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Abstract. We prove that an F-isocrystal over an abelian variety defined over a perfect field of positive characteristic has constant slopes. This recovers and extends a theorem of Tsuzuki for abelian varieties over finite fields. Our proof exploits the theory of monodromy groups of convergent isocrystals.

Contents

1. Introduction

In this article we chiefly study the behaviour of F-isocrystals over abelian varieties. Our main result is the following theorem.

Theorem 1.1 (Theorem 4.2). *Let A be an abelian variety over a perfect field k of positive characteristic. Every F-isocrystal over A has constant slopes.*

If k is a finite field, we also prove the following stronger form of Theorem 1.1.

Theorem 1.2 (Theorem 4.3). *If k is a finite field, every ι-pure F-isocrystal over A becomes constant after passing to a finite étale cover.*

Theorem 1.1 and Theorem 1.2 extend [Tsu17, Thm. 3.7] and they agree with the general expectation that families of smooth projective varieties parametrised by abelian varieties have "small monodromy". To prove them we use the theory of *monodromy groups of convergent isocrystals*. This was firstly introduced by Crew in [Cre92] and further studied in [Pál15], [LP17], [AD18], [D'Ad20a], and [D'Ad20b]. Using this theory, it is possible to prove that the category of convergent isocrystals over A, denoted by $\mathbf{Isoc}(A)$, has a rather simple structure. The key point is that the monodromy groups in this case are commutative by next proposition.

Proposition 1.3 (Proposition 4.1). *Let $\mathbf{Isoc}(A)$ be the Tannakian category of convergent isocrystals over A. The Tannaka group of $\mathbf{Isoc}(A)$ with respect to any fibre functor is commutative.*

Proposition 1.3 is proved using an Eckmann–Hilton argument, exploiting the Künneth formula for these Tannaka groups (Proposition 2.2). Theorem 1.2 then follows from a combination of the theory of weights for overconvergent F-isocrystals, as developed in [Ked06], and the global monodromy theorem, proved in [Cre92, Thm. 4.9] and [D'Ad20a, Thm. 3.4.4]. If k is not finite, we cannot rely on the global monodromy theorem, since it is false already for ordinary elliptic curves over $\bar{\mathbb{F}}_p$. To prove Theorem 1.1 we reduce instead to the case of F-isocrystals of rank 1, where the constancy is well-known.

If X is a smooth proper variety over an algebraically closed field k, we deduce from Proposition 1.3 the following result for the subcategory $\mathbf{Isoc}(X)_F \subseteq \mathbf{Isoc}(X)$ spanned by those convergent isocrystals which can be endowed with a Frobenius structure (see Definition 3.5).

Proposition 1.4 (Proposition 4.4)**.** *For every rational point x of X, the associated Albanese morphism $f : X \to \mathrm{Alb}_X$ induces a faithfully flat morphism*

$$\pi_1(\mathbf{Isoc}(X)_F, x)^{\mathrm{ab}} \xrightarrow{f_*} \pi_1(\mathbf{Isoc}(\mathrm{Alb}_X)_F, 0_{\mathrm{Alb}_X})$$

of affine group schemes, where $\pi_1(\mathbf{Isoc}(X)_F, x)$ and $\pi_1(\mathbf{Isoc}(\mathrm{Alb}_X)_F, 0_{\mathrm{Alb}_X})$ are the Tannaka fundamental groups of $\mathbf{Isoc}(X)_F$ and $\mathbf{Isoc}(\mathrm{Alb}_X)_F$ with respect to x and the identity element 0_{Alb_X}. The kernel is a finite constant group scheme isomorphic to $(\mathrm{Pic}^{\tau}_{X/k}/\mathrm{Pic}^{0,\mathrm{red}}_{X/k})^{\vee}(k)$.

This proposition is an analogue of [Lan12, Thm. 7.1] and [BdS17, Thm. 4.1]. The main tool we use here, besides Proposition 1.3, is the fact that rank 1 objects in $\mathbf{Isoc}(X)_F$ correspond to p-adic characters of the étale fundamental group of X.

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Notation

Let k be a perfect field and let K be the fraction field of the ring of Witt vectors of k. For a smooth variety X over k we denote by $\mathbf{Isoc}(X)$ the category of K-linear convergent isocrystals over X, as defined in [Ogu84]. If X is geometrically connected and η is a perfect point of X, we denote by $\pi_1(\mathbf{Isoc}(X), \eta)$ the Tannaka group of $\mathbf{Isoc}(X)$ with respect to the fibre functor induced by η (see [Cre92, §2.1]). We use an analogous notation for the other variants of $\mathbf{Isoc}(X)$ that will appear in this article. If G is an affine group scheme, we denote by G^{ab} the maximal commutative quotient, by G^{diag} the maximal pro-diagonalisable quotient, and by G^{uni} the maximal pro-unipotent quotient.

