MATHEMATISCHE ARBEITSTAGUNG 1981

UNIVERSITÄT BONN

Sonderforschungsbereich 40
Theoretische Mathematik
Beringstraße 4
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INHALT

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Mathematisches Institut
der Universität Bonn

Programm der Mathematischen Arbeitstagung 1981 (I)

Freitag, den 12.6.:

17.00 - 18.00 Uhr: M.F. Atiyah: Convexity and commuting Hamiltonians

Samstag, den 13.6.:

10.00 - 11.00 Uhr: B. Mazur: Abelian extensions of Q

12.00 - 13.00 Uhr: D. De Turck: "Manifold" of Ricci curvatures

17.00 - 18.00 Uhr: B. Malgrange: Vanishing cohomology and Bernstein polynomials

Sonntag, den 14.6.:

10.00 - 11.00 Uhr: D. Mostow: Complex reflection groups

12.00 - 13.00 Uhr: W. Fulton: Complex projective geometry (varieties of small codimension)

16.45 - 17.00 Uhr: Festlegung der nächsten Vorträge

17.00 - 18.00 Uhr: R. MacPherson: Intersection homology and nilpotent orbits

Die Vorträge finden alle im "Großen Hörsaal" (Wegelerstr. 10) statt. Erfrischungspausen mit Tee: Samstag und Sonntag 11.15-12.00 Uhr vor dem Großen Hörsaal, nachmittags ab 15.30 Uhr im Diskussionsraum Beringstraße 1. Die Post liegt während der Teepausen aus. Tischtennis im Keller des Hauses Beringstraße 4. Den Tagungsbeitrag bitte an Frau Gerber (SFB-Büro Beringstraße 4) bezahlen. Alle Teilnehmer mögen sich bitte in die Teilnehmerlisten eintragen. Teilnehmerlisten und andere Informationen liegen vor dem Diskussionsraum Beringstraße 1 aus.

Alle Tagungsteilnehmer mit ihren Damen oder Herren sind herzlich zum Empfang des Rektors eingeladen. Zeit: Montag, 15.6., 20.30 Uhr. Ort: Festsaal der Universität, Hauptgebäude; Eingang von der Straße "Am Hof" durch das Tor gegenüber Buchhandlung Röhrscheid.

Mathematisches Institut
der Universität Bonn

Programm der Mathematischen Arbeitstagung 1981 (II)

Montag, den 15.6.:

10.00 - 11.00 Uhr: J. Tate: Stark's conjecture about L-series at $s=0$

12.00 - 13.00 Uhr: W. Meyer: Gromov's work on Betti numbers

17.00 - 18.00 Uhr: K. Diederich: Complete Kähler domains

Dienstag, den 16.6.:

10.05 - 10.20 Uhr: Festlegung der restlichen Vorträge

10.20 - 11.20 Uhr: W.D. Neumann: Thurston's work

12.45 - can 21.00 Uhr: Ausflug nach Andernach. Abfahrt pünktlich um 12.45 Uhr mit Motorschiff "Carmen Silva" am Alten Zoll.

Die Vorträge finden alle im "Großen Hörsaal" (Wegelerstraße 10) statt.

Erfrischungspausen mit Tee: Montag vormittags 11.45-12.00 Uhr vor dem Großen Hörsaal, nachmittags ab 15.30 Uhr im Diskussionsraum Beringstraße 1.

Die Post liegt während der Teepausen aus. Tischtennis im Keller des Hauses Beringstraße 4. Den Tagungsbeitrag bitte an Frau Gerber (SFB-Büro, Beringstr. 4) bezahlen. Alle Teilnehmer mögen sich bitte in die Teilnehmerlisten eintragen. Teilnehmerlisten und andere Informationen liegen vor dem Diskussionsraum Beringstraße 1 aus.

Alle Tagungsteilnehmer mit ihren Damen oder Herren sind herzlich zum Empfang des Rektors eingeladen. Zeit: Montag, 15.6., 20.30 Uhr. Ort: Festsaal der Universität (Hauptgebäude), Eingang von der Straße "Am Hof" durch das Tor gegenüber Buchhandlung Röhrscheid.

Mathematisches Institut
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Programm der Mathematischen Arbeitstagung 1981 (III)

Mittwoch, den 17.6.:

10.00 - 11.00 Uhr: A. Derdzinski: Einstein metrics

12.00 - 13.00 Uhr: S. Zucker: L^2-cohomology of arithmetic groups

17.00 - 18.00 Uhr: A. Wiles: Explicit constructions of class fields

Donnerstag, den 18.6.:

10.00 - 11.00 Uhr: J. Duistermaat: Asymptotics of spherical functions

12.00 - 13.00 Uhr: M-F. Vigneras: Works of Waldspurger (automotphic forms of halfintegral weight)

17.00 - 18.00 Uhr: R. Schultz: Topological similarity of representations

Die Referenten werden gebeten, ihre Kurzfassungen bis Mittwoch, 16.30 Uhr bei Herrn Gekeler abzugeben, da wir den Tagungsbericht allen Teilnehmern noch vor ihrer Abreise aushändigen möchten.

Die Vorträge finden alle im "Großen Hörsaal" (Wegelerstraße 10) statt.

Erfrischungspausen mit Tee: Mittwoch und Donnerstag vormittags von 11.15 - 12.00 Uhr vor dem Großen Hörsaal, nachmittags ab 15.30 Uhr im Diskussionsraum Beringstraße 1.

Die Post liegt während der Teepausen aus.

Tischtennis im Keller des Hauses Beringstraße 4.

Titel: Convexity and Commuting Hamiltonians

Autor: M. F. ATIYAH

Adresse: Mathematical Institute, Oxford University

Introduction An old result of Schur asserts that if A is a Hermitian $n \times n$ matrix with eigenvalues $\lambda_1 \geq \lambda_2 \geq \quad \geq \lambda_n$ and diagonal entries $a_1 \geq a_2 \geq \quad \geq a_n$, then $\lambda_1 \geq a_1$, $\lambda_1 + \lambda_2 \geq a_1 + a_2$, $\sum \lambda_i = \sum a_i$

An equivalent formulation is that, regarding λ and a as vectors in R^n

$$a \in \text{Convex hull of } \Sigma_n \lambda$$

where the symmetric group Σ_n acts by permutation of coordinates. The converse was proved by Horn and the result generalized by Kostant [ ] to any compact Lie group G as follows.

Fix a maximal torus T of G and a bi-invariant metric on the Lie algebra $L(G)$ Let $\pi: L(G) \to L(T)$ be orthogonal projection. Recall finally that G-orbits in $L(G)$ correspond bijectively to W-orbits in $L(T)$, where W is the Weyl group. Then Kostant's theorem

π (G-orbit) = Convex hull of the W-orbit.

Symplectic generalization

Kostant's theorem has a natural generalization in the framework of symplectic geometry. Thus let M be a compact connected symplectic manifold and let a torus T^n act symplectically in M with n Hamiltonian functions $f_1, \ldots, f_n$ correspond to a basis of T. Then $f : M \to R^n$ is called the moment map and we have

THEOREM (A$_n$) $f(M)$ is the convex hull of points c_j, $j = 1, \ldots, N$

(B$_n$) $f^{-1}(c)$ is connected (or empty).

The points c_j in (A$_n$) are the images $f(Z_j)$ where Z_j are the connected components of the fixed point set of T^n.

For the proof one notes that (B$_n$) $\Rightarrow$ (A$_{n+1}$) and (B$_1$) is proven by a simple application of Morse theory

This Theorem is of special interest when M is a Kähler manifold and it can be refined in this case. The complexified torus T_c^n now acts holomorphically on M and if Y denotes an orbit, $\bar{Y}$ its closure then

$$f : \bar{Y} \longrightarrow R^n$$

has image the convex hull P of the subset of c_j for which $\bar{Y} \cap Z_j \neq \phi$. Moreover $P = \bar{Y}/T^n$ and f is a fibration over each face σ of P with fibre a torus of dimension $\dim \sigma$.

This is closely related to Mumford's theory of toroidal compactifications. Moreover Moment maps in general are related to Mumford's geometric invariant theory.

There is also an interesting infinite-dimensional example of this situation in which $M = \Omega(G)$ the loop space of a compact Lie group and $n = 1 + \text{rank } G$. The extra circle action comes from rotation of loops and the corresponding Hamiltonian is the usual Energy.

The image $f(M)$ is now the convex hull of an infinite sequence of points (the graph of the Killing form on the integral lattice in $L(T)$) and forms a polyhedral paraboloid.

Similar results have been obtained independently by Guillemin and Sternberg for applications to representation theory. There is also some overlap with the Leyden thesis of G. Heckman.

REFERENCES

[1] M. F. ATIYAH, Convexity and commuting Hamiltonians, Bull. Lond. Math. Soc. (to appear)

[2] B. Kostant, On convexity, the Weyl group and the Iwasawa decomposition, Ann. Sci. Ec. Norm. Sup. 6 (1973), 413-455.

Titel: Abelian extensions of Q

Autor: B. Mazur

Adresse: Harvard University, 1 Oxford St., Cambridge, Mass.

In my talk I described some recent results obtained in collaboration with Andrew Wiles. I said relatively little about the techniques which yield these results.

Briefly, we explicitly construct certain class fields of cyclotomic number fields as the splitting fields of certain finite subgroups of jacobians of modular curves. The construction of these finite subgroups is obtained after a close study of the so-called Eisenstein ideal and the Kubert-Lang cuspidal subgroup. The idea of obtaining detailed information about class fields by finding them as splitting fields of finite subgroups of jacobians of modular curves is due to Ribet (1).

If $\zeta(s)$ is the Riemann zeta-function, then for an odd integer $k \geq 1$,

$$\zeta(-k) = -B_{k+1}/k+1 \in Q$$

where B_n is the n-th Bernoulli number. Kummer perceived certain congruences between the $\zeta(-k)$ which have been expressed by Kubota and Leopoldt in the following form. Let p be an odd prime number. Then for each i mod p-1, where i is an odd integer $i \not\equiv -1$ mod p-1, the numbers $(1-p^k)\zeta(-k) \in Q \subseteq Q_p$ for $k \equiv i$ mod p-1 may be "p-adically interpolated" in the sense that there exists a power series with p-adic coefficients:

$$L^{(i)}(T) = a_0^{(i)} + a_1^{(i)}T + a_2^{(i)}T^2 + \dots \in Z_p[[T]]$$

such that

$$L^{(i)}((1+p^k)-1) = (1-p^k)\,\zeta(-k) \qquad (k \equiv i,\ k \geq 1)$$

Autor: B. Mazur

Ferrero and Washington have shown that, for each i, at least one of the coefficients of $L^{(i)}(T)$ is a p-adic unit. Let $\lambda^{(i)}$ be the smallest index for which this is true. Then $L^{(i)}(T)$ factors uniquely in $Z_p[[T]]$ into the product of a monic polynomial $g^{(i)}(T)$ of degree $\lambda^{(i)}$ and a unit power series $u^{(i)}(T) \in Z_p[[T]]$.

