MATHEMATISCHE ARBEITSTAGUNG 1979

UNIVERSITÄT BONN

Sonderforschungsbereich 40

Theoretische Mathematik

Wegelerstraße 10

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Mathematisches Institut
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Programm der Mathematischen Arbeitstagung 1979 (I)

Mittwoch, den 6.6.:

17.00 - 18.00 Uhr: J. Tits: On Leech's lattice and sporadic groups

Donnerstag, den 7.6.:

10.00 - 11.00 Uhr: F. Adams: G. Segal's Burnside ring conjecture

12.00 - 13.00 Uhr: F. Bogomolov: Converse Galois problems for some Chevalley groups

17.00 - 18.00 Uhr: Wang Yuan: Goldbach problem

Freitag, den 8.6.:

10.00 - 10.15 Uhr: Festlegung der nächsten Vorträge

10.15 - 11.15 Uhr: D. Vogan: Size of representations

11.30 - 12.15 Uhr: L. Berard-Bergery: A new example of Einstein manifolds

12.30 - ca. 21.30 Uhr: Ausflug nach Oberwesel. Abfahrt 12.30 Uhr mit Bussen an der Beringstr. 1 nach Koblenz. Abfahrt ca. 13.30 mit Motorschiff "Carmen Silva" ab Koblenz ca. 13.30 Uhr.

Samstag, den 9.6.:

10.00 - 11.00 Uhr: V. Kac: Infinite dimensional Lie algebras

Die Vorträge finden alle im "Großen Hörsaal" (Wegelerstraße 10) statt. Erfrischungspausen mit Tee: Mittwoch und Donnerstag 11.15-12.00 Uhr vor dem Großen Hörsaal, nachmittags ab 15.30 Uhr im Diskussionsraum Beringstraße 1. Die Post liegt während der Teepausen aus. Tischtennis im Keller des Hauses Beringstraße 4. Den Tagungsbeitrag bitte an Frau Gerber (SFB-Büro Beringstr. 4) bezahlen. Alle Tagungsteilnehmer mögen sich bitte in die Teilnehmerliste eintragen. Teilnehmerlisten und andere Informationen liegen vor dem Diskussionsraum Beringstraße 1 aus.

! Alle Tagungsteilnehmer mit ihren Damen oder Herren sind herzlich zum Empfang des
! Rektors eingeladen. Zeit: Donnerstag, den 7.6., 20.00 Uhr. Ort: Festsaal der Univer-
! sität (Hauptgebäude), Eingang von der Straße "Am Hof" durch das Tor gegenüber Buch-
! handlung Röhrscheid.

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Programm der Mathematischen Arbeitstagung 1979 (II)

Samstag, den 9.6.:

12.00 - 13.00 Uhr:	G. Mostow: New negatively curved surfaces
17.00 - 18.00 Uhr:	G. Lusztig: Representations of Hecke algebras

Sonntag, den 10.6.:

10.00 - 10.15 Uhr:	Festlegung der restlichen Vorträge
10.15 - 11.15 Uhr:	B. Gross: Conjectures of Stark and Tate
12.00 - 13.00 Uhr:	Wu-chung Hsiang: Topological space form problems
17.00 - 18.00 Uhr:	E. Looijenga: Singularities and generalised root systems

Montag, den 11.6.:

10.00 - 11.00 Uhr:	Parshin: Zeta functions and K-theory

Die Vorträge finden alle im "Großen Hörsaal" (Wegelerstraße 10) statt.

Erfrischungspausen mit Tee: Samstag, Sonntag und Montag 11.15-12.00 Uhr vor dem Großen Hörsaal, nachmittags ab 15.30 Uhr im Diskussionsraum Beringstraße 1.

Die Post liegt während der Teepausen aus.

Tischtennis im Keller des Hauses Beringstraße 4.

Den Tagungsbeitrag bitte an Frau Gerber (SFB-Büro, Beringstraße 4) bezahlen.

Alle Tagungsteilnehmer mögen sich bitte in die Teilnehmerliste eintragen. Teilnehmerlisten und andere Informationen liegen vor dem Diskussionsraum Beringstraße 1 aus.

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Programm der Mathematischen Arbeitstagung 1979 (III)

Additional lecture: Sonntag, 10.6., 15.00 - 15.30 Uhr:

M.-F. Vigneras: Isospectral but not isometric Riemannian surfaces

Montag, den 11.6.:

12.00 - 13.00 Uhr: Min-Oo: Curvature deformations relating to the Yang-Mills fields

17.00 - 18.00 Uhr: G. Harder: Cohomology and values of L-functions

Dienstag, den 12.6.:

10.15 - 11.15 Uhr: A. Todorov: Moduli of Kählerian K3-surfaces

12.00 - 13.00 Uhr: R.P. Langlands: On orbital integrals for real groups

17.00 - 18.00 Uhr: J.-P. Serre: The monster game

! Die Referenten werden nochmals gebeten, ihre Kurzfassungen bis Montag, 10.00 Uhr
! bei Herren Schwermer abzugeben, da wir den Tagungsbericht allen Teilnehmern
! noch vor ihrer Abreise aushändigen möchten.

Die Vorträge finden alle im "Großen Hörsaal" (Wegelerstraße 10) statt.

Erfrischungspausen mit Tee: Montag und Dienstag vormittags von 11.15 - 12.00 Uhr vor dem großen Hörsaal, nachmittags ab 15.30 Uhr im Diskussionsraum Beringstr. 1.

Die Post liegt während der Teepausen aus.

Tischtennis im Keller des Hauses Beringstraße 4.

Titel: On Leech's lattice and sporadic groups

Autor: J. Tits

Adresse: Collège de France, Paris

Leech's lattice can be characterized as a free $\mathbb{Z}$-module of rank 24 endowed with an integral, even quadratic form of determinant one which does not represent 2. Conway's classical construction of the lattice involves the Steiner system $S(24,5,8)$. Here, we describe another simple construction based on properties of the icosahedron.

Let G be the binary icosahedral group, embedded as usual in the real quaternion algebra $\mathbb{H}$. Let R (resp. K) denote the subring (resp. the sub-$\mathbb{Q}$-algebra) of $\mathbb{H}$ generated by G. The algebra K is isomorphic with the quaternion algebra $\mathbb{Q}(\sqrt{5})(i,j,k)$ and R is a maximal order in it. Set $\tau = \frac{1+\sqrt{5}}{2}$, let $\lambda : \mathbb{Q}(\sqrt{5}) \to \mathbb{Q}$ be the $\mathbb{Q}$-linear mapping defined by $\lambda(a+b\tau) = a$ for $a, b \in \mathbb{Q}$, choose an element ζ of order 3 in G and set $e = \zeta + \tau$. In the right vector space K^3 endowed with the hermitian form $h : (x,y) \mapsto \sum \bar{x}_i y_i$

(where $x = (x_1, x_2, x_3)$ etc.), we define

$$\Lambda = \{x \in R^3 \mid \Sigma x_i \equiv e x_1 \equiv e x_2 \equiv e x_3 \pmod 2\}.$$

The vectors $(2,0,0)$, $(\bar{e},\bar{e},0)$, $(1,e,1)$ form a basis of the (free) R-module Λ. It readily follows that

$$(1) \qquad \Lambda = \{x \in K^3 \mid h(x,\Lambda) \subset 2R\}.$$

From that relation, one easily deduces that Λ, considered as a 24-dimensional $\mathbb{Z}$-module endowed with the quadratic form $q : x \mapsto \lambda(h(x,x))$, is Leech's lattice (i.e., has the characteristic properties of that lattice recalled at the beginning).

Let R' be the subring of R generated by an element of order 3 of G. If X is any subring of R, we write Λ_X to refer to Λ considered as an X-module. Then:

$\mathrm{Aut}(\Lambda_{\mathbb{Z}}, q) = \cdot 0$ is a central extension of Conway's simple group $\cdot 1$ by $\mathbb{Z}/2\mathbb{Z}$;

$\mathrm{Aut}(\Lambda_{R'}, q)$ is a central extension of Suzuki's sporadic simple group by $\mathbb{Z}/6\mathbb{Z}$;

$\mathrm{Aut}(\Lambda_R, h)$ is a central extension of Hall-Janko's simple group of order 604 800 by $\mathbb{Z}/2\mathbb{Z}$; it is generated by the quaternion unitary reflections $r_a : x \mapsto x - \frac{1}{2} a . h(a,x)$ with $a \in \Lambda$ and $h(a,a) = 4$ (that those reflections map Λ onto itself follows from (1)).

We finally recall that, besides those three sporadic groups, nine others (out of the 26 "known" ones) occur in $\cdot 0$ as quotients of stabilizers of simple configurations, namely Conway's groups $\cdot 2$ and $\cdot 3$, the groups of Higman-Sims and McLaughlin, and the five Mathieu groups.

References.

J. H. Conway. A perfect group of order 8,315,553,613,086,720,000. Bull. Lond. Math. Soc., 1 (1969), 79-88.

— . A characterisation of Leech's lattice, Inventiones Math. 7 (1969), 137-142.

— . Three lectures on exceptional groups. In Finite simple groups, ed. by M. B. Powell and G. Higman, Academic Press, 1971, 215-247.

J. Tits. Résumés de cours. Annuaire du Collège de France, 77e année, 1976-1977, 57-66; 78e année, 1977-1978, 80-81.

— Quaternions over $\mathbb{Q}(\sqrt{5})$, Leech's lattice and the sporadic group of Hall-Janko. Journal of Algebra, to appear.

Titel: G. Segal's Burnside Ring Conjecture

Autor: J. F. Adams

Adresse: D.P.M.M.S., Mill Lane, Cambridge, U.K.

Let $A(G)$ be the Burnside ring of the finite group G; it is defined like the Grothendieck ring $R(G)$, but instead of representations one uses actions of G on finite sets. Let $A(G)^\wedge$ be its completion with respect to the topology given by powers of the augmentation ideal. Considerations of transfer [1] lead one to a natural transformation

$$A(G)^\wedge \longrightarrow \pi^0(BG),$$

where π^0 means stable cohomotopy. Segal conjectured [3] that this transformation is iso, and that $\pi^i(BG)$ is zero for $i > 0$. The case $G = Z_2$ has recently been proved by W.H. Lin [4]. His proof involves calculations which are somewhat demanding. A different and perhaps more enlightening proof has been found by Davis and Mahowald [2]. Their proof can be further simplified.

Autor: J. F. Adams Seite Nr.: 2

[1] J. F. Adams, "Infinite Loop Spaces", Princeton University Press, 1978, Chap. IV.

[2] D. M. Davis and M. E. Mahowald, "A Splitting Theorem in Homological Algebra", submitted to Math. Proc. Cambridge Phil. Soc.

[3] E. Laitinen, "On the Burnside ring and stable cohomotopy of a finite group", Aarhus University Publication, 1978.

[4] W. H. Lin, "Homotopy of the Spectrum $P_{-\infty}^{\infty}$ and Stable Cohomotopy of RP^{∞}", submitted to Math. Proc. Cambridge Phil. Soc.

Titel: Converse Galois problems over cyclotomic fields

Autor: F. Bogomolov

Adresse: Moscow, USSR, Math. Inst. of the Academy of Science.
V-333 ul. Vavilova 42. Dept. of Algebra

This report is devoted to the results of G. Bely [1] on the "converse Galois problem" of constructing an extension K' of a number field K with a given Galois group B. Of course the case $K = \mathbb{Q}$ is the one we really want. But is much easier to deal with the maximal abelian extension $\mathbb{Q}_{ab}$ of the field $\mathbb{Q}$. The main result of [1] is the construction of such extensions of Q_{ab} for groups from the following list of Chevalley groups:

1) $GL(n,q)$, $SL(n,q)$, $PSL(n,q)$
2) $SO(2n+1,q)$, q odd
3) $CSp(2n,q)$, $Sp(n,q)$, $PSp(2n,q)$, q odd and $q \neq 9$ if n is even
4) $U(n,q)$, $SU(n,q)$, $PSU(n,q)$, q odd and $q \neq 81$ if n is even.

