if x is any other prime element of R,

$\theta(yz) = \theta(y) + \theta(z)$ for all $y, z \in R-\{0\}$.

Cf. Weinberger, Proc. Symp. Pure Math. 24 (1973), 321-332 for the case $S = S_\infty$.

The Riemann hypotheses enter in this theorem via Artin's conjecture on primes with prescribed primitive roots (Hooley, Crelle 225 (1967), 209-220); one observes that $x \in R_1 - R_0$ if and only if x is a prime element of R having a unit of R as a primitive root.

Without unproved assumptions the situation is completely different. For $x \in R-\{0\}$, let the norm $N(x)$ of x be defined by $N(x) = |R/Rx|$. We call R norm-euclidean if N is a euclidean algorithm on R.

Situation: any R of the type described above which is known to be euclidean is actually norm-euclidean.

Problem: change this situation. Candidates might be $\mathbb{Z}[\sqrt{14}]$ and $\mathbb{Z}[e^{2\pi i/32}]$, which are expected to be euclidean because of theorem 2, but can be shown not to be norm-euclidean.

In the classical case $S = S_\infty$, most methods to prove that a particular R is norm-euclidean depend on embedding R as a lattice in the $\mathbb{R}$-vector space $K \otimes_{\mathbb{Q}} \mathbb{R}$; the norm N can be extended by multiplicativity and continuity to the entire space, and R is norm-euclidean if and only if

(1) for all $x \in K$ there exists $y \in R$ such that $N(x-y) < 1$;

it seems that in all cases in which this is known to be true one actually has

(2) for all $x \in K \otimes_{\mathbb{Q}} \mathbb{R}$ there is $y \in R$ such that $N(x-y) < 1$.

If (1) and (2) are equivalent properties, then it can be shown that the problem whether the ring of algebraic integers in a given number field is norm-euclidean is decidable.

In the case $S = S_\infty$, there are 311 non-isomorphic rings R known which are norm-euclidean; the largest degree (ten) among these has $R = \mathbb{Z}[e^{2\pi i/11}]$. About the finiteness of the number of examples almost nothing is known. The most important result in this direction is a theorem of Davenport stating that, up to isomorphism, only finitely many K with $|S_\infty| \leq 2$ have a norm-euclidean ring of algebraic integers. This result has been used to determine all norm-euclidean rings of integers in quadratic number fields.

In the general case, where S is not required to be equal to S_∞, these techniques can be generalized; one should just replace $K \otimes_{\mathbb{Q}} \mathbb{R}$ by $\prod_{\underline{p} \in S} K_{\underline{p}}$, where $K_{\underline{p}}$ is the completion of K at $\underline{p}$. As a generalization of Davenport's theorem one finds that, up to isomorphism, there are only finitely many norm-euclidean rings R with $|S| \leq 2$ which are not contained in any of the fields $\mathbb{Q}$, $\mathbb{Q}(\sqrt{-d})$, d = 3, 4, 7, 8, 11, 15, 20; and each of these seven fields contains infinitely many such R ($|S| \leq 2$).

A theorem of O'Meara (Crelle <u>217</u> (1965), 79-108) asserts that for any K there exists S such that R is norm-euclidean. A quantitative version of his result is as follows.

<u>Theorem 3.</u> For any number field K there exists an integer $B \geq 0$, such that for any choice of B+1 different elements ω_0, ..., ω_B of the ring A of algebraic integers of K, the ring $R = A[\prod_{0 \leq i < j \leq B} (\omega_i - \omega_j)^{-1}]$ is norm-euclidean.

For B one can take the Minkowski constant $[\frac{n!}{n^n} \cdot (\frac{4}{\pi})^s \cdot \sqrt{|d|}]$, with n and d denoting the degree and the discriminant of K over $\mathbb{Q}$, respectively, s is the number of complex archimedean places of K, and [r] denotes the largest integer $\leq$ r.

If one manages to find ω_0, ..., ω_B such that all differences $\omega_i - \omega_j$ $(i \neq j)$ are units in A, then clearly $R = A$. This remark has been used to find more than 130 norm-euclidean rings of integers in number fields.

Notice that the set S belonging to R in theorem 3 consists of S_∞ and those finite places $\underline{p}$ dividing at least one of the differences $\omega_i - \omega_j$ $(i \neq j)$. In particular, any finite place $\underline{p}$ of norm $\leq$ B belongs to S. Thus the following problem is left open:

<u>Problem.</u> Let K be a number field, and let T be a finite set of places of K disjoint from S_∞; does there exist S with $S \cap T = \emptyset$ such that R is norm-euclidean?

Only in a few non-trivial cases (e.g. $K = \mathbb{Q}(\sqrt{-5})$, $K = \mathbb{Q}(\sqrt{14})$) the answer is known to be "yes".

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Title: The Classical Problem of Plateau

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Let $\mathcal{A} = \mathrm{Emb}(S^1, \mathbb{R}^n)$ be sufficiently smooth embeddings of S^1 into $\mathbb{R}^n$. For $\alpha \in \mathcal{A}$ let $\Gamma^\alpha = \alpha(S^1)$. The classical problem of Plateau asks for solutions of the following systems of non-linear P.D.E. We ask for a map $u : D \to \mathbb{R}^n$ such that

$$* \begin{cases} (1) & \Delta u = 0 \\ (2) & u_x \cdot u_y = 0 \\ (3) & \|u_x\|^2 = \|u_y\|^2 \\ (4) & u : S^1 \to \Gamma^\alpha \text{ a homeomorphism} \end{cases}$$

Equations $*$ $\Rightarrow$ mean curvature of $u(D)$ is zero and if u is an immersion that the Laplace-Beltrami operator $\Delta_\beta u = 0$. These equations arise as the Euler-Lagrange equations of the following variational problem.

Let $\eta^\alpha = \{u \mid \Delta u = 0,\ u : S^1 \to \Gamma^\alpha,\ u|S^1 \sim \alpha\}$

Here $\sim$ means homotopic to. Let $E : \eta^\alpha \to \mathbb{R}$ be $E(u) = \frac{1}{2}\int_D \nabla u \cdot \nabla u$ The critical points of

E which are homeomorphisms on S^1 are the solutions to $(*)$.

<u>Rmk</u>. E is equivariant under the action of the non-compact three dimensional Lie group of conformal transformations of the disc to itself.

It is also important to note that a solution of * may not be an immersion. The points where a solution fails to be an immersion are called a branch points. There are finitely many such points and they also have a finite order. If u has p interior branch points of orders $\lambda_1 \ldots \lambda_p$ and q bdry. branch points of half orders $\nu_1 \ldots \nu_q$ set $\Lambda = (\lambda_1 \ldots \lambda_p)$, $\nu = (\nu_1 \ldots \nu_q)$, $|\Lambda| = \sum \lambda_i$, $|\nu| = \sum \nu_i$. In such a case we say that u is of type Λ, ν and denote such surfaces by $\mathcal{M}_{\Lambda,\nu}$.

We raise the following questions

(1) How many solutions to * are there? finitely many? infinitely many?

(2) What does the set of solutions look like?

(3) Can they be counted in any way

(4) How does the flatness of the ambient space ($\mathbb{R}^n$) enter the picture.

(5) Are the existence of branch points stable under perturbation? Can we perturb away branch points?

(6) How do minimal surfaces bifurcate as you change the wire? Can these be described?

Not much has been known about finiteness and what is known has been very recent, mainly through the work of F. Tomi.

Our point of departure with classical theory is to apply methods of global analysis to the problem.

<u>Thm</u>. There exist a smooth vector field $\bar{X}^\alpha$ on η^α and a connection K_α on η^α such that

(a) The zeros of $\bar{X}^\alpha$ are precisely the critical points of E

(b) $\nabla_\alpha \bar{X}^\alpha(u)$ = identity + completely continuous

(c) $\bar{X}^\alpha$ is equivariant w.r.t the conformal group.

This $\bar{X}^\alpha$ arises as the solution of the following elliptic system of mixed type, namely $\bar{X}^\alpha(u) : D \to \mathbb{R}^n$ satisfying

$$(1) \quad \Delta \bar{X}^\alpha(u) = 0$$

$$(2) \quad \frac{\partial \bar{X}^\alpha}{\partial n}(u) \cdot T_1(u) = \frac{\partial u}{\partial n} \cdot T_1(u)$$

$$(3) \quad \bar{X}^\alpha(u) \cdot T_j(u) = 0 \qquad j = 2, \dots n$$

where for each $p \in \Gamma_1^\alpha$ $T_j(p)$ is an orthonormal basis of $\mathbb{R}^n$ with $T_1(p) \in T_p \Gamma^\alpha$ and $p \mapsto T_j(p)$ smooth.

