ON SOME SYSTEMS OF DIFFERENCE EQUATIONS. Part 2.

L.A.Gutnik

Table of contents

§2.0. Foreword.

Here I continue translation in English of my paper [59].

Acknowledgments. I express my deepest thanks to Professors B.Z.Moroz, I.I. Piatetski-Shapiro,
A.G.Aleksandrov, P.Bundshuh and S.G.Gindikin for help and support.

§2.1. Introduction.

Let

$$|z| \geq 1, -3\pi/2 < \arg(z) \leq \pi/2, \log(z) = \ln(|z|) + i\arg(z).$$

Then $\log(-z) = \log(z) - i\pi$, if $\Re(z) > 0$ and $\log(z) = \log(-z) - i\pi$, if $\Re(z) < 0$. The followimg functions are considered in [75]:

$$f_{l,1}(z, \nu) = \sum_{k=0}^{\nu} (-1)^{(\nu+k)l} (z)^k \binom{\nu}{k}^{2+l} \binom{\nu + k}{\nu}^{2+l}, \tag{1}$$

where $l = 0,\ 1,\ 2,\ \nu \in [0, +\infty) \cap \mathbb{Z}$ (see (2) in [75]);

$$f_{l,2}(z, \nu) = \sum_{t=1+\nu}^{+\infty} z^{-t} (R(t, \nu))^{2+l} = \tag{2}$$

$$\sum_{t=1}^{+\infty} z^{-t} (R(t, \nu))^{2+l},$$

where $l = 0,\ 1,\ 2,\ \nu \in [0, +\infty) \cap \mathbb{Z}$,

(3)
$$R(t,\nu) = \frac{\prod\limits_{j=1}^{\nu}(t-j)}{\prod\limits_{j=0}^{\nu}(t+j)}.$$

(see (7), (8) and (4) in [75]);

(4)
$$f^{\vee}_{l,3}(z,\nu) = f_{l,3}(z,\nu) = (\log(z))f_{l,2}(z,\nu) + f_{l,4}(z,\nu)$$

where

(5)
$$f_{l,4}(z,\nu) = -\sum_{t=1+\nu}^{+\infty} z^{-t}\left(\frac{\partial}{\partial t}(R^{2+l})\right)(t,\nu) =$$
$$-\sum_{t=1}^{+\infty} z^{-t}\left(\frac{\partial}{\partial t}(R^{2+l})\right)(t,\nu),$$

$l = 0,\ 1,\ 2,\ \nu \in [0, +\infty) \cap \mathbb{Z}$, (see (9), (13) and (12) in the first part);

(6)
$$f_{l,5}(z,\nu) =$$
$$2^{-1}(\log(z))^2 f_{l,2}(z,\nu) + (\log(z))f_{l,4}(z,\nu) + f_{l,6}(z,\nu) =$$
$$= -2^{-1}(\log(z))^2 f_{l,2}(z,\nu) + +(\log(z))f_{l,3}(z,\nu) + f_{l,6}(z,\nu),$$

where $l = 1,\ 2,\ \nu \in [0, +\infty) \cap \mathbb{Z}$,

(7)
$$f_{l,6}(z,\nu) = 2^{-1}\sum_{t=\nu+1}^{\infty} z^{-t}\left(\left(\frac{\partial}{\partial t}\right)^2 (R^{2+l})\right)(t,\nu) =$$
$$2^{-1}\sum_{t=1}^{\infty} z^{-t}\left(\left(\frac{\partial}{\partial t}\right)^2 (R^{2+l})\right)(t,\nu),$$

(8)
$$f^{\vee}_{l,5}(z,\nu) = -i\pi f_{l,3}(z,\nu) + f_{l,5}(z,\nu)$$

where $l = 1,\ 2,\ \nu \in [0, +\infty) \cap \mathbb{Z}$, (see (14), (16)-(19) in [75]);

(9)
$$f_{l,7}(z,\nu) =$$
$$6^{-1}(\log(z))^3 f_{l,2}(z,\nu) + 2^{-1}(\log(z))^2 f_{l,4}(z,\nu) + (\log(z))f_{l,6}(z,\nu) + f_{l,8}(z,\nu) =$$
$$-3^{-1}(\log(z))^3 f_{l,2}(z,\nu) + 2^{-1}(\log(z))^2 f_{l,3}(z,\nu) + f_{l,8}(z,\nu) +$$
$$(\log(z))(f_{l,5}(z,\nu) + 2^{-1}(\log(z))^2 f_{l,2}(z,\nu) - (\log(z))f_{l,3}(z,\nu)) =$$
$$6^{-1}(\log(z))^3 f_{l,2}(z,\nu) - 2^{-1}(\log(z))^2 f_{l,3}(z,\nu) + (\log(z))f_{l,5}(z,\nu) + f_{l,8}(z,\nu),$$

where where $l = 2,\ \nu \in [0, +\infty) \cap \mathbb{Z}$,

(10)
$$f_{l,8}(z,\nu) = -6^{-1}\sum_{t=\nu+1}^{\infty} z^{-t}\left(\left(\frac{\partial}{\partial t}\right)^3 (R^{2+l})\right)(t,\nu) =$$
$$-6^{-1}\sum_{t=1}^{\infty} z^{-t}\left(\left(\frac{\partial}{\partial t}\right)^3 (R^{2+l})\right)(t,\nu),$$

(11)
$$f^{\vee}_{l,7}(z,\nu) = f_{l,7}(z,\nu) + (2\pi^2/3)f_{l,3}(z,\nu).$$

where $l = 2$, $\nu \in [0,+\infty)\cap\mathbb{Z}$, (see (21), (23)-(26) in [75]). Let

$$\mathfrak{K}_0 = \{1,\ 2,\ 3\},\ \mathfrak{K}_1 = \{1,\ 2,\ 3,\ 5\},\ \mathfrak{K}_2 = \{1,\ 2,\ 3,\ 5,\ 7\},$$

(12)
$$f^{\vee}_{l,k}(z,-\nu-1) = f^{\vee}_{l,k}(z,\nu),$$

where $\nu \in [0,+\infty)\cap\mathbb{Z}$, $l = 0,\ 1,\ 2$, $k \in \mathfrak{K}_,l$; so these functions are defined for all the $\nu \in \mathbb{Z}$, and (12) holds for all the $\nu \in \mathbb{Z}$. Let further $\nu \in [0,+\infty)\cap\mathbb{Z}$,

(13)
$$P^*_0(\nu;w) = \nu^3 - 4\nu^2 w + 8\nu w^2 - 12w^3,\ Q^*_0(\nu,w) =$$
$$16\nu + 12w,$$

(14)
$$P^*_1(\nu;w) = -\nu^5 + 6\nu^4 w - 18\nu^3 w^2 + 38\nu^2 w^3 - 66\nu w^4 +$$
$$102w^5,\ Q^*_1(\nu;w) = -146\nu^2 - 240\nu w - 102w^2,$$

(15)
$$P^*_2(\nu;w) = \nu^7 - 8\nu^6 w + 32\nu^5 w^2 - 88\nu^4 w^3 + 192\nu^3 w^4 -$$
$$360\nu^2 w^5 + 608\nu w^6 - 952w^7,\ Q^*_2(\nu;w) = 1408\nu^3 + 3640\nu^2 w + 3200\nu w^2 + 952w^3,$$

(16)
$$D_l(z,\nu,w) = z(w-\nu)^{2+l}(w+\nu+1)^{2+l} - w^{4+2l},$$

where $l = 0,\ 1,\ 2$, $\nu \in \mathbb{Z}$, and w is independent variable. Clearly,

(17)
$$D_l(z,\nu,w) = D_l(z,-\nu-1,w)$$

for $l = 0,\ 1,\ 2$, $\nu \in \mathbb{Z}$. Let further $\delta := z\frac{\partial}{\partial z}$. Then (see [75])

(18)
$$D_l(z,\nu,\delta)f^{\vee}_{l,k}(z,\nu) = 0,$$

where $\nu \in \mathbb{Z}$, $l = 0,\ 1,\ 2$, $k \in \mathfrak{K}_l, |z| > 1$. Further we have (see [75])

(19)
$$(\delta+\nu+1)^{2+l}f^{\vee}_{l,k}(z,\nu) = (\delta-\nu-1)^{2+l}f^{\vee}_{l,k}(z,\nu+1),$$

where $\nu \in [0,+\infty)\cap\mathbb{Z}$, $l = 0,\ 1,\ 2$, $k \in \mathfrak{K}_l, |z| > 1$,

(20)
$$(\delta+\nu)^{2+l}f^{\vee}_{l,k}(z,\nu-1) = (\delta-\nu)^{2+l}f^{\vee}_{l,k}(z,\nu),$$

where $\nu \in \cap\mathbb{Z}$, $l = 0,\ 1,\ 2$, $k \in \mathfrak{K}_l, |z| > 1$. Finally the following equalities was established in [75]:

(21)
$$\nu^{3+2l}f^{\vee}_{l,k}(z,\nu-1) -$$
$$(P^*_l(\nu;\delta) + zQ^*_l(\nu;\delta+1)(\delta-\nu)^{2+l})f^{\vee}_{l,k}(z,\nu) = 0,$$

where $\nu \in \mathbb{Z}$, $l = 0,\ 1,\ 2$, $k \in \mathfrak{K}_l, |z| > 1$,

(22)
$$(-(\nu+1))^{3+2l}f^{\vee}_{l,k}(z,\nu+1) -$$
$$(P^*_l(-\nu-1;\delta) + zQ^*_l(-\nu-1;\delta+1)(\delta+\nu+1)^{2+l})f^{\vee}_{l,k}(z,\nu) = 0,$$

where $\nu \in \mathbb{Z}$, $l = 0,\ 1,\ 2$, $k \in \mathfrak{K}_l, |z| > 1$.

§2. Pass to the system of difference equtions. The case $|z| > 1$.