In fact, in [1] a specific finite abelian extension L_G of $\mathbb{Q}$ is described over which the Galois group G can be realized.

We describe the main steps of the proof and also some other ideas of Bely. We start with a general construction. Consider the field $\mathbb{Q}(t)$ of rational functions of one variable t over $\mathbb{Q}$ and the maximal extension K of this field which is ramified only at $t=0, 1, \infty$. The field K includes $\bar{\mathbb{Q}}(t)$ and is a Galois extension of $\mathbb{Q}(t)$. We obtain the following exact sequence for $G_K = Gal(K/Q(t))$:

$$(1) \qquad 1 \to F \to G_K \to Gal(\bar{\mathbb{Q}}/\mathbb{Q}) \to 1 .$$

The group F is the "geometrical" Galois group of all finite coverings of $\mathbb{P}^1$ ramified only at $0,1,\infty$. Therefore F is the completion of the free group on two generators $\pi_1(P^1-\{0,1,\infty\})$ in the topology induced by the subgroups of finite index in π_1. The exact sequence (1) is constructed by means of a homomorphism $\tau: Gal(\bar{\mathbb{Q}}/\mathbb{Q}) \to Aut\, F/Inn\, F$, where $Inn\, F$ is the subgroup of inner automorphisms of F. If we denote the natural cyclic decomposition subgroups of F at the points $0, 1, \infty$ by $\langle x\rangle$, $\langle y\rangle$, $\langle xy\rangle$, respectively, then the main property of the image $\tau(\sigma)$ for $\sigma \in Gal(\bar{\mathbb{Q}}/\mathbb{Q})$ is that $\langle x^\sigma\rangle \sim \langle x\rangle$, $\langle y^\sigma\rangle \sim \langle y\rangle$, $\langle (xy)^\sigma\rangle \sim \langle xy\rangle$ where $\sim$ means conjugation. So if we define A as the subgroup of elements $g \in Aut\, F$ satisfying

i) $g(x) = x^\alpha$, $\alpha \in \hat{\mathbb{Z}}^*$ ($\hat{\mathbb{Z}}$ completion of $\mathbb{Z}$, $\hat{\mathbb{Z}}^*$ = invertible elements),
ii) $g(y) = uy^\alpha u^{-1}$ with $u \in [F,F]$, iii) $g(xy) \sim (xy)^\alpha$,

then $A \cap \mathrm{Inn}\, F = 1$ and τ factors through $A \subset \mathrm{Aut}\, F \to \mathrm{Aut}\, F/\mathrm{Inn}\, F$. We obtain the following theorem.

Theorem 1: The group G_K is the semidirect product of $\mathrm{Gal}(\bar{\mathbb{Q}}/\mathbb{Q})$ and F constructed by means of a homomorphism $\tilde{\tau}: \mathrm{Gal}(\bar{\mathbb{Q}}/\mathbb{Q}) \to A$. The homomorphism $\tilde{\tau}$ is a monomorphism.

Conjecture (G. Bely): The image of $\tilde{\tau}$ is not too far from A (maybe a subgroup of finite index, or even all of A).

There is also a geometrical result about the subfields of K.

Theorem 2: Iff the curve X is defined over a finite extension of $\mathbb{Q}$ there exists a morphism $\varphi: X \to P^1$ defined over $\mathbb{C}$ and ramified only over the points $0, 1, \infty$.

This means that any field $L(X)$, $[L:\mathbb{Q}] < \infty$, is a subfield of K. From the proof we can conclude that an even stronger result holds: if $f: X \to P^1$ is any morphism whose ramification points are algebraic, then we can choose a polynomial $g: P^1 \to P^1$ such that $gf: X \to P^1$ has only $0, 1, \infty$ as ramification points. The proof is obtained by using two induction processes. The first gives us a ϕ' all of whose ramification points in P^1 are rational. The second uses a function of the type $t \to t^a(t-1)^b$ on P^1 which maps the 4 points $0, 1, \frac{a}{a+b}, \infty$ to the 3 points $0, c, \infty$ and is ramified at only these 3 points; using it we can reduce the number of rational ramification points to three.

We now define two special classes of finite groups.

I) The finite group B lies in the class Γ' iff B has two generators a, b such that if a' and b' are two other generators with $a' \sim a$, $b' \sim b$, $a'b' \sim ab$ then there exists an automorphism σ of B taking a to a' and b to b'.

II) $B \in \Gamma$ if we can choose σ to be an inner automorphism.

Let $\Lambda(B,a,b)$ be the subgroup of finite index of $\mathbb{Z}$ consisting of those elements $\alpha \in \hat{\mathbb{Z}}^*$ with $a^\alpha \sim a$, $b^\alpha \sim b$, $(ab)^\alpha \sim ab$ in B and let L_B be the fixed field of $\Lambda(B,a,b)$ under the isomorphism $\mathrm{Gal}(\mathbb{Q}_{ab}/\mathbb{Q}) = \hat{\mathbb{Z}}^*$. Define $L_B' \supset L_B$ similarly but with the extra condition $a^\alpha = a$.

Theorem 3: a) If $B \in \Gamma'$, then there exists a covering $\phi: X \to P^1$ ramified only at $0, 1, \infty$ and defined over L_B such that $\mathrm{Gal}(X/P^1) = B$.

b) If $B \in \Gamma$, then we can choose ϕ in such a way that all of the automorphisms of the "geometric" Galois group are defined over Q_{ab}

c) If also the center of B is a direct summand of the centralizer of the element a in B, then these automorphisms are defined over L'_B.

Corollary: Under the assumption b) (resp. c)), there exists a normal extension of $\mathbb{Q}_{ab}$ (resp. L'_B) with B as Galois group.

The proof of the theorem uses standard arguments of the Galois theory for G_K and the description of the elements of A. The corollary follows immediately using Hilbert's irreducibility criterion.

Now we have to prove that the groups from the list lie in the class . This is done using the following theorem.

Theorem 4 (Sufficient criteria). Suppose that V is a vector space of finite dimension over some algebraically closed field $\bar{k}$ (in our case, $\bar{k}$ will be of characteristic 0), $D = \mathrm{Aut}\, V$, H a finite group with two generators a and b, and $\rho : H \to D$* an irreducible representation such that for some $\zeta \in \bar{k}$ the matrix $\rho(a) - \zeta \cdot 1$ has rank 1. Then $H \in \Gamma'$. If also the normalizer of $\rho(H)$ in D is the product of H and the centralizer of H in D, then $H \in \Gamma$.

To apply this theorem we have to choose generators for the groups in the list by means of their standard embedding in matrix groups, using [2].

References:

[1] G. Bely, On Galois extensions of the maximal cyclotomic field (in Russian), Izvestiya Akad. Nauk. USSR (1979), vol. 43

[2] R. Steinberg, "Lectures on Chevalley Groups", Lecture Notes, Yale University

* ρ- monomorphism

Titel: On Sieve method and Goldbach problem.

Autor: Wang Yuan

Adresse: Institute of Mathematics, Academia Sinica, Peking, China

The historical origin of sieve method is the well-known "sieve of Eratosthenes". Eratosthenes noted that the prime numbers between $n^{1/2}$ and n can be isolated by removing from the sequence $2, 3, \cdots, n$ every number which is multiple of a prime not exceeding $n^{1/2}$. Let $\pi(x)$ denote the number of primes $\leq x$ and $\Pi = \prod_{p \leq n^{1/2}} p$, where p denotes prime. Then

$$1 + \pi(n) - \pi(n^{1/2}) = \sum_{a \leq n} \sum_{d|(a,\Pi)} \mu(d) = \sum_{d|\Pi} \mu(d) \left[\frac{n}{d}\right],$$

where $\mu(d)$ denotes the Möbius function. If we use $\frac{n}{d} + \vartheta$ to instead of $[\frac{n}{d}]$, then it will caused an error term

$$O(2^{\pi(\sqrt{n})})$$

in above formula, so the sieve of Eratosthenes with its large error term is almost useless.

It was a great achievement when Viggo Brun in 1919 devised his new sieve method and applied it successfully to several difficult and important problems in number theory. In 1947, A. Selberg gave another sieve method which is much simple and leads to more precise results than the more complicated Brun's method. Indeed these methods represent an indispensable tool in number theory.

The essence of the methods of Brun and Selberg is to use some inequalities to instead of

$$\sum_{d|n} \mu(d) = \begin{cases} 1, & \text{if } n = 1, \\ 0, & \text{if } n \neq 1. \end{cases}$$

in order to decrease the error term. For example, for any given set of real numbers λ_d's with $\lambda_1 = 1$, then

$$\sum_{d|n} \mu(d) \leq \left(\sum_{d|n} \lambda_d\right)^2.$$

Chose suitable λ_d's, then we obtain the Selberg's upper bound method.

Two famous conjectures are connected with the sieve method.

(a) Goldbach conjecture. Every ~~large~~ even integer n (≥ 4) is a sum of two primes.

(b) prime twins conjecture. There exists infinitely many prime twins $(p, p+2)$.

These two problems may be treated similarly, so we state only the Goldbach conjecture.

Let $A = \{a_\nu\}$ be a set of integers. Let P denote a set of r primes $p_1 < \cdots < p_r$. Further let $S(A, P)$ denote the number of elements in A which is unsifted by the sequence of P. Take $A = \{\nu(n-\nu),\ \nu = 1, 2, \cdots, n\}$ and P the set of all primes $\leq n^{\frac{1}{l+1}}$, where l is a natural number. Suppose that we can obtain a positive lower estimation for $S(A, P)$ when n is large. Then it follows that

(a′) Every large even integers n is the sum of two numbers each being a product of at most l prime factors.

The proposition (a′) is denoted by (l, l). Similarly, we may define (l, m) for $l \neq m$.

Brun was the first who proved $(9, 9)$. Brun's method and his results were improved by several mathematicians, for examples

$(7, 7)$ (H. Rademacher, 1924),

$(6, 6)$ (T. Estermann, 1932),

$(5, 7)$, $(4, 9)$, $(3, 15)$, $(2, 366)$ (G. Ricci, 1937),

$(5, 5)$, $(4, 4)$ (A. A. Buchstab, 1938–1940),

(a, b) $(a+b \leq 6)$ (P. Kuhn, 1953–1954).

Wang Yuan proved in 1956–1957

$(3, 4)$, $(3, 3)$, (a, b) $(a+b \leq 5)$, $(2, 3)$.

in which (3,3) was obtained by A. I. Vinogradov independently in 1956 and (2,3) was announced by Selberg in 1949, but no proof of (2,3) had been appeared.

If we take $A = \{n-p,\ p \le n\}$, where p denotes prime number and P the set of all primes $\le n^{\frac{1}{\ell+1}}$, then a positive lower estimation of $S(A,P)$ leads the proposition $(1,\ell)$.

In 1932, Estermann first proved (1,6) under the assumption of GRH (grand Riemann hypothesis). Without any unproved hypothesis, A. Rényi proved $(1,c)$ in 1948, where c is a constant. In Rényi's proof, a mean value theorem of $\pi(x;k,\ell)$ is proved by means of the so called large sieve of Linnik and Rényi that may be used to instead of quasi-RH, namely

$$(1) \qquad \sum_{k \le x^{\delta}} \max_{(\ell,k)=1} \left| \pi(x,k,\ell) - \frac{li\,x}{\varphi(k)} \right| = O\left(\frac{x}{\log^{3} x}\right),$$

where $\pi(x;k,\ell) = \sum\limits_{\substack{p \le x \\ p \equiv \ell \pmod k}} 1$, $li\,x = \int_2^x \frac{dt}{\log t}$, $\varphi(k)$ the Euler function and δ a certain positive constant. Notice that $\pi(x;k,\ell)$ should be replaced by a sum with a weight in his original paper. If the formula (1) holds for $\delta = \frac{1}{2} - \varepsilon$ for any pre assigned positive number ε, it may be used to instead of the GRH in the proof of Estermann's result (1,6).

Wang Yuan improved the 6 to 3, that is, (1,3) under the assumption of GRH or (1) with $\delta = \frac{1}{2} - \varepsilon$.