Iwasawa provided a conjectural arithmetic interpretation of the zeroes of $L^{(i)}(T)$ in the p-adic unit disk (equivalently, of the polynomial $g^{(i)}(T)$).

Namely, if $\mu_N \subset Q$ is the group of N-th roots of 1, let A_n denote the p-part of the ideal class group of the field $Q(\mu_{p^{n+1}})$. Let X denote the Pontrjagin dual of the discrete p-torsion group $\varinjlim A_n$. By the work of Iwasawa and Ferrero-Washington, one knows that X is a free Z_p module of finite rank. There is a natural action of $\mathrm{Gal}(Q(\mu_{p^\infty})/Q) \cong Z_p^*$ on X. Since Z_p^* admits a canonical splitting

$$0 \to \Gamma \longrightarrow Z_p^* \underset{\omega}{\rightleftarrows} \mathbb{F}_p^* \longrightarrow 0$$

where $Z_p^* \to \mathbb{F}_p^*$ is reduction mod p, ω is its unique lifting, and where Γ is topologically generated by $\gamma = 1+p$, the natural action of Z_p^* on X is retrievable if one retains the action of $\mathbb{F}_p^*$ induced by the lifting ω, and the (commuting) action of γ. Decomposing X as a direct sum of eigenspaces for the action of $\mathbb{F}_p^*$ we have: $X = \bigoplus_i X(\omega^i)$ where i runs through integers mod p-1, and where $X(\omega^i)$ is the subgroup of X consisting of elements such that the action of $\mathbb{F}_p^*$ is via the character ω^i.

Let $f^{(i)}(T)$ be the characteristic polynomial of the endomorphism $\gamma - I$ acting on $X(\omega^i)$. Then our main theorem asserts that

$$f^{(i)}(T) \doteq g^{(i)}(T)$$

for odd indices i mod $p-1$ such that $i \not\equiv -1$, establishing the conjecture of Iwasawa.

Prior to our work, Iwasawa had established that

$$\sum_{\substack{i \text{ odd} \\ i \not\equiv -1}} \deg f^{(i)}(T) = \sum_{\substack{i \text{ odd} \\ i \not\equiv -1}} \deg g^{(i)}(T)$$

by means of the classical analytic formula. To prove Iwasawa's conjecture,then, it suffices to show that

$$g^{(i)}(T) \quad \text{divides} \quad f^{(i)}(T)$$

for every odd index i ($i \not\equiv -1$). The problem is therefore one of constructing enough unramified p-abelian extensions of $Q(\mu_{p^m})$ and, at the same time, keeping track of the behavior of their Galois groups under the natural action of $Gal(Q(\mu_{p^m})/Q)$.

My talk concluded with a discussion of the application of our work to three classical problems: the "Herbrand-Ribet" problem, the conjecture of Gras, and the size of the higher K-groups $K_{2k}(Z)$ where $k \geq 1$ is an odd integer.

Reference:

(1) Ribet, K. A modular construction of unramified p-extensions of $Q(\mu_p)$. Inv. Math., 34, 151-162 (1976)

Titel: The "manifold" of Ricci curvatures

Autor: Dennis M. DeTurck

Adresse: Courant Institute of Math. Sci., New York, NY 10012

Ever since Gauss proved his Theorema Egregium, the question of precisely determining to what extent the curvature of a manifold determines its geometry has been considered. A fundamental question is to determine which functions (tensors) can be curvature functions of Riemannian (or other) metrics. For scalar curvature, this problem has been almost completely resolved (see [K] and its bibliography). Here, we examine the corresponding problem for Ricci curvature, i.e., we consider the question: "Given a symmetric tensor R_{ij}, can a metric g be found so that R is the Ricci curvature tensor of g?" This question is also of importance in general relativity. Unlike the situation for scalar curvature, however, even the local version of this problem is difficult to resolve. The problem boils down to the question of existence of solutions for the equation

$$\mathrm{Ric}(g) = R \tag{1}$$

where Ric is the (second-order quasilinear) system of partial differential operators that map metrics to their Ricci tensors (see [D1]). So, the system (1) seems determined in the sense that there are the same number of equations as unknowns. However, upon examining the linearization of the Ricci equation:

$$\mathrm{Ric}'(g)h \equiv \frac{d}{dt}\mathrm{Ric}(g+th)\Big|_{t=0} = \frac{1}{2}\Delta_L h - \delta^* \delta\, G(h) \tag{2}$$

one finds that the Ricci operator is degenerate in the sense that every direction is characteristic for the operator at every point. In (2), Δ_L is the Lichnerowicz Laplacian (an elliptic operator), δ is the divergence mapping of symmetric tensors to 1-forms, and δ^* is its L^2-adjoint (the symmetrized covariant derivative, see [BE]). The "gravitation operator" G is an invertible self-adjoint algebraic operator defined by $G(h) = h - \frac{1}{2}(\mathrm{tr}\ h)g$. Note that $G(\mathrm{Ric}(g))$ is the stress-energy tensor of general relativity.

The degeneracy of the Ricci operator is related to the fact that it is invariant under the action of the group of diffeomorphisms, i.e., that

$$\text{(3)} \qquad \mathrm{Ric}(\varphi^{*}g) = \varphi^{*}\mathrm{Ric}(g)$$

for any diffeomorphism φ. If φ_t is a one-parameter family of diffeomorphisms with φ_0= identity, then the derivative of (3) at t=O yields the Bianchi identity for Ricci curvature:

$$\text{(4)} \qquad -\delta G(\mathrm{Ric}(g)) = \mathrm{Ric}(g)^{a}_{b,a} - \tfrac{1}{2}\mathrm{Ric}(g)^{a}_{a,b} = 0$$

If we are to solve equation (1) for a specific R, then a necessary condition on R is that there must exist metrics with respect to which $\delta G(R)=0$. That this is a genuine restriction on the choices of R can be shown by examples (see [D1,D2]) of R's for which no such metrics exist, even locally. However, the following local theorems hold:

Existence theorem. (a) [D2] If R_{ij} is analytic and R^{-1} exists in a neighborhood of a point p, then there exists an analytic metric of any desired signature such that Ric(g)=R in a neighborhood of p.

(b) [D2] If R_{ij} is smooth and R^{-1} exists in a neighborhood of a point p, then there exists a smooth Riemannian metric g such that Ric(g)=R in a neighborhood of p.

(c) [D3] Under the hypotheses of (b), the Cauchy problem (initial value problem) for Ric(g)=R with Lorentz g is solvable locally in time, provided the initial data satisfy certain necessary compatibility conditions.

In the above, "smooth" means $C^{k,\alpha}$ or H^s.

Regularity theorem. [DK] If g is a Riemannian metric such that Ric(g)=R is $C^{k,\alpha}$ (analytic) and R^{-1} exists, then g is $C^{k,\alpha}$ (analytic).

There also are special local theorems for Einstein metrics (i.e., metrics for which Ric(g)=cg for some constant c.)

Einstein metric theorem. (a) (Existence [G]) Given the 2-jet of a metric g at p such that $Ric(g)=cg$ at p, we can find an analytic Einstein metric in a neighborhood of p with this 2-jet.

(b) (Regularity [DK]) In appropriately chosen coordinate systems, Einstein metrics are real analytic.

These last results indicate that the Bianchi identity is the only obstruction to local solvability of equation (1), since $\delta G(cg)$ is necessarily zero. A main ingredient in the proofs of the first two theorems is the observation that the system

$$Ric(g) - \delta^* R^{-1} \delta G(R) = R \tag{5}$$

is elliptic. Moreover, equation (1) implies equation (5), by the Bianchi identity (4). Since elliptic equations are easier (not impossible) to deal with analytically, we search for conditions under which equation (5) implies equation (1).

Perturbation lemma. Suppose $Ric(g_O)=R_O$ on a compact manifold M with $b_1(M)=0$, and R_O^{-1} exists. Then, for g sufficiently close to g_O, and R to R_O, equation (5) implies equation (1).

To prove this, we take δG of both sides of (5), noting that the 1-form $u=R^{-1}\delta G(R)$ satisfies a perturbation of the Hodge-deRham Laplace equation. This equation will not have 0 as an eigenvalue if R and g are close enough to R_O and g_O. Thus, u must be zero.

A consequence of the perturbation lemma is that on manifolds with $b_1(M)=0$, we can use elliptic PDE theory to analyze the local structure of the image of Ric (i.e., the Banach manifold of Ricci curvatures). In particular, we have a smooth manifold structure if the linearization of (5) (which is essentially $\frac{1}{2}\Delta_L$) is nice. As an example, we can use techniques from bifurcation theory to prove the following:

Proposition. The manifold of Ricci curvatures on S^n in a neighborhood of the Ricci curvature of the canonical metric is a codimension-one hypersurface.

Thus, not every Ricci candidate near the standard Ricci curvature on S^n is the Ricci curvature of a metric near the standard one. The normal direction is given (at the canonical curvature) by the canonical metric.

To conclude, we remark that similar techniques can be used to analyze the equations

$$\mathrm{Ric}(g) = T + fg$$

where f is a scalar function either of $x \in M$ or else of x and tr T, as in the Einstein equations of general relativity.

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Titel: Vanishing cohomology and Bernstein-Sato polynomial.

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Let X be an open set $\subset \mathbb{C}^{n+1}$ (coordinates $x_1, \dots, x_{n+1}$) and f a holomorphic function $X \to \mathbb{C}$ (coordinate t); suppose that 0 is the only one critical value of f; denote by $\mathcal{O}_X$ (resp. $\mathcal{D}_X$) the sheaf of holomorphic functions on X (resp. the sheaf of linear differential operator on X with coefficients in $\mathcal{O}_X$) If M is any $\mathcal{D}_X$-Module, i.e. a $\mathcal{O}_X$-Module provided with an integrable connection, $DR(M)$ denotes the complex

$$0 \to M \xrightarrow{d} M \underset{\mathcal{O}_X}{\otimes} \Omega^1_X \xrightarrow{d} \cdots \xrightarrow{d} M \underset{\mathcal{O}_X}{\otimes} \Omega^n_X \to 0$$

(where d is defined by the connection given on M).

It is known that the functor $M \mapsto DR(M)$ is an equivalence of categories between

a) The holonomic $\mathcal{D}_X$-Modules ~~and~~ with regular singularities.

b) The subcategory of $D_{b,\mathrm{cons}}(X)$ of complexes satisfying the "middle perversity condition" of Goreski-Mac-Pherson; here $D_{b,\mathrm{cons}}(X)$ means the derived category of the category of bounded complexes of ~~[illegible]~~ $\mathbb{C}_X$-Modules with analytically constructible cohomology.