Let

$$\mu = \nu(\nu + 1) \tag{23}$$

where $\nu \in \mathbb{Z}$. Then in view of (16

$$D_l(z, \nu, w) = z(w^2 + w - \mu)^{2+l} - w^{4+2l}, \tag{24}$$

where $l = 0, 1, 2,$

$$D_0(z, \nu, w) = z\mu^2 - 2zw\mu - 2zw^2\mu + zw^2 + 2zw^3 + (z - 1)w^4,$$

$$D_1(z, \nu, w) = -z\mu^3 + 3zw\mu^2 + 3zw^2\mu^2 -$$
$$3zw^2\mu - 6zw^3\mu - 3zw^4\mu + zw^3 + 3zw^4 + 3zw^5 + (z - 1)w^6,$$
$$D_2(z, \nu, w) = z\mu^4 - 4zw\mu^3 - 4zw^2\mu^3 +$$
$$6zw^2\mu^2 + 12zw^3\mu^2 + 6zw^4\mu^2 -$$
$$4zw^3\mu - 12zw^4\mu - 12zw^5\mu - 4zw^6\mu +$$
$$zw^4 + 4zw^5 + 6zw^6 + 4zw^7 + (z - 1)w^8.$$

Let

$$b_{0,1}(z;\nu) = -(z-1)^{-1}z\mu^2 = -(z-1)^{-1}z(\nu^2 + 2\nu^3 + \nu^4),$$
$$b_{0,2}(z;\nu) = -(z-1)^{-1}2z(-\mu) = (z-1)^{-1}2z(-\nu - \nu^2),$$
$$b_{0,3}(z;\nu) = -(z-1)^{-1}z(1 - 2\mu) = -(z-1)^{-1}z(1 - 2\nu - 2\nu^2),$$
$$b_{0,4}(z;\nu) = -(z-1)^{-1}2z,$$
$$b_{1,1}(z;\nu) = -(z-1)^{-1}z(-\mu^3) = -(z-1)^{-1}z(-\nu^3 - 3\nu^4 - 3\nu^5 - \nu^6),$$
$$b_{1,2}(z;\nu) = -(z-1)^{-1}3z\mu^2 = -(z-1)^{-1}z(3\nu^2 + 6\nu^3 + 3\nu^4),$$
$$b_{1,3}(z;\nu) = -(z-1)^{-1}z(3\mu^2 - 3\mu) = -(z-1)^{-1}z(-3\nu + 6\nu^3 + 3\nu^4),$$
$$b_{1,4}(z;\nu) = -(z-1)^{-1}z(1 - 6\mu) = -(z-1)^{-1}z(1 - 6\nu - 6\nu^2),$$
$$b_{1,5}(z;\nu) = -(z-1)^{-1}z(3 - 3\mu) = -(z-1)^{-1}z(3 - 3\nu - 3\nu^2),$$
$$b_{1,6}(z;\nu) = -(z-1)^{-1}3z,$$
$$b_{2,1}(z;\nu) = -(z-1)^{-1}z(\mu^4) = -(z-1)^{-1}z(\nu^4 + 4\nu^5 + 6\nu^6 + 4\nu^7 + \nu^8),$$
$$b_{2,2}(z;\nu) = -(z-1)^{-1}z(-4\mu^3) = -(z-1)^{-1}z(-4\nu^3 - 12\nu^4 - 12\nu^5 - 4\nu^6),$$
$$b_{2,3}(z;\nu) = -(z-1)^{-1}z(-4\mu^3 + 6\mu^2) = -(z-1)^{-1}z\times$$
$$(-4\nu^3 - 12\nu^4 - 12\nu^5 - 4nu^6 + 6\nu^2 + 12\nu^3 + 6\nu^4) =$$
$$-(z-1)^{-1}z(6\nu^2 + 8\nu^3 - 6\nu^4 - 12\nu^5 - 4\nu^6),$$
$$b_{2,4}(z;\nu) = -(z-1)^{-1}z(-4\mu + 12\mu^2) = -(z-1)^{-1}z(-4\nu + 8\nu^2 + 24\nu^3 + 12\nu^4),$$
$$b_{2,5}(z;\nu) = -(z-1)^{-1}z(1 - 12\mu + 6\mu^2) = -(z-1)^{-1}z(1 - 12\nu - 6\nu^2 + 12\nu^3 + 6\nu^4),$$
$$b_{2,6}(z;\nu) = -(z-1)^{-1}z(4 - 12\mu) = -(z-1)^{-1}z(4 - 12\nu - 12\nu^2),$$

$$b_{2,7}(z;\nu)=-(z-1)^{-1}z(6-4\mu)=-(z-1)^{-1}z(6-4\nu-4\nu^2),$$
$$b_{2,8}(z;\nu)=-(z-1)^{-1}4z.$$

Let $l=0,\,1,\,2$ and $B_l(z;\nu)$ denotes $(4+2l)\times(4+2l)$- matrix having the following propeties:

a) it has on the intersection of its first $3+2l$ rows and last $3+2l$ columns the unit $(3+2l)\times(3+2l)$-matrix,

b) its elements on the intersection of its first $3+2l$ rows and first column is equal to 0; the unit $(3+2l)\times(3+2l)$-matrix,

c) the k-th element of its last row is equal to $b_{l,k}$ for $k=1,\,...,\,4+2l$.

Let further $X_{l,k}(z;\nu)$, where $l=0,\,1,\,2$ and $k\in\mathfrak{K}_l$, $|z|>1$, be the columnn with $4+2l$ elements, i-th of which is equal to $\delta^{i-1}f^{\vee}_{l,k}$ for $i=1,\,...,\,4+2l$. Then

$$\delta X_{l,k}(z;-\nu-1)=X_{l,k}(z;\nu), \tag{25}$$

$$\delta X_{l,k}(z;\nu)=B_l(z;\nu)X_{l,k}(z;\nu), \tag{26}$$

where $l=0,\,1,\,2$ and $k\in\mathfrak{K}_l$, $|z|>1,\nu\in\mathbb{Z}$. Let $\tau=\nu+1/2$; then $tau^2=\mu+1/4$. Since, in view of (13) – (15),

$$Q^*_0(\nu,w+1)=12+16\nu+12w,$$

$$Q^*_1(\nu;w+1)=$$
$$-102-240\nu-146\nu^2-204w-240\nu w-102w^2,$$
$$Q^*_2(\nu;w+1)=952+3200\nu+3640\nu^2+1408\nu^3+$$
$$2856w+6400\nu w+3640\nu^2w+2856w^2+3200\nu w^2+952w^3,$$

it follows that

$$Q^*_0(\nu,w+1)(w-\nu)^2=$$
$$12\nu^2+16\nu^3-24\nu w-20\nu^2w-8\nu w^2+12w^3=$$
$$12(\tau-1/2)^2+16(\tau-1/2)^3-(24(\tau-1/2)-$$
$$20(\tau-1/2)^2)w-8(\tau-1/2)w^2+12w^3=$$
$$\sum_{k=0}^{3}q^*_{0,k}(\nu)w^k,$$

where

$$q^*_{0,0}(\nu)=-2-12\mu+(4+16\mu)\tau,$$
$$q_{0,1}(\nu)^ast=2-20\mu-4\tau,$$
$$q_{0,2}(\nu)^*=16-8\tau,$$
$$q_{0,3}(\nu)^*=12,$$

$$Q^*_1(\nu,w+1)(w-\nu)^3=102\nu^3+240\nu^4+146\nu^5+(-306\nu^2-516\nu^3-198\nu^4)w+$$
$$(306\nu+108\nu^2-180\nu^3)w^2+(-102+372\nu+268\nu^2)w^3+$$
$$(-204+66\nu)w^4-102w^5=$$

$$\sum_{k=0}^{5} q^*_{1,k}(\nu)w^k,$$

where

$$q^*_{1,0}(\nu) = -4 - 38\mu - 125\mu^2 + (8 + 60\mu + 146\mu^2)\tau,$$
$$q^*_{1,1}(\nu) = 6 + 72\mu - 198\mu^2 + (-12 - 120\mu)\tau,$$
$$q^*_{1,2}(\nu) = -9 + 378\mu + (18 - 180\mu)\tau,$$
$$q^*_{1,3}(\nu) = -154 + 268\mu + 104\tau,$$
$$q_{1,4}(\nu) = -237 + 66\tau,$$
$$q^*_{1,5}(\nu) = -102,$$
$$Q^*_2(\nu, w+1)(w-\nu)^4 = 952\nu^4 + 3200\nu^5 + 3640\nu^6 + 1408\nu^7 +$$
$$(-3808\nu^3 - 9944\nu^4 - 8160\nu^5 - 1992\nu^6)w +$$
$$(5712\nu^2 + 7776\nu^3 - 904\nu^4 - 2912\nu^5)w^2 +$$
$$(-3808\nu + 4336\nu^2 + 12416\nu^3 + 4360\nu^4)w^3 +$$
$$(952 - 8224\nu - 4824\nu^2 + 2240\nu^3)w^4 +$$
$$(2856 - 5024\nu - 3448\nu^2)w^5 + (2856 - 608\nu)w^6 + (952)w^7 =$$
$$\sum_{k=0}^{7} q^*_{2,k}(\nu)w^k,$$

where

$$q_{2,0}(\nu)^* = -8 - 104\mu - 524\mu^2 - 1288\mu^3 + (16 + 176\mu + 728\mu^2 + 1408\mu^3)\tau,$$
$$q_{2,1}(\nu)^* = 16 + 248\mu + 1492\mu^2 - 1992\mu^3 + (-32 - 432\mu - 2184\mu^2)\tau,$$
$$q_{2,2}(\nu)^* = -28 - 480\mu + 6376\mu^2 +$$
$$(56 + 848\mu - 2912\mu^2)\tau,$$
$$q_{2,3}(\nu)^* = 44 - 5568\mu + 4360\mu^2 + (-88 + 3696\mu)\tau,$$
$$q_{2,4}(\nu)^* = 1532 - 8184\mu + (-1160 + 2240\mu)\tau,$$
$$q_{2,5}(\nu)^* = 3644 - 3448\mu - 1576\tau,$$
$$q_{2,6}(\nu)^* = 3160 - 608\tau,$$
$$q_{2,7}(\nu)^* = 952.$$

Let

$$q^\vee_{0,0}(\nu) = -2 - 12\mu,\ q^\wedge_{0,0}(\nu) = 4 + 16\mu,$$
$$q_{0,1}(\nu)^\vee = 2 - 20\mu,\ q_{0,1}(\nu)^\wedge = -4,$$
$$q_{0,2}(\nu)^\vee = 16,\ q_{0,2}(\nu)^\wedge = -8,$$
$$q_{0,3}(\nu)^\vee = 12,\ q_{0,3}(\nu)^\wedge = 0,$$
$$q^\vee_{1,0}(\nu) = -4 - 38\mu - 125\mu^2,\ q^\wedge_{1,0}(\nu) = 8 + 60\mu + 146\mu^2,$$
$$q^\vee_{1,1}(\nu) = 6 + 72\mu - 198\mu^2,\ q^\wedge_{1,1}(\nu) = -12 - 120\mu),$$
$$q^\vee_{1,2}(\nu) = -9 + 378\mu,\ q^\wedge_{1,2}(\nu) = 18 - 180\mu,$$

$$q^{\vee}_{1,3}(\nu) = -154 + 268\mu,\ q^{\wedge}_{1,3}(\nu) = 104,$$
$$q^{\vee}_{1,4}(\nu) = -237,\ q_{1,4}(\nu)^{\wedge} = 66,$$
$$q^{\vee}_{1,5}(\nu) = -102,\ q^{\wedge}_{1,5}(\nu) = 0,$$
$$q_{2,0}(\nu)^{\vee} = -8-104\mu-524\mu^2-1288\mu^3,\ q_{2,0}(\nu)^{\wedge} = 16+176\mu+728\mu^2+1408\mu^3,$$
$$q_{2,1}(\nu)^{\vee} = 16 + 248\mu + 1492\mu^2 - 1992\mu^3,\ q_{2,1}(\nu)^{\wedge} = -32 - 432\mu - 2184\mu^2,$$
$$q_{2,2}(\nu)^{\vee} = -28 - 480\mu + 6376\mu^2,\ q_{2,2}(\nu)^{\wedge} = 56 + 848\mu - 2912\mu^2,$$
$$q_{2,3}(\nu)^{\vee} = 44 - 5568\mu + 4360\mu^2,\ q_{2,3}(\nu)^{\wedge} = -88 + 3696\mu,$$
$$q_{2,4}(\nu)^{\vee} = 1532 - 8184\mu,\ q_{2,4}(\nu)^{\wedge} = -1160 + 2240\mu,$$
$$q_{2,5}(\nu)^{\vee} = 3644 - 3448\mu,\ q_{2,5}(\nu)^{\wedge} = -1576,$$
$$q_{2,6}(\nu)^{\vee} = 3160,\ q_{2,6}(\nu)^{\wedge} = -608,$$
$$q_{2,7}(\nu)^{\vee} = 952,\ q_{2,7}(\nu)^{\wedge} = 0.$$