In 1961, M. B. Barban proved that (1) holds for $\delta = \frac{1}{6}$. In 1962, Pan proved independently that (1) is true for $\delta = \frac{1}{3}$. By the combination of Wang's argument of the proof of (1,3), he derived (1,5). Barban and Pan also proved (1) with $\delta = 3/8$ and derived

(1,4). Finally, E. Bombieri and A. I. Vinogradov established independently the formula (1) with $\delta = \frac{1}{2} - \varepsilon$, so they obtained (1,3). More precisely, Bombieri's formula may be stated as follows

$$\sum_{k \le x^{1/2}/(\log x)^B} \max_{(l,k)=1} \left| \pi(x,k,l) - \frac{\operatorname{li} x}{\varphi(k)} \right| = O\left(\frac{x}{(\log x)^A}\right),$$

where A is a given positive constant and $B = B(A)$. Although Bombieri's result is slightly stronger than that of Vinogradov but Bombieri's formula has many other important applications in number theory.

In 1966, Chen[1] gave the previous argument of the proof of (1,3) an important improvement. So he proved (1,2), that is

Theorem 1 (Chen) Every large even integer is a sum of a prime and a product of at most 2 primes.

There are several simplified proofs of Chen's theorem of which one was given by Pan Cheng-Dong, Ding Xia Xi and Wang[2] in 1975. It may be sketched as follows.

Let n denote an even integer and $S(n, q, n^{\frac{1}{c}})$ denote the number of primes such that

$$2 \le p \le n, \quad n-p \not\equiv 0 \pmod{p'}, \quad n-p \equiv a \pmod{q}, \quad 2 \le p' < x^{1/c}, \ p' \nmid q$$

Let

$$M = S(n, 2, n^{\frac{1}{10}}) - \frac{1}{2} \sum_{n^{\frac{1}{10}} < p \le n^{\frac{1}{3}}} S(n, 2p, n^{\frac{1}{10}}) - \frac{1}{2} \sum_{\substack{(P_1,2) \\ P_3 \le n/P_1P_2 \\ n-p = P_1P_2P_3}} 1,$$

where P, P_1, P_2, P_3 are primes and $(P_{1,2})$ denotes the condition $n^{\frac{1}{10}} < p \le n^{1/3} \le P_2 \le \left(\frac{n}{P_1}\right)^{1/2}$. Then $M > 0$ for sufficiently large n implies the (1,2), since there exists p such that $n-p$ has at most 1 prime factor in the interval $[n^{\frac{1}{10}}, n^{1/3})$ and 1 prime factor $> n^{1/3}$ or $n-p$ has only the prime factors $> n^{1/3}$. The first and second terms of M may be evaluated by the sieve method of Selberg and the mean value theorem of Bombieri. The third term may be estimated by a mean value theorem similar to those of Bombieri.

Let $2 \le y \le x$. Let $\pi(y, a, q, l) = \sum_{\substack{p \le y/a \\ ap \equiv l \pmod q}} 1$. Then we have

Theorem 2. For any given positive constant A and positive number $\varepsilon (< 1)$, the estimation

$$I = \sum_{q \le x^{1/2} \log^{-B} x} \max_{y \le x} \max_{(l,q)=1} \left| \sum_{\substack{A_1 < a \le A_2 \\ (a,q)=1}} f(a) \left(\pi(y,a,q,l) - \frac{li \frac{y}{a}}{\varphi(q)} \right) \right| = O\left(\frac{x}{\log^A x}\right)$$

holds for $\log^{2B} x < A_1 \le A_2 < y^{1-\varepsilon}$, where $|f(a)| \le 1$, $B = A + 7$ and the constant implied by the symbol "O" depends only on ε and A.

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Titel:
Size of representations

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Let G be a topological group. An irreducible unitary representation of G is a continuous homomorphism of G into the group of unitary operators on a Hilbert space, such that no non-trivial closed subspace is invariant under all the operators in the image. Such representations can often be realized in some concrete way; for example if G acts in a nice way as automorphisms of some hermitian vector bundle $\mathcal{V}$ on a manifold M with a measure, then it acts on the Hilbert space $L^2(M,\mathcal{V})$ of square integrable sections of $\mathcal{V}$. Gelfand suggested the problem of attaching to abstract representations some invariants which, for representations of this particular kind, would be just the dimensions of M and $\mathcal{V}$. (This might be thought of as a step toward realizing abstract representations in a nice way.)

We discuss here the notion of Gelfand-Kirillov dimension for representations of semisimple Lie groups, and show that it has several properties suggested by the preceding motivation. Detailed formulations of most of the definitions and results may be found in (1) and (4). We attach to a representation $(\pi,\mathcal{H})$ of such a group G a finitely generated graded module for the symmetric algebra S(g) of the Lie algebra of G; then the Gelfand-Kirillov dimension $d(\pi)$ and the multiplicity $m(\pi)$ are the transcendence degree and multiplicity of this module. Then $d(\pi)$ is half the GK dimension of the enveloping algebra U(g) modulo the kernel I_π of the differentiated representation, as one would expect from the strucure of the algebra of differential operators on a manifold. If $L \in U(g)$ is a Laplacian defined with respect to a positive form on g invariant under a maximal compact subgroup, then the eigenvalues of $\pi(L)$ have asymptotic properties exactly like those of the eigenvalues of the Laplacian on a compact Riemannian manifold of dimension $d(\pi)$. Finally, in analogy with the formula

$$\dim(\pi) = \operatorname{tr}(\pi(1))$$

for finite dimensional representations, the singularity of the distribution character of π at 1 is exactly measured by $d(\pi)$.

The lecture concluded, following Joseph (3), by attaching to π a "characteristic polynomial" $p(\pi)$ on a Cartan subalgebra of g. This polynomial measures an analogue of dim $\mathcal{V}$ in the setting of the first paragraph, and is related to several more technical questions in representation theory and harmonic analysis (primitive ideals in U(g), Fourier inversion of unipotent orbital integrals, and possibly the Springer correspondence .)

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Titel: NEW EXAMPLES OF EINSTEIN MANIFOLDS

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An Einstein manifold is a Riemannian manifold whose Ricci tensor is proportional to the metric. (Notice that, up to homotheties, only the sign of the factor of proportionality matters, i.e. +, 0, − .)

Until recently, very few examples of compact (or even complete) Einstein manifolds were known, namely :

- manifolds with constant sectional curvature,
- irreducible Riemannian symmetric spaces,
- some other homogeneous spaces (including a non-canonical S^{4n+3} due to G. Jensen). Notice that the Einstein homogeneous manifolds are not classified.

The first compact nonhomogeneous examples of Einstein manifolds came from the solution of the Calabi conjecture : "if a complex compact manifold M has a Kähler metric and $c_1^{\mathbb{R}}(M)$ is zero (resp. negative definite), then M has a Kähler Einstein metric with zero (resp. negative) factor (S.T. Yau, T. Aubin) (see [2] for a survey of the proofs and references therein). Notice that the analogous conjecture with a positive definite $c_1^{\mathbb{R}}(M)$ fails : the complex surface F_1 ($\mathbb{C}P^2$ with one point blown up) has $c_1^{\mathbb{R}}(M)$ positive definite and no Kähler Einstein metric (known to E. Calabi).

Very recently, Don Page (cf [1]) has given an hermitian Einstein metric with positive factor on F_1. His proof uses the Kerr-de Sitter Lorentzian metric and concepts borrowed from general relativity. I want to give another description of this example : there is an action of $U(2)$ on F_1, such that the principal orbits are 3-spheres with only two 2-spheres as exceptional orbits. So F_1 may be viewed as $S^3 \times [a,b]$ with the two components of the boundary contracted along the Hopf fibration $S^3 \to S^2$. Moreover, by $U(2)$-invariance, the metric on $S^3 \times]a,b[$ must split as $\alpha_t + dt^2$ where t parametrizes $[a,b]$ and α_t is a $U(2)$-invariant metric on S^3. Such metrics on S^3 depend upon two parameters, the sizes $f(t)$ and $g(t)$ respectively of the fiber and of the basis of the Hopf fibration. Using B. O'Neill's formulas (cf [3]) for the two Riemannian submersions : $(S^3,\alpha_t) \to (S^2, g^2(t).\mathrm{can})$ and $(S^3 \times]a,b[, \alpha_t + dt^2) \to (]a,b[, dt^2)$, one easily finds that $\alpha_t + dt^2$ is Einstein if and only if :

$$(1) \quad f\,g^3 g'' - g^3 g' f' + f^3 = 0$$

$$(2) \quad f\,g^3 g'' - f g^2 g'^2 + g^4 f'' - g^3 g' f' + 4\, f g^2 - 2\, f^3 = 0 .$$

These equations have many solutions, but in order to get a metric on F_1, one needs boundary conditons (roughly, the Taylor expansion of g (resp. f) at a and b is "even" (resp. "odd" with first derivative one)). Then one is left with only one solution up to homotheties : this is Don Page's metric.

Now the same methods apply to other groups, other (codimension one) principal orbits and other dimensions. Among others, one has the following results:

1. The compact Einstein 4-manifolds with an isometry group of dimension bigger than or equal to 4 are S^4, $\mathbb{C}P^2$, $S^2\times S^2$, T^4 with their usual metrics, $\mathbb{C}P^2 \# \overline{\mathbb{C}P}^2$ with Don Page's metric and some of their discrete quotients. (Notice that F_1 is topologically $\mathbb{C}P^2 \# \overline{\mathbb{C}P}^2$ and also the total space of the unique nontrivial S^2-bundle over S^2, but this space has many complex structures.)

2. There are hermitian Ricci-flat non-flat U(2)-invariant metrics on $\mathbb{R}^4$ and the total spaces of the line bundles $\mathcal{O}(0)$, $\mathcal{O}(-1)$, $\mathcal{O}(-2)$ over S^2.

3. There is a family of hermitian Einstein U(2)-invariant metrics with negative factor on $\mathbb{R}^4$ and all the $\mathcal{O}(-p)$.

4. There is an hermitian Einstein U(n+1)-invariant metric on $\mathbb{C}P^{n+1} \# \overline{\mathbb{C}P}^{n+1}$ and families of such metrics on the total space of line bundles over $\mathbb{C}P^n$.

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Titel: Infinite-dimensional Lie algebras as an underlying structure

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Some time ago the celebrated Macdonald identities [1] were interpreted as a Weyl denominator formula for certain infinite-dimensional Lie algebras [2], [3]. A specialization of these identities in the unity element gives a θ-series type expansion for $\eta(q)^{\dim G}$ over the root lattice of the complex simple group G [1]. This formula embraces most of the known expansions of this particularly simple type. However there is an analogous Atkin's expansion for $\eta(q)^{26}$ [34] but there is no simple groups of dimension 26.

It was noticed not long ago that most of the irreducible linear groups with a particularly simple orbit structure appear as a "part" of a $\mathbb{Z}_m$-gradation of a finite-dimensional simple Lie algebra [5]. The most interesting exception is $Spin_{13}$ [6] which should appear ~~as~~ a "part" of a finite-dimensional Lie algebra of type F_6, which does not exist.

Recently McKay noticed that one of the coefficients in the q-series of the modular invariant $j(q) = q^{-1} + 744 + 196884q + 21493760q^2 + \ldots$

is the dimension of the sum of lowest non-trivial and 1-dimensional representations of the Fisher-Griess Monster group M, and Thompson found that the later coefficients are also dimensions of some representations of the group M (see [7]) Moreover, it is suggested in [7] that there is a sequence of M-modules V_k such that $q^{-1} + \sum_{k \geq 1} (\operatorname{tr} \sigma|_{V_k}) q^k$ is the q-series of a modular function of weight 0 for any $\sigma \in M$. Even more, all these functions are Hauptmoduls and ~~all~~ most of them appear in a natural way in connection with the Leech lattice.

McKay also discovered the first several coefficients of $(q j(q))^{\frac{1}{3}}$ are the dimensions of some representations of the exceptional Lie group E_8.

In this talk I want to give an explanation of this phenomenon [8]. The underlying structure is the infinite-dimensional Lie algebra $\hat{E}_8^{(1)}$ and the root lattice of E_8 plays the same role as the Leech lattice does for the Monster. I beleive that this gives an evidence of the existence of the underlying structures for the mysterious phenomena mentioned above (e.g. there exist structures with the "root lattices" isomorphic to the Atkin and Leech lattices).