(Kashiwara - Mebkhout - Deligne)

Let, in particular $\psi(f)$ the "Complex of vanishing cohomology" of X along $f^{-1}(0)$, defined by Deligne in S.G.A. 7-XIV, and let T be the action of the monodromy on $\psi(f)$. In this work, we construct a $\mathcal{D}_X$-Module N with regular singularities, concentrated on $f^{-1}(0)$, and provided with an action of T such that one has an isomorphism (in the derived category)

$$(\psi(f), T) \xrightarrow{\sim} (DR(N[1]), T).$$

The construction of N is related with the study of the Bernstein-Sato polynomial. As a corollary, this results interprets the B-S polynomial (slightly modified) in terms of the minimal polynomial of the action of T on $\psi(f)$. In particular, this interprets the roots of the B-S polynomial as logarithms of the eigenvalues of the action of T on $H^p(X_{t,x_0})$, $0 \leq p \leq n$, $x_0 \in f^{-1}(0)$, X_{t,x_0} the Milnor fiber of the "Milnor tube" centred at x_0 (the result in the opposite way was already known by a previous work of the author).

The details will be published in the Comptes-Rendus of the conference on D-Modules, intersection homology, etc... to be held next summer in Luminy. The result has been also obtained independantly by I. Bernstein (unpublished).

Titel: Complex Reflection Groups and Non-arithmetic Monodromy

Autor: G. D. Mostow

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In a paper which seems to have escaped the notice of researchers in lattice subgroups [4], Picard generalized to two variables Schwarz's realization of the disc as a branched covering of complex projective 1-space $\mathbb{P}^1$. Picard defined generalized hypergeometric functions

$$W_{g,h}(x,y) = \int_g^h u^{\lambda_1-1}(u-1)^{\lambda_2-1}(u-x)^{\lambda_3-1}(u-y)^{\lambda_4-1}\,du$$

$$g,h \in \{0,1,\infty,x,y\}$$

Quotients of these yield a multivalued map f of $M'' = \mathbb{P}^1 \times \mathbb{P}^1 - 7$ lines to the ball in $\mathbb{C}^2$, the 7 lines being

$$x = \begin{cases} 0 \\ 1 \\ \infty \end{cases}, \quad y = \begin{cases} 0 \\ 1 \\ \infty \end{cases}, \quad x = y.$$

The fundamental group $\pi_1(M'')$ operates on $\mathbb{B}^2$ via monodromy. A necessary condition that the map f have a single-valued holomorphic inverse is

$$\text{(INT)} \qquad \lambda_i + \lambda_j - 1 = 1/\text{integer}$$

$$2 - \sum_{j \neq i} \lambda_i = 1/\text{integer} \quad 1 \le i < j \le 4$$

Picard asserts that these conditions are sufficient that the monodromy group operate discontinuously on the ball, but his line of proof has a serious drawback. A correct proof can be given along a somewhat different line (cf [3]).

Picards map f can be described for n variables as follows (cf. Deligne - Mostow [3]):
Let X be a Riemann surface ~~mapping~~ onto $\mathbb{P}^1$ with ramification at $n+3$ points $x_0, x_1, \cdots x_{n+2}$, and with $\mathrm{Aut}(X|\mathbb{P}^1) = \mathbb{Z}/N$. Since we shall be interested in varying the complex structure of X, which be preserved by the diagonal action of $PGL(2)$ on $(\mathbb{P}^1)^{n+2}$, we can restrict ourselves to the set M'' of $n+3$ points with $x_0 = \infty$, $x_1 = 0$, $x_2 = 1$.
For $m \in M''$ define the curves X_m by the equation

$$v = u^{\frac{a_1}{N}-1}(u-1)^{\frac{a_2}{N}-1}(u-x_3)^{\frac{a_3}{N}-1}\cdots(u-x_{n+2})^{\frac{a_{n+2}}{N}-1}$$

where $0 < a_i < N$, $\gcd(a_1, a_2 \cdots N) = 1$.
Then $\mathrm{Aut}(X/\mathbb{P}^1) = \mathrm{Gal}(\mathbb{C}(v,u) \mid \mathbb{C}(u)) = \mathbb{Z}_N$, call this group G. Set $\lambda_i = \frac{a_i}{N}$ $(i = 1, 2, \cdots n+2)$.
By hypothesis $0 < \lambda_i < 1$. Assume in addition

$$\sum_{i=1}^{n+2} \lambda_i < n+1 \quad \text{and set} \quad \lambda_0 = n+1 - \sum_{i=1}^{n+2} \lambda_i$$

Then $v\,du$ is a holomorphic 1-form on X_m. The Galois group G acts on each curve X_m and hence on

$$H^1(X_m, \mathbb{C}) = \bigoplus_{\chi} H^1_\chi \qquad (\chi = \text{character of } G).$$

Let χ_0 denote the character by which v transforms. Let Ω^1 denote the space of holomorphic 1-forms on X. Then $\Omega^1 = \oplus\, \Omega^1_\chi$. It is clear that $\dim \Omega^1_{\chi_0} = 1$. One shows also

$$\dim H^1_\chi = n+1, \qquad \text{if order } \chi = N.$$

There is a well defined isomorphism between $H_1(X_m, \mathbb{Z})$ and $H_1(X_{m'}, \mathbb{Z})$ for nearby points m, m' in M''. This defines a flat connection which allow us to regard $\{H_1(X_m, \mathbb{Z}), m \in M''\}$ as a locally constant sheaf and $\{H^1(X_m, \mathbb{C}), m \in M''\}$ as well. The Picard map is nothing more than

$$m \longmapsto [\text{line } \Omega^1_{\chi_0} \to \text{projective space of } H^1_{\chi_0}(X_m)]$$

Cup product on $H^1(X_m, \mathbb{C}) = \Omega^1 + \overline{\Omega}^1$ gives a hermitian form $\frac{1}{i}\varphi \wedge \overline{\varphi}$ which is definite on Ω^1 and $\overline{\Omega}^1$ of opposite signs. The restriction of this form to $H^1_{\chi_0}$ has signature $(n,1)$ since $\dim \Omega^1_{\chi_0} = 1$.

Fix $m_0 \in M''$. The flat connection gives a well defined homomorphism $\mu : \pi_1(M'', m_0) \to \operatorname{Aut} H^1(X_{m_0}, \mathbb{C})$ let $\mu_0 = p \circ \mu$ where p is the projection of $H^1(X_{m_0}, \mathbb{C})$ onto its summand $H^1_{\chi_0}(X_{m_0}, \mathbb{C})$. By abuse of notation, we denote also by μ and μ_0 the homomorphisms into the projective linear groups. The images under μ and μ_0 are actually in projective unitary groups since monodromy preserves cup product; denote these images by $\Gamma(\lambda)$ and $\Gamma(\lambda)_0$ respectively.

Theorem: Assume that for all (i,j) such that $\lambda_i + \lambda_j \geq 1$,

$$\lambda_i + \lambda_j - 1 = \frac{1}{n_{ij}} \qquad (0 \leq i < j \leq n+2)$$

with n_{ij} an integer. Then $\Gamma(\lambda)_0$ is a (discrete) lattice subgroup of $PU(n,1)$.

Theorem: For all λ such that $\Gamma(\lambda)_0$ is a non-arithmetic lattice, the group $\Gamma(\lambda)$ is not a lattice in its Zariski closure $\overline{\Gamma(\lambda)}$, i.e. $\overline{\Gamma(\lambda)}_{\mathbb{R}} / \Gamma(\lambda)$ does not have finite measure.

The number of values of λ (up to permutations), for which $\Gamma(\lambda)_0$ is discrete (resp. a non arithmetic lattice) is given by the following table

n	# discrete	# non-arithmetic
2	27	7
3	7	1
4	1	0
5	1	0
>5	0	0

Remark: For all (i,j) with $\lambda_i + \lambda_j > 1$, the monodromy γ_{ij} induced by a simple circuit around the line $x_i = x_j$ of $(\mathbb{P}^1)^{n+2}$ is a complex reflection in the ball B^n of order n_{ij} if $n_{ij} < \infty$; if $n_{ij} = \infty$ then γ_{ij} is unipotent and the group $\Gamma(\lambda)_0$ is not cocompact. The groups $\Gamma(\lambda)_0$ in $PU(2,1)$ are different from the groups generated by complex reflections defined in [2].

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Titel: Complex projective geometry (varieties of small codimension)

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Some elementary geometric constructions, related to a basic connectedness theorem, have led to several striking discoveries regarding algebraic varieties and their mappings to complex projective spaces. A detailed survey, written jointly with R. Lazarsfeld, will appear soon in a SLN proceedings of a Midwest Algebraic Geometry Conference.

Let X^n be a compact, irreducible, n-dimensional variety, $f: X^n \to \mathbb{P}^m \times \mathbb{P}^m$ a generically finite-to-one mapping. If $n \geq m$ it is an ancient but important fact that $f(X)$ must meet the diagonal Δ. The connectedness theorem (Fulton–Hansen, Annals, 1979) states that if $n > m$, $f^{-1}(\Delta)$ must be connected. (A proof had been given by Barth in 1969). Deligne's refinement (Sém Bour 543, 1979) states that, if $n > m$ and X is locally analytically irreducible, then $\pi_1(f^{-1}\Delta)$ surjects onto $\pi_1(X)$; if X were not compact, the same holds provided Δ is replaced by a neighborhood. Higher homotopy analogues, based on cases proved by Lefschetz, Zariski, Lê, Hamm, and Cheniot, were conjectured by Deligne, and have since been proved and generalized to singular varieties by Goresky and MacPherson.

Let $f: X^n \to \mathbb{P}^m$, $m < 2n$. The connectedness theorem applies to the induced mapping $X \times X \to \mathbb{P}^m \times \mathbb{P}^m$. If f is unramified, Δ_X is open as well as closed in $(f\times f)^{-1}(\Delta)$; therefore $(f\times f)^{-1}(\Delta) = \Delta_X$, i.e., f must be one-to-one. (Variations of this fundamental insight of Hansen are used in the other applications of the connectedness theorem.) One Consequence is that any irreducible subvariety of $\mathbb{P}^m$ of dimension $> \frac{1}{2} m$ must be simply connected.

If $f: X^n \to \mathbb{P}^n$ is a finite branched covering of degree d, Gaffney and Lazarsfeld (Inv. Math, 1980) showed that $\min(d, n+1)$ sheets must come together at some point. If X is normal and $d \leq n$, then X must be simply connected.

Zariski's conjecture on the fundamental group of the complement of a plane node curve was proved using similar methods (Fulton, Annals, 1980 for the algebraic fundamental group; Deligne, Sém Bour 543 for the topological π_1.

Let $X^n \subset \mathbb{P}^N$ be non-singular. F.L. Zak (letter, Dec. 1979; unpublished manuscript) showed that for any linear subspace L^m not containing X,

$$\dim \{ x \in X \mid T_x X \subset L \} \leq m-n .$$

In particular, if X is not a linear subspace, the Gauss map is everywhere finite. For $N < 2n$ (resp. $N < 2n-1$) every hyperplane section of X is reduced (resp. normal). Zak applies this to prove a conjecture of Hartshorne: if $n > \frac{2}{3}(N-1)$, X is not a non-trivial projection from a larger projective space. He also shows that the three known examples with $n = \frac{2}{3}(N-1)$ are the only examples. Recently, Zak announced a proof of at least the codimension 2 and 3 part of the celebrated conjecture that varieties of small codimension are complete intersections.