The plausibility of counting the number of minimal surfaces arises ~~the~~ from (b) above since for a Banach space E, $GL(E)$ the

general linear group and $GL_c(E) =$
$GL(E) \cap \{id + \text{comp. cont}\}$ has $\pi_0(GL_c(E)) = Z_2$
whereas for "most" E, $GL(E)$ is contractible.
But we don't know when the solution set is
finite. Let $\eta = \bigcup_{\alpha \in \mathcal{a}} \eta^\alpha$ be a "surface fibre
bundle over the embeddings $\mathcal{a}$ with canonical
projection map π. How do the solutions to $*$
look like in this bundle.

<u>Thm</u>. The set of all minimal surfaces in η
is the union of submanifolds (a strata, pieced
together in a specific way) $\mathcal{M}_{\Lambda,\nu}$. Moreover we
can restrict the bundle map π to $\mathcal{M}_{\Lambda,\nu}$
obtaining a map $\pi_{\Lambda,\nu}$. This map is a nonlinear
(proper after factoring out the conformal group)
Fredholm map of index

$$3 + 2p + q + 2|\Lambda|(2-n) + 2|\nu|(2-n).$$

<u>Cor 1</u> $\exists$ an open dense set $\mathcal{a}_0 \subset \mathcal{a}$ such
that for $\alpha \in \mathcal{a}_0$ there is a finite # of
solutions. If $n > 3$ all $\alpha \in \mathcal{a}_0$ have the
property that all minimal surfaces spanning
them are immersed. If $n=3$ all surfaces
spanning wires in $\mathcal{a}_0$ have no bdry. branch
points.

Cor 2. In $\mathbb{R}^3$ the index of projection on the stratum of minimal surfaces with simple branch points is zero. This implies [c.f. [1]] the stability of singularities of solutions in $\mathbb{R}^3$ but not in $\mathbb{R}^n$, $n>3$.

To count the surfaces there are several possible approaches.

(1) One can define an invariant say the Euler-characteristic $\chi(\Sigma^\alpha)$ for each α and compute it

(2) Use modern degree theory methods

(3) Use Morse theory.

Both (1) and (2) are possible, (3) looks likely. The well known Morse theory of Palais and Smale does not apply to this problem. Another M.T has been developed [5] and all the axioms of this theory (save one) have been checked. For (1) see [6]. I will describe (2) which uses the ideas of [7]. For ~~simplicity~~ simplicity I consider only the case $n>3$ because since degree is generic we need only consider the stratum of immersed surfaces. Using prop. (b) of Σ^α it follows that $\mathcal{M}_{0,0}$ is an ∞-dim. ∞-codim. oriented subvariety of $\mathcal{N}$ in the sense of [7], and $\pi_{0,0}: \mathcal{M}_{0,0} \to \mathcal{a}$ is Fredholm of index 0 (after factoring out the conformal group action). Moreover $\pi_{0,0}$ is proper and has a Brouwer degree

Thm $\deg \pi_{0,0} = 1$.

Cor This gives a new degree theoretic proof of the existence of a minimal surface.

How does this tie up with variational theory?

Let $\alpha \in \mathcal{A}_0$ and $u_1, \ldots u_N$ the finite # of "non-degenerate" minimal surfaces spanning α.

Let β_i = Morse index of the Hessian of E at u_i. Each β_i is a finite non-negative integer.

Thm $$\deg \pi = \sum (-1)^{\beta_i} = 1.$$

Cor If there exist two relative minima there exists a third unstable surface. Moreover if Γ^α is a curve with k-relative minima then arbitrarily close to Γ^α there exists a curve $\Gamma^{\alpha'}$ with k-n.d. minima and $k-1$ unstable minimal surfaces.

Question (6) has begun to be answered in recent work by the author & M. Beeson in which the cusp catastrophe of Thom has been observed in the bifurcation of Enneper's surface

Bonn. Seminar 1977.

Title: Projective modules of some finite groups of Lie type.

Name of author: Walter Feit
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Let $G = Sp_4(2^m)$ or $Sz(2^{2s+1})$. Let Φ_ϕ be the projective character corresponding to the principal Brauer character of G (in characteristic 2) and let $c_{\phi\phi}$ be the corresponding Cartan invariant. The purpose of this talk is to give formulas for $\Phi_\phi(1)$ and $c_{\phi\phi}$. In case $G = Sz(2^{2s+1})$ these results were first obtained by L. Chastkofsky and later it was realized that similar (and simpler) arguments could also handle $G = Sp_4(2^m)$. Aside from the groups $SL_2(p^n)$ no such results were available for any other class of groups of Lie type before this work was begun.

The following formulas can be obtained.

Let $T_n = \left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{2}\right)^n$. Then

$$\Phi_\phi(1) = q^4(q^4 - q^2 T_{2m} + 1) = q^2(q^2 - q T_m + (-1)^m) q^2 (q^2 + q T_m + (-1)^m)$$

for $G = Sp_4(q)$, $q = 2^m$.

$$= q^2 - q T_{2s+1} - 1 \qquad \text{for } G = Sz(q),\ q = 2^{2s+1}.$$

Observe that if the formula for $G = Sp_4(2^m)$ is "twisted" then in case in m is even it yields the formula for $G = Sp_4(2^{m/2})$ and in case $m = 2s+1$ it yields the formula for $Sz(2^{2s+1})$.

Let $q = 2^n$. Define

$K_n = \alpha^n + \beta^n + \gamma^n$ where $(x-\alpha)(x-\beta)(x-\gamma) = x^3 - 3x^2 - x + 5$.

Let

$$f(n) = q^3 + q^2 + q + (-1)^n 2q + qK_n - 2q(q+1)T_n .$$

Then

$$\begin{aligned} c_{\phi\phi} &= f(2m) \quad \text{for } G = Sp_4(2^{m}) \\ &= f(2s+1) \quad \text{for } G = Sz(2^{2s+1}). \end{aligned}$$

These results in particular imply that

$$\lim_{n\to\infty} \frac{c_{\phi\phi}}{|G|_2^{3/2}} = 1 \qquad \text{for } \underset{n=2s+1}{G = Sz(2^{2s+1})} \text{ or } \underset{n=m}{G = Sp_4(2^m)}$$

A similar argument can be used to show that if $G = SL_3(2^m)$ or $SU_3(2^m)$ then

$$\Phi_\phi(1) = 8^m(6^m - 5^m).$$

So far it has not been possible to evaluate $c_{\phi\phi}$ for these latter groups. It can be shown however that

$$\lim_{m\to\infty} \frac{c_{\phi\phi}}{q^m} \geq 1 .$$

Observe that $|G_2|_2 = 8^m$.

In the process of proving these results it is possible to get formulas concerning $\Phi_i(1)$ for any projective indecomposable character Φ_i. It seems to be much more difficult to get any information concerning c_{ij} in general.

1

Title: Hyperbolicity in dynamics and geometry

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Bibliography:

Given a group Γ with an automorphism $\alpha : \Gamma \circlearrowleft$ one can canonically construct a dynamical system $X \circlearrowleft A$ with the following property: if Γ is realized as a fundamental group of a space Y and α is induced by a homeomorphism $B : Y \circlearrowleft$ then there is a canonical morphism $(B : Y \circlearrowleft) \to (A : X \circlearrowleft)$. That connection between algebra and dynamics can be exploited both ways. In particular one can study Γ by means of dynamics. When $(X \supset A)$ is hyperbolic the picture one obtains is essentially clear.

There is an analogues relation in geometry between properties of the fundamental group of a Riemannian manifold V, dynamics of its geodesic flow G and the "fuzzy" geometry of the universal covering of V. Again the simpliest case is hyperbolic (V of negative curvature) when the fundamental group uniquely determins topology of G. Not all hyperbolic groups can be realized by compact manifolds of negative curvature but in general geometry is more suggestive than pure combinatorial analysis based on the notion of a group with small ~~cancellation~~ cancellation.

Combinatorial complexity of nonhyperbolic groupsis reflected in the structere of the Morse filtration in the loop (or free loop) space of V. Say, non solvability of the word problem in

$\pi_1(V)$ ~~implies~~ obviously implies existence of infinitely many contractable simple closed geodesics in V.

Title: An Application of Residues to Algebraic Geometry

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Bibliography: to appear

(1) Local Theory For $f_1(z), \dots, f_n(z)$ holomorphic in $\|z\| < \varepsilon$ with origin as isolated common zero, $g(z)$ holomorphic in $\|z\| < \varepsilon$ and $\omega = \frac{g(z)\,dz}{f} \underset{\mathrm{def}}{=} \frac{g(z)\,dz_1 \wedge \dots \wedge dz_n}{f_1(z) \cdots f_n(z)}$ the local residue is

$$\boxed{\operatorname{Res}_{\{0\}} \omega = \left(\frac{1}{2\pi\sqrt{-1}}\right)^n \int_\Gamma \omega}$$

where $\Gamma = \{ |f_i(z)| = \delta \}$ with orientation $d(\arg f_1) \wedge \dots \wedge d(\arg f_n) \geq 0$. This residue was studied by B. Segre (1930's) and is an analytic version of the Grothendieck residue symbol. Two elementary properties are (a) Setting $J_f(z) = \frac{\partial(f_1, \dots, f_n)}{\partial(z_1, \dots, z_n)}$, in the non-degenerate case where $J_f(0) \neq 0$ then $\operatorname{Res}_{\{0\}} \omega = \frac{g(0)}{J_f(0)}$ (b) If $f_i' = \sum_j a_{ij} f_j$ have the origin as isolated common zero and $\Delta = \det a_{ij}$, $\omega' = \frac{\Delta g\, dz}{f'}$ then $\operatorname{Res}_{\{0\}} \omega = \operatorname{Res}_{\{0\}} \omega'$ (transformation formula).