Let L be a finite-dimensional complex simple Lie algebra of rank n of one of the types A_n, D_n, E_6, E_7, E_8. Let H be the Cartan subalgebra of L, $\alpha_1, \dots, \alpha_n$ be simple roots, Q be the lattice generated by $\alpha_1, \dots, \alpha_n$, θ be the highest root, $t_1, \dots, t_n$ be the basis of H dual to $\alpha_1, \dots, \alpha_n$. Let A be the Cartan matrix of L, h be the Coxeter number and $\langle \; , \; \rangle$ be the Killing form. Let $\mathcal{L}$ be the complex Lie group with the Lie algebra L.

Define an infinite-dimensional complex Lie algebra $\hat{L}$ as a complex space $(\mathbb{C}[t,t^{-1}] \otimes_{\mathbb{C}} L) + \mathbb{C}c$ with the following bracket:

$$[g_1 + a_1 c, g_2 + a_2 c] = [g_1, g_2] + \frac{1}{2h} \langle \frac{dg_1}{dt}, g_2 \rangle c,$$

where $g_1, g_2 \in \mathbb{C}[t, t^{-1}] \otimes_{\mathbb{C}} L$, $a_1, a_2 \in \mathbb{C}$.

There exists a unique irreducible $\hat{L}$-module V for which there is a non-zero vector v such that

$$(\mathbb{C}[t] \otimes_{\mathbb{C}} L)(v) = 0 \quad \text{and} \quad c(v) = v.$$

The module V admits a unique $\mathbb{Z}_+$-gradation $V = \bigoplus_{k \in \mathbb{Z}_+} V_k$ such that $V_0 = \mathbb{C}v$, $\dim V_k < \infty$, and $(t^s \otimes g) V_k \subset V_{s+k}$, $g \in L \subset \hat{L}$, $s \in \mathbb{Z}_+$.

Introduce the character of the module V as a function on $\mathcal{L}$ with the values in q-series as follows:

$$(\operatorname{ch} V)(\sigma) = \sum_{k \geq 0} (\operatorname{tr} \sigma)|_{V_k} q^k, \quad \sigma \in \mathcal{Z}.$$

Let $\sigma = \exp 2\pi i \sum_s z_s t_s \in \mathcal{Z}$; set $q_1 = q e^{-2\pi i {}^t\gamma z}$.

Then we have the following formula [3], [8]:

$$(1) \quad q_1^{-\frac{n}{24}} (\operatorname{ch} V)(\sigma) = \eta(q_1)^{-n} \sum_{\gamma \in Q} q_1^{\frac{1}{2} {}^t\gamma A \gamma} e^{2\pi i {}^t\gamma z}$$

where $\eta(q) = q^{\frac{1}{24}} \prod_{k \geq 1} (1 - q^k)$ is the Dedekind η-function. Setting $\sigma = e$ in (1) we obtain:

$$q^{-\frac{n}{24}} \sum_{k \geq 0} (\dim V_k) q^k = \frac{\theta_Q(q)}{\eta(q)^n}, \text{ where}$$

$\theta_Q(q)$ is the classical θ-series of the lattice Q.

In the case $L = E_8$ we obtain:

$$q^{-\frac{1}{3}} \sum_{k \geq 0} (\dim V_k) q^k = j(q)^{\frac{1}{3}}$$

This follows from the usual formula for $j(q)$. The latter formula explains McKay's phenomenon.

Formula (1) also shows that if σ is an element of finite order in $\mathcal{Z}$, then the right-hand side of (1) is the q-series of a modular function of weight 0.

In this connection, I would like to suggest the following conjecture. Suppose that n divides 24, say $24 = ns$. Denote by $f_\sigma(q)$ the right-hand side of (1) where q_1 is replaced by q^s. Then $f_\sigma(q)$ is the q-series of a Hauptmodul if and only if $\sigma \in \mathcal{Z}(Q)$. (i.e. σ is a conjugate of a rational element).

This suggests that the Monster might be a subgroup of a "canonically" defined infinite group $\hat{M}^{(\mathbb{C})}$, which has a "canonical" irreducible module V with an M-invariant $\mathbb{Z}_+$-gradation $V = \oplus V_k$ such that $q^{-1} + \sum_{k>0} (\dim V_k) q^k = j(q) - 744$.

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1

Titel: A new class of negatively curved surfaces

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It has been conjectured that a compact Kähler manifold M of strictly negative sectional curvature has as its simply connected cover the bi-holomorphic image of the ball in $\mathbb{C}^n$, $n = \dim_{\mathbb{C}} M$. The corresponding conjecture for positive sectional curvature is the Frankel conjecture:

A compact Kähler manifold of positive sectional curvature is biholomorphic to complex projective space.

This Frankel conjecture was proved in $\dim_{\mathbb{C}} 2$ case by Andreotti-Frankel, in $\dim_{\mathbb{C}} 3$ by Mabuchi and in general by Mori [1] using char p algebraic geometry and by Siu-Yau [4] using complex analyticity of harmonic maps.

By contrast, the negative sectional curvature conjecture is false, for the new class of negatively curved M with $\dim_{\mathbb{C}} M = 2$ that is defined as follows.

Let $V = \mathbb{C}e_1 + \mathbb{C}e_2 + \mathbb{C}e_3$, $\eta = e^{\pi i/p}$, $H(z,w) \equiv \langle z, w\rangle$ the hermitian form on V with

$$\langle e_1, e_2\rangle = \langle e_2, e_3\rangle = \langle e_3, e_1\rangle = \frac{1}{2\sin\frac{\pi}{p}}\varphi$$

$$\langle e_i, e_i\rangle = 1, \quad (i = 1,2,3)$$

where $\varphi \in \mathbb{C}$ with $|\varphi| = 1$, $p = 3, 4, 5$. Define for $z \in V$

$$R_i : z \longrightarrow z + (\eta^2 - 1)\langle z, e_i\rangle e_i, \quad (i = 1,2,3)$$

Γ_{ij} = group generated by $\{R_i, R_j\}$ $(i \neq j)$

Each group Γ_{ij} is finite of order $24\left(\frac{p}{6-p}\right)^2$ with relations:

$$R_i^p = 1 \quad (i = 1,2,3)$$

$$R_i R_j R_i = R_j R_i R_j, \quad i \neq j;$$

and $(R_1R_2R_1)^2$ is of order $\frac{2p}{6-p}$ and generates the center of Γ_{ij}.

Set $\Gamma = \Gamma(\varphi)$ = group generated by $\{R_1, R_2, R_3\}$

The hermitian form H is of type $(++-)$ for $\arg(\varphi^3) < 3\left(\frac{\pi}{2} - \frac{\pi}{p}\right)$ and $\Gamma \subset U(H)$, the unitary group of H.

Set

$$V^- = \{z \in V;\ \langle z, z\rangle < 0\}$$

$$\mathbb{C}h^2 = V^-/\mathbb{C}^* \hookrightarrow \mathbb{C}P^2$$

$\mathbb{C}h^2$ is hermitian hyperbolic space with the $U(H)$ invariant metric

$$d(z_1, z_2) = \cosh^{-1}\left(\frac{\langle z_1, z_2\rangle\langle z_2, z_1\rangle}{\langle z_1, z_1\rangle\langle z_2, z_2\rangle}\right)^{\frac{1}{2}}$$

Choose $p_0 \in V$ so that

$\langle p_0, e_1 \rangle = \langle p_0, e_2 \rangle = \langle p_0, e_3 \rangle$

For any $\gamma \in \Gamma$ set

$\gamma^+ = \{ x \in \mathbb{C}h^2 ;\ d(x,p_0) \le d(\gamma x, p_0) \}$

$\hat{\gamma} = \{ x \in \mathbb{C}h^2 ;\ d(x,p_0) = d(\gamma x, p_0) \}$

Clearly $\gamma \hat{\gamma} = \hat{\gamma}^{-1}$.

Set $F_{ij} = \bigcap_{\gamma \in \Gamma_{ij}} \gamma^+$

$F = F_{12} \cap F_{23} \cap F_{31}$

$\tilde{\gamma} = \hat{\gamma} \cap F$, for $\hat{\gamma}$ containing a 3-face of F.

Theorem The 3-faces of F are

$E_1(F) = \{ \tilde{R}_i^{\pm 1}, (R_i \tilde{R}_j)^{\pm 1}, (R_i \tilde{R}_j R_i)^{\pm 1} ;\ i \ne j,\ i,j = 1,2,3 \}$

Theorem For each $\text{codim}_{\mathbb{R}} 1$ face $\tilde{\gamma}$ of F

$\gamma \cdot \tilde{\gamma} = \tilde{\gamma}^{-1}$

Lemma Γ is discrete $\Longleftrightarrow$ F satisfies

(CD2): For each $\text{codim}_{\mathbb{R}} 2$ face e, write

$e = \tilde{\gamma}_0 \cap \tilde{\gamma}_1^{-1}$

$\gamma_1 e = \tilde{\gamma}_1 \cap \tilde{\gamma}_2^{-1}$

$\vdots$, $i = 1, 2, \dots$,

where $\tilde{\gamma}_i \in E_1(F)$

Then $F \cap \gamma_1 \gamma_2 \cdots \gamma_n F$ has a non-empty interior

implies $F = \gamma_1 \gamma_2 \cdots \gamma_n F$.

Theorem (CD2) holds $\Longleftrightarrow$ $\frac{\pi}{2} - \frac{\pi}{p} = \arg(\varphi^3) = \frac{2\pi}{\text{integer}}$

(i.e. both conditions hold)

Cor. $\Gamma(\varphi)$ is discrete for only finitely many φ.

~~Fo~~ Form the topological direct product $\Gamma \times F$ and form the space

$$Y = \Gamma \times F / \equiv$$

defining $(\gamma, x) \equiv (\gamma', x')$ if

(1) $\gamma x = \gamma' x'$ with $x, x' \in \partial F$ and $\gamma^{-1}\gamma' \in E_1(F)$

or (2) $x \in \operatorname{Int} F$ and $\gamma^{-1}\gamma' \in \operatorname{Aut} F$

Let $\sigma : Y \to \mathbb{C}h^2$ be the map induced by $(\gamma, x) \to \gamma x$. Clearly Γ is discrete on Y.

Choose Γ_0 a torsion-free subgroup of Γ with Γ/Γ_0 finite. Set $k = \# \Gamma/\Gamma_0$

<u>Theorem</u> If $\frac{\arg \varphi}{\pi} \in \mathbb{Q}$ (the rational numbers) and $|\arg \varphi^3| < \frac{\pi}{2} - \frac{\pi}{p}$, then Y has a complex analytic structure such that the map $\sigma : Y \to \mathbb{C}h^2$ is holomorphic.

Set $M = \Gamma_0 \backslash Y$. Then M is a compact complex analytic manifold. For a more detailed announcement of the foregoing cf [2].

Together with Y. T. Siu (cf [3]) the following results are proved:

1. ~~M is an algebraic surface~~

2. There is a Kaehler metric on M such that the sectional curvature is strictly negative, provided σ order $\eta \cdot \varphi^3$ where $t = \frac{\arg(\varphi^3)}{\pi}$,

$\boxed{t = \frac{1}{2} - \frac{1}{p} - \frac{2}{\sigma}}$ [illegible].