Titel: Intersection homology and nilpotent orbits

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Part I: Intersection Homology (joint work with Mark Goresky)

Let X be an n-dimensional complex algebraic variety. Choose a Whitney stratification and let S_k be the union of the k dimensional strata. Let L be a local system of coefficients on S_n. We define a sheaf of cochain complexes on X called the sheaf of intersection chains $\underline{IC}^{\cdot}(L)$ as follows:

$$\underline{IC}^{i}(U,L) = \left\{ \begin{array}{l} 2n-i \text{ chains with closed supports} \\ \text{on } U \text{ with values in } L \text{ that meet} \\ \text{each } S_k, \text{ for } k<n, \text{ in a set of dimension} \\ \text{at most } n-i+k-1 \text{ and whose boundary} \\ \text{meets each } S_k, \text{ for } k<n, \text{ in a set of} \\ \text{dimension at most } n-i+k-2 \end{array} \right.$$

for U open in X. The intersection homology of X, $IH^{i}(X)$, is the i^{th} cohomology group of the cochain complex $\underline{IC}^{\cdot}(X, \mathbb{Q})$ where $\mathbb{Q}$ is the trivial local system with fiber the rational numbers.

The group $IH^i(X)$ is independent of the stratification used in its construction. It is a topological invariant of X but not a homotopy invariant. The evident map

$$IH^i(X) \longrightarrow H_{2n-i}(X)$$

is an isomorphism if X is nonsingular.

For some questions, intersection homology is more useful than ordinary homology for the study of singular varieties in that some theorems on the ordinary homology of nonsingular varieties remain true for the intersection homology of singular varieties. Some examples:

$\boxed{1}$ Poincaré duality. There is a nonsingular intersection pairing for X compact

$$IH^i(X) \times IH^{2n-i}(X) \longrightarrow \mathbb{Q}$$

$\boxed{2}$ Lefschetz hyperplane theorem. If X is closed in a complex projective N space and H is a generic hyperplane, then there is a map

$$IH^i(X) \longrightarrow IH^i(X \cap H)$$

which is an isomorphism for $i > n+1$ and injective for $i = n+1$

[3] Hard Lefschetz theorem (due to O. Gabber) For X projective, there is a map

$$IH^{n-i}(X) \xrightarrow{(\frown [H])^i} IH^{n+i}(X)$$

which is an isomorphism

[4] Morse Theory Among proper restrictions of ambient real valued C^∞ functions, there is an open dense structurally stable set for which for all $v \in \mathbb{R}$ and small ϵ

either $IH^i\left(f^{-1}(-\infty, v+\epsilon), f^{-1}(-\infty, v-\epsilon)\right) = 0$ for all i

(in which case we say v is not a critical value)

or $IH^i\left(f^{-1}(-\infty, v+\epsilon), f^{-1}(-\infty, v-\epsilon)\right) = 0$ for all but one i

(which we call the Morse index).

Other constructions of $\underline{IC}^{\cdot}$ have been given:

(1) Deligne has given a construction which is valid for algebraic varieties in all characteristics

(2) Cheeger and Zucker have given constructions as L^2 differential forms on S_n for particular varieties and metrics

(3) Brylinski-Kashiwara and Beilinson-Bernstein have given a construction in terms of holonomic modules over the ring of holomorphic differential operators with regular singularities.

A remarkable property of intersection homology sheaves is the following:

Theorem (Deligne, Beilinson, Bernstein)

Let $f: X \to Y$ be any projective map. Then there exist inclusions of closed subvarieties $j^\alpha: V_\alpha \hookrightarrow Y$, local systems L_α on the largest strata of V_α, and integers n_α so that

$$f_* \underline{\underline{IC}}^{\cdot}(\mathbb{Q}) \underset{q.i.}{=} \bigoplus_\alpha j^\alpha_* \underline{\underline{IC}}^{\cdot}(L^\alpha)[n_\alpha]$$

where $\underset{q.i.}{=}$ means a quasi-isomorphism.

Part II Nilpotent orbits (joint work with Walter Borho)

Let G be a semisimple complex Lie group, $\mathfrak{g}$ its Lie algebra, $\mathcal{N} \subset \mathfrak{g}$ the variety of nilpotent elements and $\pi: \tilde{\mathcal{N}} \to \mathcal{N}$ the Springer resolution. Then if we apply the above theorem to π, we obtain

$$\pi_* \underline{\underline{IC}}^{\cdot}(\mathbb{Q}) \underset{q.i.}{=} \bigoplus_{(x,\varphi)} j^x_* \underline{\underline{IC}}^{\cdot}(L_\varphi)[-2d_x] \otimes V_{(x,\varphi)}$$

where j^* includes $\overline{O}_x$ in N where O_x is an orbit of G by the adjoint action, L_φ are certain irreducible local systems on O_x, [redacted] $2d_x$ is the complex codimension of O_x in N, and $V_{(x,\varphi)}$ are certain rational vector spaces. Springer has defined an action of the Weyl group W on $\pi_* \underline{\underline{IC}}^{\cdot}(\mathbb{Q})$. This action is identified by the following theorem, which was essentially conjectured by Lusztig:

<u>Theorem</u> W acts on $\pi_* \underline{\underline{IC}}^{\cdot}(\mathbb{Q})$ by acting on the vector spaces $V_{(x,\varphi)}$. The representation on $V_{(x,\varphi)}$ is irreducible, and every irreducible representation of W occurs, exactly once.

This correspondence between irreducible representations of W and pairs $(x\ \varphi)$ is the Springer correspondence. A number of new results about it follow from this point of view, see [3].

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Titel: Stark's conjectures on $L(s,\chi)$ at $s=0$

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Stark's papers, in Advances in Math, on $L(s,\chi)$ at $s=1$, I (1971), II (1975), III (1976), and IV (1980), contain some remarkable conjectures about the leading term of the Taylor expansion of Artin's L-functions $L(s,\chi)$ at $s=0$. Let

K/k be a finite Galois extension of number fields;
$G = \mathrm{Gal}(K/k)$ the Galois group;
S a finite set of places of k containing the ∞ places
G_v a decomposition subgroup of G for a place $v \in S$
χ a character of G, V_χ a $\mathbb{C}[G]$-module realizing χ.
$L_S(s,\chi)$ the Artin L-series with Euler factors at the S removed
$r(\chi) = -\dim V_\chi^{G} + \sum_{v\in S} \dim V_\chi^{G_v}$ = order of zero of $L_S(s,\chi)$ at $s=0$;

Stark defines a "regulator" $R(\chi)$, which depends on a choice of "Artin S-units" in K, as the determinant of a certain $r(\chi)\times r(\chi)$-matrix whose entries are linear forms in logs of absolute values of S-units of K with algebraic coefficients coming from the representation V_χ. He puts

$$\theta(\chi) = \lim_{s\to 0} \frac{L_S(s,\chi)}{s^{r(\chi)}\, R(\chi)}$$

and makes

<u>Main Conjecture</u> (cf. Stark II) $\theta(\chi^\alpha) = \theta(\chi)^\alpha$ <u>for every automorphism</u> α <u>of</u> $\mathbb{C}$. <u>In particular</u>, $\theta(\chi)$ <u>is algebraic and contained in the field generated by the character values</u> $\chi(\sigma)$, $\sigma\in G$.

This is true if χ has rational values — by much algebra and class field theory one reduces that case to the case of the trivial character for G and subgroups of G

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and uses then the classical formula for the residue of the zeta function of k, switched from $s=1$ to $s=0$ via the functional equation; Stark's regulator is the classical one in case of the trivial character.

There are two p-adic analogs of the main conjecture, one by Serre at $s=1$, and one by B. Gross at $s=0$. These two analogs are unrelated — there is no functional equation for p-adic L-functions.

For $r(\chi)=0$ the main conjecture reads $L_S(0,\chi^\alpha) = L_S(0,\chi)^\alpha$ and was proven by Siegel.

What I have called Stark's main conjecture is a generalization to arbitrary Artin L-functions of the classical class number formula, taken only mod $\mathbb{Q}^*$; another conjecture of his, more special but more precise, is a vast (conjectural) generalization of Kronecker's limit formula and a big step of unexpected sort in the direction of Hilbert's 12th problem — the generation of class fields by special values of analytic functions.

Suppose G is abelian, $\#S \geq 3$ ($\#S \geq 2$ is essential; ≥ 3 simplifies statements), S contains the archimedean places and places ramified in K, and suppose S contains one place v which splits completely in K. Let $\| \ \|$ denote an extension to K of the normalized v-adic absolute value, and let W denote the number of roots of unity in K. Let $\hat{G}$ denote the character group of G.

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<u>Conjecture</u> <u>St (K/k, S)</u> (cf. Stark IV): <u>There is</u> <u>an element</u> $\varepsilon \in K^*$ <u>such that</u>

(a) $L'_S(0,\chi) = -\frac{1}{W}\sum_{\sigma\in G}\chi(\sigma)\log\|\varepsilon^\sigma\|$, for all $\chi \in \hat{G}$

(b) $\|\varepsilon\|_{v'} = 1$ for each place v' of K not above v

(c) $K(\varepsilon^{1/W})$ is abelian over k.

That such an ε exists for some integer $W>0$ is a consequence of the Main conjecture; that it exists for W = number of roots of unity in K is the new thing. Note that (a) and (b) determine ε up to a root of unity.

For $k = \mathbb{Q}$ or imaginary quadratic and v the archimedean place, Stark proves this conjecture in IV, the ε being a cyclotomic, or an elliptic-modular unit.

If k has more than 1 complex place the conjecture is trivial because $L'_S(0,\chi)=0$ for all χ and we can take $\varepsilon = 1$. For other ground fields it is a total mystery. For real quadratic k Stark, and also Shintani who had similar ideas independently, have found some remarkable numerical confirmation of the conjecture, finding for several choices of k and of quartic or sextic or octic K/k, a unit $\varepsilon \in K$ such that (b) and (c) hold, and (a) holds to an accuracy of 10 or 15 decimals.

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Conjecture $St(K/k, S)$ is true if K/k is quadratic, or more generally, if G is generated by decomposition groups G_v of order 2, of places $v \in S$. Then the characters χ are rational and it is a nice exercise.