Then
$$q_{l,k}(\nu)^{*} = q_{l,k}(\nu)^{\vee} + q_{l,k}(\nu)^{\wedge}\tau$$
for $l = 0,\ 1,\ 2,\ k = 0,\ ...,\ 3 + 2l$. In view of (13) – (15),

(27) $$P^{*}_{l}(\nu; w) = \sum k = 0^{3+2l} p^{*}_{l,k}(\nu)w^k,$$

for $l = 0,\ 1,\ 2,$ where

$$p^{*}_{0,0}(\nu) = \nu^3 = (\tau - 1/2)^3 =$$
$$-1/8 - (3/2)(\mu + 1/4) + (\mu + 1/4 + 3/4)\tau = -1/2 - (3/2)\mu + (1 + \mu)\tau,$$
$$p^{*}_{0,1}(\nu) = -4\nu^2 = -4(\tau - 1/2)^2 = -2 - 4\mu + 4\tau,$$
$$p^{*}_{0,2}(\nu) = 8\nu = -4 + 8\tau, p^{*}_{0,3}(\nu) = -12,$$
$$p^{*}_{1,0}(\nu) = -\nu^5 = -(\tau - 1/2)^5 = (1/2)(1 + 5\mu + 5\mu^2) + (-1 - 3\mu - \mu^2)\tau,$$
$$p^{*}_{1,1}(\nu) = 6\nu^4 = 3 + 12\mu + 6\mu^2 + (-6 - 12\mu)\tau,$$
$$p^{*}_{1,2}(\nu) = -18\nu^3 = 9 + 27\mu + (-18 - 18\mu)\tau,$$
$$p^{*}_{1,3}(\nu) = 38\nu^2 = 19 + 38\mu - 38\tau,$$
$$p^{*}_{1,4}(\nu) = -66\nu = 33 - 66\tau, p^{*}_{1,5}(\nu) = 102,$$
$$p^{*}_{2,0}(\nu) = \nu^7 = \nu^5\mu - \nu^6 =$$
$$(1/2)(-\mu - 5\mu^2 - 5\mu^3) + (\mu + 3\mu^2 + \mu^3)\tau - \mu\nu^4 + \nu^5 =$$
$$(1/2)(-\mu - 5\mu^2 - 5\mu^3) + (\mu + 3\mu^2 + \mu^3)\tau+$$
$$(-1/2)\mu - 2\mu^2 - \mu^3 + (\mu + 2\mu^2)\tau+$$
$$(-1/2)(1 + 5\mu + 5\mu^2) + (1 + 3\mu + \mu^2)\tau =$$
$$(1/2)(-1 - 7\mu - 14\mu^2 - 7\mu^3) + (1 + 5\mu + 6\mu^2 + \mu^3)\tau,$$
$$p^{*}_{2,1}(\nu) = -8\nu^6 = -4 - 24\mu - 36\mu^2 - 8\mu^3 + (8 + 32 + 24\mu^2)\tau,$$
$$p^{*}_{2,2}(\nu) = 32\nu^5 = -16 - 80\mu - 80\mu^2 + (32 + 96\mu + 32\mu^2)\tau,$$

$$p^*_{2,3}(\nu) = -88\nu^4 = -44 - 176\mu - 88\mu^2 + (88 + 176\mu)\tau,$$
$$p^*_{2,4}(\nu) = 192\nu^3 = -96 - 288\mu + (192 + 192\mu)\tau,$$
$$p^*_{2,5}(\nu) = -360\nu^2 = -180 - 360\mu + 360\tau,$$
$$p^*_{2,6}(\nu) = 608\nu = -304 + 608\tau,$$
$$p^*_{2,7}(\nu) = -952.$$

Let

$$p^\vee_{0,0}(\nu) = -1/2 - (3/2)\mu,\ p^\wedge_{0,0}(\nu) = (1 + \mu),$$
$$p^\vee_{0,1}(\nu) = -2 - 4\mu,\ p^\wedge_{0,1}(\nu) = 4,$$
$$p^\vee_{0,2}(\nu) = -4,\ p^\wedge_{0,2}(\nu) = 8,$$
$$p^\vee_{0,3}(\nu) = -12,\ p^\wedge_{0,3,}(\nu) = 0,$$
$$p^\vee_{1,0}(\nu) = (1/2)(1 + 5\mu + 5\mu^2),\ p^\wedge_{10,}(\nu) = -1 - 3\mu - \mu^2,$$
$$p^\vee_{1,1}(\nu) = 3 + 12\mu + 6\mu^2,\ p^\wedge_{1,1}(\nu) = -6 - 12\mu,$$
$$p^\vee_{1,2}(\nu) = 9 + 27\mu,\ p^\wedge_{1,2}(\nu) = -18 - 18\mu,$$
$$p^\vee_{1,3}(\nu) = 19 + 38\mu,\ p^\wedge_{1,3}(\nu) = -38,$$
$$p^\vee_{1,4}(\nu) = 33,\ p^\wedge_{1,4}(\nu) = -66,$$
$$p^\vee_{1,5}(\nu) = 102,\ p^\wedge_{1,5}(\nu) = 0,$$
$$p^\vee_{2,0}(\nu) = (1/2)(-1 - 7\mu - 14\mu^2 - 7\mu^3),\ p^\wedge_{2,0}(\nu) = 1 + 5\mu + 6\mu^2 + \mu^3,$$
$$p^\vee_{2,1}(\nu) = -4 - 24\mu - 36\mu^2 - 8\mu^3,\ p^\wedge_{2,1}(\nu) = 8 + 32 + 24\mu^2,$$
$$p^\vee_{2,2}(\nu) = -16 - 80\mu - 80\mu^2,\ p^\wedge_{2,2}(\nu) = 32 + 96\mu + 32\mu^2,$$
$$p^\vee_{2,3}(\nu) = -44 - 176\mu - 88\mu^2,\ p^\wedge_{2,3}(\nu) = 88 + 176\mu,$$
$$p^\vee_{2,4}(\nu) = -96 - 288\mu,\ p^\wedge_{2,4}(\nu) = 192 + 192\mu,$$
$$p^\vee_{2,5}(\nu) = -180 - 360\mu,\ p^\wedge_{2,5}(\nu) = 360,$$
$$p^\vee_{2,6}(\nu) = -304,\ p^\wedge_{2,6}(\nu) = 608,$$
$$p^\vee_{2,7}(\nu) = -952,\ p^\wedge_{2,7}(\nu) = 0.$$

Then

$$p_{l,k}(\nu)^* = p_{l,k}(\nu)^\vee + p_{l,k}(\nu)^\wedge\tau$$

for $l = 0,\ 1,\ 2,\ k = 0,\ ...,\ 3 + 2l$. We denote by

$$\alpha^*_{l,1}(z;\nu),\ \alpha^\vee_{l,1}(z;\nu),\ \alpha^\wedge_{l,1}(z;\nu)$$

for $l = 0,\ 1,\ 2,\ k = 0,\ ...,\ 3 + 2l$ the row with $4 + 2l$ elements, $(k + 1)$-th of which is equal respectively

$$p^*_{l,k}(\nu) + zq^*_{l,k}(\nu),$$
$$p^\vee_{l,k}(\nu) + zq^\vee_{l,k}(\nu),$$
$$p^\wedge_{l,k}(\nu) + zq^\wedge_{l,k}(\nu),$$

where $k = 0, ..., 3+2l$. Let

(28)
$$\alpha^*_{l,i+1}(z;\nu) = \delta\alpha^*_{l,i}(z;\nu) +$$
$$\alpha^*_{l,i}(z;\nu)B_l(z;\nu),$$

(29)
$$\alpha^\vee_{l,i+1}(z;\nu) = \delta\alpha^*_{l,i}(z;\nu) +$$
$$\alpha^\vee_{l,i}(z;\nu)B_l(z;\nu),$$

(30)
$$\alpha^\wedge_{l,i+1}(z;\nu) = \delta\alpha^\wedge_{l,i}(z;\nu) +$$
$$\alpha^\wedge_{l,i}(z;\nu)B_l(z;\nu),$$

where $l = 0, 1, 2, i = 0, ..., 3+2l$. Clearly,

$$\alpha^*_{l,k}(z;\nu) = \alpha^\vee_{l,k}(z;\nu) + \alpha^\wedge_{l,k}(z;\nu)\tau$$

for $l = 0, 1, 2, k = 0, ..., 3+2l$. We denote by

$$a^*_{l,i,k}(z;\nu),\ a^\vee_{l,i,k}(z;\nu),\ a^\wedge_{l,i,k}(z;\nu)$$

for $l = 0, 1, 2, i = 1, ..., 4+2l, k = 1, ..., 4+2l$ the k-th elements of the rows respecively