Autor: Mostow Seite Nr.: 5

1. M is an algebraic surface
2. There is a Kähler metric on M such that the sectional curvature is strictly negative provided

$$\frac{\arg(\varphi^3)}{\pi} = \frac{1}{2} - \frac{1}{p} - \frac{2}{\sigma}$$

where σ = order $\bar{\eta}^{*}i\,\bar{\varphi}^3$ $\quad (\eta = e^{\pi\sqrt{-1}/p})$

3. Let $t = \frac{\arg \varphi^3}{\pi}$, p = order $\bar{\eta}^{*} i\, \varphi^3$. Then

$$\frac{c_1^2(M)}{c_2(M)} = \frac{\frac{3}{8}\left(\frac{3}{2} - \frac{6}{p} + \frac{6}{p^2} - 2t^2\right) + 5\,\frac{m-1}{4p}\left(\frac{1}{2} - \frac{1}{p} + t\right)}{\frac{1}{3} - \frac{1}{p} + \frac{1}{N} - \left(\frac{1}{2} - \frac{1}{p}\right)\left(\frac{1}{\rho} + \frac{1}{\sigma}\right) + \frac{1}{\rho\sigma}}$$

where $N = 24\left(\frac{p}{6-p}\right)^2$, $\frac{\pi}{2} - \frac{\pi}{p} + \arg\varphi^3 = \frac{2m\pi}{\rho}$.

$m = 1$ only in the cases

$p = 3$, $\sigma = 6, 7, 8, 9, 10, 12$

$p = 4$, $\sigma = 4, 5, 6, 8$

$p = 5$, $\sigma = 4, 5$

The ratio $\frac{c_1^2}{c_2} \leq 3$ and equals 3 if and only if $m = 1$. For $m > 1$, Y is not biholomorphic to the ball.

<u>Remark:</u> For $m > 1$, M is not diffeomorphic to a locally symmetric space. This is the first such example ~~of a 4~~ 4 dimensional compact Riemannian manifold.

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Representations of Hecke algebras

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This is a joint work with D. Kazhdan.

We consider the following ~~problem~~

Problem. Let E be a complex irreducible representation of the symmetric group $\mathfrak{S}_n$. Construct a basis $e_1, \dots, e_N$ of E and a collection $(n_{ij})_{1 \le i \ne j \le N}$ of ~~non~~ integers $n_{ij} = n_{ji} \geq 0$ with the following property

If $s \in \mathfrak{S}_n$ is a simple reflection and $1 \le i \le N$, then either

$$s e_i = -e_i$$

or

$$s e_i = e_i + \sum_{\substack{j \\ s e_j = -e_j}} n_{ij} e_j .$$

For example if $E = \{(x_1, \dots, x_n) \in \mathbb{C}^n \mid x_1 + \dots + x_n = 0\}$ with the standard $\mathfrak{S}_n$-action, the basis $e_1 = (1, -1, 0, \dots 0)$, $e_2 = (0, 1, -1, 0 \dots 0), \dots$ has the required property.

~~In general the existence of such a basis is a consequence of deep results of Joseph on primitive ideals in enveloping algebras.~~ We will sketch an ~~We will sketch an alternative~~ approach which actually gives a stronger result.

Let (W, S) be a Coxeter group with length function l.

Let $\mathcal{H}_0$ be the $\mathbb{Z}[q]$-algebra with basis T_w $(w \in W)$ and multiplication defined by

$$T_w T_{w'} = T_{ww'} \quad \text{if} \quad l(ww') = l(w) + l(w')$$

$$(T_s + 1)(T_s - q) = 0 \quad \text{if} \quad s \in S$$

The Hecke algebra $\mathcal{H}$ is defined to be $\mathcal{H} = \mathcal{H}_0 \underset{\mathbb{Z}[q]}{\otimes} A$ where $A = \mathbb{Z}[q^{1/2}, q^{-1/2}]$.

The ring $\mathcal{H}$ has a natural involution

$$\overline{\sum p_w(q) T_w} = \sum p_w(q^{-1}) T_{w^{-1}}^{-1} .$$

Let [redacted] $\le$ [redacted] be the standard partial order on W. Thus $y \le w \Leftrightarrow \exists$ reduced decomposition $w = s_{i_1} \cdots s_{i_k}$ such that y is obtained from w by dropping some of the s_{i_α}.

Theorem. (a) For any $w \in W$, there exists a unique element $C_w \in \mathcal{H}$ such that $\overline{C_w} = C_w$ and

$$C_w = \sum_{y \le w} \varepsilon_y \varepsilon_w \, q^{\frac{\ell(w)}{2} - \ell(y)} P_{y,w}(q^{-1}) T_y$$

($\varepsilon_y = (-1)^{\ell(y)}$) where $P_{y,w}(q)$ is a polynomial in q of degree $\le \frac{1}{2}(\ell(w) - \ell(y) - 1)$ if $y < w$ and $P_{w,w}(q) = 1$.

(b) $T_s C_w = -C_w$ if $sw < w$

$$T_s C_w = q C_w + q^{1/2} C_{sw} + q^{1/2} \sum_{\substack{z < w \\ \varepsilon_z = -\varepsilon_w \\ sz < z}} \mu(z,w) C_z$$

where $\mu(z,w) =$ coefficient of $q^{\frac{1}{2}(\ell(w) - \ell(z) - 1)}$ in $P_{z,w}$.

(c) If W is a Weyl group or an affine Weyl group then $P_{y,w}$ have positive coefficients.

Specializing q to 1, we get a basis as desired for the regular representation of W (if W is a Weyl group). If W is the symmetric group, there exist two left W-submodules ~~subspaces~~ $I_1 \subset I_2$ of $\mathbb{C}[W]$ such that I_1 and I_2 are spanned by a subset of $\{C_w\}$ and such that I_2/I_1 is an [redacted] irreducible representation of $\mathfrak{S}_n$ given in advance. Clearly I_2/I_1 has a basis as required.

The proof of Theorem 1 is elementary, except for the proof of (a). That is based on a cohomological interpretation of $P_{y,w}$.

Let X be a complex algebraic variety of dimension n. Goresky and Macpherson associate to X some new invariants $\mathbb{H}^i(X)$, $\mathbb{H}^i_c(X)$ which they call middle cohomology groups (with rational coefficients). They satisfy Poincaré duality: $\dim \mathbb{H}^i(X) = \dim \mathbb{H}_c^{2n-i}(X)$. In the case where X is non-singular they are just the usual cohomology groups.

For example $\mathbb{H}_i(X)$ is defined as follows. Consider a stratification $X = X_0 \supset X_1 \supset X_2 \supset \dots \supset X_n$ where $X_i - X_{i+1}$ is non-singular, $\operatorname{codim}_{\mathbb{C}} X_i = i$. Consider k-cycles formed of k simplices which intersect $X_i - X_{i+1}$ in a set of dimension $\leq k-i$. We say that two such k-cycles are "homologous" if their difference is the boundary of a $(k+1)$-chain formed of $(k+1)$ simplices which intersect $X_i - X_{i+1}$ in a set of dimension $\leq k-i+1$. This defines $\mathbb{H}_i(X)$.

Let G be a semisimple algebraic group, B_0 a Borel subgroup, $T_0 \subset B_0$ a maximal torus and let W be the corresponding Weyl group. The set $\mathcal{B}$ of Borel subgroups has a natural structure of projective variety on which G acts transitively: $(g, B) \to gBg^{-1}$. The set $\mathcal{B}^{T_0} \subset \mathcal{B}$ of T_0-invariant points is in 1-1 correspondence with W: $w \leftrightarrow wB_0w^{-1}$. Given two points B_1, B_2 in $\mathcal{B}$ we say that B_1, B_2 are in relative position w ($w \in W$) if for some $g \in G$ we have $B_1^g = B_0$, $B_2^g = wB_0w^{-1}$. (We then write $B_1 \xrightarrow{w} B_2$)

For any $w \in W$, we denote by $\mathcal{B}_w$ the set of all $B \in \mathcal{B}$ such that $B_0 \xrightarrow{w} B$. Its closure $\bar{\mathcal{B}}_w$ is called a Schubert variety. Given $y, w \in W$ we have $y \leq w \Leftrightarrow \bar{\mathcal{B}}_y \subset \bar{\mathcal{B}}_w$.

For such y, w let $V_{y,w}$ be the set of all $B \in \mathcal{B}$ such that $B_0 \xrightarrow{\leq w} B \xleftarrow{w_0 y} w_0 B_0 w_0^{-1}$. ($w_0$ = longest element)

Then $\dim \bar{\mathcal{B}}_w = l(w)$, $\dim \bar{\mathcal{B}}_y = l(y)$, $\dim V_{y,w} = l(w) - l(y)$.

Moreover, there is an isomorphism of $\mathcal{B}_y \times V_{y,w}$ onto an open neighbourhood of $\mathcal{B}_y$ in $\bar{\mathcal{B}}_w$.

Theorem 2. $\mathbb{H}^{2i+1}(V_{y,w}) = 0$ and

$$P_{y,w} = \sum_i \dim \mathbb{H}^{2i}(V_{y,w})\, q^i.$$

Thus, the polynomials $P_{y,w}$ are a measure for the singularities of $\bar{\mathcal{B}}_w$ in the neighbourhood of a point in $\mathcal{B}_y$.

We now state a conjecture on ∞-dimensional representations of the semisimple Lie algebra $\mathfrak{g}$.

Let $\mathfrak{h}$ be a Cartan subalgebra, $\mathfrak{b}$ a Borel subalgebra containing $\mathfrak{h}$. Let $\rho : \mathfrak{h} \to \mathbb{C}$ be the $\frac{1}{2}$ sum of positive roots, (W, S) the Weyl group.

For each $w \in W$, let M_w be the Verma module with heighest weight $-w(\rho) - \rho$ and let L_w be its unique irreducible quotient.

Conjecture:
$$\begin{cases} \operatorname{ch} L_w = \sum_{y \leq w} \varepsilon_y \varepsilon_w P_{y,w}(1) \operatorname{ch} M_y \\ \operatorname{ch} M_w = \sum_{y \leq w} P_{w_0 w, w_0 y}(1) \operatorname{ch} L_y. \end{cases}$$

Similarly, the characters of the irreducible finite dimensional representations of a semisimple group in char. $p > 0$ are given conjecturally in terms of the polynomials $P_{y,w}$ where $y, w \in$ affine Weyl gr.

Titel: The Conjectures of Stark and Tate

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Let k be a number field and let F be a finite Galois extension of k with group $G=\mathrm{Gal}(F/k)$. Let V be a finite dimensional complex representation space for G, and let $L(V,s)$ be the Artin L-series corresponding to this representation. Define $r(V)$ and $L(V)$ by

$$L(V,s) \sim L(V)\, s^{r(V)} \quad \text{as} \quad s \to 0.$$

If V is the trivial representation of G, then $L(V,s) = \zeta_k(s)$, $r(V) = r_1+r_2-1$, and $L(V) = -hR/w$ by Dirichlet's class-number formula.

In general, $r(V) = \sum_{w|\infty} \dim V^{G_w} - \dim V^G$, where G_w is a decomposition group for w in G. Much less is known concerning the arithmetical nature of $L(V)$. Stark has conjectured its value as the determinant of a matrix whose entries are linear forms in logarithms of units of F.

To be more precise, let $U = \mathcal{O}_F^*$ be the unit group, let X be the free abelian group generated by the infinite places v of F, and let Y be the subgroup of elements of degree 0 in X. Consider the G-homomorphism:

$$\begin{aligned} U &\xrightarrow{\lambda} \mathbb{R}X = \mathbb{R}\otimes X \\ \varepsilon &\longmapsto \sum_v \log\|\varepsilon\|_v \cdot v \, . \end{aligned}$$

The image of λ lies in $\mathbb{R}Y$ by the product formula; by Dirichlet's theorem λ induces an isomorphism over $\mathbb{R}$:

$$\mathbb{R}U \xrightarrow[\sim]{\lambda} \mathbb{R}Y.$$

We may therefore choose a G-isomorphism of the rational spaces:

$$\mathbb{Q}U \xleftarrow[\sim]{\varphi} \mathbb{Q}Y,$$

and define the quantities:

$$R(V,\varphi) = \det\left(1\otimes\lambda\varphi \,\middle|\, (V\otimes\mathbb{C}Y)^G\right)$$

$$A(V,\varphi) = L(V) \,/\, R(V,\varphi).$$

Conjecture (Stark-Tate). $A(V,\varphi)$ is an algebraic number which lies in the field E generated by the character values of V. For all $\sigma\in \mathrm{Aut}(E)$: $A(V,\varphi)^\sigma = A(V^\sigma,\varphi)$.