In (II) Stark treats only the case in which the splitting place v in S is archimedean. Consideration of the case $v = \mathfrak{P}$ as non-archimedean leads to the following:

Suppose still G is abelian. Suppose T is a set of places of k containing the archimedean ones, the ones ramified in K, and at least 2 places ($T \neq \emptyset$ is essential, but $\# T \geq 2$ simplifies statements). For $\sigma \in G$, let

$$\zeta_T(s, \sigma) = \sum_{\substack{(\mathfrak{a}, K/k) = \sigma \\ (\mathfrak{a}, T) = 1}} (N\mathfrak{a})^{-s}$$

be the partial zeta function for σ and prime to T, and put

$$\xi_T = \sum_{\sigma \in G} \zeta_T(0, \sigma)\sigma^{-1}, \qquad \in \mathbb{Q}[G].$$

Then the union of the Conjectures $St(K/k, T \cup \{\mathfrak{P}\})$ for all $\mathfrak{P} \notin T$ such that $\mathfrak{P}$ splits in K is equivalent to the following

Conjecture B-S: For each ideal $\mathfrak{A}$ of K there is an element $\varepsilon \in K^*$ such that a) $\mathfrak{A}^{W\xi_T} = (\varepsilon)$

b) $\|\varepsilon\|_{v'} = 1$ for each archimedean place v' of K.

c) $K(\varepsilon^{1/W})$ is abelian over k.

Titel: On Stark's conjectures

Autor: J. Tate

Adresse:

The idea (a), that $W\varepsilon_T$ annihilates the ideal class group of K, is due to Brumer; the extra features (b) and (c) come from Stark — hence the name B-S, i.e., Brumer-Stark.

For $k = \mathbb{Q}$, conjecture B-S is true. One can take ε to be a Jacobi sum, (a) is Stickelberger's factorization of it (b) is well known, and (c) is true because $\varepsilon^{1/W}$ is a Gauss sum.

If k is not totally real, then B-S is trivial (with $\varepsilon = 1$), but for k totally real $\neq \mathbb{Q}$ it is a mystery.

Mazur proposed considering the function-field analog of B-S. Deligne proved this by a skillful use via "1-motives" of Weil's idea that the L-function (in the variable $u = q^{-s}$) in that case is the characteristic polynomial of the Frobenius endomorphism acting on l-adic cohomology.

For Deligne's proof, and details of some other things mentioned here, see the forthcoming (Birkhäuser) notes of my course at Orsay this year.

All in all, the evidence for the conjectures: numerical, internal consistency, special cases classical but non-trivial, and function field analog, seems to me overwhelming.

Let F be a field and V a manifold,

$b_i(V) := \dim H_i(V, F)$.

The proof of the following theorem of M. Gromov is given:

<u>Theorem</u>

Given $n \in \mathbb{N}$. Then there exists a constant $c(n)$ such that any complete connected n-dim Riemannian manifold V with sectional curvature $K \geq 0$ satisfies

$$\sum_{i=0}^{n} b_i(V) \leq c(n).$$

If the sectional curvature of V satisfies $K \geq -\varkappa$ with $\varkappa > 0$ and if $\operatorname{diam} V = D < \infty$, then

$$\sum_{i=0}^{n} b_i(V) \leq c(n)^{(1+\varkappa D)}$$

<u>Remark</u>: For the n-dim torus T^n one has $\sum_{i=0}^{n} b_i(T^n) = 2^n$. It is a conjecture that one can choose $c(n) = 2^n$ in general, however the current estimate of Gromov yields

$$c(n) \leq \left[(n+1)\ J(n)\right]^{k_0}$$

where $k_0 \leq 100^n$, $J(n) = 2^M$ and $M = 8^n\, 10^{n^2+4n}$.

In a given dimension one can find manifolds V with $\sum_{i=0}^{n} b_i(V)$ arbitrary large: If V is the connected sum of k copies of $S^p \times S^{n-p}$, then $\sum_{i=0}^{n} b_i(V) = 2k+2$. For k sufficiently large, V cannot admit a metric with nonnegative sectional curvature.

For $K \geq 0$ Bochner has given the estimate $b_1(V; \mathbb{R}) \leq n$ with equality if and only if V is a flat torus, c.f. [1].

For the proof of the Theorem Gromov constructs an analog of a Morse theory for the distance function d_x from a point $x \in V$, $d_x(y) := dist(x,y)$

Since preprints are readily available, c.f. [2], we will not give any details in this abstract.

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Titel: On complete Kähler domains

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H.Grauert asked in 1956, [3] , for the first time the question how it is possible to characterize Stein manifolds by means of complete Kähler metrics. He proved:

Theorem 1. Let X be a Stein manifold and $A \subset X$ a closed complex-analytic variety. Then $X \setminus A$ carries a complete Kähler metric.

Remark. Since $X \setminus A$ is not Stein for codimension $A \gtrsim 2$, this shows that the existence of a complete Kähler metric on a domain in $\mathbb{C}^n$ does not imply that this domain is Stein.

But H.Grauert also proved:

Theorem 2. Let $D \subset \mathbb{C}^n$ be a domain with smooth, real-analytic boundary. If there exists a complete Kähler metric in D, then D is Stein.

After a brief report on further important results in this direction, especially on the theorems of R.E.Greene and H. Wu, the most recent progress in this field was discussed.

S.Y.Cheng and S.T.Yau announced the following very strong existence statement:

Theorem 3. On any bounded domain of holomorphy in $\mathbb{C}^n$ there is a complete Kähler-Einstein metric.

The sufficiency statement for this additional curvature condition was recently announced by Mok:

Theorem 4. Any bounded domain of holomorphy in $\mathbb{C}^n$ with a complete Kähler-Einstein metric is Stein.

What can be said without imposing further curvature conditions? After T.Ohsawa generalized in [5] Grauert's theorem 2 to the case of domains with smooth C^1-boundaries, the author showed together with P.Pflug

Theorem 5. Suppose D is a complete Kähler domain on a complex manifold X. Put $A := \mathring{\bar{D}} \setminus D$. Then one has:

a) If D is locally Stein at all $p \in A$, then D is everywhere locally Stein.

b) If A is a removable singularity set for $\mathcal{O}(D)$, then $\mathring{\bar{D}}$ is locally Stein.

Remark. Notice that it follows in the situation of a) resp. b) that D resp. $\mathring{\bar{D}}$ is Stein if X is known to be Stein.

An interesting question is which thin sets A can appear in the situation of thm.5. T.Ohsawa proved in [6]:

Theorem 6. Let D and A be as in theorem 5. If A is a C^1-submanifold of real codimension 2 then A is complex-analytic.

For the case of higher codimensions it is proved in [2]

Theorem 7. Let D and A be as in thm. 5. If A is a real-analytic variety of real codimension $\geqq 2$, then A is complex-analytic.

Surprisingly, the assumption of real-analyticity in codimension $\geqq 3$ in general cannot be weakened to C^∞, because one has, [2]:

Theorem 8. Let $d \geqq 3$ be any integer. Then there is a closed C^∞-submanifold A of a ball B with codimension d, which is not complex-analytic and for which there is a complete Kähler metric on $B \setminus A$.

The nature of these non-complex-analytic examples is not yet clear. But it can be proved:

Theorem 9. Let $A \subset B \subset \mathbb{C}^n$ be a C^∞-submanifold with $\mathbb{C} \otimes T_pA = \mathbb{C}^n$ for all $p \in A$. Then $B \setminus A$ does not carry a complete Kähler metric.

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Titel: THURSTON'S WORK ; GEOMETRY OF 3-MANIFOLDS.

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Since a 1 hour talk on "Thurston's work" must make choices, I will restrict myself to the status and implications of his "monster conjecture" below. I thus will not describe his work on surface homeomorphisms, beyond saying that it is well exposed in the final reference below.

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THURSTON'S CONJECTURE: Every closed irreducible 3-manifold M has a "geometric decomposition". That is, there is an embedded surface $T \subset M$ such that each component of $M-T$ has a "geometric structure" (a complete locally homogeneous riemannian metric of finite volume). Moreover the minimal such T is unique up to isotopy.

T will automatically be a union of tori and klein bottles T_i, each incompressible in M (that is, $\pi_1(T_i) \to \pi_1(M)$ is injective.

Thus if $\pi_1(M)$ is finite the conjecture is $M \cong S^3/\Gamma$, $\Gamma \subset SO(4)$ acting freely. Thus Thurston's Conjecture includes a) The Poincaré Conjecture
b) The 3-dim. spherical space form problem.

It also includes the Nielsen Conjecture, now a Theorem, (Kerkhoff 1979) that finite extensions of Fuchsian groups are Fuchsian, and much more.

Positive work on it has led to a proof of the Smith Conjecture on unknottedness of fixed point sets (Meeks, Yau, Gordon, Bass, Thurston, ...)

Thurston has pointed out that there are 8 simply connected homogeneous spaces $X = G/K$ which can occur as the universal cover of a finite volume M^3. Here G = Isometries (X) K = Isometries (X, point). The possible X, classified by the identity component of K, are:

$K_0 = SO(3)$	S^3	$\mathbb{E}^3$	$\mathbb{H}^3$
$K_0 = SO(2)$	$S^2 \times \mathbb{E}^1$	N	$\mathbb{H}^2 \times \mathbb{E}^1$, $\widetilde{PSL(2,\mathbb{R})}$
$K_0 = \{1\}$		S	

N and S are respectively a nilpotent & solvable Lie group.

If M^3 has a geometric structure then $M = \text{int}\,\bar{M}$, $\bar{M}$ compact, $\partial\bar{M}$ = union of tori. Moreover if X is the relevant geometry

1) $X = \mathbb{H}^3 \Rightarrow \bar{M}$ is <u>simple</u> (every incompressible T^2 in M is isotopic to a boundary component)

2) $X = S \iff M$ fibers over S^1 with fiber T^2 and the monodromy $A \in SL(2,\mathbb{Z})$ has $|\text{tr}\,A| > 2$

3) X other $\iff M$ is Seifert fibered, $M \neq D^2 \times S^1$, $T^2 \times I$, I-bundle over Klein bottle.

In case 3) the relevant geometry is given by

$e \backslash \chi$	>0	$=0$	<0
$=0$	$S^2 \times \mathbb{E}$	$\mathbb{E}^3$	$\mathbb{H}^2 \times \mathbb{E}$
$\neq 0$	S^3	N	$\widetilde{PSL}$

$\text{vol}(M) = 4\pi^2 \frac{\chi^2}{|e|}$ if this is defined

Here if M is Seifert fibered over F with Seifert invariant $(b, (\alpha_1, \beta_1), \dots, (\alpha_n, \beta_n))$ the invariants e and χ are

$$\chi(M \to F) = \chi(F) - \textstyle\sum_i'(\alpha_i - 1)/\alpha_i \qquad (\chi(F) = \text{euler char})$$

$$e(M \to F) = -b - \textstyle\sum \beta_i/\alpha_i \qquad (M \text{ closed})$$

$= $ arbitrary, depending on peripheral structure on M (M not closed).

There is a pretty correspondence in cases 2) & 3) between classification of the geom. structures and classification of certain complex surfaces.

The last few sentences are a compilation of results of Seifert, Threlfall, Raymond, Macbeath, Neumann, maybe others.