2. Künneth formula

In this section we want to prove the Künneth formula for the fundamental group of convergent isocrystals. The main ingredient is the following existence theorem.

Theorem 2.1 ([LP17, §8][1])**.** *For a smooth morphism $f : Y \to X$ of smooth proper varieties, the functor $f^* : \mathbf{Isoc}(X) \to \mathbf{Isoc}(Y)$ admits a right adjoint f_*. The formation of f_* is compatible with base change with respect to morphisms $Z \to X$ where Z is smooth and proper.*

Proposition 2.2. *Let X and Y be two smooth proper connected varieties endowed with the choice of rational points x and y. The projections of the product $X \times Y$ to the two factors induce an isomorphism*

$$\pi_1(\mathbf{Isoc}(X \times Y), (x, y)) \xrightarrow{\sim} \pi_1(\mathbf{Isoc}(X), x) \times \pi_1(\mathbf{Isoc}(Y), y).$$

Proof. We denote by $q : X \times Y \to X$ the projection to the first factor and by $i : x \times Y \hookrightarrow X \times Y$ the natural inclusion. These morphisms induce the following cartesian diagram

$$\begin{tikzcd} x \times Y \arrow[r, hook, "i"] \arrow[d, two heads, "q"] & X \times Y \arrow[d, two heads, "q"] \\ x \arrow[r, hook, "i"] & X. \end{tikzcd}$$

Moreover, we get the following sequence of affine group schemes over K

$$1 \to \pi_1(\mathbf{Isoc}(Y), y) \xrightarrow{\alpha} \pi_1(\mathbf{Isoc}(X \times Y), (x, y)) \xrightarrow{\beta} \pi_1(\mathbf{Isoc}(X), x) \to 1, \tag{2.2.1}$$

where α is induced by i^* and β by q^*. We use [DE20, Thm. A.13] to show that (2.2.1) is an exact sequence. First, note that the projection $X \times Y \twoheadrightarrow Y$ and the closed immersion $X \times y \hookrightarrow X \times Y$ induce respectively a retraction for α and a section for β. This shows that α is a closed immersion, β is faithfully flat, and $i^* : \mathbf{Isoc}(X \times Y) \to \mathbf{Isoc}(X)$ is essentially surjective, thus observable. It is also clear by construction that $\beta \circ \alpha$ is trivial.

It remains to show that for every convergent isocrystal $\mathcal{M}$ over $X \times Y$, there exists $\mathcal{N} \subseteq \mathcal{M}$, such that $i^*\mathcal{N}$ is the maximal trivial subobject of $i^*\mathcal{M}$. We claim that we can take as $\mathcal{N}$ the convergent isocrystal $q^*q_*\mathcal{M}$ equipped with the adjunction morphism $q^*q_*\mathcal{M} \to \mathcal{M}$. Indeed, by the compatibility of the formation of direct image with base change given by Theorem 2.1, we have a natural isomorphism $i^*q^*q_*\mathcal{M} \simeq q^*q_*i^*\mathcal{M}$. Combining this with the fact that $q_*i^*\mathcal{M} = H^0(X, i^*\mathcal{M})$, we deduce that $i^*q^*q_*\mathcal{M}$ is the maximal trivial subobject of $i^*\mathcal{M}$. In addition, since i^* is an exact $\otimes$-functor, this also implies that $q^*q_*\mathcal{M} \to \mathcal{M}$ is an injective morphism. This concludes the proof of the exactness of (2.2.1). For symmetry reasons, we deduce that the analogue sequence where X and Y are exchanged is also exact. Combining these two facts, we get the desired result. □

Remark 2.3. If X and Y are projective one can alternatively recover Proposition 2.2 from [LP17, Thm. 7.1]. A variant of Proposition 2.2 is also proven in [DTZ18, Thm. III].

[1]Note that Theorem 2.1 can be also obtained as a consequence of [DTZ18] or [Xu19].

3. Isocrystals with commutative monodromy

This section is an interlude on convergent isocrystals with commutative monodromy. The main result in this section is Proposition 3.4. Thanks to Proposition 4.1, over abelian varieties every convergent isocrystal has this property.