This conjecture is known to be true when:

1. $r(V) = 0$ (Siegel).
2. $E = \mathbb{Q}$ (Ono, Lichtenbaum, Tate).

3. V is abelian, $k = \mathbb{Q}$ or $k = \mathbb{Q}(\sqrt{-d})$ (Kronecker).

When $r(V) = 1$ this conjecture suggests an explicit analytic method for the generation of the field F.

The prescription for defining a regulator at $s=0$ works equally well at any negative integer. At $s = -n$ one replaces $U = K_1\mathcal{O}$ by the group $K_{2n+1}\mathcal{O}$, and Dirichlet's theorems by the results of Borel. An interesting case is to consider is that of Dirichlet L-series; at $s = -1$ the conjecture is true by results of Bloch. One can also define regulators for p-adic L-series, but there one gets essentially different conjectures at $s = 0$ and $s = 1$.

Titel: Topological Space Form Problems

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I shall report on a joint work with F. T. Farrell. Let me start off by asking a very simple question.

Question 1. Let M^n be a closed manifold $K(\Gamma, 1)$, i.e. $\pi_i M^n = 0$ for $i > 1$ and $\pi_1 M^n = \Gamma$. Let $f: N^n \to M^n$ be a homotopy equivalence. Is f homotopic to a homeomorphism?

If both M^n, N^n are locally symmetric spaces of rank >1, G. D. Mostow actually proved that they are isometric. Question 1 is also related to a conjecture of Novikov:

Novikov's Conjecture Let $L_*(M^n) \in H^{4*}(M^n, \mathbb{Q})$ be the total L-genus of M^n and let

$$g : M^n \to K(\pi, 1)$$

be the natural map induced by a homomorphism of $\pi_1 M^n \to \pi$. Define

$$D(x)(M^n) = \langle L(M^n) \cup g^*(x), [M^n] \rangle \in \mathbb{Q}.$$

$L(x)$ is a homotopy invariant.

For the special case $M^n = K(\Gamma, 1)$ and $g = id$, we can reduce Novikov's conjecture to the following question.

Autor: Wu-chung Hsiang

Question 2 Let M^n be a closed manifold which is a $K(\Gamma,1)$; then

$$\theta: [M^n \times D^k, \partial; G/Top, *] \longrightarrow L_{n+k}(\Gamma, w_1(M^n))$$

is a rational monomorphism.

Question 1 says that θ is an isomorphism.

Loday has defined a map [2]

$$\lambda: h_*(B\pi; \underline{K}(\mathbb{Z})) \longrightarrow K_*(\mathbb{Z}[\pi])$$

where $h_*(\ ; \underline{K}(\mathbb{Z}))$ denotes the generalized homology theory with coefficients in the spectrum $\underline{K}(\mathbb{Z})$. We can ask the analogous question of Question 1 for algebraic K-theory.

Question 3 If M^n is a closed manifold which is a $K(\Gamma,1)$, then

$$\lambda_*: h_*(M^n; \underline{K}(\mathbb{Z})) \longrightarrow K_*(\mathbb{Z}[\pi])$$

is an isomorphism. In particular, we should have $Wh(\Gamma) = \tilde{K}_0(\Gamma) = 0$.

Let us now provide some evidence for questions 1, 3 possibly to be true. Let $E(n)$ be the group of rigid motions of n-dim Euclidean space and let Γ be a torsion-free uniform discrete subgroup of $E(n)$. The Riemannian ~~m~~ manifold $M^n = R^n/\Gamma$ is the usual flat manifold and Γ is called a Bieberbach group. Bieberbach group Γ is fitted into a short exact sequence

$$(1) \quad 1 \to A \to \Gamma \to G \to 1$$

where A is a normal abelian subgroup and G is a finite group.

<u>Theorem A</u> Let M^n $(n>1)$ be a closed connected Riemannian flat manifold. Let N^n be a topological manifold and let $f: N^n \to M^n$ be a homotopy equivalence. Then, f is homotopic to a homeomorphism.

<u>Theorem B</u> If M^n is a closed Riemannian flat manifold, then $Wh(\pi_1 M^n) = \tilde{K}_0(\pi_1 M^n) = 0$

The finite group G is called the holonomy group of the flat Riemannian manifold. If $|G|$ is of odd order, Theorem A was proved in [1]. We just completed the proof of Theorem A without holonomy restriction recently. The proof is based on the result of Ferry-Chapman-Quinn and the induction theory of Swan-Dress.

It seems to us that there is some hope to prove Theorem A for M^n to be a negatively curved manifold. Of course, Question 2 was answered affirmatively for the negatively curved manifold case by Miscenko [4].

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Titel: ISOSPECTRAL BUT NOT ISOMETRIC RIEMANNIAN SURFACES.

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Gelfand [1] asked the following question: Does the spectrum of a Riemannian surface X (for the laplacian) determine the geometry of X?

It is indeed the case if X is a torus (Berger Gauduchon Mazet [2]) but not if X is a Riemannian surface of genus $g > 2$. Let us write $X = H_2/\Gamma$ where:

H_2 = upper half-plane with hyperbolic metric

Γ = subgroup of $PSL(2,\mathbb{R})$, discrete, cocompact without elements of finite order.

Then we have the following dictionary:

Geometry of X <—> conjugacy class of Γ in $PGL(2,\mathbb{R})$ = group of isometries of H_2

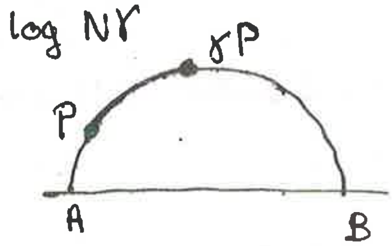
Spectrum of X <—> Spectrum of lengths of closed geodesics

$\Updownarrow$

{numbers of conjugacy classes of Γ with given norm}

What is the norm of $\gamma \neq 1$, $\gamma \in \Gamma$? Such a γ being hyperbolic is conjugate to the homography $z \longrightarrow mz$, $m > 1$. Let $N(\gamma) = m$.

A geometrical interpretation of the norm:



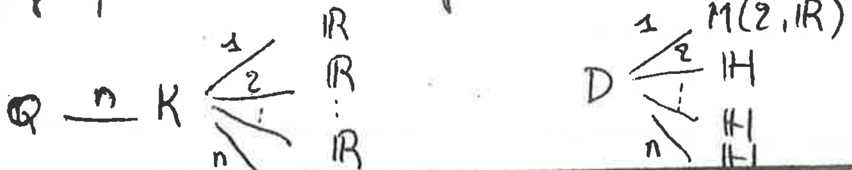
A, B = real fixed points of γ
If P belongs to the geodesic between A and B, then for the hyperbolic distance d, $d(P,\gamma P) = \operatorname{Inf}_{Q \in H_2} d(Q,\gamma Q)$ and up to some constant is equal to $\log N\gamma$.

Take for Γ quaternion groups, and remember the fact that quaternion algebras over number fields K, with class number $h_K = 0 \pmod 2$, (can) have

1) Maximal orders which are not conjugate
2) Maximal orders are always locally conjugate

1) will imply the construction of Riemannian surfaces H_2/Γ and H_2/Γ', Γ and Γ' associated to 2 maximal non conjugate orders, which are not isometric. But they have the same spectrum by 2).

The construction of Γ:
Start with a number field K which is totally real and with a quaternion field D over K, such that for one imbedding of K in $\mathbb{R}$, D gives $M(2,\mathbb{R})$ and for the others D imbedds in the field $\mathbb{H}$ of quaternions of Hamilton.



~~Recall~~ that $D \hookrightarrow M(2,\mathbb{R})$, and choose a maximal order O in D. Then

$$\Gamma = (O \cap SL(2,\mathbb{R}))/\{\mp 1\}$$

is discrete, cocompact in $PSL(2,\mathbb{R})$.

For further details, see Vigneras [3]

Remarks

1) For all $n \geq 2$, this construction permits to find irreducibles, riemannian, compact, connected manifolds of dimension n, isospectral and not isometric.

2) All the examples being arithmetic possess the common property: if X and X' are isospectral, they have finite isometric coverings

3) For $n=3$, there exist not isomorphic discrete sub-groups of $SL(2,\mathbb{C})$ with the same number of conjugacy classes with given norm

4) The Selberg zeta function. Define a prime p of Γ as a primitive conjugacy class $(\{\gamma\} \neq \{\gamma'\}^m, m>1)$ and

$$Z_p(s) = \prod_{k\geq 0} (1 - Np^{-k-s}) \qquad \mathrm{Re}\, s > 1$$

and let $Z(s) = \prod_p Z_p(s)$ be the Selberg zeta function of Γ. Groups which are not conjugate in $PGL(2,\mathbb{R})$ can have the same Selberg zeta function. If $\{\frac{1}{2} + ir = \rho\}$ are the non trivial zeroes of $Z(s)$ then the spectrum of the laplacian is $\{\lambda = \frac{1}{4} + r^2\}$

More remarks on the Selberg zeta function for PSL(2,Z)

For the remaining primitive conjugacy classes of PSL(2, Z) let us define (Reference, Vigneras [4]):

$$Z_1(s) = \left[\Gamma_2(s)^2 \ \Gamma(s)^{-1} (2\pi)^s\right]^{-1/6} \quad \text{(class) } 1$$

where Γ_2 = double-gamma function

$$Z_{e_2}(s) = \left[1 + tg\left(\frac{\pi s}{2} - \frac{\pi}{4}\right)\right]^{1/2} \quad \text{elliptic order 2}$$

$$Z_{e_3}(s) = \left[1 + \sqrt{3}\ tg\left(\frac{\pi s}{3} - \frac{\pi}{6}\right)\right]^{2/3} \quad \text{elliptic order 3}$$

$$Z_{par}(s) = \zeta(2s-1) \quad \text{parabolic}$$

where ζ = Riemann zeta function.

Then the modified Selberg zeta function

$$Z^*(s) = \prod_p Z_p(s) \cdot Z_{par}(s) \cdot Z_{e_3}(s) \cdot Z_{e_2}(s) \cdot Z_1(s)$$

defines an entire function on $\mathbb{C}$ satisfying the functional equation $Z^*(s) = Z^*(1-s)$ with zeroes located on the critical line $\operatorname{Re} s = \frac{1}{2}$ (The Riemann hypothesis is true for $Z^*(s)$.)

Recently Neunhöffer computed the first zeroes $\rho = \frac{1}{2} + ir$, $r < 21$ and it seems that

8,03974
11,24921 } first 3 zeroes of $L(s, (\frac{-3}{\cdot}))$
15,70462
20,45577 } fifth zero of $L(s, (\frac{-3}{\cdot}))$

are zeroes of $Z^*(s)$. The 4^{th} zero 18,26200 of the Dirichlet L serie $L(s, (\frac{-3}{\cdot}))$ is strangly missing

It appears also that ...
14,13473
the first zero of the Riemann zeta function would be a zero of $Z^*(s)$. For references see Audrey Terras's book [5] and the forthcoming computations of Pierre Cartier [6].

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Titel: Singularities and generalised root systems

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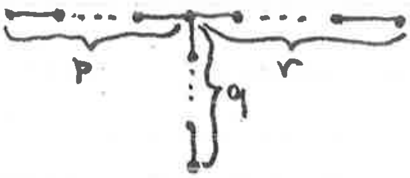
Let $(c_{\lambda\mu})$ be an integral $l \times l$ matrix satisfying (i) $c_{\lambda\lambda} = 2$, (ii) $c_{\lambda\mu} < 0$ if $\lambda \neq \mu$ and (iii) $c_{\lambda\mu} = 0 \Rightarrow c_{\mu\lambda} = 0$. Such a matrix is called a Cartan matrix. Choose a real vector space V of dim $l +$ corank $(c_{\lambda\mu})$ and linearly independent elements $\alpha_1, \dots, \alpha_l \in V$ (the fundamental roots) and $\alpha_1^\vee, \dots, \alpha_l^\vee \in V^*$ (the fundamental dual roots) such that $\langle \alpha_\lambda, \alpha_\mu^\vee \rangle = c_{\lambda\mu}$. Define the fundamental Weyl chamber $C := \{x \in V : \langle x, \alpha_\lambda^\vee \rangle > 0\}$. The fundamental reflections $s_\lambda : V \circlearrowleft$, $s_\lambda(x) = x - \langle \alpha_\lambda^\vee, x \rangle \alpha_\lambda$, generate the Weyl group $W \subset \mathrm{Aut}(V)$. Let I denote the W-orbit of $\bar{C}$. It can be shown that I is a convex cone and that W acts properly discontinuously on $\mathring{I}$.