Thurston's Conjecture is <u>true for</u> M <u>sufficiently large</u> (i.e. containing an incompressible surface) by the <u>Jaco-Shalen/Johanssen Splitting Theorem</u> (such an M can be decomposed along incompressible tori and Klein bottles into Seifert pieces and simple pieces; this predates Thurston's Conjecture and helped motivate it) and

<u>THURSTON HYPERBOLIZATION THEOREM</u>. If $M = \mathrm{int}\,\bar{M}$, $\bar{M}$ compact, irreducible, simple, $\partial\bar{M}$ = tori. Then if $\bar{M}$ is sufficiently large (e.g. $\partial\bar{M} \neq \emptyset$) M has a complete finite volume hyperbolic structure.

The proof is still appearing. The idea is: cut M again & again along incompressible surfaces to get a collection of disks (Possible by Haken). Put on a $\mathbb{H}^3$-structure. Now paste back together again, at each pasting step wiggling the $\mathbb{H}^3$-structure so that you can paste geometrically. Wild things happen - formerly totally geodesic surfaces get crumpled along geodesic curves, and so on - but it can be done.

The set $\mathcal{S}$ of all complete Hyperbolic 3-manifolds of finite volume has wonderful structure. The ingredients are a result of Jørgensen plus Thurston's refinement of Gromov's proof of Mostow rigidity. $\mathcal{S}$ has a topology such that the volume function

$$\mathrm{vol} : \mathcal{S} \longrightarrow \mathbb{R}_+$$

is proper, finite to one. Its image is well ordered of order type ω^ω: possible volumes

$$v_1 < v_2 < \dots < v_\omega < v_{\omega+1} < \dots < v_{2\omega} < \dots < v_{3\omega} < \dots\dots < v_{\omega^2} < \dots < v_{\omega^3} < \dots$$

v_1: smallest poss. volume

v_ω: smallest volume with 1 cusp

v_{ω^2}: Smallest Volume with 2 cusps

This is well described in Thurston's Notes and in the Bourbaki Exposé of Gromov.

Titel: Einstein metrics

Autor: Andrzej Derdziński

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We consider the problem which Einstein metrics g ($Ric = \frac{1}{4} Scal.g$) on compact 4-manifolds can be obtained from Kähler metrics by conformal deformations. Except for spaces of constant curvature, all known examples of compact Einstein 4-manifolds can be obtained essentially this way.

For a Riemannian 4-manifold (M,g), the Bach tensor [2] is the symmetric (0,2) tensor field B, given in local coordinates by

$$B_{ij} = \nabla^r \nabla^s W_{rijs} + \frac{1}{2}(Ric)^{rs} W_{rijs},$$

W being the Weyl conformal tensor. Condition $B = 0$ is conformally invariant and it holds, for M compact, if and only if g is a critical point for the conformally invariant functional $g \mapsto \int_M |W|^2$.

Thus, $B = 0$ whenever g is locally conformally Einsteinian, as well as when M is compact, oriented and g is self-dual [1], i.e., $W^- = 0$.

QUESTION. When is a Kähler 4-manifold (M, g) locally conformally Einsteinian?

An answer (in a generic case) can be obtained as follows. For Kähler 4-manifolds, $B=0$ if and only if

(*) $2\nabla^2 Scal + Scal.\, Ric = \varphi g$

for some function φ. From (*) it follows that

(a) $J(\nabla Scal)$ is a holomorphic Killing field, J being the complex structure (i.e., this is an extremal Kähler metric in the sense of Calabi)

(b) $Scal^{-2} g$ is an Einstein metric

(defined wherever $\mathrm{Scal} \neq 0$).

(c) $\mathrm{Scal}^3 + 6\,\mathrm{Scal}.\Delta\mathrm{Scal} - 12|\nabla\mathrm{Scal}|^2 =$ $= \text{constant}.$

On the other hand, for any Kähler 4-manifold we have $W^+ = \mathrm{Scal}.A$, A being a parallel tensor field, which, viewed as an endomorphism of Λ^+, has eigenvalues $1/12$, $-1/24$, $-1/24$, the single one corresponding to the Kähler form ω. Thus, we have:

For a Kähler 4-manifold (M,g), at points with $W^+ \neq 0$, the following statements are equivalent [4]:

(i) g locally conformally Einsteinian

(ii) $B = 0$

(iii) $(*)$ holds

(iv) (a) and (c) hold.

On the other hand, there is the related question: Which oriented

Einstein four-manifolds are locally conformally Kählerian (in a manner compatible with the orientation)? An obvious necessary condition is that our Einstein metrics are to be special, i.e., the self-dual part $\tilde{W}^+$ of the Weyl tensor of such a metric $\tilde{g}$, viewed as an endomorphism of Λ^+, must have less than 3 distinct eigenvalues at each point.

LEMMA [4]. If an orientable Riemannian 4-manifold $(M, \tilde{g})$ is not special for any orientation, then there exists an open dense set $\mathcal{U}_0 \subset M$ such that any Killing field X defined in an open connected $\mathcal{U} \subset \mathcal{U}_0$, must either vanish identically, or be non-zero everywhere in $\mathcal{U}$.

The known examples of compact Einstein four-manifolds are essentially the following ones:

1) Kähler-Einstein manifolds
2) Spaces of constant curvature
3) The metric of D. Page [6]: a Hermitian, non-Kählerian Einstein metric on $\mathbb{C}P^2 \# (-\mathbb{C}P^2)$ with $\mathrm{Isom}^0 \cong U(2)$; the principal orbits or Isom^0 are 3-dimensional.

Thus, all these examples are special (the last one, e.g., in view of the lemma). It turns out that generically (at points with $W^+ \neq 0$) any special Einstein metric is locally conformally Kählerian. This local result has also the following global version:

THEOREM [4]. Let $(M, \tilde{g})$ be a compact, oriented, special Einstein 4-manifold. Replacing M by a 2-fold Riemannian cover, if necessary, we have only 3 possible cases:

(I) $\tilde{W}^+ = 0$ (an anti-self-dual Einstein manifold)

(II) $\tilde{\nabla}\tilde{W}^+ = 0$, $\tilde{W}^+ \neq 0$ (a Kähler-Einstein manifold).

(III) $\tilde{\nabla}\tilde{W}^+ \neq 0$. In this case $M = S^2 \times S^2$ or $M = \mathbb{C}P^2 \# (-k\mathbb{C}P^2)$, $0 \leq k \leq 8$, $\tilde{g}$ is Hermitian, globally conformal to a Kähler metric g, which satisfies (*), $\mathrm{Scal}_g > 0$, and $J(\nabla^g \mathrm{Scal}_g)$ is a nontrivial holomorphic field, preserving both $\tilde{g}$ and g.

The class (III) above is non-empty (the example of Page; the corresponding extremal Kähler metric was constructed by Calabi).

Idea of proof. If g is a Kähler metric conformal to $\tilde{g}$, then we must have $\tilde{g} = (24)^{-1} \mathrm{Scal}_g^{-2}\, g$, which gives $g = 24\, \xi^{2/3} \tilde{g}$ (the factor 24 is taken for convenience), where ξ is determined by $\tilde{g}$ in such a way that the eigenvalues of $\tilde{W}^+$ are $2\xi, -\xi, -\xi$ ($\xi = = \mathrm{Scal}_g^3$). The naturally defined metric g is Kählerian, namely,

$$\nabla^g(24\,\xi^{2/3}\tilde{\omega}) = 0,$$

$\tilde{\omega} \in \Lambda^+$ being a unit 2-form with $\tilde{W}^+\tilde{\omega} = = 2\xi\tilde{\omega}$. Thus $X = J(\nabla^g \mathrm{Scal}_g) = = -\frac{1}{24}\tilde{\omega}(\tilde{\nabla}\xi^{-1/3})$ is a Killing field for $\tilde{g}$ and therefore it is global, which implies $\xi \neq 0$ everywhere in M. Thus, g is defined globally. The statement of ($\underline{III}$) follows now from the classification of complex surfaces admitting holomorphic vector fields [3] together with the inequality $|\tau(M)| < < \frac{2}{3}\chi(M)$ valid for any compact non-Ricci-flat Einstein 4-manifold [5].

Autor: A. Derdziński Seite Nr.: 8

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Titel: L_2 cohomology of arithmetic groups.

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Let M be a Riemannian manifold, and put

$$L^i_{(2)}(M, \mathbb{C}) = \{\mathbb{C}\text{-valued } i\text{-forms on } M \text{ such that } \varphi \text{ and } d\varphi \text{ are square-summable}\}.$$

Then $L^*_{(2)}(M, \mathbb{C})$ is a sub-complex of the full de Rham complex of M, and its cohomology, $H^*_{(2)}(M, \mathbb{C})$, is called the L_2-cohomology of M. In the above, we are using forms with distribution coefficients, though C^∞ forms may also be used.

The following are basic properties of L_2 cohomology:

(1) For each i, there is a mapping

$$H^i_{(2)}(M, \mathbb{C}) \to H^i(M, \mathbb{C})$$

that is, a priori, neither injective nor surjective.

(2) If M is compact (with boundary), then the mapping in (1) is an isomorphism for all i.

(3) The L_2 cohomology depends on the metrics only to the extent that the L_2 spaces do; if there is a uniform estimate $C g_2 \le g_1 \le C' g_2$ for two metrics g_1, g_2 on M, then g_1 and g_2 give rise to the same L_2 cohomology.

(4) [Hodge Theorem] If $dL^{i-1}_{(2)}$ is a closed sub-

space of L_2 (e.g., if M is compact), then

$$H^i_{(2)}(M, \mathbb{C}) \simeq \{L_2 \text{ harmonic } i\text{-forms which satisfy (implicit) Neumann boundary conditions}\}.$$

In the above, we may replace $\mathbb{C}$ by the locally constant sheaf $\mathbb{V}$ of horizontal sections of a metrized flat vector bundle (the metric need not be flat).

One would like to impart a topological interpretation to L_2 cohomology groups on non-compact manifolds.

Let G be the set of real points of a semi-simple algebraic group over $\mathbb{Q}$, viewed as a Lie group. Let K be a maximal compact subgroup of G, and denote by X the associated symmetric space G/K. If Γ is a torsion-free arithmetic subgroup of G, then $\Gamma \backslash X$ is a manifold, in general non-compact. There is a G-invariant metric on X, which descends to $\Gamma \backslash X$.

Let V be a complex vector space on which G acts. There is a corresponding flat bundle on $\Gamma \backslash X$, with a natural metric determined by an admissible inner product on V. If one wants to know about $H^*(\Gamma, V) \simeq H^*(\Gamma\backslash X, \mathbb{V})$, one can look for the L_2 harmonic forms, which can "in principle" be determined from the representation theory of $L_2(\Gamma\backslash G)$ [Borel-Wallach, Zuckerman]. However, these faithfully represent classes only in the L_2 cohomology. Thus, the nature of the mappings

$$(5) \qquad H^i_{(2)}(\Gamma\backslash X, \mathbb{V}) \longrightarrow H^i(\Gamma\backslash X, \mathbb{V})$$

is of interest.