(2) Residue theorem For $E \to M$ a rank-n holomorphic vector bundle over a compact, complex n-manifold M and given $s \in H^0(\mathcal{O}(E))$ with isolated zero-locus Z, $\psi \in H^0(\mathcal{O}(K \otimes \det E))$ with K = canonical bundle we define

$$\operatorname{Res}_p \left(\frac{\psi}{s}\right) = \operatorname{Res}_{\{0\}} \omega$$

where $e_1, \dots, e_n$ is holomorphic frame for E near p, $z_1, \dots, z_n$ are holomorphic coordinates centered at p, and $s = f_1(z) e_1 + \dots + f_n(z) e_n$, $\psi = g(z)\, dz_1 \wedge \dots \wedge dz_n \otimes e_1 \wedge \dots \wedge e_n$, and $\omega = \frac{g\,dz}{f}$. By (b) this is well defined, and by (a) $\operatorname{Res}_p \left(\frac{\psi}{s}\right) = 0 \Leftrightarrow \psi(p) = 0$ if p is a simple zero. The global residue theorem is

$$\boxed{(*) \qquad \sum_{p \in Z} \mathrm{Res}_p \left(\frac{\psi}{s}\right) = 0}$$

Of course, it is proved by Stokes' theorem. For a geometric
in interpretation, given a line bundle $L \to M$ with
complete linear system $|L|$, a zero-cycle Z has the
Cayley-Bacharach property relative to $|L|$ in case
any $D \in |L|$ passing thru $Z-p$ contains all of Z.
If $|L|$ induces a projective embedding this implies
the points of Z are not in general position. $(*)$ means
that $(s) = Z$ has Cayley-Bacharach property relative
to $|K \otimes \det E|$. For $M = \mathbb{P}^2$ and $\mathcal{O}(E) = \mathcal{O}(m) \oplus \mathcal{O}(n)$ this is
the Cayley-Bacharach theorem ($\Rightarrow$ Pascal theorem when $m=n=3$).

③ Generalized Castelnuovo Bound. Suppose E is induced
by a holomorphic mapping $f: M \to G(n,N)$ = Grassmanian
of $(N-n)$-planes in $\mathbb{C}^N$, and $\varphi: M \to \mathbb{P}^r$ is given by
$|K \otimes \det E|$. In the diagram

$$\begin{array}{ccc} M & \xrightarrow{f} & G(n,N) \\ \downarrow & & \\ \mathbb{P}^r & & \end{array} \qquad \left\{ \begin{array}{l} f^{-1}(\text{Schubert cycle}) = \\ (s) \text{ for } s \in H^0(\mathcal{O}(E)) = \\ p_1 + \cdots + p_d \end{array} \right.$$

the residue theorem $(*)$ gives relations on the $\varphi(p_i)$.
Differentiating $(*)$ k times with respect to $s \in H^0(\mathcal{O}(E))$
gives linear relations on the k^{th} osculating spaces to
$\varphi(p_i)$. In particular, taking $E = \underbrace{L \oplus \cdots \oplus L}_{n\text{-times}}$ where L is the
pullback of the hyperplane bundle under birational
morphism $M \to \mathbb{P}^N$ (different N) gives inequalities

$$\boxed{(**) \qquad \dim |K + mL| \leq \kappa(n, N, d, m) \qquad 0 \leq m \leq n}$$

together with determination of cases where equality holds.
For $m=0$ we have a bound on $h^{n,0}(M)$ (e.g. $h^{n,0} = 0$ for
$d \leq n(N-n)+1$). In case $|K|$ is biregular (not only birational)
we obtain $h^{n,0}(M) \leq \frac{(-1)^n c_1^n}{n+1} + C$ ($C = 7/3$ for $n=2$).

Title: WIEDERSEHENSMANNIGFALTIGKEITEN

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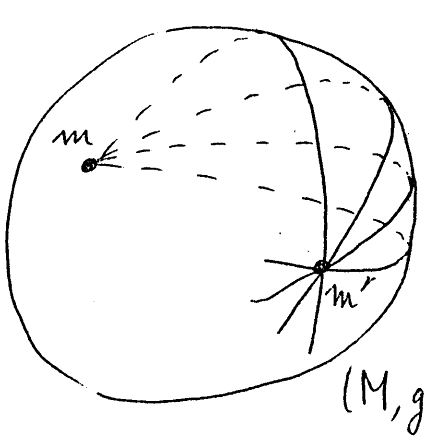
Leon W. GREEN Auf Wiedersehensflächen, Annals of Mathematics, 78, 289-299 (1963)

Alan WEINSTEIN On the volume of manifolds all of whose geodesics are closed, Journal of Differential Geometry 9, 513-517 (1974)

Soit (M,g) une variété riemannienne, où M désigne une variété C^∞ de dimension n et g la structure riemannienne considérée sur M ; soit en particulier (S^n, can) la sphère $S^n \subset \mathbb{R}^{n+1}$ munie de sa structure riemannienne canonique.

Définition On dit que (M,g) est une Wiedersehensmannigfaltigkeit si elle possède la propriété suivante : pour tout point m de M, toutes les géodésiques issues de m repassent au bout du temps π par un seul point m', $m \neq m'$, et ne se rencontrent pas entre m et m'.



Il est facile de voir que M est alors difféomorphe à S^n. En 1920 Blaschke conjectura que tout Wiedersehensfläche (Wiedersehensmannigfaltigkeit de dimension $n=2$) est isométrique à (S^2, can). Cette conjecture fut démontrée par Leon Green en 1961. Ce que l'on peut maintenant démontrer est le :

Théorème 0. Toute (M,g) de dimension n paire qui est une Wiedersehensmannigfaltigkeit est isométrique à (S^n, can)

Si $Vol(g)$ désigne le volume d'une variété riemannienne (M,g) et $\alpha(n)$ le volume de (S^n, can), le théorème ci-dessus sera la composition des deux ci-dessous.

Théorème 1 (Weinstein) Pour toute Wiedersehensmannigfaltigkeit on a $Vol(g) = \alpha(n)$.

En fait ce théorème est valable sous une hypothèse plus faible

Théorème 2. Pour une Wiedersehensmannigfaltigkeit de dimension n quelconque on a toujours $Vol(g) \geq \alpha(n)$ et, en outre, $Vol(g) = \alpha(n)$ si et seulement si (M,g) est isométrique à (S^n, can).

La démonstration du théorème 2 se fait ainsi : on calcule $Vol(g)$ en coordonnées polaires de centre $m \in M$, puis l'on fait la moyenne de ce calcul pour m parcourant M. Il s'agit alors d'une intégration sur le fibré unitaire UM de (M,g), intégration que l'on réarrange, grâce à Fubini, en une intégrale d'intégrales curvilignes le long des géodésiques de (M,g). Après quelques manipulations, qui utilisent essentiellement : ① le théorème de Liouville sur l'invariance de la mesure canonique de UM par son flot géodésique, ② le fait que l'élément de volume en coordonnées polaires est un déterminant de champs de Jacobi, ③ le fait que les champs de Jacobi d'une variété riemannienne vérifient un système de Sturm-Liouville, après ces manipulations, dis-je, on est ramené à démontrer l'inégalité suivante :

pour tout entier $n \geq 2$, pour toute fonction f strictement positive sur l'intervalle ouvert $]0,\pi[$, on a :

$$\int_{x=0}^{x=\pi}\int_{y=x}^{y=\pi}\int_{t=x}^{t=\pi} \frac{f(x)f(y)}{f^2(t)} \sin^{n-2}(y-x)\,dy\,dx\,dt \geqslant \int_{x=0}^{x=\pi}\int_{y=x}^{y=\pi} \sin^{n-1}(y-x)\,dy\,dx .$$

La démonstration de cette inégalité est due à Terry L. Kazdan.

Quant au cas de l'égalité, il s'étudie en suivant en arrière les manipulations (2) et (3) mentionnées plus haut.

$\square$

Title: Inequalities for Chern numbers of surfaces of general type

Name of author: A. Van de Ven

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Starting point is the following fact:

- Minimal surfaces of general type X with given $c_1^2(X)$, $c_2(X)$ have an algebraic $\mathbb{C}$-space of finite type as moduli space.

Thus there are two questions:

1) For which pairs of integers (p,q) does exist at least one minimal surface of general type X with $c_1^2(X) = p$ and $c_2(X) = q$?