$$\alpha^*_{l,i}(z;\nu),\ \alpha^\vee_{l,i}(z;\nu),\ \alpha^\wedge_{l,i}(\nu).$$

Then,

$$a^\vee_{0,1,1}(z;\nu) = (1/2)(-5-27\mu+(z-1)(-4-24\mu)),$$
$$a^\vee_{0,1,2}(z;\nu) == (1/2)(-48\mu+(z-1)(4-40\mu)),$$
$$a^\vee_{0,1,3}(z;\nu) = -4+z16 = (1/2)(24+(z-1)32),$$
$$a^\vee_{0,1,4}(z;\nu) = -12+z12 = (1/2)(z-1)24,$$
$$a^\wedge_{0,1,1}(z;\nu) = 1+\mu+z(4+16\mu) = 5+17\mu+(z-1)(4+16\mu),$$
$$a^\wedge_{0,1,2}(\nu) = 4+z(-4) = (z-1)(-4),$$
$$a^\wedge_{0,1,3}(z;\nu) = 8+z(-8) = (z-1)(-8),$$
$$a^\wedge_{0,1,4}(z;\nu) = 0,$$
$$a^\vee_{1,1,1}(z;\nu) = (1/2)(-7-71\mu-245\mu^2+(z-1)(-8-76\mu-250\mu^2)),$$
$$a^\vee_{1,1,2}(z;\nu) = (1/2)(18+168\mu-384\mu^2+(z-1)(12+144\mu-396\mu^2)),$$
$$a^\vee_{1,1,3}(z;\nu) = (1/2)(810\mu+(z-1)(-18+756\mu)),$$
$$a^\vee_{1,1,4}(z;\nu) = (1/2)(-270+612\mu+(z-1)(-308+536\mu)),$$
$$a^\vee_{1,1,5}(z;\nu) = (1/2)(-408+(z-1)(-474\mu)),$$
$$a^\vee_{1,1,6}(z;\nu) = -(1/2)(z-1)204$$
$$a^\wedge_{1,1,1}(z;\nu) = 7+57\mu+145\mu^2+(z-1)(8+60\mu+146\mu^2),$$
$$a^\wedge_{1,1,2}(z;\nu) = -18-132\mu+(z-1)(-12-120\mu),$$
$$a^\wedge_{1,1,3}(z;\nu) = -198\mu+(z-1)(18-180\mu),$$

$$a^{\wedge}_{1,1,4}(z;\nu) = 66 + (z-1)(104),$$
$$a^{\wedge}_{1,1,5}(z;\nu) = (z-1)66,$$
$$a^{\wedge}_{1,1,6}(z;\nu) = 0,$$
$$a^{\vee}_{2,1,1}(z;\nu) = (1/2)(-17 - 215\mu - 1062\mu^2 - 2583\mu^3)+$$
$$(1/2)(z-1)(-16 - 208\mu - 1048\mu^2 - 2576\mu^3),$$
$$a^{\vee}_{2,1,2}(z;\nu) = (1/2)(24 + 448\mu + 2912\mu^2 - 4000\mu^3)+$$
$$(1/2)(z-1)(32 + 496\mu + 2984\mu^2 - 3984\mu^3),$$
$$a^{\vee}_{2,1,3}(z;\nu) = (1/2)(-88 - 1120\mu + 12592\mu^2)+$$
$$(1/2)(z-1)(-56 - 960\mu + 12752\mu^2),$$
$$a^{\vee}_{2,1,4}(z;\nu) = (1/2)(-11488\mu + 8544\mu^2)+$$
$$(1/2)(z-1)(88 - 11136\mu + 8720\mu^2),$$
$$a^{\vee}_{2,1,5}(z;\nu) = (1/2)(2872 - 16944\mu) + (z-1)(3064 - 16368\mu),$$
$$a^{\vee}_{2,1,6}(z;\nu) = (1/2)(6928 - 7616\mu) + (z-1)(7288 - 6896\mu)),$$
$$a^{\vee}_{2,1,7}(z;\nu) = (1/2)(5712) + (z-1)(6320)),$$
$$a^{\vee}_{2,1,8}(z;\nu) = (1/2)(z-1)(1904),$$
$$a^{\wedge}_{2,1,1}(z;\nu) = 17 + 181\mu + 734\mu^2 + 1409\mu^3+$$
$$(z-1)(16 + 176\mu + 728\mu^2 + 1408\mu^3),$$
$$a^{\wedge}_{2,1,2}(z;\nu) = -24 - 400\mu - 2160\mu^2 + (z-1)(-32 - 432\mu - 2184\mu^2),$$
$$a^{\wedge}_{2,1,3}(z;\nu) = 88 + 944\mu - 2880\mu^2 + (z-1)(56 + 848\mu - 2912\mu^2),$$
$$a^{\wedge}_{2,1,4}(z;\nu) = 3872\mu + (z-1)(-88 + 3696\mu),$$
$$a^{\wedge}_{2,1,5}(z;\nu) = -968 + 2432\mu + (z-1)(-1160 + 2240\mu),$$
$$a^{\wedge}_{2,1,6}(z;\nu) = -1216 + (z-1)(-1576),$$
$$a^{\wedge}_{2,1,7}(z;\nu) = (z-1)(-608),$$
$$a^{\wedge}_{2,1,8}(z;\nu) = 0,$$

and, in view of (28) – (30),

$$a^{\vee}_{0,2,1}(z;\nu) = (1/2)(-4 - 24\mu - 24\mu^2 + (z-1)(-4 - 24\mu - 24\mu^2)),$$
$$a^{\vee}_{0,2,2}(z;\nu) = (1/2)(-1 - 19\mu + (z-1)(-16)),$$
$$a^{\vee}_{0,2,3}(z;\nu) = (1/2)(8 + (z-1)(12 + 8\mu)),$$
$$a^{\vee}_{0,2,4}(z;\nu) = (1/2)(8(z-1)),$$
$$a^{\vee}_{0,3,1}(z;\nu) = (1/2)(-4 - 24\mu - 32\mu^2 + (z-1)(-4 - 24\mu - 32\mu^2)),$$
$$a^{\vee}_{0,3,2}(z;\nu) = (1/2)(-4 - 24\mu - 24\mu^2 + (z-1)(-4 - 24\mu - 24\mu^2)),$$
$$a^{\vee}_{0,3,3}(z;\nu) = (1/2)(3 + 5\mu + (z-1)(4 + 8\mu)),$$
$$a^{\vee}_{0,3,4}(z;\nu) = (1/2)((z-1)(4 + 8\mu)),$$
$$a^{\vee}_{0,4,1}(z;\nu) =$$

$$(1/2)(-4-24\mu-36\mu^2-8\mu^3+(z-1)(-4-24\mu-8\mu^3)),$$

$$a^{\vee}_{0,4,2}(z;\nu)=$$

$$(1/2)(-8-40\mu-40\mu^2+(z-1)(-8-40\mu-40\mu^2)),$$

$$a^{\vee}_{0,4,3}(z;\nu)=(1/2)(-4-16\mu-8\mu^2+(z-1)(-4-16\mu-8\mu^2)),$$

$$a^{\vee}_{0,4,4}(z;\nu)=(1/2)(-1-3\mu),$$

$$a^{\wedge}_{0,2,1}(z;\nu)=4+16\mu+(z-1)(4+16\mu),$$

$$a^{\wedge}_{0,2,2}(z;\nu)=1+17\mu+(z-1)(4+16\mu),$$

$$a^{\wedge}_{0,2,3}(z;\nu)=-8+(z-1)(-12),$$

$$a^{\wedge}_{0,2,4}(z;\nu)=8(z-1),$$

$$a^{\wedge}_{0,3,1}(z;\nu)=4+16\mu+8\mu^2+(z-1)(4+16\mu+8\mu^2),$$

$$a^{\wedge}_{0,3,2}(z;\nu)=4+16\mu+(z-1)(4+16\mu),$$

$$a^{\wedge}_{0,3,3}(z;\nu)=-3+\mu+(z-1)(-4),$$

$$a^{\wedge}_{0,3,4}(z;\nu)=(z-1)(-4),$$

$$a^{\wedge}_{0,4,1}(z;\nu)=4+16\mu+12\mu^2+(z-1)(4+16\mu+12\mu^2),$$

$$a^{\wedge}_{0,4,2}(z;\nu)=8+24\mu+8\mu^2+(z-1)(8+24\mu+8mu^2),$$

$$a^{\wedge}_{0,4,3}(z;\nu)=4+8\mu+(z-1)(4+8\mu),$$

$$a^{\wedge}_{0,4,4}(z;\nu)=1+\mu,$$

$$a^{\vee}_{1,2,1}(z;\nu)=$$

$$(1/2)(-8-76\mu-250\mu^2-204\mu^3+(z-1)(-8-76\mu-250\mu^2-204\mu^3)),$$

$$a^{\vee}_{1,2,2}(z;\nu)=(1/2)(5+73\mu-29\mu^2+(z-1)(4+68\mu-34\mu^2)),$$

$$a^{\vee}_{1,2,3}(z;\nu)=(1/2)(312\mu+228\mu^2+(z-1)(-6+288\mu+216\mu^2)),$$

$$a^{\vee}_{1,2,4}(z;\nu)=(1/2)(-104+122\mu+(z-1)(-122+68m),$$

$$a^{\vee}_{1,2,5}(z;\nu)=(1/2)(-132+(z-1)(-170-76\mu)),$$

$$a^{\vee}_{1,2,6}(z;\nu)=(1/2)(z-1)(-66),$$

$$a^{\vee}_{1,3,1}(z;\nu)=$$

$$(1/2)(-8-76\mu-250\mu^2-270\mu^3+(z-1)(-8-76\mu-250\mu^2-270\mu^3)),$$

$$a^{\vee}_{1,3,2}(z;\nu)=(1/2)(-4-8\mu-86\mu^2-204\mu^3+(z-1)(-4-8\mu-86\mu^2-204\mu^3)),$$