Notation 3.1. Let $F : X \to X$ be the absolute Frobenius of X. For a positive integer n, we write $\mathbf{F}^n\text{-}\mathbf{Isoc}(X)$ for the category of convergent F^n-isocrystals and $\mathbf{F}^\infty\text{-}\mathbf{Isoc}(X)$ for 2-$\varinjlim_n \mathbf{F}^n\text{-}\mathbf{Isoc}(X)$. If $(\mathcal{M}, \Phi_\mathcal{M})$ is a convergent F^n-isocrystal, we write $(\mathcal{M}, \Phi_\mathcal{M}^\infty)$ for its image in $\mathbf{F}^\infty\text{-}\mathbf{Isoc}(X)$. By [Ber96, Thm. 2.4.2], the category $\mathbf{F}^n\text{-}\mathbf{Isoc}(X)$ is equivalent to the category of F^n-isocrystals over the absolute crystalline site of X.

Lemma 3.2. *Suppose k algebraically closed and let G be a pro-diagonalisable group over K. If V is an irreducible K-linear representation of G, then V is of rank 1. The same is true if K is replaced by $\mathbb{Q}_p^{\mathrm{ur}}$.*

Proof. By [Lan52, Thm. 10], the field K is a C_1 field, which implies that $\mathrm{Br}(K) = 0$. Thanks to Schur's lemma, $\mathrm{End}_G(V)$ is a division algebra over K, thus we have $\mathrm{End}_G(V) = K$. In turn, this implies that $\mathrm{End}_G(V \otimes_K \bar{K}) = \bar{K}$, so that $V \otimes_K \bar{K}$ is an irreducible representation of $G \otimes_K \bar{K}$. Since $G \otimes_K \bar{K}$ is isomorphic to the product of a split torus and a constant commutative finite group, we deduce that $V \otimes_K \bar{K}$ is of rank 1, as we wanted. The second part can be proven in the same way thanks to the fact that $\mathbb{Q}_p^{\mathrm{ur}}$ is a C_1 field by [Lan52, Thm. 12]. □

Lemma 3.3. *Suppose k algebraically closed and let $(\mathcal{M}, \Phi_\mathcal{M})$ be a convergent F^n-isocrystal over X. If $(\mathcal{M}, \Phi_\mathcal{M}^m)$ is irreducible for every $m > 0$, then $\mathcal{M}$ is irreducible.*

Proof. Let $\mathcal{N} \subseteq \mathcal{M}$ be an irreducible subobject. Since $\Phi_\mathcal{M}$ permutes the isomorphism classes of the irreducible subobjects of $\mathcal{M}$, we deduce that $(F^n)^*\mathcal{N} \simeq \mathcal{N}$ for n big enough. Therefore, $\mathcal{N}$ can be endowed with some F^n-structure $\Phi_\mathcal{N}$. Write $(\mathcal{P}, \Phi_\mathcal{P}^\infty)$ for $(\mathcal{N}, \Phi_\mathcal{N}^\infty)^\vee \otimes (\mathcal{M}, \Phi_\mathcal{M}^\infty)$. If $\mathcal{T} \subseteq \mathcal{P}$ is the maximal trivial subobject of $\mathcal{P}$, it defines a subobject $(\mathcal{T}, \Phi_\mathcal{T}^\infty) \subseteq (\mathcal{P}, \Phi_\mathcal{P}^\infty)$. Up to replacing $\Phi_\mathcal{N}$ with $p^s\Phi_\mathcal{N}^r$ for some $(s, r) \in \mathbb{Z} \times \mathbb{Z}_{>0}$, we may assume that one of the slopes of $(\mathcal{T}, \Phi_\mathcal{T}^\infty)$ is 0. Since $(\mathcal{T}, \Phi_\mathcal{T}^\infty)$ comes from $\mathrm{Spec}(k)$, we deduce that $(\mathcal{T}, \Phi_\mathcal{T}^\infty)$ has a non-trivial global section in $\mathbf{F}^\infty\text{-}\mathbf{Isoc}(X)$. This implies that there exists a non-zero morphism $(\mathcal{N}, \Phi_\mathcal{N}^\infty) \to (\mathcal{M}, \Phi_\mathcal{M}^\infty)$. Since $(\mathcal{M}, \Phi_\mathcal{M}^\infty)$ is irreducible, we deduce that $(\mathcal{N}, \Phi_\mathcal{N}^\infty) = (\mathcal{M}, \Phi_\mathcal{M}^\infty)$. In turn, this implies that $\mathcal{M}$ is irreducible, as we wanted. □