2) Find for these (p,q) the structure of the corresponding moduli space.

Miyaoka proved recently that for each surface of general type X the inequality $c_1^2(X) \leq 3\, c_2(X)$ holds. This also follows for such surfaces with ample canonical bundle from recent work of Aubin and Yau concerning the Calabi-conjecture. The results of Aubin and Yau also imply that there i only one complex strukture on $\mathbb{P}_2$.

Title : Representations of Semisimple Lie Groups

Name of Author : Gregg J. Zuckerman

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See [3] for review of progress on classification of irreducible unitary reps. of real semisimple groups. Full classification known for

a. $SO(n,1)$, $n \geq 2$. Hirai (1962)

b. $SU(n,1)$ Kraljevic (1972)

c. Complex groups of rank 2. Duflo (1976)

d. $GL(n, \mathbb{R})$? Speh (1977). (at least up to $n = 4$)

An Example:

$G = SO(4,2)$ conformal group of space-time (compactified):

$S^3 \times S^1$ (projective real null quadric in $\mathbb{R}(4,2)$).

$\cong G/P$,

$P \cong SO(3,1)\, \mathbb{R}^\times \ltimes \mathbb{R}^4$.

(maximal parabolic subgroup)

$\Omega^2(G/P)$ smooth 2-forms on G/P.

Maxwell's equations $\omega \in \Omega^2(G/P)$,

$$d\omega = d{*}\omega = 0.$$

Photon rep.: smooth solutions $\mathcal{M}$ to Maxwell's eqn., completed w.r.t. an $SO(4,1)$ invariant positive definite inner product defined on Cauchy data: pair of 1-forms on S^3: we show $SO(4,2)$ leaves inner product invariant.

We discuss:

a. action of center of enveloping algebra on $\mathcal{M}$

b. restriction to $K = SO(4) \times SO(2)$.

c. Reducibility w.r.t. $SO(4,2)$ (4 components)

d. Relative Lie algebra cohomology: $H^*(\mathfrak{g}, \mathfrak{k}, \mathcal{M})$; $H^{**}(\mathfrak{g}, \mathfrak{k},$ components of $\mathcal{M})$ (Hodge numbers) (see [4], [5])

e. relation to unitary principal series, unitary induction; use of center of K.

f. relation to (holomorphic) discrete series (see [1]) (also [2]).

g. relation to metaplectic rep. of $Sp(8, \mathbb{R})$.

h. Formula: let Γ be cocompact discrete subgroup of $G = SO(4,2)$. Let $h^{p,q}(\Gamma) = \dim H^{p,q}({}_K\backslash G/_\Gamma)$: then,

$$(*) \quad h^{2,0}(\Gamma) = \dim \operatorname{Hom}_G(\mathcal{M}^+, L^2(G/_\Gamma)),$$

where $\mathcal{M}^+$ = positive frequency solutions in $\mathcal{M}$. (see also [5])

i. Kahzdan's methods (see [5]) show there exist $\Gamma \subseteq SO(4,2)$ with r.h.s. of $(*)$ arbitrarily large.

j. Analogy with special representation of a p-adic simple group; Casselman's "vanishing" theorem (see [5]); Matsushima's vanishing theorem for $h^{p,0}(\Gamma)$, $p < rk_{\mathbb{R}} G$, in real hermitian case.

Finally, we discuss recent progress on classification of irreducible unitary reps. π with $H^*(\mathfrak{g}, \mathfrak{k}, \pi) \neq 0$ (see [4], [5].); we take G real and simple:

Theorem (see [4], also [5]): If π is nontrivial,

$$H^k(\mathfrak{g}, \mathfrak{k}, \pi) = 0 \text{ if}$$

$$k < rk_{\mathbb{R}}\, G.$$

Remark: Photon rep. of $SO(4,2)$ realizes the rank.

We have explicit constructions of π with $H^{rk}(\mathfrak{g}, \mathfrak{k}, \pi) \neq 0$ for

i) $SU(p,q)$ (see [5])
ii) $SO(n,1)$ (see [5])
iii) $SL(n, \mathbb{R})$
iv) $SL(n, \mathbb{C})$.

Title: Space Form Problems

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We work in the topological category (TOP) for convenience. Our starting point is the

Existence Theorem: Let Γ be a finite group, such that
(1) Γ has cohomological period $2d$,
(2) every subgroup of order $2p$ (p = prime) is cyclic. Then there exists a free action by Γ on S^{2rd-1} with $r = 1$ or 2.

Remark: if Γ is metacyclic, i.e. $\Gamma_2 = \{1\}$ or cyclic, $r = 1$.

In this paper we begin the classification of the space forms, S^{2rd-1}/Γ.

Example: Algebraic K-theory implies that there are free actions by the binary tetrahedral group T_1^* on S^{4k-1} ($k \geq 2$), homotopically distinct from the unique linear action.

Notation: Γ is of Type (mn) if Γ is an extension of the form $1 \to \mathbb{Z}/m \to \Gamma \to \mathbb{Z}/n \to 1$. We call the extension faithful, if $\mathbb{Z}/n \to \mathrm{Aut}(\mathbb{Z}/m)$ is a monomorphism.

As usual p and q denote prime numbers.

Homotopy Type invariants

Let $(r, |\Gamma|)=1$ and (r,Σ) be the projective ideal in $\mathbb{Z}\Gamma$ generated by r and the sum of the group elements. Then

(i) Type $(p\cdot q^n)$, faithful, $(r,\Sigma)\sim 0$ in $\tilde{K}^0(\mathbb{Z}\Gamma)$ if $r\equiv s^{q^n}(p)$,

and (ii) Type $(p\cdot q^2)$, non-faithful, $(r,\Sigma)\sim 0$ in $\tilde{K}^0(\mathbb{Z}\Gamma)$ if $r\equiv s^{q}(p)$.

Remark: In case (ii) it follows easily from representation theory that $(r,\Sigma)\sim 0$ if $r\equiv s^{q}(|\Gamma|)$. The interest of the arithmetic calculation is that reduction modulo q^2 is irrelevent.

Simple Homotopy Type invariants.

Even more unsatisfactory.

If Γ is of Type $(m\cdot q)$, necessarily faithful, $SK_1(\mathbb{Z}\Gamma)=0$. Hence we may define Reidemeister torsions at q and the prime factors of m.

Let Y represent a fixed simple homotopy type. For an arbitrary group Γ we have the following invariants:

Normal invariant ν^N, consisting of a stable TOP bundle over Y and a map $\varphi: S^{N+\dim Y}\to Y^\nu$ of degree 1. This splits as $\nu_{(odd)}$ and $\nu_{(2)}$; $\nu_{(odd)}$ is detected by $\widetilde{KO}(Y)\otimes\mathbb{Z}[\frac{1}{2}]$.

Multisignature ρ. This is defined absolutely, but arises naturally as the element in $\mathbb{R}_{\pm\mathrm{Herm}}(\Gamma)$ determined by the (skew) Hermitian form on $H^{\dim Y+1/2}(\tilde{W},\mathbb{R})$, for W a normal cobordism between M_1 and M_2 simple homotopy equivalent to Y.

An easy argument with representations of metacyclic groups shows that ρ determines $\nu_{(odd)}$.

Recall from (HF VI, Thm. 2.4.3) that for Γ of odd order

(i) $L^{\bullet}_i(\Gamma)=0$, $i=$ odd,

(ii) $L^s_i(\Gamma)$ is ~~detected by~~ determined by ρ, $i\equiv 0(4)$,

(iii) $L^s_i(\Gamma)$ is determined by the classical Arf invariant, $i\equiv 2(4)$.

Putting all this together we have :

<u>Classification Theorem</u> : If Γ is metacyclic of odd order, and $M_1 \overset{s}{\simeq} M_2$, then $M_1 \cong M_2$ if and only if $\rho(M_1) = \rho(M_2)$. In particular, for (pq)-groups, $M_1 \cong M_2$ iff M_1 and M_2 have equal signatures and Reidemeister torsions at the primes p and q.

(Even for the binary dihedral groups - Type $(p4)$, non-faithful, the prime 2 introduces complications. Thus inside a fixed simple homotopy type, classification is in terms of ρ, $\nu_{(2)}$ and an, as yet badly understood, 2-torsion element in L_0.)

<u>Application</u>

Using the homotopically exotic actions of the Type (pq^2) groups (non-faithful), together with work of J. F. Adams on maps between classifying spaces, one can show that, if $F = STOP(S^{2q-1})/SO(2q)$, then $\pi_{2q-1} F$ is not p-divisible for infinitely many primes p. In particular, $\pi_5\left(\dfrac{STOP(S^5)}{SO(6)}\right) \neq 0$, and hence (using smoothing theory) that $SDiff(S^5)$ is not homotopy equivalent to $SO(6)$. This is the lowest dimension in which I know this to hold.