$$a^{\vee}_{1,3,3}(z;\nu)=(1/2)(-1+163\mu+385\mu^2+(z-1)(-2+158\mu+380\mu^2)),$$

$$a^{\vee}_{1,3,4}(z;\nu)=(1/2)(-56-16\mu+228\mu^2+(z-1)(-62-40m+216m^2),$$

$$a^{\vee}_{1,3,5}(z;\nu)=(1/2)(-76-152\mu+(z-1)(-94-206\mu)),$$

$$a^{\vee}_{1,3,6}(z;\nu)=(1/2)(z-1)(-38-76\mu),$$

$$a^{\vee}_{1,4,1}(z;\nu)=(1/2)(-8-76\mu-250\mu^2-308\mu^3-76\mu^4+$$

$$(1/2)(z-1)(-8-76\mu-250\mu^2-308\mu^3-76\mu^4),$$

$$a^{\vee}_{1,4,2}(z;\nu)=$$

$$(1/2)(-12-84\mu-222\mu^2-246\mu^3+(z-1)(-12-84\mu-222\mu^2-246\mu^3)),$$
$$a^{\vee}_{1,4,3}(z;\nu)=(1/2)(-6+36\mu+180\mu^2+24\mu^3+(z-1)(-6+36\mu+180\mu^2+24\mu^3)),$$
$$a^{\vee}_{1,4,4}(z;\nu)=(1/2)(-25-29\mu+145\mu^2+(z-1)(-26-34m+140m^2),$$
$$a^{\vee}_{1,4,5}(z;\nu)=(1/2)(-36-108\mu+(z-1)(-42-132\mu-12\mu^2)),$$
$$a^{\vee}_{1,4,6}(z;\nu)=(1/2)(z-1)(-18-54\mu),$$
$$a^{\vee}_{1,5,1}(z;\nu)=(1/2)(-8-76\mu-250\mu^2-326\mu^3-130\mu^4+$$
$$(1/2)(z-1)(-8-76\mu-250\mu^2-326\mu^3-130\mu^4),$$
$$a^{\vee}_{1,5,2}(z;\nu)=(1/2)(-20-160\mu-418\mu^2-392\mu^3-76\mu^4)+$$
$$(1/2)(z-1)(-20-160\mu-418\mu^2-392\mu^3-76\mu^4)),$$
$$a^{\vee}_{1,5,3}(z;\nu)=$$
$$(1/2)(-18-102\mu+150\mu^2-60\mu^3+(z-1)(-18-102\mu-150\mu^2-60\mu^3)),$$
$$a^{\vee}_{1,5,4}(z;\nu)=$$
$$(1/2)(-14-52\mu-4\mu^2+24\mu^3+(z-1)(-14-52m-4m^2+24\mu^3)),$$
$$a^{\vee}_{1,5,5}(z;\nu)=(1/2)(-13-53\mu-29\mu^2+(z-1)(-14-58\mu-34\mu^2)),$$
$$a^{\vee}_{1,5,6}(z;\nu)=(1/2)(z-1)(-6-24\mu-12\mu^2),$$
$$a^{\vee}_{1,6,1}(z;\nu)=(1/2)(-8-76\mu-250\mu^2-332\mu^3-154\mu^4-12\mu^5$$
$$(1/2)(z-1)(-8-76\mu-250\mu^2-332\mu^3-154\mu^4-12\mu^5),$$
$$a^{\vee}_{1,6,2}(z;\nu)=(1/2)(-28-236\mu-650\mu^2-646\mu^3-170\mu^4)+$$
$$(1/2)(z-1)(-28-236\mu-650\mu^2-646\mu^3-170\mu^4)),$$
$$a^{\vee}_{1,6,3}(z;\nu)=$$
$$(1/2)(-38-280\mu-622\mu^2-416\mu^3-40\mu^4+$$
$$(1/2)(z-1)(-38-280\mu-622\mu^2-416\mu^3-40\mu^4),$$
$$a^{\vee}_{1,6,4}(z;\nu)=$$
$$(1/2)(-26-166\mu-286\mu^2+108\mu^3+(z-1)(-26-166\mu-286\mu^2+108\mu^3)),$$
$$a^{\vee}_{1,6,5}(z;\nu)=$$
$$(1/2)(-10-56\mu-74\mu^2-12\mu^3+(z-1)(-10-56\mu-74\mu^2-12\mu^3)),$$
$$a^{\vee}_{1,6,6}(z;\nu)=(1/2)(-1-5\mu-5\mu^2+(z-1)(-2-10\mu-10\mu^2),$$
$$a^{\vee}_{2,2,1}(z;\nu)=(1/2)(-16-208\mu-1048\mu^2-2576\mu^3-1904\mu^4)+$$
$$(1/2)(z-1)(-16-208\mu-1048\mu^2-2576\mu^3-1904\mu^4),$$
$$a^{\vee}_{2,2,2}(z;\nu)=(1/2)(15+281\mu+1922\mu^2+1049\mu^3)+$$
$$(1/2)(z-1)(16+288\mu+1936\mu^2+1056\mu^3),$$
$$a^{\vee}_{2,2,3}(z;\nu)=(1/2)(-32-512\mu+4240\mu^2+3616\mu^3+$$
$$(1/2)(z-1)(-24-464\mu+4312\mu^2+3632\mu^3),$$
$$a^{\vee}_{2,2,4}(z;\nu)=(1/2)(-4640\mu-1536\mu^2+(z-1)(32-4480\mu-1376\mu^2)),$$

$$a^{\vee}_{2,2,5}(z;\nu) = (1/2)(1160 - 5008\mu - 2880\mu^2 + (z-1)(1248 - 4656\mu - 2704\mu^2)),$$
$$a^{\vee}_{2,2,6}(z;\nu) = (1/2)(2544 - 992\mu + (z-1)(2736 - 416\mu)),$$
$$a^{\vee}_{2,2,7}(z;\nu) = (1/2)(1824 + (z-1)(2184 + 720\mu)),$$
$$a^{\vee}_{2,2,8}(z;\nu) = (1/2)(z-1)(608),$$
$$a^{\vee}_{2,3,1}(z;\nu) = (1/2)(-16 - 208\mu - 1048\mu^2 - 2576\mu^3 - 2512\mu^4$$
$$+(1/2)(z-1)(-16 - 208\mu - 1048\mu^2 - 2576\mu^3 - 2512\mu^4),$$
$$a^{\vee}_{2,3,2}(z;\nu) = (1/2)(80\mu + 888\mu^2 + 912\mu^3 - 1904\mu^4)+$$
$$(1/2)(z-1)(80\mu + 888\mu^2 + 912\mu^3 - 1904m^4),$$
$$a^{\vee}_{2,3,3}(z;\nu) = (1/2)(-9 - 183\mu + 2856\mu^2 + 7113\mu^3)+$$
$$(1/2)(z-1)(-8 - 176\mu + 2600\mu^2 + 7120\mu^3),$$
$$a^{\vee}_{2,3,4}(z;\nu) = (1/2)(-2560\mu - 4432\mu^2 + 3616\mu^3)+$$
$$(1/2)(z-1)(8 - 2512\mu - 4360\mu^2 + 3632\mu^3),$$
$$a^{\vee}_{2,3,5}(z;\nu) = (1/2)(640 - 2000\mu - 7888\mu^2)+$$
$$(1/2)(z-1)(672 - 1840\mu - 7728\mu^2),$$
$$a^{\vee}_{2,3,6}(z;\nu) = (1/2)(1464 + 1872\mu - 2880\mu^2)+$$
$$(1/2)(z-1)(1552 + 2224\mu - 2704\mu^2),$$
$$a^{\vee}_{2,3,7}(z;\nu) = (1/2)(1080 + 2160\mu + (z-1)(1272 + 2736\mu)),$$
$$a^{\vee}_{2,3,8}(z;\nu) = (1/2)(z-1)(360 + 720\mu),$$
$$a^{\vee}_{2,4,1}(z;\nu) = (1/2)(-16 - 208\mu - 1048\mu^2 - 2576\mu^3 - 2872\mu^4 - 720\mu^5)+$$
$$(1/2)(z-1)(-16 - 208\mu - 1048\mu^2 - 2576\mu^3 - 2872m^4 - 720\mu^5),$$
$$a^{\vee}_{2,4,2}(z;\nu) = (1/2)(-16 - 128\mu - 160\mu^2 - 224\mu^3 - 1536\mu^4)+$$
$$(1/2)(z-1)(-16 - 128\mu - 160\mu^2 - 224\mu^3 - 1536\mu^4),$$
$$a^{\vee}_{2,4,3}(z;\nu) = (1/2)(-8 - 96\mu + 1328\mu^2 + 5152\mu^3 + 976\mu^4)+$$
$$(1/2)(z-1)(-8 - 96\mu + 1328\mu^2 + 5152\mu^3 + 976\mu^4),$$
$$a^{\vee}_{2,4,4}(z;\nu) = (1/2)(-1 - 1225\mu - 3214\mu^2 + 2105\mu^3)+$$
$$(1/2)(z-1)(-1248\mu - 3200\mu^2 + 2112\mu^3),$$
$$a^{\vee}_{2,4,5}(z;\nu) = (1/2)(312 - 800\mu - 5680\mu^2 - 704\mu^3)+$$
$$(1/2)(z-1)(320 - 752\mu - 5608\mu^2 - 688\mu^3),$$
$$a^{\vee}_{2,4,6}(z;\nu) = (1/2)(752 + 1664\mu - 1952\mu^2)+$$
$$(1/2)(z-1)(784 + 1824\mu - 1792\mu^2),$$
$$a^{\vee}_{2,4,7}(z;\nu) = (1/2)(576 + 1728\mu) + (z-1)(664 + 2080\mu + 176\mu^2)),$$
$$a^{\vee}_{2,4,8}(z;\nu) = (1/2)(z-1)(192 + 576\mu\mu),$$
$$a^{\vee}_{2,5,1}(z;\nu) = (1/2)(-16 - 208\mu - 1048\mu^2 - 2576\mu^3 - 3064\mu^4 - 1296\mu^5)+$$
$$(1/2)(z-1)(-16 - 208\mu - 1048\mu^2 - 2576\mu^3 - 3064\mu^4 - 10296\mu^5),$$