Next, put $\Omega := \{x + iy \in V_{\mathbb{C}} : y \in \mathring{I}\}$. This domain is invariant under W and also under the translations in the root lattice $Q := \mathbb{Z}\alpha_1 + \dots + \mathbb{Z}\alpha_l$. So the semi-direct product $\tilde{W} := W.Q$ acts as an affine transformation group on Ω. This action is also properly discontinuous. Hence the orbit space $M := \Omega/\tilde{W}$ is in a natural way a normal analytic variety. It turns out that M is non-singular.

Theorem. There exists a Stein manifold $\hat{M}$ containing M such that $\hat{M} - M$ is a codimension ≥ 2 subvariety of $\hat{M}$ (note that then $\hat{M}$ is characterised by these properties). The subvariety $\hat{M} - M$ is naturally equipped with a stratification into (non-closed) analytic submanifolds.

Moreover D entends to a divisor $\hat{D}$ containing $\hat{M}-M$.

Now let $p \leq q \leq r$ be positive integers with $\frac{1}{p}+\frac{1}{q}+\frac{1}{r} \leq 1$. To the so-called $T_{p,q,r}$ diagram, drawn below, there is associated a Cartan matrix of seize $l := p+q+r-2$: number the vertices from 1 up to l and put $c_{\lambda\lambda}=2$, $c_{\lambda\mu}=-1$ (resp. 0) if the vertices labelled λ and μ are (resp. aren't) connected. Let $M_{p,q,r}$, $\hat{M}_{p,q,r}$, $\hat{D}_{p,q,r}$ be the associated spaces.



<u>Theorem</u>. Let $p \leq q \leq r$ be positive integers with $\frac{1}{p}+\frac{1}{q}+\frac{1}{r}=1$ and let $\lambda \in \mathbb{C}$ be such that $X_0 := \{x^p+y^q+z^r+\lambda xyz=0\} \subset \mathbb{C}^3$ has an isolated singular point at $0 \in \mathbb{C}^3$. Then blowing up $0 \in X_0$ yields an elliptic curve E as an exceptional divisor. Choose $\tau \in \mathcal{H}$ such that $E \cong \mathbb{C}/\mathbb{Z}+\tau\mathbb{Z}$. Then $\hat{M}_{p,q,r}-M_{p,q,r} \cong \mathcal{H}$ (canonically) and the germ of the discriminant of the semi-universal deformation of $(X_0,0)$ is isomorphic to the germ of $(\hat{M}_{p,q,r}, \hat{D}_{p,q,r})$ at $\tau \in \mathcal{H} \hookrightarrow \hat{M}_{p,q,r}$. Moreover the monodromy group of $(X_0,0)$ is isomorphic to $\tilde{W}/(\text{centre of } \tilde{W})$. Here the centre of $\tilde{W}$ is infinite cyclic.

<u>Theorem</u>. Let now $\frac{1}{p}+\frac{1}{q}+\frac{1}{r}<1$ and set $X_0=\{x^p+y^q+z^r+xyz=0\} \subset \mathbb{C}^3$. Then the discriminant of the semi-universal deformation of $(X_0,0)$ is isomorphic to the germ of $(\hat{M}_{p,q,r}, \hat{D}_{p,q,r})$ at the unique zero-dimensional stratum of $\hat{M}_{p,q,r}-M_{p,q,r}$. Moreover, the monodromy group of $(X_0,0)$ is isomorphic to $\tilde{W}$ (the centre of $\tilde{W}$ is trivial).

—#—

Titel: Zeta - functions and K-theory

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The talk is devoted to a discussion of the following conjectures about the relation between algebraic K-groups defined by Quillen and zeta-functions of algebraic varieties introduced by Weil. Let X be such variety, proper and smooth, over finite field $\mathbb{F}_q$ and of dimension d. It's zeta-function $\zeta_X(s) = Z_X(t)$ equals to a product $\prod_i P_i(t)^{(-1)^{i+1}}$ of characteristical polynomials of Frobenius map. Introduce Zariski sheaves $\underline{K}_m$ of Quillen K-groups. Then we assume that groups $H^p(X,\underline{K}_m)$ are finite if $p < m$, finitely generated if $p = m$ and trivial if $p > m$ (the last statement is known only).

Conjecture. $|P_{2i}(q^n)| = \# H^{d-i}(X,\underline{K}_{2n+d+i-1}),\ i \geq 0\ (n > 0)$

$$|P_{2i-1}(q^n)| = \# H^{d-i}(X,\underline{K}_{2n+d+i-2}),\ i \geq 1\ (n > 0)$$

This conjecture is consistent with the blowings of varieties and with the multiplication of them by $\mathbb{P}_m$. It can be proved for d=0; $P_1(q)$ when d= 1; $X=\mathbb{P}_d$ and for some surfaces X which is birationally equivalent to $\mathbb{P}_2$ over closure of $\mathbb{F}_q$. The above conjectures gives us a behavior of $\zeta_X(s)$ in s=-n. Consider now a situation in s=0 and d=2. Then we have

Conjecture.

$$\zeta_X(s) \underset{s \to 0}{\sim} (-1)^{\rho} \frac{\# H^2(X,K_2)_{tors} \# H^0(X,K_2)}{\# H^1(X,K_2) \cdot (q^2 - 1)} (1 - q^{-s})^{-1}$$

$$\rho = \mathrm{rk}\ NS(X)$$

This is true for ratinal surfaces, some forms of P_2 and for products $\mathbb{P}_1 \times C$.

Titel: Curvature deformations and Yang-Mills fields

Autor: Min-Oo

Adresse: Bonn University

This is a report on joint work with Ernst Ruh.

The general problem is the deformation of connections to connections having good curvature properties. "Good" in this context means that the study of the manifold and vector bundle in question is reduced to an algebraic problem often solved classically.

<u>Example 1</u> (Space form problem)

Good connections are the riemannian connections with constant sectional curvature.

<u>Example 2</u> (Lie groups)

A good connection is provided by the parallelization by left invariant vector fields. The Maurer-Cartan equation can be interpreted as the vanishing of the corresponding Cartan curvature.

In [3] we obtain the following result by curvature deformation. As a corollary we obtain theorems generalizing the well known stability results on spherical space forms to all compact symmetric spaces.

<u>Theorem.</u> Let $\mathfrak{g}$ be a compact semi-simple Lie algebra. There exists a constant $A>0$ such that if $\omega : TP \to \mathfrak{g}$ is a parallelization of a compact manifold P with $\|\Omega\| < A$, where $\Omega = d\omega + [\omega,\omega]$, then there exists $\bar{\omega} : TP \to \mathfrak{g}$ with $\|\omega - \bar{\omega}\| < c\|\Omega\|$ satisfying the Maurer-Cartan equation $\bar{\Omega} = d\bar{\omega} + [\bar{\omega},\bar{\omega}] = 0$. As a consequence $P \underset{\text{diffeo}}{\cong} \Gamma^{G}$, where Γ is a finite subgroup of the simply connected Lie group G of $\mathfrak{g}$.

The techniques of curvature deformation utilized in the proof of this theorem also apply to the study of Yang-Mills fields. We refer to [1] for a self contained account of the 4-dimensional riemannian geometry related to the Yang-Mills fields.

The Y-M functional is defined by

$$\omega \longmapsto \int_M \|\Omega\|^2 \quad , \text{ where}$$

M is an oriented 4-dimensional riemannian manifold. Ω is the curvature of a connection on a principal bundle P over M with structure group G, a compact Lie group.

The critical points of this functional, i.e., connections with $\delta\Omega = 0$, are called Y-M fields.
Let $\Omega = \Omega_+ + \Omega_-$ be the splitting of the curvature according to the ± 1 eigen spaces of the Hodge $*$ operator on 2-forms. The 1st Pontrijagin class gives a lower bound for the Y-M functional and self dual ($\Omega_- = 0$) connections, if they exist, give the absolute minimum of the functional.

Self dual Y-M fields on S^4 are classified by complex algebraic methods in a paper of Atiyah, Hitchin, Drinfeld and Manin. Our following theorem is a stability result on approximate solutions of the Y-M equation.

Theorem (C^0-version)

Let ω be a connection on P satisfying the inequality

$$|\Omega_-| < C_0 (K - |W_-|) \quad \text{at all points of } M.$$

(K = scalar curvature, W_- = anti self dual Weyl tensor of M)

then there exits a self dual connection $\bar{\omega}$ (i.e. $\bar{\Omega}_- = 0$) satisfying $\|\omega - \bar{\omega}\| < c\|\Omega_-\|$.

In the L_2-version the same conclusion holds under the assumption $\|\Omega_-\|_{L_2} < c_1 \min_M (K - |W_-|)$. In this case the constant c_1 depends on M.

Remark: This result is a qualitative version of Theorem 5.4 of [2], where the attention is restricted to harmonic fields.

To prove the Theorem let $\omega(0)$ be the initial connection with curvature $\Omega(0)$. Define a vector field $\dot{\omega} = -\delta_- G\, \Omega(0)$ on the affine space of connections, where G is the Green's operator of $\Delta = d_-\delta_-$. By estimates on elliptic equations, $\dot{\omega}$ is in the same Sobolev space as ω. We deform our connection by integrating the vector field $\dot{\omega}$ on the integral curve $\omega(t)$ through $\omega(0)$. We obtain $\Omega_-(t) = (1-t)\,\Omega_-(0)$ and the Bochner formula for $\Delta = d_-\delta_-$ together with estimates on elliptic operators yields a uniform bound on $\|\dot{\omega}\|_{1,p}$ along this integral curve in case $|t| \leq 1$. We define $\bar{\omega} = \omega(1)$ to obtain the connection $\bar{\omega}$ of the Theorem.

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Titel: Periods of Eisenstein classes and values of L-functions

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1 Damerell, L- functions of elliptic curves with complex multiplication, I, II, Acta arithmetica, 17 (1970) and 19, (1971)

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The subject of the talk was to discuss a special case of the folwing general problem. Let Γ be an arithmetic subgroup of a reductive real Lie group G_∞. Then the arithmetic group Γ acts on the symmetric space $X = G_\infty/K$. If the quotient space $\Gamma\backslash X$ is not compact we can construct cohomology classes on $\Gamma\backslash X$ by starting from a cohomology class ψ at infinity. These classes are represented by differential forms $E(\psi, s_\psi)$ which are Eisenstein series and which will be called Eisenstein classes. On the other hand one can construct homology classes on $\Gamma\backslash X$. To get homology classes we embed lower dimensional groups M_∞ into G_∞, such that $\Gamma_M = \Gamma \cap M_\infty$ is still arithmetic and cocompact. Then the fundamental cycle

$\Gamma_M \backslash X_M \longrightarrow \Gamma\backslash X$ defines a homology class.

Now we ask whether we can construct cases where these cycles and an Eisenstein class sit in the same dimension. In this case we can ask for the value of the Eisenstein class on the cycle which amounts to the evaluation of the Integral

$$\int_{\Gamma_H \backslash X_M} E(\psi, s_\psi)$$

This is the general problem. We discussed the special case where $G_\infty = PGL_2(\mathbb{C})$ and Γ is a very special type of congruence subgroups in $PGL_2(\mathbb{Z}[i])$. In this case the Eisenstein classes are associated to certain Grössencharaktere φ of type $(1,0)$ on $\mathbb{Q}[i]$. The cycles are obtained from the units in quadratic extensions $E/\mathbb{Q}[i]$. Then the period integrals can be expressed in terms of ratios

$$\frac{L_E(\chi \cdot \varphi \circ N, 1/2)}{L_{\mathbb{Q}[i]}(\varphi^2, 1)}$$

where χ is a certain ideal class character on E. For the precise statement we refer to the following preprint which will appear in the Proceedings of the 1979 Colloquium in Bombay

G. Harder Period Integrals of Cohomology Classes which are represented by Eisenstein Series.

Titel: MODULI OF KAHLERIAN K-3 SURFACES.