C

Title: Surface Tension phenomena and geometry

Name of author: R Finn

Address: Stanford Univ.

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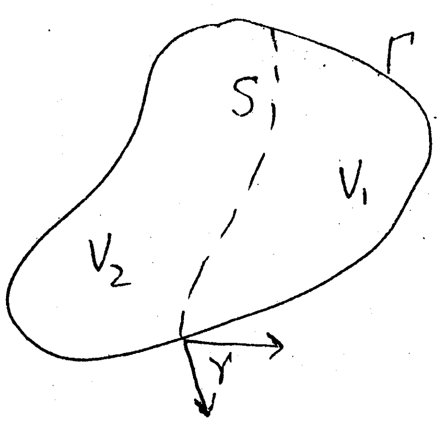
An equilibrium surface interface S between two adjacent fluids V_1, V_2, is characterized by the condition that the associated energy be stationary. If the fluid lies in a container Γ in a vertically directed gravity field, one finds the equation



$$n H_S = \kappa u + \lambda$$

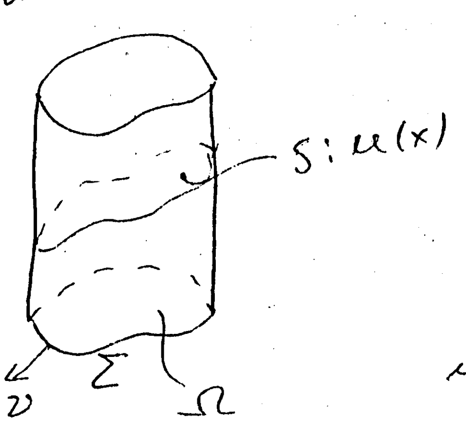
where H_S is the mean curvature of S, λ a Lagrange parameter, u is vertical height on S, $n = \dim(S)$, physically $n=2$.

On Γ, one finds

$$\gamma \equiv \text{const.}$$

κ and γ are physical constants. Once they are given, the problem becomes that of finding a surface of prescribed mean curv, meeting Γ in prescribed angle.

In this generality, little is known and the solns are in genl not unique. But in special cases some information can be obtained. Suppose, with gravity $g \equiv 0$, we seek a surface $u(x)$ in a cylindrical container with base section Ω. Then



$$\operatorname{div} Tu = \frac{\Sigma}{\Omega}\cos\gamma \quad \text{in } \Omega$$

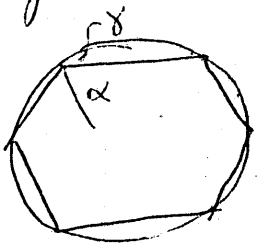
$$\nu \cdot Tu = \cos\gamma \quad \text{on } \Sigma$$

with $Tu = \dfrac{Du}{\sqrt{1+|Du|^2}}$,

$\Sigma = \text{meas}(\Sigma)$, etc.

We find a particular soln, if Σ is a sphere; then S is a lower spherical cap.

The same soln works if Ω is the interior of an inscribed regular polygon $(n=2)$, since a plane cuts a sphere with constant angle. In this case, $\alpha+\gamma = \pi/2$.



A dilation of the sphere yields a family of solutions with $\alpha+\gamma \geq \pi/2$, all of which are lower spherical caps. We have, however,

<u>Thm</u> If Σ contains an angle 2α with $\alpha+\gamma < \pi/2$, then $\nexists$ soln in Ω.

This result has been verified by experiment; if $\alpha+\gamma \geq \pi/2$ the spherical solns are observed, if $\alpha+\gamma < \pi/2$ the fluid flows out along the corner to ∞ (top of the container), One has also

<u>Thm</u> Given Σ, Ω, $\exists\ \hat{\Sigma}, \hat{\Omega}$ with $\hat{\Sigma}$ analytic and arbitrarily close to Σ, s.t. $\nexists$ soln in $\hat{\Omega}$.

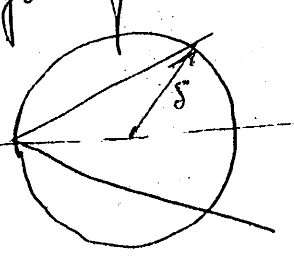
However:

<u>Thm</u> given Σ, $\exists\ \gamma_0(\Sigma)$, $0 \le \gamma_0 < \pi/2$, s.t. if $\gamma_0 < \gamma \le \pi/2$, then $\exists$ soln. If $\gamma < \gamma_0$, then $\nexists$ soln.

We have also

<u>Thm</u> $\exists$ soln if and only if $\exists$ vector field $\vec{v}$ in Ω, $\operatorname{div} \vec{v} \equiv \frac{\Sigma}{\Omega}$, $\vec{v} \cdot \nu = 1$ on Σ, $|\vec{v}| < \frac{1}{\cos\gamma}$ in Ω.

The ~~above~~ above results are for $g = 0$. If $g > 0$, existence is known for quite genl bdries. But consider Ω with corner (conical pt), suppose indicated domain lies interior to Ω.



Thm If $\alpha + \gamma \geq \pi/2$ then

$$|u| < \frac{h}{\kappa\delta} + \delta$$

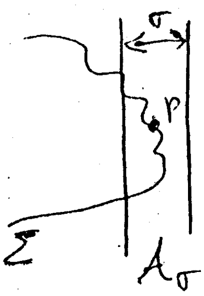
If $\alpha + \gamma < \pi/2$, then $u \to \infty$ in corner. Thus, the soln is discontinuous in α. This has been verified exptlly.

For symmetric soln u^s in a ball, the bdry value u^s_Σ satisfies

$$u^s_\Sigma > \sqrt{\tfrac{2}{\kappa}(1-\sin\gamma)}$$

indpt of radius. For genl Σ, this leads to

Thm given $\varepsilon > 0$, $\exists\ \sigma(\varepsilon)$ s.t. if Σ lies on one side of a plane, then for all $p \in \Sigma \cap A_\sigma$, $u(p) > \sqrt{\tfrac{2}{\kappa}(1-\sin\gamma)} - \varepsilon$.



The result depends in no other way on the shape of Σ.

The following result is proved with the aid of a particular capillary boundary prob.

<u>Thm</u> $\exists\ R_0 = 0{,}5654062332\ldots$ with the property: if $u(x)$ has mean curv. $= 1$ in $x_1^2 + x_2^2 = |x|^2 < R^2$ and if $R > R_0$, then $|Du(0)| < C(R) < \infty$, as $R \to 1$, $C(R) \to 0$, and $\hat{u}(x) \equiv u(x) - u(0)$ tends in every compact subset to a lower hemisphere.

For further results, see the indicated literature.

Title: Results of S.T.Yau on a conjecture by E.Calabi

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Let (M,J,g,ω) be a compact Kähler manifold of complex dimension m , i.e. J is the complex structure, g the Hermitian metric and ω the Kähler form of type $(1,1)$.

It is well known that the Ricci curvature Ric of the metric is Hermitian, the form $\gamma_\omega(.,.) = \mathrm{Ric}(.,J.)$ is of type $(1,1)$ and is closed. The cohomology class that γ_ω defines, denoted by $[\gamma_\omega]$, is precisely the first Chern class of (M,J), $c_1(M,J)$.

In 1954 E.Calabi presented the following conjectures (cf. [3] and [4]) :

Conjecture I : Let $\tilde{\gamma}$ be a form of type $(1,1)$ which is closed and such that $[\tilde{\gamma}] = c_1(M,J)$.

Does there exist one and only one Kähler form $\tilde{\omega}$ defining the same class as ω such that $\gamma_{\tilde{\omega}} = \tilde{\gamma}$?

<u>Conjecture II</u> : Suppose furthermore that $c_1(M,J) = \lambda[\omega]$ (we then say that c_1 is positive or negative according to the sign of λ). Does there exist a Kähler metric $\tilde{\omega}$ so that $[\tilde{\omega}]=[\omega]$ and

$$\gamma_{\tilde{\omega}} = \lambda . \tilde{\omega}$$

(such a metric is called Kähler-Einstein)?

These conjectures can be considered as optimistic since they claim that the topological constraints of a Kähler metric are only the known ones.

The situation is the following :

In [4] E.Calabi proved uniquemess and existence when $\tilde{\gamma}$ is close to γ_ω (this was clarified by Ochiai in 1974). T.Aubin in 1970 (cf [1]) gave a partial result. S.T.Yau proves completely Conjecture I in [6].

Conjecture II was proved for $\lambda<0$ by T.Aubin in [2]. The case $\lambda = 0$ falls under Conjecture I. Is still open for case $\lambda > 0$ (and according to some people it is likely to fail uniqueness).

Applications to analytic geometry

<u>Corollary 1</u> . Let (M,J,ω) be a compact Kähler manifold. If $c_1(M,J) = 0$ and $c_2(M,J) = 0$, then M is covered by a torus and in particular all its Chern classes vanish.

<u>Corollary 2</u> . The complex projective space has only one complex structure for which there is a Kähler form.