Proposition 3.4. *Let $(\mathcal{M}, \Phi_\mathcal{M})$ be a convergent F^n-isocrystal over a geometrically connected variety X over a perfect field k. If $G(\mathcal{M}, \eta)$ is commutative for some perfect point η, the slopes of $(\mathcal{M}, \Phi_\mathcal{M})$ are constant.*

Proof. By [Cre92, (2.1.10)] we may assume k algebraically closed. In addition it is enough to look at the induced convergent F^∞-isocrystal $(\mathcal{M}, \Phi_\mathcal{M}^\infty)$. We may further assume that $(\mathcal{M}, \Phi_\mathcal{M}^\infty)$ is irreducible. By Lemma 3.3, we deduce that $\mathcal{M}$ is irreducible as well. Therefore, Lemma 3.2 implies that $\mathcal{M}$ is of rank 1. The result then follows from [Ked16, Thm. 3.12]. □

We end this section with a result that we will need in Proposition 4.4.

Definition 3.5. For a smooth connected variety X we write $\mathbf{Isoc}(X)_F$ for the smallest strictly full abelian $\otimes$-subcategory of $\mathbf{Isoc}(X)$ closed under subquotients containing all the convergent isocrystals which can be endowed with a Frobenius structure. If k is algebraically closed and η is a geometric point of X, we denote instead by $\mathbf{Isoc}_{\mathbb{Q}_p^{\mathrm{ur}}}(X,\eta)_F$ the category constructed in [D'Ad20b, Def. 3.2.6].

Proposition 3.6. *If k is an algebraically closed, there is an isomorphism*

$$\pi_1(\mathbf{Isoc}_{\mathbb{Q}_p^{\mathrm{ur}}}(X,\eta)_F,\eta)^{\mathrm{diag}} \xrightarrow{\sim} \pi_1(\mathbf{LS}(X,\mathbb{Q}_p^{\mathrm{ur}}),\eta)^{\mathrm{diag}}.$$

Proof. The morphism is induced by the functor constructed in [D'Ad20b, Prop. 3.3.4]. By Lemma 3.2, the two affine groups are the Tannaka groups of the subcategory of $\mathbf{Isoc}_{\mathbb{Q}_p^{\mathrm{ur}}}(X)_F$ and $\mathbf{LS}(X,\mathbb{Q}_p^{\mathrm{ur}})$ spanned by rank 1 objects. Therefore, thanks to [*ibid.*, Prop 3.3.4], it suffices to show that every rank 1 object $(\mathcal{M},V_{\mathcal{M}})\in \mathbf{Isoc}_{\mathbb{Q}_p^{\mathrm{ur}}}(X,\eta)_F$ is *étale* (cf. [*ibid.*, Def. 3.3.3]). To prove this we notice that by [*ibid.*, Prop 3.2.4], if $(\mathcal{M},V_{\mathcal{M}})\in \mathbf{Isoc}_{\mathbb{Q}_p^{\mathrm{ur}}}(X,\eta)_F$ is a rank 1 object, the $\mathbb{Q}_p^{\mathrm{ur}}$-structure $V_{\mathcal{M}}$ is induced by some F^n-structure $\Phi_{\mathcal{M}}$ on $\mathcal{M}$. Therefore, by [Ked16, Thm. 3.12], the convergent F^n-isocrystal $(\mathcal{M},\Phi_{\mathcal{M}})$ has constant slopes. This yields the desired result. □

4. Isocrystals over abelian varieties

Let A be an abelian variety over a perfect field k with identity point 0_A. We first prove the following result.

Proposition 4.1. *The affine group scheme $\pi_1(\mathbf{Isoc}(A),0_A)$ is commutative.*

Proof. We want to prove that the fundamental group $\pi_1(\mathbf{Isoc}(A),0_A)$ is commutative via an Eckmann–Hilton argument (see [EH62, Thm 5.4.2]). By Proposition 2.2, the two projections of $A\times A$ to its factors induce an isomorphism

$$\pi_1(\mathbf{Isoc}(A\times A),0_A\times 0_A)\xrightarrow{\sim}\pi_1(\mathbf{Isoc}(A),0_A)\times\pi_1(\mathbf{Isoc}(A),0_A).$$