Theorem. Let i_0 be a non-negative integer. Suppose that for every $q \le i_0$ all $\underline{a}$-weights λ of $H^q(\underline{u}, V)$ satisfy $\lambda + \rho|_{\underline{a}} > 0$. Then (5) is an isomorphism for all $i \le i_0$. (Here, $\underline{m} \oplus \underline{a} \oplus \underline{u}$ is the $\mathbb{Q}$-decomposition of a minimal $\mathbb{Q}$-parabolic subalgebra of the Lie algebra of G, and ρ is one-half the sum of the positive roots.

We suspect that this result is best possible when $\mathrm{rk}_{\mathbb{Q}} G = \mathrm{rk}_{\mathbb{R}} G$.

(6) Examples: If $G = SO(2n, 1)$ [X = $(2n)$-dim. real hyperbolic space], or $G = SU(n,1)$ [X is

the complex n-ball], then we can take $i_0 = n-1$ for any V. If $G = R_{k/\mathbb{Q}}\, SL(2,k)$, where k is a totally real extension of $\mathbb{Q}$ with $[k:\mathbb{Q}] = n$ [$\Gamma\backslash X$ is a Hilbert modular variety], then $i_0 = 0$, but (5) can be shown to be an isomorphism for $i \leq n-1$.

The underlying idea behind the proof is to show that for certain open subsets U of $\Gamma\backslash X$

(7) $$H^i_{(2)}(U, V) \xrightarrow{\sim} H^i(U, \mathbb{V})$$

is an isomorphism for all $i \leq i_0$, and then to globalize by means of a spectral sequence. To do this, we must look at the behavior of the metric near the boundary of the manifold with corners $\Gamma\backslash\bar{X}$ [Borel-Serre]. The discussion is quite simple in the case $G = SO(k,1)$. Here, the boundary is a finite union of disjoint compact, connected manifolds N ($rk_{\mathbb{Q}}\, G = 1$). The sets U are collars of N in $\Gamma\backslash X$; U is differentiably $(0,1) \times N$. However, the metric on U is equivalent (in the sense of (3)) to

$$ds^2 = \left(\frac{dt}{t}\right)^2 \oplus t^2\, ds^2_N,$$

a so-called warped product. We obtain a

Künneth-type formula for the L_2 cohomology of a warped product under mild hypotheses (satisfied in our case), which gives the isomorphism in (7). In general, things are more complicated, but the argument can be adjusted.

In the examples (6), one sees that for the collars U one has in fact

$$(8) \qquad H^i_{(2)}(U, \mathbb{V}) \simeq \begin{cases} H^i(N, \mathbb{V}) = H^i(U, \mathbb{V}) & \text{if } i \leq n-1 \\ 0 & \text{if } i \geq n \end{cases}$$

Let $\Gamma\backslash X^*$ denote the space obtained from $\Gamma\backslash\overline{X}$ by collapsing the boundary components N to points. The collars then form a fundamental system of deleted neighborhoods of these points. Then (8) is the local condition which characterizes the intersection homology [Goresky-MacPherson] of a space with isolated singular points. Thus in these cases we obtain

$$(9) \qquad H^i_{(2)}(\Gamma\backslash X, \mathbb{V}) \simeq IH^i(\Gamma\backslash X^*, \mathbb{V}).$$

We have conjectured that (9) holds for all Hermitian X, with $\Gamma\backslash X^*$ the Baily-Borel-Satake compactification.

For details, etc, see the author's "L_2 cohomology of warped products and arithmetic groups" (preprint).

Titel: Explicit Construction of Class Fields.

Autor: A. Wiles

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This was an account of joint work with B. Mazur continuing his talk given earlier in these proceedings. The problem studied was the explicit construction of unramified abelian p-extensions of $\mathbb{Q}(\zeta_p)$. The extensions ~~on whose~~ on which complex conjugation acts by -1 (the 'minus' extensions) can be constructed using the p-adic Tate module of $\mathrm{Jac}(X_1(p))$. Here $X_1(p)$ is the complete modular curve associated to $\Gamma_1(p)$. This work has been generalized to cover all abelian extensions of $\mathbb{Q}$. The idea of using modular curves originated in the paper of Ribet [1].

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See also

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Titel: Asymptotics of elementary spherical functions.

Autor: J.J.Duistermaat,

Adresse: Mathematisch Instituut, Utrecht, Holland.

This is a report on joint work with J.A.C.Kolk and V.S.Varadarajan.

Let G be a real connected semisimple Lie group with finite center and G = K.A.N its Iwasawa decomposition. Let be the Lie algebra of A. Then the "Iwasawa projection" $H: G \longrightarrow \mathfrak{a}$ is defined by

1) $x \in K.\exp H(x).N$.

We wanted to study the asymptotic behaviour of oscillatory integrals of the type

2) $I_{a,\xi} = \int_K e^{i\langle H(ak),\xi\rangle} g(k)\, dk \qquad (a \in A),$

as $\xi \to \infty$ in $\mathfrak{a}$. Here $g \in C^\infty(K)$; examples are the elementary spherical functions, where $g(k) = \exp -\langle H(ak),\rho\rangle$, and more generally the matrix coefficients of the principal series representations of G. The famous asymptotics of Harish-Chandra is the one for $a \to \infty$ in A^+ (the positive Weyl chamber in A), keeping ξ fixed. It was based on the fact that these functions are the restrictions to A of the common eigenfunctions of all G-invariant differential operators on the symmetric space $K\backslash G$ (ξ represents the eigenvalues). For the elementary spherical functions and for $a \in A^+$, a careful study of Harish-Chandra's methods also yields the desired asymptotics for $\xi \to \infty$. However, following L.Cohn's approach to the asymptotics of c-functions, we preferred a direct study of the integrals 2), because it leads to more general results and the proof is much simpler. More importantly, it lead us also to some surprising insights in the geometric properties of the Iwasawa projection itself.

To start with, let us consider the <u>radial asymptotics</u>, that is we replace ξ by $\omega.\xi$ and let $\omega \to \infty$. It is well-known then that the contributions to the asymptotic expansion only come from the critical points of the phase function $F_{a,\xi}$ on K, where

3) $$F_{a,\xi}(k) = \langle H(ak), \xi \rangle .$$

This may be considered as testing the Iwasawa projection of the K-orbit $\{Kak \; ; \; k \in K\}$ of the point Ka in the symmetric space $K \backslash G$, by a linear form. The fibers of the Iwasawa projection are the N-orbits in the symmetric space, which are also called the horospheres.

<u>Theorem 1</u>. If $a = \exp X$, $X, \xi \in \mathfrak{a}$, then the critical set of $F_{a,\xi}$ is equal to

4) $$K_{X,\xi} = \bigcup_{w \in \mathcal{W}} K_X \, w \, K_\xi .$$

Here K_X, resp. K_ξ denotes the centralizer of X, resp. ξ in K. Furthermore, $\mathcal{W}$ is the Weyl group = the normalizer of $\mathfrak{a}$ in K modulo the centralizer M of $\mathfrak{a}$ in K. So the notation $w K_\xi$ makes sense because $K_\xi \supset M$. Moreover $K_X = M.K_X^o$ and $K_\xi = K_\xi^o.M$ (component problems come from M), 4) can be written as a <u>disjoint</u> union over $w \in \mathcal{W}_X \backslash \mathcal{W} / \mathcal{W}_\xi$ and finally each component of the $K_X \times K_\xi$ -orbit $K_X \, w \, K_\xi$ has the same dimension:

5) $$\dim(K_X \, w \, K_\xi) = \dim M + \sum_{\substack{\alpha \in \Delta^+, \\ \alpha(\xi) \cdot w\alpha(X) = 0}} \dim \mathfrak{g}_\alpha .$$

In particular, $K_{X,\xi}$ depends only on the set of roots α vanishing on X, resp. ξ . So testing the Iwasawa projection of the K-orbit with varying linear forms, the critical sets do not move, they only, when more roots vanish on ξ, suddenly change into higher dimensional manifolds, and there are only finitely many choices. The critical values are equal to $\langle w^{-1}(X),\xi\rangle$, $w\in\mathcal{W}$, and a little reflection makes it plausible that H(aK) must be equal to the convex hull of the finitely many points $w^{-1}(X)$, $w\in\mathcal{W}$, this is Kostant's convexity theorem. For completing the above proof one only needs the additional information that $F_{a,\xi}$ has only <u>one</u> local maximum, resp. local minimum, this follows from the description of the Hessians of $F_{a,\xi}$ given below. This proof has been carried out in detail in the thesis of Heckman (Leiden), which contains many more interesting convexity results.

<u>Theorem 2</u>. The Hessian of $F_{a,\xi}$ at each critical point is non-degenerate transversally to the critical manifold. At $w\in\mathcal{W}$ it is given by

6) $$-\tfrac{1}{2}\sum_{\alpha\in\Delta^+}\alpha(\xi).(1-e^{-2\cdot w\alpha(X)}).F_\alpha ,$$

where F_α denotes the orthogonal projection from to $k_\alpha = (\mathfrak{g}_\alpha\oplus\mathfrak{g}_{-\alpha})\cap k$.

A direct application of the method of stationary phase now gives:

<u>Corollary 3</u>. For $\omega\to\infty$ we have the asymptotic expansion

7) $$I_{a,\omega.\xi} = \sum_{\mathcal{W}_X\backslash\mathcal{W}/\mathcal{W}_\xi} e^{i\langle w^{-1}(X),\xi\rangle}\sum_{l=0}^{\infty}\omega^{-\frac{1}{2}n_w-l}\, c_{w,l,\xi}$$

with n_w = codimension of $K_X\,w\,K$ in K (cf. 5)) and the leading coefficient is of the form

8) $$c_{w,0,\xi} = \prod_{\alpha\in\Delta_w^+}|\alpha(\xi)|^{-\frac{1}{2}\dim\mathfrak{g}_\alpha}\,.\,c_{w,0}\,.$$

Here Δ_w^+ is the set of positive roots which vanish neither on ξ nor on $w^{-1}(X)$, $c_{w,0}$ is a constant which does not depend on ξ and can be expressed explicitly as an integral of g over $K_X\, w\, K_\xi$, which manifold does not change as long as the set of roots vanishing on ξ remains the same.

The important thing to observe is that the order of growth $-\frac{1}{2} n_w$ jumps up (becomes less negative) if ξ enters the intersection of more root hyperplanes $\operatorname{Ker} \alpha$, this change in asymptotic behaviour is also reflected by the singular behaviour of the leading coefficient as ξ approaches $\operatorname{Ker} \alpha$, cf. 8). This dependence of the asymptotics on the position of ξ in $\mathfrak{a}$ is an example of so-called caustic behaviour. However here it is of a very non-generic kind, because the caustic set is a union of hyperplanes instead of a catastrophe set à la Thom. Away from the caustics the expansions are locally uniform in ξ, and near the caustics one can estimate from above by what happens at the caustic. However, it is usually very hard to write down uniform estimates which are sharp in every direction, and therefore we are quite proud to present the following obvious guess as a theorem:

<u>Theorem 4</u>. Fix a compact subset A_0 of A. For any $w \in \mathfrak{w}$, let $\Delta_w^+(A_0)$ be the set of positive roots which vanish nowhere on $w^{-1}(A_0)$. Then we can find a C^m-norm ν on $C^m(K)$, $m = \frac{1}{2} \dim N$, such that for all $a \in A_0$, $g \in C^m(K)$, and all $\xi \in \mathfrak{a}$:

$$9) \quad |I_{a,\xi}| \leq \nu(g) \cdot \sum_{w \in \mathfrak{w}} \prod_{\alpha \in \Delta_w^+(A_0)} (1 + |\alpha(\xi)|)^{-\frac{1}{2}\dim \mathfrak{g}_\alpha}.$$

An important tool in the study of $F_{a,\xi}$ is its "infinitesimal version"

10) $f_{X,\xi}(k) = \frac{d}{dt} F_{\exp tX,\xi}(k)_{t=0} = \langle X, \mathrm{Ad}\, k(\xi)\rangle$.