Applications to Riemannian geometry

<u>Corollary 3</u> . Any hypersurface of degree n+1 in $\mathbb{C}P^n$ has a metric with vanishing Ricci curvature which is not flat $(n>2)$.

<u>Corollary 4</u> . Any hypersurface of degree at least n+2 in $\mathbb{C}P^n$ has an Einstein metric with negative Ricci curvature but has no metric with negative sectional curvature.

From these corollaries one can deduce also examples of non-locally homogeneous Einstein manifolds and of manifolds having SU(n) as holonomy groups (up to now the only known examples were non-compact). In particular the K3 surfaces provide examples of deformable Einstein structures.

Monge-Ampère equation

We reduce the problem to a PDE one in the case of Conj. I.

We denote by d' and d'' the parts of type (1,0) and (0,1) respectively of the exterior differential.

We define $\tilde{\omega} = \omega + \sqrt{-1}\, d'd''\varphi$.

Since $[\tilde{\gamma}] = [\gamma_\omega] = c_1(M,J)$, then there exists F such that $\tilde{\gamma} = \gamma_\omega + \sqrt{-1}\, d'd'' F$.

If in a chart we set $\tilde{g} = \det \tilde{\omega}$, it is well known that $\gamma_{\tilde{\omega}} = \sqrt{-1}\, d'd'' \log \tilde{g}$. Therefore in a chart the equation to solve is

$$\log \tilde{g} - \log g = F \quad ,$$

which globally reads

$$(*) \qquad (\omega + \sqrt{-1}\, d'd''\varphi)^m = e^F \omega^m .$$

For the second conjecture II, the equation is

$$(\omega + \sqrt{-1}\, d'd''\varphi)^m = e^{\lambda\varphi + F}\omega^m .$$

These equations are of Monge-Ampère type.

S.T.Yau solves the equation $(*)$ by the continuity method. Fix F in $C^{k-1,\alpha}$ $(k \geqslant 3 \ , \ 0 < \alpha < 1)$ such that $\int_M e^F \omega^m = \int_M \omega^m$. We look at the family of equations

$$(*_t) \qquad (\omega + \sqrt{-1}\, d'd''\varphi)^m = e^{tF}\omega^m$$

for t in $[0,1]$.

We set $\mathcal{O} = \left\{ \varphi \,\middle|\, \varphi \in C^{k+1,\alpha} , \int_M \varphi\omega^m = 0, \omega + \sqrt{-1}\, d'd''\varphi \text{ is a metric} \right\}$

We consider $A = \{ t \mid t \in [0,1] , \exists\, \varphi \in \mathcal{O} \text{ solution of } (*_t) \}$

We want to prove that $A = [0,1]$.

We know that 0 is in A .

The openness of A follows from the implicit function theorem applied to the map $C : \mathcal{O} \to C^{k-1,\alpha}$ where

$$C(\varphi) = (\omega + \sqrt{-1}\, d'd''\varphi)^m / \omega^m$$

since the differential of C reduces to a Laplacian for a Kähler metric for which everything is known.

For the closedness, we need to estimate (in terms of t_oF) the $C^{k+1,\alpha}$-norm of a solution of $(*_{t_o})$.

This is the hard and tricky part. In fact a bound on the $C^{2,\alpha}$-norm is enough by the usual Schauder estimate and the technique of linearizing $(*)_{t_o}$.

A $C^{2,\alpha}$-estimate of a solution φ is deduced from a uniform estimate of φ, gradφ and $\Delta\varphi$ (this is the beautiful contribution of S.T.Yau) and an extra estimate on the third order derivatives of φ.

Title: Ideals of determinants and Young diagrams

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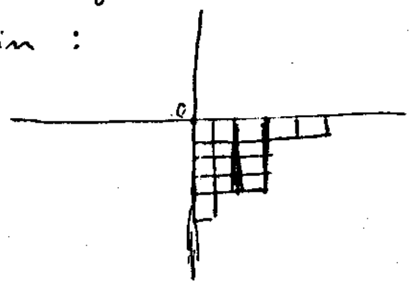
Bibliography: C. DE CONCINI, D. EISENBUD, C. PROCESI - SAME TITLE TO APPEAR, & C. DE CONCINI, C. PROCESI - A CHARACTERISTIC FREE APPROACH TO INVARIANT THEORY - ADVANCES IN MATHEMATICS - 1976.

The object under study is the space $A_{n,m}$ of $n \times m$ matrices with entries in a field K. On $A_{n,m}$ we consider the natural action of the group $G = Gl(n,K) \times Gl(m,K)$. The orbit structure is particularly simple, the only invariant being the rank, the G invariant subvarieties are thus the varieties V_i formed by matrices of rank $< i$ and defined by the vanishing of $i \times i$ minors. It is known that in fact these minors generate the ideal of functions vanishing on V_i. Here we want to describe more generally the G-invariant ideals in the coordinate ring R of $A_{n,m}$. Changing notations $R = S(E \otimes F)$, E, F two vector spaces, $G = Gl(E) \times Gl(F)$. It is then known that, if char $K = 0$, $S(E \otimes F) = \bigoplus_\sigma L_\sigma(E) \otimes L_\sigma(F)$ where σ runs over the set of all Young diagrams and L_σ denotes the Schur module associated. If $n = \min(\dim E, \dim F)$ one can restrict to the set Y of those diagrams σ with at most n columns. We indicate $M_\sigma = L_\sigma(E) \otimes L_\sigma(F)$, M_σ is an irreducible G module. The description just given of $R = S(E \otimes F)$ implies immediately that there is a 1-1 correspondence between G-invariant subspaces of R and subsets T of Y, let us indicate by $M_T = \bigoplus_{\sigma \in T} M_\sigma$. Our aim is first of all to study the subsets corresponding to ideals of R.

Let us consider the Young diagrams as subsets in the 4^{th} quadrant of a Cartesian plane with vertex in the origin:



then it makes sense, given two diagrams σ, τ, to say that $\sigma \geq \tau$ and to form $\sigma \cup \tau$ and $\sigma \cap \tau$ which will be again Young diagrams, we have:

<u>Theorem</u> M_T is an ideal if and only if T satisfies the following condition:

If $\sigma \in T$ and $\tau \geq \sigma$ then $\tau \in T$.

In view of the theorem we will call such a set an ideal in Y. Given any subset $S \subseteq Y$, S generates an ideal $\{S\} = \{\tau \in Y \mid \exists \sigma \in S \text{ and } \tau \geq \sigma\}$

Of course every ideal is finitely generated.

Clearly the union of ideals in Y is an ideal and it corresponds in R to the sum of the corresponding ideals: $M_{T_1 \cup T_2} = M_{T_1} + M_{T_2}$, similarly for the intersection, moreover if $T_1 = \{\sigma_1, \ldots, \sigma_k\}$, $T_2 = \{\tau_1, \ldots, \tau_h\}$ we have $T_1 \cap T_2 = \{\sigma_i \cup \tau_j\}$..

Before proceeding we define the notion of product of two diagrams $\sigma \cdot \tau$. It is the diagram obtained adding the columns of σ and τ or else taking all the rows of σ and τ and placing them in order so to make a diagram.

We can now define the notions of radical, prime, primary ideals of Y.

1) $\sqrt{T} = \{\sigma \in Y \mid \sigma^m \in T \text{ for some } m\}$.

2) T is prime if $\sigma\tau \in T$ and $\tau \notin T \Rightarrow \sigma \in T$

3) T is primary if $\sigma\tau \in T$, $\tau \notin \sqrt{T} \Rightarrow \sigma \in T$.

Theorem 1) $\sqrt{M_T} = M_{\sqrt{T}}$

2) M_T is prime $\iff$ T is prime.

3) M_T is primary $\iff$ T is primary.

In practise:

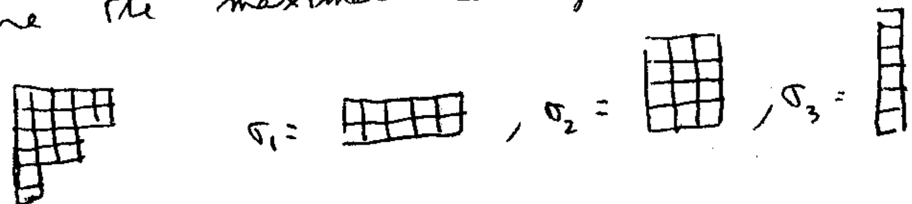
1) If σ is a diagram and τ is its first row it is clear that $\sigma \leq \tau^m$ for some m so $\tau \in \sqrt{\{\sigma\}}$ and in fact in general $\sqrt{T} = \{\tau\}$ where τ is the smallest first row of elements of T. If τ has k elements we call $\tau = k$ and $I_\tau = I_k$ (the ideal of $k \times k$ minors).

2) $\{k\}$ are the only prime ideals.