If $m : A\times A\to A$ is the multiplication map of A, the morphism

$$\widetilde{m}_* : \pi_1(\mathbf{Isoc}(A),0_A)\times\pi_1(\mathbf{Isoc}(A),0_A)\xrightarrow{\sim}\pi_1(\mathbf{Isoc}(A\times A),0_A\times 0_A)\xrightarrow{m_*}\pi_1(\mathbf{Isoc}(A),0_A).$$

endows $\pi_1(\mathbf{Isoc}(A),0_A)$ with the structure of a group object in the category of affine group schemes. This implies that $\pi_1(\mathbf{Isoc}(A),0_A)$ is commutative, as we wanted. □

Theorem 4.2. *If A is an abelian variety over a perfect field k of positive characteristic, every F^n-isocrystal over A has constant slopes.*

Proof. Let $(\mathcal{M},\Phi_{\mathcal{M}})$ be an F^n-isocrystal over A. By Proposition 4.1, the monodromy group $G(\mathcal{M},0_A)$, being a quotient of $\pi_1(\mathbf{Isoc}(A),0_A)$, is commutative. Thanks to Proposition 3.4, we deduce that the slopes of $(\mathcal{M},\Phi_{\mathcal{M}})$ are constant. This ends the proof. □

Theorem 4.3. *If k is a finite field, every ι-pure F^n-isocrystal over A becomes constant after passing to a finite étale cover.*

Proof. By [D'Ad20a, Cor. 3.5.2], if $(\mathcal{M},\Phi_{\mathcal{M}})$ is a ι-pure F^n-isocrystal over A, then $\mathcal{M}$ is semi-simple. Therefore, thanks to [*ibid.*, Cor. 3.4.5], the neutral component $G(\mathcal{M},\eta)^\circ$ is a semi-simple algebraic group. Combining this with Proposition 4.1, we deduce that $G(\mathcal{M},\eta)^\circ$ is trivial. Therefore, by [*ibid.*, Prop. 3.3.4], after passing to a finite étale cover of A, the isocrystal $\mathcal{M}$ becomes trivial. This yields the desired result. □

Finally, we get an additional consequence of Proposition 4.1, which was inspired by the work in [Lan12] and [BdS17].

Proposition 4.4. *Let X be a smooth connected proper variety over an algebraically closed field k and let x be a k-point of X. If $f : X \to \mathrm{Alb}_X$ is the Albanese morphism mapping x to 0_{Alb_X}, the induced morphism*

$$\pi_1(\mathbf{Isoc}(X)_F, x)^{\mathrm{ab}} \xrightarrow{f_*} \pi_1(\mathbf{Isoc}(\mathrm{Alb}_X)_F, 0_{\mathrm{Alb}_X})$$

is faithfully flat. Moreover, the kernel is a finite constant group scheme over K isomorphic to $C := (\mathrm{Pic}^{\tau}_{X/k}/\mathrm{Pic}^{0,\mathrm{red}}_{X/k})^{\vee}(k)$.

Proof. Write G for $\pi_1(\mathbf{Isoc}(X)_F, x)^{\mathrm{ab}}$ and H for $\pi_1(\mathbf{Isoc}(\mathrm{Alb}_X)_F, 0)$. By Proposition 4.1, the affine group scheme H is commutative, therefore both G and H decompose as a product of a pro-diagonalisable affine group and a commutative pro-unipotent one. By [KL81, Lem. 5], the morphism $\pi_1^{\text{ét}}(X, x) \to \pi_1^{\text{ét}}(\mathrm{Alb}_X, 0_{\mathrm{Alb}_X})$ is surjective and the kernel is isomorphic to C. Combining Proposition 3.6 and [D'Ad20b, Prop. 3.3.2], this implies that $G^{\mathrm{diag}} \to H^{\mathrm{diag}}$ is faithfully flat with kernel C. It remains to study the morphism $G^{\mathrm{uni}} \to H^{\mathrm{uni}}$.

By [DE20, Thm. 5.4], there is an equivalence between the category of unipotent convergent isocrystals over X (resp. A) and the full subcategory of $\mathbf{Isoc}(X)_F$ (resp. $\mathbf{Isoc}(\mathrm{Alb}_X)_F$) of unipotent objects. By the discussion after [CLS99, Prop. 3.2.1], the Lie algebra of G^{uni} (resp. H^{uni}) is then dual to $H^1_{\mathrm{rig}}(X)$ (resp. $H^1_{\mathrm{rig}}(\mathrm{Alb}_X)$). Thanks to [Ill79, Rmq. II.3.11.2], we deduce that $G^{\mathrm{uni}} \to H^{\mathrm{uni}}$ is an isomorphism, as we wanted. □

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