$$a^{\vee}_{2,5,2}(z;\nu) = (1/2)(-32-336\mu-1208\mu^2-2032\mu^3-2104\mu^4-720\mu^5)+$$
$$(1/2)(z-1)(-32-336\mu-1208\mu^2-2032\mu^3-2104\mu^4-720\mu^5),$$
$$a^{\vee}_{2,5,3}(z;\nu) = (1/2)(-24-224\mu+16\mu^2+2240\mu^3+1744\mu^4)+$$
$$(1/2)(z-1)(-24-224\mu+16\mu^2+2240\mu^3+1744\mu^4),$$
$$a^{\vee}_{2,5,4}(z;\nu) = (1/2)(-8-576\mu-1872\mu^2+352\mu^3+976\mu^4)+$$
$$(1/2)(z-1)(-8-576\mu-1872\mu^2+352\mu^3+976\mu^4),$$
$$a^{\vee}_{2,5,5}(z;\nu) = (1/2)(127-279\mu-3062\mu^2-2039\mu^3)+$$
$$(1/2)(z-1)(128-272\mu-3048\mu^2-2032\mu^3),$$
$$a^{\vee}_{2,5,6}(z;\nu) = (1/2)(328+1024\mu-560\mu^2-704\mu^3)+$$
$$(1/2)(z-1)(336+1072\mu-488\mu^2-688\mu^3),$$
$$a^{\vee}_{2,5,7}(z;\nu) = (1/2)(264+1056\mu+528\mu^2+(z-1)(296+1216\mu+688\mu^2)),$$
$$a^{\vee}_{2,5,8}(z;\nu) = (1/2)(z-1)(88+352\mu+176\mu^2),$$
$$a^{\vee}_{2,6,1}(z;\nu) = (1/2)(-16-208\mu-1048\mu^2-2576\mu^3-3152\mu^4-1648\mu^5-176\mu^6)+$$
$$(1/2)(z-1)(-16-208\mu-1048\mu^2-2576\mu^3-3152\mu^4-1648\mu^5-176\mu^6),$$
$$a^{\vee}_{2,6,2}(z;\nu) = (1/2)(-48-544\mu-2256\mu^2-4256\mu^3-3760\mu^4-1312\mu^5)+$$
$$(1/2)(z-1)(-48-544\mu-2256\mu^2-4256\mu^3-3760\mu^4-1312\mu^5),$$
$$a^{\vee}_{2,6,3}(z;\nu) = (1/2)(-56-560\mu-1720\mu^2-1552\mu^3-8\mu^4-16\mu^5)+$$
$$(1/2)(z-1)(-56-560\mu-1720\mu^2-1552\mu^3-8\mu^4-16\mu^5),$$
$$a^{\vee}_{2,6,4}(z;\nu) = (1/2)(-32-448\mu-1504\mu^2-928\mu^3+608\mu^4)+$$
$$(1/2)(z-1)(-32-448\mu-1504\mu^2-928\mu^3+608\mu^4),$$
$$a^{\vee}_{2,6,5}(z;\nu) = (1/2)(32-144\mu-1400\mu^2-1680\mu^3-80\mu^4)+$$
$$(1/2)(z-1)(32-144\mu-1400\mu^2-1680\mu^3-80\mu^4),$$
$$a^{\vee}_{2,6,6}(z;\nu) = (1/2)(111+441\mu-30\mu^2-615\mu^3)+$$
$$(1/2)(z-1)(112+448\mu-16\mu^2-608\mu^3),$$
$$a^{\vee}_{2,6,7}(z;\nu) = (1/2)(96+480\mu+480\mu^2)+$$
$$(1/2)(z-1)(104+528\mu+552\mu^2+16\mu^3),$$
$$a^{\vee}_{2,6,8}(z;\nu) = (1/2)(z-1)(32+160\mu+160\mu^2),$$
$$a^{\vee}_{2,7,1}(z;\nu) = (1/2)(-16-208\mu-1048\mu^2-2576\mu^3-3184\mu^4-1808\mu^5-336\mu^6)+$$
$$(1/2)(z-1)(-16-208\mu-1048\mu^2-2576\mu^3-3184\mu^4-1808\mu^5-336\mu^6),$$
$$a^{\vee}_{2,7,2}(z;\nu) = (1/2)(-64-752\mu-3304\mu^2-6704\mu^3-6272\mu^4-2320\mu^5-176\mu^6)+$$
$$(1/2)(z-1)(-64-752\mu-3304\mu^2-6704\mu^3-6272\mu^4-2320\mu^5-176\mu^6),$$
$$a^{\vee}_{2,7,3}(z;\nu) = (1/2)(-104-1104\mu-4168\mu^2-6640\mu^3-4088\mu^4-688\mu^5)+$$
$$(1/2)(z-1)(-104-1104\mu-4168\mu^2-6640\mu^3-4088\mu^4-688\mu^5),$$
$$a^{\vee}_{2,7,4}(z;\nu) = (1/2)(-88-880\mu-2968\mu^2-3760\mu^3-1320\mu^4-16\mu^5)+$$

$$(1/2)(z-1)(-88-880\mu-2968\mu^2-3760\mu^3-1320\mu^4-16\mu^5),$$

$$a^{\vee}_{2,7,5}(z;\nu)=(1/2)(-32-368\mu-1336\mu^2-1648\mu^3-432\mu^4)+$$

$$(1/2)(z-1)(-32-368\mu-1336\mu^2-1648\mu^3-432\mu^4),$$

$$a^{\vee}_{2,7,6}(z;\nu)=(1/2)(16+48\mu-136\mu^2-368\mu^3-80\mu^4)+$$

$$(1/2)(z-1)(16+48\mu-136\mu^2-368\mu^3-80\mu^4),$$

$$a^{\vee}_{2,7,7}(z;\nu)=(1/2)(23+137\mu+202\mu^2+41\mu^3)+$$

$$(1/2)(z-1)(24+144\mu+216\mu^2+48\mu^3),$$

$$a^{\vee}_{2,7,8}(z;\nu)=(1/2)(z-1)(8+48\mu+72\mu^2+16\mu^3),$$

$$a^{\vee}_{2,8,1}(z;\nu)=$$

$$(1/2)(-16-208\mu-1048\mu^2-2576\mu^3-3192\mu^4-1856\mu^5-408\mu^6-16\mu^7)+$$

$$(1/2)(z-1)(-16-208\mu-1048\mu^2-2576\mu^3-3192\mu^4-1856\mu^5-408\mu^6-16\mu^7),$$

$$a^{\vee}_{2,8,2}(z;\nu)=(1/2)(-80-960\mu-4352\mu^2-9248\mu^3-9264\mu^4-3840\mu^5-448\mu^6)+$$

$$(1/2)(z-1)(-80-960\mu-4352\mu^2-9248\mu^3-9264\mu^4-3840\mu^5-448\mu^6),$$

$$a^{\vee}_{2,8,3}(z;\nu)=$$

$$(1/2)(-168-1856\mu-7520\mu^2-13600\mu^3-10600\mu^4-2816\mu^5-112\mu^6)+$$

$$(1/2)(z-1)(-168-1856\mu-7520\mu^2-13600\mu^3-10600\mu^4-2816\mu^5-112\mu^6),$$

$$a^{\vee}_{2,8,4}(z;\nu)=(1/2)(-192-1952\mu-7040\mu^2-10688\mu^3-6208\mu^4-896\mu^5)+$$

$$(1/2)(z-1)(-192-1952\mu-7040\mu^2-10688\mu^3-6208\mu^4-896\mu^5),$$

$$a^{\vee}_{2,8,5}(z;\nu)=(1/2)(-128-1200\mu-3848\mu^2-4848\mu^3-1992\mu^4-112\mu^5)+$$

$$(1/2)(z-1)(-128-1200\mu-3848\mu^2-4848\mu^3-1992\mu^4-112\mu^5),$$

$$a^{\vee}_{2,8,6}(z;\nu)=(1/2)(-48-416\mu-1184\mu^2-1216\mu^3-320\mu^4)+$$

$$(1/2)(z-1)(-48-416\mu-1184\mu^2-1216\mu^3-320\mu^4),$$

$$a^{\vee}_{2,8,7}(z;\nu)=(1/2)(-8-64\mu-160\mu^2-128\mu^3-16\mu^4)+$$

$$(1/2)(z-1)(-8-64\mu-160\mu^2-128\mu^3-16\mu^4),$$

$$a^{\vee}_{2,8,8}(z;\nu)=(1/2)(-1-7\mu-14\mu^2-7\mu^3),$$

$$a^{\wedge}_{1,2,1}(z;\nu)=8+60\mu+146\mu^2+(z-1)(8+60\mu+146\mu^2),$$

$$a^{\wedge}_{1,2,2}(z;\nu)=-5-63\mu+145\mu^2+(z-1)(-4-60\mu+146\mu^2)),$$

$$a^{\wedge}_{1,2,3}(z;\nu)=-312\mu+(z-1)(6-300\mu),$$

$$a^{\wedge}_{1,2,4}(z;\nu)=104-198\mu+(z-1)(122-180m),$$

$$a^{\wedge}_{1,2,5}(z;\nu)=132+(z-1)(170),$$

$$a^{\wedge}_{1,2,6}(z;\nu)=(z-1)(66),$$

$$a^{\wedge}_{1,3,1}(z;\nu)=8+60\mu+146\mu^2+66\mu^3+(z-1)(8+60\mu+146\mu^2+66\mu^3),$$

$$a^{\wedge}_{1,3,2}(z;\nu)=4+94\mu^2+(z-1)(4+94\mu^2)),$$

$$a^{\wedge}_{1,3,3}(z;\nu) = 1 - 165\mu - 53\mu^2 + (z-1)(2 - 162\mu - 52\mu^2),$$
$$a^{\wedge}_{1,3,4}(z;\nu) = 56 - 96\mu + (z-1)(62 - 84\mu),$$
$$a^{\wedge}_{1,3,5}(z;\nu) = 76 + (z-1)(94 + 18\mu),$$
$$a^{\wedge}_{1,3,6}(z;\nu) = (z-1)(38),$$
$$a^{\wedge}_{1,4,1}(z;\nu) = 8 + 60\mu + 146\mu^2 + 104\mu^3 + (z-1)(8 + 60\mu + 146\mu^2 + 104\mu^3),$$
$$a^{\wedge}_{1,4,2}(z;\nu) = 12 + 60\mu + 126\mu^2 + 66\mu^3 + (z-1)(12 + 60\mu + 126\mu^2 + 66\mu^3),$$
$$a^{\wedge}_{1,4,3}(z;\nu) = 6 - 48\mu - 72\mu^2 + (z-1)(6 - 48\mu - 72\mu^2),$$
$$a^{\wedge}_{1,4,4}(z;\nu) = 25 - 21\mu - 53\mu^2 + (z-1)(26 - 18\mu - 52\mu^2),$$
$$a^{\wedge}_{1,4,5}(z;\nu) = 36 + 36\mu + (z-1)(42 + 48\mu),$$
$$a^{\wedge}_{1,4,6}(z;\nu) = (z-1)(18 + 18\mu),$$
$$a^{\wedge}_{1,5,1}(z;\nu) = 8 + 60\mu + 146\mu^2 + 122\mu^3 + 18\mu^4 +$$
$$(z-1)(8 + 60\mu + 146\mu^2 + 122\mu^3 + 18\mu^4),$$
$$a^{\wedge}_{1,5,2}(z;\nu) = 20 + 120\mu + 218\mu^2 + 116\mu^3 + (z-1)(20 + 1200\mu + 218mu^2 + 116\mu^3),$$
$$a^{\wedge}_{1,5,3}(z;\nu) = 18 + 66\mu + 54\mu^2 + 12\mu^3 + (z-1)(18 + 66\mu + 54\mu^2 + 12\mu^3),$$
$$a^{\wedge}_{1,5,4}(z;\nu) = 14 + 24\mu - 16\mu^2 + (z-1)(14 + 24\mu - 16\mu^2),$$
$$a^{\wedge}_{1,5,5}(z;\nu) = 13 + 27\mu + \mu^2 + (z-1)(14 + 30\mu + 2\mu^2),$$
$$a^{\wedge}_{1,5,6}(z;\nu) = (z-1)(6 + 12\mu),$$
$$a^{\wedge}_{1,6,1}(z;\nu) = 8 + 60\mu + 146\mu^2 + 128\mu^3 + 30\mu^4 +$$
$$(z-1)(8 + 60\mu + 146\mu^2 + 128\mu^3 + 30\mu^4),$$
$$a^{\wedge}_{1,6,2}(z;\nu) = 28 + 180\mu + 346\mu^2 + 202\mu^3 + 18\mu^4 +$$
$$(z-1)(28 + 180\mu + 346\mu^2 + 202\mu^3 + 18\mu^4),$$
$$a^{\wedge}_{1,6,3}(z;\nu) = 38 + 204\mu + 290\mu^2 + 92\mu^3 + (z-1)(38 + 204\mu + 290\mu^2 + 92\mu^3),$$
$$a^{\wedge}_{1,6,4}(z;\nu) = 26 + 114\mu + 110\mu^2 + 12\mu^3 + (z-1)(26 + 114\mu + 110\mu^2 + 12\mu^3),$$
$$a^{\wedge}_{1,6,5}(z;\nu) = 10 + 36\mu + 22\mu^2 + (z-1)(10 + 36\mu + 22\mu^2),$$
$$a^{\wedge}_{1,6,6}(z;\nu) = 1 + 3\mu + \mu^2 + (z-1)(2 + 6\mu + 2\mu^2),$$
$$a^{\wedge}_{2,2,1}(z;\nu) =$$
$$16 + 176\mu + 728\mu^2 + 1408\mu^3 + (z-1)(16 + 176\mu + 728\mu^2 + 1408\mu^3),$$
$$a^{\wedge}_{2,2,2}(z;\nu) = -15 - 251\mu - 1450\mu^2 + 1409\mu^3 +$$
$$(z-1)(-16 - 256\mu - 1456\mu^2 + 1408\mu^3),$$
$$a^{\wedge}_{2,2,3}(z;\nu) = 32 + 448\mu - 5072\mu^2 + (z-1)(24 + 416\mu - 5096\mu^2),$$
$$a^{\wedge}_{2,2,4}(z;\nu) = 4640\mu - 2880\mu^2 + (z-1)(-32 + 4544\mu - 2912\mu^2),$$
$$a^{\wedge}_{2,2,5}(z;\nu) = -1160 + 6112\mu + (z-1)(-1248 + 5936\mu),$$
$$a^{\wedge}_{2,2,6}(z;\nu) = -2544 + 2432\mu + (z-1)(-2736 + 2240\mu),$$
$$a^{\wedge}_{2,2,7}(z;\nu) = -1824 + (z-1)(-2184\mu),$$