3) If $\sqrt{T} = \{k\}$, T is primary means that if $\sigma \in T$ and we remove from σ all the rows of length $< k$ the resulting diagram σ' is still in T. Since T must have a diagram with first row of length k this means that $T = \{\sigma_1, \sigma_2, \dots, \sigma_i\}$ where all the rows of each σ_j have length $\geq k$ and σ_1 is rectangular with rows of length k.

As for primary decomposition first if $T = \{\sigma\}$ then $\{\sigma\} = \{\sigma_1\} \cap \{\sigma_2\} \cap \dots \cap \{\sigma_h\}$ where σ_i are the maximal rectangles contained in σ.

e.g. $\sigma =$, $\sigma_1 =$, $\sigma_2 =$, $\sigma_3 =$



More generally one can procede by induction on the total number of rows.

If $T = \{\sigma_1, \dots, \sigma_h\}$ say that σ_1 contains the rectangle τ with least base and largest height among the σ_i's. $T_1 = \{\tau, \sigma_2, \ldots, \sigma_n\}$ is primary and $T = T_1 \cap T'$, $T' = \{\sigma_1', \sigma_2, \ldots, \sigma_n\}$, σ_1' is obtained from σ_1 removing the rows of lowest length. The continue on T'.

We then mention the theory of complete ideals. If $\sigma \in Y$ has columns of length $h_1, h_2, \ldots, h_n$ we set $\gamma(\sigma) = (\gamma_1(\sigma), \gamma_2(\sigma), \ldots, \gamma_n(\sigma))$ with $\gamma_i(\sigma) = \sum_{j=i}^{n} h_j$. We then say $\sigma \geq \tau$ if $\gamma_i(\sigma) \geq \gamma_i(\tau)$ for all i.

<u>Theorem</u> The ideal of functions vanishing on V_i with order $\geq p$ (the symbolic power $I_i^{(p)}$) is M_T where $T = \{\sigma \mid \gamma_i(\sigma) \geq p\}$.

To relate to determinants again, if σ has rows of length $k_1 \geq k_2 \geq \cdots \geq k_r$ we set $D_\sigma = I_{k_1} \cdot I_{k_2} \cdot \ldots \cdot I_{k_r}$ and then we have.

i) $D_\sigma = \sum_{\tau \geq \sigma} M_\tau$

ii) $D_\sigma = I_1^{(\gamma_1(\sigma))} \cap I_2^{(\gamma_2(\sigma))} \cap \cdots \cap I_n^{(\gamma_n(\sigma))}$ is a primary decomposition by symbolic powers.

iii) D_σ is the integral closure of I_σ.

More generally one can classify all the integrally closed ideals. Any such ideal is of the form $\sum_{i=1}^{t} D_{\sigma_i}$ where the σ_i's have the property that: If $\tau \in Y$ and $\gamma(\tau) \geq$ convex combination of $\gamma(\sigma_i)$ then

$\tau \geq \sigma_i$ for some i. In terms of Young diagrams the integral closure $\overline{T}$ of an ideal T is $\overline{T} = \{\tau \in Y \mid \exists\, m$ and $\sigma_1, \sigma_2, \dots, \sigma_m \in T$ with $\tau^m \geq \sigma_1 \sigma_2 \cdots \sigma_m \}$. The main technique to prove the theorems is the straightening law of Doubilet, Rota, Stein. Given the minors of the matrix of indeterminates $X = \begin{vmatrix} x_{11} & x_{12} & \cdots & x_{1m} \\ \vdots & \vdots & & \\ x_{n1} & x_{n2} & \cdots & x_{nm} \end{vmatrix}$

they can be displayed by double rows: $(i_k \cdots i_1 \mid j_1 \cdots j_k)$ where $j_1 \cdots j_k$ are the column indeces, $i_1 \cdots i_k$ the row indeces of the minor. Then one can construct various quadratic equations among minors which dominate the algebra in R, in particular if one displays a product of minors in a double tableau:

$$\left|\begin{array}{cccc|cccc} i_{k_1} & \cdots & i_2 & i_1 & j_1 & j_2 & \cdots & j_{k_1} \\ t_{k_2} & \cdots & t_2 & t_1 & s_1 & s_2 & \cdots & s_{k_2} \\ & \cdots & & & & \cdots & & \\ & \cdots & & & & \cdots & & \\ & & \vdots & \vdots & \vdots & \vdots & & \\ & & v_{k_s} & v_1 & u_1 & u_{k_s} & & \end{array}\right|$$

one has that the double tableaux which are standard on both sides are a basis of R, the use of this basis and the quadratic equations give together with some simple representation theory the ingredients needed for all the proofs.

Title: Kodaira Dimensions of Open Complex Manifolds

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The notion of Kodaira dimension of compact complex manifolds plays an important role in classification theory (see [1], [6]). The purpose of this note is to introduce the notion of Kodaira dimension for open complex manifolds.

§1. Open Complex Manifolds

Let X be a complex manifold of dimension n and Ω^n the sheaf of germs of holomorphic n-forms on X. An element $\omega \in H^0(X, (\Omega^n)^{\otimes m})$ can be written locally as

$$\omega = f(z)(dz_1 \wedge \cdots \wedge dz_n)^m,$$

using local coordinates $(z_1, \ldots, z_n)$. We associate with ω the continuous (n,n)-form $(\omega \wedge \bar{\omega})^{1/m}$, given locally by

$$(\omega \wedge \bar{\omega})^{1/m} = |f(z)|^{2/m} \prod_{i=1}^{n} (\sqrt{-1}/2\pi) dz_i \wedge d\bar{z}_i .$$

Define the quasi norm $\| \ \|$ of $H^0(X,(\Omega^n)^{\otimes m})$ as follows

$$\|\omega\| = \int_X (\omega\wedge\bar{\omega})^{1/m}.$$

Though $\| \ \|$ may not be a norm. We have the following facts

(i) $\|\omega\|=0$ if and only if $\omega=0$,

(ii) $\|c\omega\| = |c|\|\omega\|$,

(iii) $\|\omega_1+\omega_2\| \leqq C_m(\|\omega_1\|+\|\omega_2\|)$, $(C_1=1, C_m=2^{(2/m)-1})$.

<u>Definition</u> 1. $F_m(X)=\{\omega\in H^0(X,(\Omega^n)^{\otimes m}) \mid \|\omega\|<\infty\}$

By the property (iii), $F_m(X)$ is vector subspace. We put

$$\gamma_m = \dim_C F_m(X).$$

<u>Definition</u> 2. The Kodaira dimension $\kappa(X)$ of X is defined as follows

$$\kappa(X)=\begin{cases} \max\limits_{m>0} \max\limits_{\substack{L\subset F_m(X)\\ \text{finite dimensional vector subspace}}} \{\text{rank of the Jacobian matrix of } \Phi_L\} \\ -\infty \quad \text{if } \gamma_m=0 \text{ for all } m>0, \end{cases}$$

where Φ_L is the meromorphic map of X into $\mathbb{P}_N$ $(N+1=\gamma_m)$ defined by a basis $\omega_0,\dots,\omega_N$ of L as follows

$$X \ni z \longrightarrow (\omega_0(z):\dots:\omega_N(z))\in\mathbb{P}_N.$$

Note that $\kappa(X)$ takes one of the values of $-\infty,0,1,\dots,n$. We list some properties of $\kappa(X)$.

(i) $\kappa(X)$ is a bimeromorphic invariant of X (in the sense of Remmert),

(ii) if X contains Y as an open subset. Them $\kappa(X)\leqq\kappa(Y)$.

(iii) if Z is an analytic subset of X with codimension$\geqq 2$, then $\kappa(X-Z)=\kappa(X)$.

(iv) $\kappa(\mathbb{C})=-\infty$, $\kappa(\mathbb{C}^*)=-\infty$. Further $\kappa(\mathbb{C}\times Y)=-\infty$.

In case X is a complex space, we define $\kappa(X)$ to be $\kappa(X^*)$, using a desingularization X^* of X.

§2. Complements of Divisors.

We assume that X has a smooth compactification $\overline{X}$ in the sense that $\overline{X}$ is compact complex manifold and $D=\overline{X}-X$ is a divisor with at most normal crossings. We have

<u>Theorem</u> 1. $F_m(X)\cong H^0(\overline{X},(\Omega^n)^{\otimes m}\otimes[D]^{m-1})$.

The proof is based on the fact that if $f(z)(dz)^m$ is an element of $F_m(\Delta^*)$, where Δ^* is the punctured unit disc, then the Laurent expansion of $f(z)$ becomes as $\sum_{k=-(m-1)}^{\infty}a_k z^k$.

<u>Example</u> 1. Let D be a hypersurface in P_n with at most normal crossings. Then $\kappa(P_n-D)=n$ if d(the degree of D)$>n+1$, $\kappa(P_n-D)=-\infty$ if $d\leq n+1$.