This can be regarded as testing the Ad K-orbit of ξ in the orthogonal complement $\mathfrak{s}$ ("symmetric matrices") of $\mathfrak{k}$ in $\mathfrak{g}$, by a linear form; because $X \in \mathfrak{a}$ we are only testing the orthogonal projection of Ad K(ξ) to $\mathfrak{a}$ by linear forms. The $f_{X,\xi}$ have the same qualitative properties as the $F_{\exp X,\xi}$, in fact there is an automorphism Φ of the tangent bundle of K, depending smoothly on X and not depending on ξ, which maps $df_{X,\xi}$ to $dF_{\exp X,\xi}$. Unfortunately we could not decide whether there is a diffeomorphism of K depending smoothly on X and ξ, which maps $f_{X,\xi}$ to $F_{\exp X,\xi}$, such a reduction to the simpler function $f_{X,\xi}$ would have reduced the complications in the analysis considerably. For example, the $f_{X,\xi}$ are obviously left K_X- and right K_ξ-invariant. In contrast, $F_{\exp X,\xi}$ is left K_X- invariant but not always right K_ξ-invariant, although for ξ in the closure of the positive Weyl chamber it is right K_ξ-invariant; a confusing situation.

The same proof applies to show that also the orthogonal projection to $\mathfrak{a}$ of Ad K(ξ) is equal to the convex hull of $\mathfrak{w}(\xi)$, it is this convexity theorem (also due to Kostant), and moreover restricted to the case that the group G is complex, which was cited in the lecture of Atiyah. In fact, if G is complex, then $i\mathfrak{a} = \mathfrak{t}$ is the Lie algebra of a maximal torus T in K, $i\mathfrak{s} = \mathfrak{k}$ and Ad K(ξ) can be identified with the Ad K -orbit if $i\xi$ in $\mathfrak{k}$.

Functions like $f_{X,\xi}$ had been considered before by Bott for compact (= complex) groups, and by Takeuchi and Kobayashi for general real semisimple G. Using a suitable Riemannian metric on the flag manifold $K/K_\xi \simeq \mathrm{Ad}\ K(\xi)$, they identified the stable manifolds for the gradient flow of $f_{X,\xi}$ with the Bruhat cells (=Schubert cells), which in turn can be used to show that $f_{X,\xi}$ has the minimal number of critical points when it is Morse. Or, in Kuiper's terminology, the embedding $k \mapsto \mathrm{Ad}\ k(\xi)$ of K/K_ξ in $\mathfrak{s}$ is tight. For $G = SL(3,R)$, ξ singular, this is the Veronese embedding of the projective plane, the orthogonal projection to $\mathfrak{a}$ yields the equilateral triangle which appeared in the movie of Banchoff, which he showed at the International Congress in Helsinki.

Similar results could be obtained for oscillatory integrals on conjugacy classes in G, but there is no time left to explain these here.

Titel: On Works of Waldspurger.

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In 1972, Shimura defined a map from cusp-forms of half-integral weight to cusp forms of even weight (Springer L.N. 320). In 1975, Shintani constructed a map in the inverse direction, using theta series associated to an indefinite quadratic form with three variables (Nagoya Math J. 58). In 1979, Waldspurger studied this application in an adelic language as in Jacquet-Langlands (Springer LN 114 (1970)) and proved the following results (1) J. de Math pures et appliquées 59, 2) same journal to appear):

- let ψ be a non trivial additive character of $F \backslash A$, where F is a number field and A the adeles of F, A_0 the space of the cuspidal automorphic forms on $GL(2,F)\backslash GL(2,A)$ with trivial central character, $\tilde{S}$ the metaplectic covering of order 2 of $SL(2)$, $\tilde{A}_0$ the space of the genuine cuspidal automorphic forms on $\tilde{S}_F \backslash \tilde{S}_A$, orthogonal to the distinguished ones (as defined by Piatetski-Shapiro and Gelbart). For every irreducible subspace V of A_0, and every irreducible subspace T of $\tilde{A}_0$, let $T(\psi,V) \subset \tilde{A}_0$ and $V(\psi,T) \subset A_0$ the images of V and T obtained by the Shintani construction, and its adjoint. Let $L(V,s)$ be the L-function associated to V, with functional equation in $s \to 1-s$.

THEOREM 1.

- $T(\Psi, V)$ is irreducible
- $V(\Psi, T)$ is irreducible

THEOREM 2.

The following properties are equivalent:

1) $T(\Psi, V) \neq 0$
2) the space of Ψ-Fourier coefficients $\tilde{W}(\Psi, T(\Psi, V))$ is not zero
3) $L(V, 1/2) \neq 0$

The following properties are equivalent:

1) $V(\Psi, T) \neq 0$
2) $\tilde{W}(\Psi, T) \neq 0$

THEOREM 3. The map $T \longrightarrow T(\Psi, V)$ is a bijection from the set of irreducible subspaces T of $\tilde{A}_0$ such that $\tilde{W}(\Psi, T) \neq 0$ on the set of irreducible subspaces V of A_0 such that $L(V, 1/2) \neq 0$. The inverse map is $V \longrightarrow V(\Psi, T)$.

THEOREM 4 (Multiplicity one). The weak multiplicity one theorem is true for $\tilde{A}_0$. The strong multiplicity one theorem is false for $\tilde{A}_0$.

This means that if 2 irreducible subspaces T_1 and T_2 of $\tilde{A}_0$ defined 2 representations $\tilde{\rho}_1$ and

$\tilde{P}_2$ of the Hecke Algebra, locally isomorphic everywhere then $T_1 = T_2$. It is not possible to replace everywhere by almost everywhere.

Waldspurger translated his results in the classical language of modular forms and give a complete description of the Shimura and Shintani maps. He deduced from this the following striking result.

Let ϕ be a new form of weight k, with trivial character and ε-factor equal to 1. Let $X(\phi)$ be the set of Dirichlet quadratic characters η, such that the ε-factor of the twist $\phi\eta$ is 1, satisfying some local conditions at the primes dividing the conductor of ϕ. Let D be the discriminant of the quadratic field defined by η. One introduces a Dirichlet character χ_0 satisfying some explicit conditions at primes $p \mid N$ and K_ϕ the field generated over $\mathbb{Q}$ by the values of χ_0, and the Fourier coefficient of ϕ.

THEOREM 5. There exist $\Omega \in \mathbb{C}^\times$ such that

1) $L(\phi\eta, k/2)\ \varepsilon(\chi_0\eta)\ |D|^{(k-1)/2} = \Omega\ a^2(|D|)$

2) $a(|D|)$ is an algebraic integer in K_Φ

3) $a(|D|)$ is the $|D|$-th Fourier coefficient of a cusp-form F of weight $\frac{k+1}{2}$ associated to ϕ by the Shintani-correspondance.

Titel: Topological similarity of representations

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Background References

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The focal point of this report is the following:

THEOREM. Let G be a finite cyclic group of odd order, and let $S,T\colon G \longrightarrow GL(n,R)$ be representations of G. Then the following are equivalent:

(i) There is an invertible $n \times n$ matrix A such that $T(g) = AS(g)A^{-1}$, all $g \in G$.

(ii) There is a homeomorphism A of R^n so that the identity in (i) holds.

Of course, (i) $\Longrightarrow$ (ii) is trivial. Recently two somewhat different proofs of the converse were announced. One is due to W.-C. Hsiang and W. Pardon, and the other is due to I. Madsen and M. Rothenberg.

Standard machinery yields the following:

COROLLARY. The conclusion of the Theorem is valid for (a) finite groups with order not divisible by four, (b) compact Lie groups with an odd number of components, provided the representations are continuous.

Historical Remarks. Equivalence of (i) and (ii) had been known in many special cases:

(1) n at most 2 (essentially due to Poincaré).

(2) representations for which $g \neq 1$ implies g leaves only 0 fixed. (This follows from the Atiyah-Bott formula).

(3) p- groups for p an odd prime (using Sullivan's orientation class for topological n-plane bundles

with respect to real K-theory away from 2).

(4) semifree representations— a vector is either fixed by all of G or by no element except 1.
(Hsiang and Pardon, unpublished extension of (2)).

In contrast, if G is cyclic of order 4k, with k at least 2, examples of Cappell and Shaneson show that the statements in the Theorem are NOT equivalent. One can combine the Theorem with further results of Cappell and Shaneson to obtain the following result:

Theorem. Suppose that RTop(G) is formed from RO(G) by identifying topologically equivalent representations. Given a subfield K of the reals, define RK(G) to be the irreducible representations over K, and let RK map to RTop in the obvious way. Then one can find an algebraic extension of Q such that the composite from RK(G) to RTop(G) is a raional isomorphism. If G is a 2-group, thenone can take K=Q.

Remarks on the proofs of the Theorem stated at the beginning.

Both proofs involve a cumbersome induction involving the stratification of a G-manifold by orbit types If one concentrates on the case G = Z+pq, where p and q are distinct odd primes,the inductive process can be mostly bypassed ahd the geometry becomes clearer (this was the course chosen for the lecture).

The key to the Hsiang-Pardon proof is contained in the following result (and it is also crucial to the semifree case):

INTERMEDIATE THEOREM. Let G be cyclic of odd order, and let V and W be topologically equivalent representations. Write $V = k + V_o$, where G acts trivially on the first summand and has 0 for the fixed point set of the second, and write W likewise. Then there is a PL G-action on a homotopy sphere H with the following properties:

(i) The action is combinatorially locally smoothable

(ii) The fixed point set of each subgroup is a PL sphere.

(iii) The fixed point set of G consists of two points,aand the local representations at these points are a power of two times V_o and W_o.

With this in hand, the proof concludes by establishing a PL variant of the Atiyah-Bott formula.

The key idea behind the Madsen-Rothenberg proof is the construction of an equivariant KO-theoretic orientation for locally linear G-equivariant n-plane bundles. Of course, one wants to have the same sorts of formal properties as one has for Sullivan's orientation in the case where no group action is present. The construction of the equivariant orientation is a technically nontrivial generalization, and the methods shed valuable new light on the topological properties of manifolds with "nice" group actions.