$$a^{\wedge}_{2,2,8}(z;\nu) = (z-1)(-608),$$
$$a^{\wedge}_{2,3,1}(z;\nu) = 16 + 176\mu + 728\mu^2 + 1408\mu^3 + 608\mu^4 +$$
$$(z-1)(16 + 176\mu + 728\mu^2 + 1408\mu^3 + 608\mu^4),$$
$$a^{\wedge}_{2,3,2}(z;\nu) = -80\mu - 728\mu^2 + 384\mu^3 +$$
$$(z-1)(-80\mu - 728\mu^2 + 384\mu^3),$$
$$a^{\wedge}_{2,3,3}(z;\nu) = 9 + 165\mu - 2898\mu^2 - 1023\mu^3 +$$
$$(z-1)(8 + 160\mu - 2904\mu^2 - 1024\mu^3),$$
$$a^{\wedge}_{2,3,4}(z;\nu) = 2560\mu - 688\mu^2 + (z-1)(-8 + 2528\mu - 712\mu^2),$$
$$a^{\wedge}_{2,3,5}(z;\nu) = -640 + 3280\mu + 768\mu^2 + (z-1)(-672 + 3184\mu + 736\mu^2),$$
$$a^{\wedge}_{2,3,6}(z;\nu) = -1464 + 1056\mu + (z-1)(-1552 + 880\mu),$$
$$a^{\wedge}_{2,3,7}(z;\nu) = -1080 - (z-1)(-1272 - 192\mu),$$
$$a^{\wedge}_{2,3,8}(z;\nu) = (z-1)(-360),$$
$$a^{\wedge}_{2,4,1}(z;\nu) = 16 + 176\mu + 728\mu^2 + 1408\mu^3 + 968\mu^4 +$$
$$(z-1)(16 + 176\mu + 728\mu^2 + 1408\mu^3 + 968\mu^4),$$
$$a^{\wedge}_{2,4,2}(z;\nu) = 16 + 96\mu + 352\mu^3 + 608\mu^4$$
$$(z-1)(16 + 96\mu + 352\mu^3 + 608\mu^3),$$
$$a^{\wedge}_{2,4,3}(z;\nu) = 8 + 80\mu - 1472\mu^2 - 2080\mu^3 +$$
$$(z-1)(8 + 80\mu - 1472\mu^2 - 2080\mu 3),$$
$$a^{\wedge}_{2,4,4}(z;\nu) = 1 + 1253\mu + 710\mu^2 - 1023\mu^3 +$$
$$+(z-1)(1248\mu + 704\mu^2 - 1024\mu^3),$$
$$a^{\wedge}_{2,4,5}(z;\nu) = -312 + 1424\mu + 2208\mu^2 +$$
$$(z-1)(-320 + 1392\mu + 2184\mu^2),$$
$$a^{\wedge}_{2,4,6}(z;\nu) = -752 - 160\mu + 768\mu^2 + (z-1)(-784 - 256\mu + 736\mu^2),$$
$$a^{\wedge}_{2,4,7}(z;\nu) = -576 - 576\mu + (z-1)(-664 - 752\mu),$$
$$a^{\wedge}_{2,4,8}(z;\nu) = (z-1)(-192 - 192\mu),$$
$$a^{\wedge}_{2,5,1}(z;\nu) = 16 + 176\mu + 728\mu^2 + 1408\mu^3 + 1160\mu^4 + 192\mu^5 +$$
$$(z-1)(16 + 176\mu + 728\mu^2 + 1408\mu^3 + 1160\mu^4 + 192\mu^5),$$
$$a^{\wedge}_{2,5,2}(z;\nu) = 32 + 272\mu + 728\mu^2 + 992\mu^3 + 808\mu^4$$
$$(z-1)(32 + 272\mu + 728\mu^2 + 992\mu^3 + 808\mu^4),$$
$$a^{\wedge}_{2,5,3}(z;\nu) = 24 + 176\mu - 320\mu^2 - 1344\mu^3 - 160\mu^4 +$$
$$(z-1)(24 + 176\mu - 320\mu^2 - 1344\mu^3 - 160\mu^4),$$
$$a^{\wedge}_{2,5,4}(z;\nu) = 8 + 560\mu + 768\mu^2 - 800\mu^3 +$$
$$+(z-1)(8 + 560\mu + 768\mu^2 - 800\mu^3),$$
$$a^{\wedge}_{2,5,5}(z;\nu) = -127 + 533\mu + 1742\mu^2 + 129\mu^3$$

$$(z-1)(-128+528\mu+1736\mu^2+128\mu^3),$$
$$a^{\wedge}_{2,5,6}(z;\nu)=-328-368\mu+640\mu^2+(z-1)(-336-400\mu+616\mu^2),$$
$$a^{\wedge}_{2,5,7}(z;\nu)=-264-528\mu+(z-1)(-296-624\mu-32\mu^2),$$
$$a^{\wedge}_{2,5,8}(z;\nu)=(z-1)(-88-176\mu),$$
$$a^{\wedge}_{2,6,1}(z;\nu)=16+176\mu+728\mu^2+1408\mu^3+1248\mu^4+368\mu^5+$$
$$(z-1)(16+176\mu+728\mu^2+1408\mu^3+1248\mu^4+368\mu^5),$$
$$a^{\wedge}_{2,6,2}(z;\nu)=48+448\mu+1456\mu^2+2048\mu^3+1264\mu^4+192\mu^5+$$
$$(z-1)(48+448\mu+1456\mu^2+2048\mu^3+1264\mu^4+192\mu^5),$$
$$a^{\wedge}_{2,6,3}(z;\nu)=56+448\mu+936\mu^2+352\mu^3-56\mu^4+$$
$$(z-1)(56+448\mu+936\mu^2+352\mu^3-56\mu^4),$$
$$a^{\wedge}_{2,6,4}(z;\nu)=32+384\mu+800\mu^2-32\mu^3-160\mu^4+$$
$$(z-1)(32+384\mu+800\mu^2-32\mu^3-160\mu^4),$$
$$a^{\wedge}_{2,6,5}(z;\nu)=-32+208\mu+920\mu^2+384\mu^3+$$
$$(z-1)(-32+208\mu+920\mu^2+384\mu^3),$$
$$a^{\wedge}_{2,6,6}(z;\nu)=-111-219\mu+246\mu^2+129\mu^3+$$
$$+(z-1)(-112-224\mu+240\mu^2+128mu^3),$$
$$a^{\wedge}_{2,6,7}(z;\nu)=-96-288\mu-96\mu^2+(z-1)(-104-320\mu-120\mu^2),$$
$$a^{\wedge}_{2,6,8}(z;\nu)=(z-1)(-32-96\mu-32\mu^2),$$
$$a^{\wedge}_{2,7,1}(z;\nu)=16+176\mu+728\mu^2+1408\mu^3+1280\mu^4+464\mu^5+32\mu^6+$$
$$(z-1)(16+176\mu+728\mu^2+1408\mu^3+1280\mu^4+464\mu^5+32\mu^6),$$
$$a^{\wedge}_{2,7,2}(z;\nu)=64+624\mu+2184\mu^2+3328\mu^3+2128\mu^4+432\mu^5+$$
$$(z-1)(64+624\mu+2184\mu^2+3328\mu^3+2128\mu^4+432\mu^5),$$
$$a^{\wedge}_{2,7,3}(z;\nu)=104+896\mu+2584\mu^2+2848\mu^3+1016\mu^4+64\mu^5+$$
$$(z-1)(104+896\mu+2584\mu^2+2848\mu^3+1016\mu^4+64\mu^5),$$
$$a^{\wedge}_{2,7,4}(z;\nu)=88+704\mu+1736\mu^2+1344\mu^3+168\mu^4+$$
$$(z-1)(88+704\mu+1736\mu^2+1344\mu^3+168\mu^4),$$
$$a^{\wedge}_{2,7,5}(z;\nu)=32+304\mu+792\mu^2+544\mu^3+32\mu^4+$$
$$(z-1)(32+304\mu+792\mu^2+544\mu^3+32\mu^4),$$
$$a^{\wedge}_{2,7,6}(z;\nu)=-16-16\mu+136\mu^2+128\mu^3+$$
$$(z-1)(-16-16\mu+136\mu^2+128mu^3),$$
$$a^{\wedge}_{2,7,7}(z;\nu)=-23-91\mu-66\mu^2+\mu^3+(z-1)(-24-96\mu-72\mu^2),$$
$$a^{\wedge}_{2,7,8}(z;\nu)=(z-1)(-8-32\mu-24\mu^2)$$
$$a^{\wedge}_{2,8,1}(z;\nu)=16+176\mu+728\mu^2+1408\mu^3+1288\mu^4+496\mu^5+56\mu^6+$$
$$(z-1)(16+176\mu+728\mu^2+1408\mu^3+1288\mu^4+496\mu^5+56\mu^6),$$

$$a^{\wedge}_{2,8,2}(z;\nu) = 80 + 800\mu + 2912\mu^2 + 4704\mu^3 + 3280\mu^4 + 800\mu^5 + +32\mu^6 +$$
$$(z-1)(80 + 800\mu + 2912\mu^2 + 4704\mu^3 + 3280\mu^4 + 800\mu^5 + +32\mu^6),$$
$$a^{\wedge}_{2,8,3}(z;\nu) = 168 + 1520\mu + 4816\mu^2 + 6336\mu^3 + 3160\mu^4 + 400\mu^5 +$$
$$(z-1)(168 + 1520\mu + 4816\mu^2 + 6336\mu^3 + 3160\mu^4 + 400\mu^5),$$
$$a^{\wedge}_{2,8,4}(z;\nu) = 192 + 1568\mu + 4288\mu^2 + 4480\mu^3 + 1472\mu^4 + 64\mu^5 +$$
$$(z-1)(192 + 1568\mu + 4288\mu^2 + 4480\mu^3 + 1472\mu^4 + 64\mu^5 +),$$
$$a^{\wedge}_{2,8,5}(z;\nu) = 128 + 944\mu + 2216\mu^2 + 1792\mu^3 + 344\mu^4 +$$
$$(z-1)(128 + 944\mu + 2216\mu^2 + 1792\mu^3 + 344\mu^4),$$
$$a^{\wedge}_{2,8,6}(z;\nu) = 48 + 320\mu + 640\mu^2 + 384\mu^3 + 32\mu^4 +$$
$$(z-1)(48 + 320\mu + 640\mu^2 + 384\mu^3 + 32\mu^4),$$
$$a^{\wedge}_{2,8,7}(z;\nu) = 8 + 48\mu + 80\mu^2 + 33\mu^3 +$$
$$(z-1)(8 + 48\mu + 80\mu^2 + 32\mu^3),$$
$$a^{\wedge}_{2,8,8}(z;\nu) = 1 + 5\mu + 6\mu^2 + \mu^3.$$