Iitaka [2] defines the logarithmic Kodaira dimension $\overline{\kappa}(X)$ of X, using the vector space $H^0(\overline{X},(\Omega^n)^{\otimes m}\otimes[D]^m)$. We have the relation:

(i) $\overline{\kappa}(X)\geq\kappa(X)$,

(ii) $\overline{\kappa}(X)=n$ if and only if $\kappa(X)=n$,

(iii) if $\kappa(X)\geq 0$, then $\overline{\kappa}(X)=\kappa(X)$.

Iitaka in [2]proves that $\overline{\kappa}(X)$ is a proper birational invariant of X. But there exist two compactifications of $(\mathbb{C}^*)^2$ such that the logarithmic Kodaira dimensions differ.

§3. Quasi-Projective manifolds with $\kappa(X)=\dim X$.

Let $X=\overline{X}-D$ be the same as in [2]. Let

$$V_m=\sup_{\omega\in S}\{\mu_m\omega\wedge\overline{\omega}\},\qquad (\mu_m=[(\sqrt{-1})^{n^2}/(2\pi)^n]^m)$$

where the supremum is taken for $S=\{\omega\in F_m(X)\mid \|\omega\|=1\}$. We define

$v_m=V_m^{1/m}$.

Lemma 1. $\int_X v_m<\infty$.

So we can define a hermitian inner product on $F_m(X)$ by the following formula

$$(\omega_1,\omega_2)=\int_X\frac{\mu_m\omega_1\wedge\overline{\omega}_2}{v_m^{m-1}}.$$

<u>Proposition</u> 1. Let g be a bimeromorphic automorphism of X. Then $(g^*\omega_1, g^*\omega_2)=(\omega_1,\omega_2)$

Thus we get a unitary representation

$$\rho_m : \mathrm{Bim}(X) \longrightarrow U(F_m(X)).$$

<u>Theorem</u> 2. Let X be a complex manifold with a smooth compactification $\overline{X}$. Assume that $\overline{X}$ is a Moišezon space. Then the image $\rho_m(\mathrm{Bim}_{alg}(X))$ is a finite group, where $\mathrm{Bim}_{alg}(X)$ is the intersection of Bim(X) and $\mathrm{Bim}(\overline{X})$.

When X is compact, this result was obtained by Nakamura and Ueno (see [6]).

<u>Theorem</u> 3. Let X be a quasi-projective manifold of dimension n. Assume that $\bar{\kappa}(X)=n$. Then X satisfies

(i) Any non-degenerate holomorphic map $f:\Delta^{*}\times\Delta^{n-1} \longrightarrow X$ can be extended to a meromorphic map from Δ^n to any compactification of X.

(ii) Every biholomorphic automorphism of X extends as a bimeromorphic automorphism of any compactification of X.

(iii) X is measure hyperbolic,

(iv) Let Aut(X) be the group of automorphisms of X. Then Aut(X) is a finite group.

These properties show that in this case X behaves like a projective algebraic manifold of general type.

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a) Let Ω be any open set in $\mathbb{R}^n$ and

$$A: \quad \mathcal{E}^p(\Omega) \longrightarrow \mathcal{E}^q(\Omega) \qquad (\mathcal{E}(\Omega) = C^\infty(\Omega))$$

a differential operator given by a $q \times p$ matrix of differential polynomials with constant coefficients. We denote by $\mathcal{H}(\Omega)_A$ the space $\{u \in \mathcal{E}^p(\Omega) \mid Au = 0\}$. We will make the assumption for A to be elliptic in the sense that $\forall\, u \in \mathcal{D}'(\Omega)^p$ $Au = 0$ implies that u is real analytic. If $\mathcal{O}_A$ is the sheaf of germs of solution of $Au=0$ one recognize that $\mathcal{O}_A$ is a Hausdorff space.

This enable us to define the "envelope of regularity" of a given domain Ω with respect to A as it is done for the classical envelope of holomorphy. A domain $\Omega \subset \mathbb{R}^n$ is called a domain of regularity if it coincide with its envelope. The theorem of Cartan Thullen extends to this situation as follow:

A domain $\Omega \subset \mathbb{R}^n$ is a domain of regularity iff for every sequence $\{x_\nu\} \subset \Omega$ with $\mathrm{dist}(x_\nu, \partial\Omega) \to 0$ $\exists\, u \in \mathcal{H}(\Omega)_A$ such that $\sup |u(x_\nu)| = \infty$.

b) Consider $\mathbb{R}^n \subset \mathbb{C}^n$ where $z = x+iy$ are coordinates so that $\mathbb{R}^n = \{y = 0\}$. For $\forall\, \tilde{\Omega}$ open in $\mathbb{C}^n$ we consider the operator

$$\tilde{A}: \quad \mathcal{E}^p(\tilde{\Omega}) \xrightarrow[\bar{\partial}]{A(\frac{\partial}{\partial z})} \begin{matrix}\mathcal{E}^q(\tilde{\Omega}) \\ \oplus \\ (\mathcal{E}^{01}(\tilde{\Omega}))^p\end{matrix} \qquad \text{"} \tilde{A} = A \oplus \bar{\partial} \text{"}$$

and call it the $\bar\partial$ suspension of A. Any $\bar\partial$-suspension is always elliptic.

Given $\Omega \subset \mathbb{R}^n$ $\exists$ a neighborhood U of Ω in $\mathbb{C}^n$ so that

$$\mathcal{H}(U)_{A\oplus\bar\partial} \xrightarrow{\sim} \mathcal{H}(\Omega)_A$$

this shows that the envelope of regularity of A is known if it is known the envelope of regularity of the $\bar\partial$-suspension.

Any domain of regularity of a $\bar\partial$ suspended operator is a particular domain of holomorphy.

Assume in particular that $p=1$ so that $A\oplus\bar\partial$ looks like

$$(*)\quad \begin{cases} \varphi_1\left(\frac{\partial}{\partial z}\right) u = 0 \\ \cdots \\ \varphi_\ell\left(\frac{\partial}{\partial z}\right) u = 0 \\ \bar\partial u = 0 \end{cases}$$

Let $\mathfrak{b} = \mathbb{C}[\xi_1, \dots, \xi_n](\varphi_1(\xi), \dots, \varphi_\ell(\xi))$ $\quad(\xi = (\xi_1, \dots, \xi_n))$ be the ideal of polynomials associated to $(*)$ and let $\mathfrak{M}$ = homogeneous ideal of principal parts of elements of $\mathfrak{b}$. We set $V(\mathfrak{M}) = \{z \in \mathbb{C}^n \mid g(z) = 0 \ \forall g \in \mathfrak{M}\}$. For every $a \in V(\mathfrak{M}) - \{0\}$ we set $\pi_a : \mathbb{C}^n \longrightarrow \mathbb{C}$ by $\pi_a(z_1, \dots z_n) = \sum_1^n a_i z_i$

We have

Let Ω be open and convex in $\mathbb{C}^n$ then the envelope of regularity of Ω with respect to $(*)$ is the set

$$\hat\Omega = \text{Interior of} \left\{ \bigcap_{a \in V(\mathfrak{M}) - \{0\}} \pi_a^{-1} \pi_a(\Omega) \right\}$$

In particular setting $\|z\| = (\sum |z_j|^2)^{1/2}$, the usual euclidean norm we have if $B = \{\|z\| < 1\}$

$$\tilde B = \{ z \in \mathbb{C}^n \mid |\sum a_i z_i| < \|a\| \quad \forall a \in V(\mathfrak{M}) \}$$

If $z_0 \in \partial B \cap \partial \tilde B$ then $\bar z_0 \in V(\mathfrak{M})$. As a consequence we deduce the following theorem

Let Ω be a domain of regularity for (*) and let $d(z,\partial\Omega)=d(z)$ be the euclidean distance from $\partial\Omega$. Then $d(z)$ is almost everywhere differentiable and satisfy the system of equations

$$g\left(\frac{\partial d}{\partial z_1}, \dots \frac{\partial d}{\partial z_n}\right) \equiv 0 \qquad \forall\, g \in \sqrt{\mathfrak{A}}.$$

As a corollary we get:

Let Ω be as above and let $\delta(z) = -\log d(z)$. Consider $\forall\, g \in \sqrt{\mathfrak{A}}$ $\forall\, \mu \in \mathbb{C}^n$ $\forall\, z_0 \in \Omega$ the diff. operator

$$L = \sum_1^n \frac{\partial g}{\partial z_\alpha}\left(\frac{\partial \delta(z_0)}{\partial z}\right)\frac{\partial}{\partial z_\alpha} + \sum \mu_\beta \frac{\partial}{\partial \bar z_\beta}$$

At the points $z_0 \in \Omega$ where $\delta(z) \in C^2$ then we have

$$(L\bar L\, \delta)(z_0) \geq 0 .$$

This is the analog of the "Levi-form".

This describes in part the nature of a domain of regularity for a $\bar\partial$-suggested operator.

One can show that no bounded domain of regularity for (*) can exist with C^2 boundary unless (*) reduces to the set of the Cauchy-Riemann equations $\bar\partial u = 0$ only.