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The aim of this article is to prove the following theorem:

Theorem 1. Every point of SO(3,19)/SO(2)xSO(1,19) corresponds to a Kahler K-3 surface.

From theorem 1 and the following theorem proved by E.Looijenga and C. Peters in [L & P] : Let X be a K-3 surface and suppose that there exists a kahlerian K-3 surface X' and a Hodge isometry from $H^2(X',\mathbb{Z})$ onto $H^2(X,\mathbb{Z})$. Then X and X' are isomorphic, in particular, X is kahlerian, we obtain the following result: Every K-3 surface is a kahlerian one.

The proof of Theorem 1.

Definition 1. A K-3 surface is a two dimensional complex manifold with the following properties: a) it is simply connected, b)the canonical class is trivial.

It is well known fact that $H^2(X,\mathbb{Z})$ is a free abelian group of rank 22. The cup product defines in $H^2(X,\mathbb{Z})$ a scalar product with values in $\mathbb{Z}$.

Definition 2. An Euclidean lattice L we will call a free abelian group L with the scalar product in $\mathbb{Z}$.

So $H^2(X,\mathbb{Z})$ is an Euclidean lattice, which we will denote by H_X. In [Sh] chapter 10 it is proved that for every K-3 surface H_X is an even unimodular lattice with signature (3,19). In[S] it is proved that all such lattices are isomorphic. Let me fixed one of them and called it L.

Definition 3. A marked K-3 surface is called a pair (X,f), where X is a K-3 surface and $f:H_X \xrightarrow{\sim} L$ is an isomorphism of lattices.

Definition 4. An admissible Hodge structure on L of type (1,20,1) on L is defined as a filtration F: $H^{2,0} \subset H^{2,0} + H^{1,1} \subset L\otimes\mathbb{C}$ with the following properties:a) $(H^{2,0}+H^{0,2})^{\perp}=H^{1,1}$, where $H^{0,2}=\overline{H^{2,0}}$., b) w.w=0 for $w\neq 0\in H^{2,0}$, where w.w is the scalar product in $L\otimes C$ induced from L. c) $w.\bar{w}>0$, for w in $H^{2,0}$, d) dim $H^{2,0}=1$ and dim $L\otimes C=22$.

It is not difficult to prove that M=SO(3,19)/SO(2)xSO(1,19) parametrizes all admissible Hodge structures on L of type (1,19,1). M we will call the moduli space of marked K-3 surfaces.

Another interpretation of M.

From the definition of an admissible Hodge structure it follows that the Hodge filtration on $L\otimes\mathbb{C}$ of type (1,20,1),F, is uniquely determined by $H^{2,0}$. Since dim $H^{2,0}=1$, then $H^{2,0}$ corresponds to a point in $\mathbb{P}(L\otimes\mathbb{C})=P^{21}$, because $\mathbb{P}^{21}$ parametrizes all one dimensional subspaces in $L\otimes\mathbb{C}$. The condition w.w=o implies that all admissible Hodge structures F must lie on a quadric Q in $\mathbb{P}^{21}$ defined by the quadratic form that defines the scalar product in L. The condition $w.\bar{w}>0$ defines an open subset in Q. Analitically this interpretation of M can be represented by the following formulas:

$$(*)\qquad z_1^2+z_2^2+z_3^2-z_4^2-\ldots-z_{22}^2=0$$

$$(**)\qquad |z_1|^2+|z_2|^2+|z_3|^2-|z_4|^2-\ldots>0$$

It is easy to prove that M parametrizes all two dimensional subspaces in $L\otimes R$ on which the restriction of Q is strictly positive.

Next we will define the period map for marked K-3 surfaces.

Let (X,f) be a marked K-3 surface, then we put to (X,f) the admissible Hodge structure on L induced by the Hodge structure on H_X via f.

PROOF: Let y be a point in M, then as it was said before y defines a two dimensional subspace in $L\otimes R$ on which Q is strictly positive. This subspace we will denote by E_y'. Let $H_y^{1,1}$ be equal to $(E_y')^{\perp}$. Notice that $H_y^{1,1}$ is in $L\otimes R$. Let me denote by $V=\{x\in H_y^{1,1} \mid x.x>0\}$. Since the form Q has signature (1,19) on $H_y^{1,1}$ V is a disjoint union of two cones V^+ and V^-, where $V^+ \cap V^- = (0)$. Let me denote by $E_y^{1,1}(Z\ ) = H_y^{1,1}$ L. Let $D = \{ x \in H_y^{1,1} \mid x.x = -2 \}$. For $b \in D$, let s_b be the reflection of the vector space $H_y^{1,1}$:

$$s_b : x \to x + (x,b)b$$

These reflections generate a Coxeter group W operating properly and discontinuously on V^+. For the proof of this look at P&Sh . The fundamental domain of W is:

$$V_P^+ = \quad x \quad H_y^{1,1} \ (x,b) \quad 0 \text{ for every } b \quad D$$

V_P^+ is an open convex cone in $H_y^{1,1}$. The Dekart product of $V_P^+ x E_y'$ is an open set in $L \otimes R$. Let $q \in V_P^+ x E_y'$ be with the following properties: a) q has rational coordinates in $L \otimes R$ b) q is not an element of E_y'. These properties follow from the fact that all points with rational coordinates in the open subset $V_P^+ x E_y'$ is every where dence subset in $V_P^+ x E_y'$. Let me denote by E_y the three dimensional subspace in $L \otimes R$ spanned by E_y' and q. Clearly Q restricted to E_y is strictely positive. From this property of E_y it follows that the projective space $\mathbb{P}^2(E_y \otimes C)$ in $\mathbb{P}(L \otimes C)$ is contained in U defined by the inequality (**). From this fact it follows that the rational curve $P_y^1 = P^2(E_y \otimes C) \cap Q$ is contained in M, where Q is a quadric in $P(L \otimes C)$ defined by the intersection form Q of L.

<u>Construction of a family of K-3 surfaces over P_y^1.</u>

Let H_q be $\{ x \in L \otimes \mathbb{C} \mid xq = 0 \}$. Let $\mathbb{P}(H_q)$ be the projective subspace in $\mathbb{P}(L \otimes \mathbb{C})$ spanned by H_q. In [K] and [T] it is proved that all , points of $P(H_q) \cap M$ corresponds to algebraic K-3 surfaces if q has the following properties: a) $q.q > 0$ and B) q is a vector with rational coefficients. From the construction of E_y it follows that we can find $q \in E_y$ such that q has rational coordinates and of course $q.q > 0$. From the theorem mentioned above it follows that the point $x \in \mathbb{P}(H_q) \cap \mathbb{P}_y^1$ corresponds to a marked algebraic K-3 surface (X,f,w_X) , where X is an algebraic K-3 surface, f is an isomorphism of the lattices H_X and L and w_X is a form from a Hodge metric on X such that $f^{-1}(q)=w_X$. Of course we must prove that $f^{-1}(q)=w_X$ is defined by a Hodge metric on X.

Lemma 1. $w_X=f^{-1}(q)$ comes from a Hodge metrics on X.

Proof: $w_X.w_X=f(w_X).f(w_X)=q.q>0$. Next we must prove that $w_X.b>0$ for all $b\in H_X\cap H^{1,1}(X)$ such that $b.b=-2$. If this is proved then from Moichezon criterium it will follow that the Poincare dual of w_X will be an ample divisor on X. That the dual of w_X is an algebraic curve, follows from the fact that w_X has rational cooridinates in $H_X\otimes R$ and w_X is perpendicular to $H^{2,0}+H^{0,2}$. The last assertion follows from the definition of $\mathbb{P}(H_q)$ and the way we defined the point x. So if the Poincare dual of w_X is an ample divisor then it is clear that some multiple of w_X will be a form obtained from some Hodge metrics on X. Suppose that there exists b such that $w_X.b=0$,$b.b=-2$ and $b\in H_X\cap H^{1,1}(X)$. Of course we will have that $w_X(2,0).b=0$ and $b.w_X(0,2)=0$, so from here it follows that $b\in(f^{-1}(E_y))$. But this is impossible, because of the way we constructed E_y.

Q.E.D.

Lemma 2. $f^{-1}(E_y)$ defines a trivial subbundle $X\times R^3\subset\Lambda^2T^*_X(R)$. This subbundle is Λ_+, where Λ_+ is the eigen subbundle corresponding to the eigen value +1 of the Hodge operator defined by the Hodge metric which corresponds to w_X.

Proof: It is clear that E_y is spanned by $f(\mathrm{Re}\ w_X(2,0)),f(\mathrm{Im}\ w_X(2,0))$ and $q=f(w_X)$. From the fact that the first Chern class of X vanishes it follows that $\mathrm{Re}w_X(2,0)$ and $\mathrm{Im}\ w_X(2,0)$ spanned a trivial subbundle in $\Lambda^2T^*_X(R)$. From this fact we get immidiately that the subbundle spanned by $\mathrm{Re}\ w_X(2,0)$, $\mathrm{Im}\ w_X(2,0)$ and w_X is trivial. It is an easy exersize to prove that this trivial bundle is Λ_+.

Q.E.D.

From the recent results of Yau it follows that we can find Ricci flat matric on X, i.e. a Kahler-Einstein metric whose form will be in the cohomology class of w_X. This Kahler-Einstein metric induces a metric on Λ_+, with a zero curvature, so Λ_+ is a flat bundle. For the proof of these fact look at [H_1]. We can regard Λ_+ as a trivial bundle of Lie algebras of SU(2). So Λ_+ is spanned by three skew-symmetric operators on the tangent bundle I,J,&K with the following properties: $I^2=J^2=K^2=-id$ $IJ+JI=...=0$. So these three parralel forms define three almost complex structures on X. Note that $aI+bJ+cK$ is also a complex structure on X, where $a^2+b^2+c^2=1$. So any form of Λ_+ after normalization defines a complex structure on X. Applying this construction first done by N.Hitchin we get a family of K-3 surfaces $p:Z \longrightarrow P^1$. It is not difficult to prove that Z is $P(V_+)$. In [A H S] it is proved that $P(V_+)$ is a complex manifold iff the self-dual part of the Weyl tensor W_+ is zero. If the metric is Kahler-Einstein on a K-3 surface then it is easy to prove that $W_+=0$. So Z is a complex 3dim manifold and all the fibres are Kahler K-3 surfaces. From lemma 2 it follows that if we consider the period map for $p:Z \longrightarrow P^1$ in M then the image of P^1 will be P^1_y. So our theorem is proved.

Q.E.D.

Using theorem 1 and the results of Penrose and Hitchin we get the following theorem:

THEOREM 2. The moduli space of all Kahler-Einstein metrics on a K-3 surface is isomorphic to (the space of all $\mathbb{P}^2$ in $U \subset P^{21}$

where U is defined by (**) on page 2)/$G \times R_+$, where $G=SO(3,19;Z)$ and R_+ are the prositive real numbers. This moduli space has real dimension 58.

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Titel: On orbital integrals for real groups

Autor: R Langlands

Adresse: IAS, Princeton, N.J.

This lecture was a report on the work of Diana Shelstad on orbital integrals for real groups and their applications to character identities for tempered representations

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2. D. Shelstad, Characters and inner forms of a quasi-split group over $\mathbb{R}$, to appear in Comp. Math.

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4. —, Embeddings of L-groups, to appear in Can. Jour. Math.

Titel: The Monster Game

Autor: Jean-Pierre Serre

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This game relates the Fischer-Griess sporadic group M, of order $2^{46} 3^{20} 5^{9} 7^{6} 11^{2} 13^{3}.17.19.23.29.31.41.47.59.71$

$$\simeq 8 \times 10^{53},$$

with modular functions of one variable. It is entirely conjectural – and entirely convincing. Its main aspects have been described in Kac's lecture. I give a few complements, e.g. on the actual construction of "Hauptmodul(n)" and on congruence properties.

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J.-P. Serre.