We denote by

$$A^*_l(z;\nu),\ A^{\vee}_l(z;\nu),\ A^{\wedge}_l(z;\nu)$$

for $l = 0,\ 1,\ 2$ the $(4+2l)\times(4+2l)$-matrix, such that its element in i-th row and k-th column is equal respectively to the k-th elements of the rows respecively

$$a^*_{l,i,k}(z;\nu),\ a^{\vee}_{l,i,k}(z;\nu),\ a^{\wedge}_{l,i,k}(z;\nu)$$

for $i = 1,\ ...,\ 4+2l$, $k = 1,\ ...,\ 4+2l$. Clearly,

$$A^*_l(z;\nu) = A^{\vee}_l(z;\nu) + A^{\wedge}_l(z;\nu)\tau \tag{31}$$

for $l = 0, 1, 2$. Let

$$U^{\vee}_l(z,\mu) = \sum_{k=0}^{3+2l} \mu^k (R^{\vee}_l(k)) + (z-1)V^{\vee}_l(k),$$

$$U^{\wedge}_l(z,\mu) = \sum_{k=0}^{2+2l} \mu^k (R^{\wedge}_l(k)) + (z-1)V^{\wedge}_l(k)),$$

where $l = 0,\ 1,\ 2,$

$$R^{\vee}_0(0) = \begin{pmatrix} -5 & 0 & 24 & 0 \\ -4 & -1 & 8 & 0 \\ -4 & -4 & 3 & 0 \\ -4 & -8 & -4 & -1 \end{pmatrix},\ V^{\vee}_0(0) = \begin{pmatrix} -4 & 4 & 32 & 24 \\ -4 & -4 & 12 & 8 \\ -4 & 0 & 4 & 4 \\ -4 & -8 & -4 & 0 \end{pmatrix},$$

$$R^{\vee}_0(1) = \begin{pmatrix} -27 & -48 & 0 & 0 \\ -24 & -19 & 0 & 0 \\ -24 & -24 & 5 & 0 \\ -24 & -40 & -16 & -3 \end{pmatrix},\ V^{\vee}_0(1) = \begin{pmatrix} -24 & -40 & 0 & 0 \\ -24 & -16 & 8 & 0 \\ -24 & -24 & 8 & 8 \\ -24 & -40 & -16 & 0 \end{pmatrix},$$

$$R_0^{\vee}(2)=\begin{pmatrix}0&0&0&0\\-24&0&0&0\\-32&-24&0&0\\-36&-40&-8&0\end{pmatrix},\ V_0^{\vee}(2)=\begin{pmatrix}0&0&0&0\\-24&0&0&0\\-32&-24&0&0\\-36&-40&-8&0\end{pmatrix},$$

$$R_0^{\vee}(3)=\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\-8&0&0&0\end{pmatrix},\ V_0^{\vee}(3)=\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\-8&0&0&0\end{pmatrix},$$

$$R_0^{\wedge}(0)=\begin{pmatrix}5&0&0&0\\4&1&-8&0\\4&4&-3&0\\4&8&4&1\end{pmatrix},\ V_0^{\wedge}(0)=\begin{pmatrix}4&-4&-8&0\\4&0&-12&-8\\4&4&-4&-4\\4&8&4&0\end{pmatrix},$$

$$R_0^{\wedge}(1)=\begin{pmatrix}17&0&0&0\\4&1&-8&0\\16&16&1&0\\16&24&8&1\end{pmatrix},\ V_0^{\wedge}(1)=\begin{pmatrix}16&16&0&0\\16&16&0&0\\16&16&0&0\\16&24&8&0\end{pmatrix},$$

$$R_0^{\wedge}(2)=\begin{pmatrix}0&0&0&0\\0&0&0&0\\8&0&0&0\\12&8&0&0\end{pmatrix},\ V_0^{\wedge}(2)=\begin{pmatrix}0&0&0&0\\0&0&0&0\\8&0&0&0\\12&8&0&0\end{pmatrix}$$

$$R_1^{\vee}(0)=\begin{pmatrix}-7&18&0&-270&-408&0\\-8&5&0&-104&-132&0\\-8&-4&-1&-56&-76&0\\-8&-12&-6&-25&-36&0\\-8&-20&-18&-14&-13&0\\-8&-28&-38&-26&-10&-1\end{pmatrix},$$

$$V_1^{\vee}(0)=\begin{pmatrix}-8&12&-18&-308&-474&-204\\-8&4&-6&-122&-170&-66\\-8&-4&-2&-62&-94&-38\\-8&-12&-6&-26&-42&-18\\-8&-20&-18&-14&-14&-6\\-8&-28&-38&-26&-10&-2\end{pmatrix},$$

$$R_1^{\vee}(1)=\begin{pmatrix}-71&168&810&612&0&0\\-76&73&312&122&0&0\\-76&-8&163&-16&-152&0\\-76&-84&36&-29&-108&0\\-76&-160&-102&-52&-53&0\\-76&-236&-280&-166&-56&-5\end{pmatrix},$$

$$V_1^{\vee}(1)=\begin{pmatrix}-76&144&756&536&-474&0\\-76&68&288&68&-76&0\\-76&-8&158&-40&-206&-76\\-76&-84&36&-34&-132&-54\\-76&-160&-102&-52&-58&-24\\-76&-236&-280&-166&-56&-10\end{pmatrix},$$

$$R_1^{\vee}(2) = \begin{pmatrix} -245 & -384 & 0 & 0 & 0 & 0 \\ -250 & -29 & 228 & 0 & 0 & 0 \\ -250 & -86 & 385 & 228 & 0 & 0 \\ -250 & -222 & 180 & 145 & 0 & 0 \\ -250 & -418 & -150 & -4 & -29 & 0 \\ -250 & -650 & -622 & -286 & -74 & -5 \end{pmatrix},$$

$$V_1^{\vee}(2) = \begin{pmatrix} -250 & -396 & 0 & 0 & 0 & 0 \\ -250 & -34 & 216 & 0 & 0 & 0 \\ -250 & -86 & 380 & 216 & 0 & 0 \\ -250 & -222 & 180 & 140 & -12 & 0 \\ -250 & -418 & -150 & -4 & -34 & -12 \\ -250 & -650 & -622 & -286 & -74 & -10 \end{pmatrix},$$

$$R_1^{\vee}(3) = V_1^{\vee}(3) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ -204 & 0 & 0 & 0 & 0 & 0 \\ -270 & -204 & 0 & 0 & 0 & 0 \\ -308 & -246 & 24 & 0 & 0 & 0 \\ -326 & -392 & -60 & 24 & 0 & 0 \\ -332 & -646 & -416 & -108 & -12 & 0 \end{pmatrix},$$

$$R_1^{\vee}(4) = V_1^{\vee}(4) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -76 & 0 & 0 & 0 & 0 & 0 \\ -130 & -76 & 0 & 0 & 0 & 0 \\ 154 & -170 & -40 & 0 & 0 & 0 \end{pmatrix},$$

$$R_1^{\vee}(5) = V_1^{\vee}(5) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -12 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$R_1^{\wedge}(0) = \begin{pmatrix} 7 & -18 & 0 & 66 & 0 & 0 \\ 8 & -5 & 0 & 104 & 132 & 0 \\ 8 & 4 & 1 & 56 & 76 & 0 \\ 8 & 12 & 6 & 25 & 36 & 0 \\ 8 & 20 & 18 & 14 & 13 & 0 \\ 8 & 28 & 38 & 26 & 10 & 1 \end{pmatrix},$$

$$V_1^{\wedge}(0) = \begin{pmatrix} 8 & -12 & 18 & 104 & 66 & 0 \\ 8 & -4 & 6 & 122 & 170 & 0 \\ 8 & 12 & 6 & 26 & 42 & 18 \\ 8 & 12 & 6 & 25 & 36 & 0 \\ 8 & 20 & 18 & 14 & 14 & 6 \\ 8 & 28 & 38 & 26 & 10 & 2 \end{pmatrix},$$

$$R_1^{\wedge}(1) = \begin{pmatrix} 57 & -132 & -198 & 0 & 0 & 0 \\ 60 & -63 & -312 & -198 & 0 & 0 \\ 60 & 0 & -165 & -96 & 0 & 0 \\ 60 & 60 & -48 & -21 & 36 & 0 \\ 60 & 120 & 66 & 24 & 27 & 0 \\ 60 & 180 & 204 & 114 & 36 & 3 \end{pmatrix},$$

$$V_1^{\wedge}(1) = \begin{pmatrix} 60 & -120 & -180 & 0 & 0 & 0 \\ 60 & -60 & -300 & -180 & 0 & 0 \\ 60 & 0 & -162 & -84 & 18 & 0 \\ 60 & 60 & -48 & -18 & 48 & 18 \\ 60 & 120 & 66 & 24 & 30 & 12 \\ 60 & 180 & 204 & 114 & 36 & 6 \end{pmatrix},$$

$$R_1^{\wedge}(2) = \begin{pmatrix} 145 & 0 & 0 & 0 & 0 & 0 \\ 146 & 145 & 0 & 0 & 0 & 0 \\ 146 & 94 & -53 & 0 & 0 & 0 \\ 146 & 126 & -72 & -53 & 0 & 0 \\ 146 & 218 & 54 & -16 & 1 & 0 \\ 146 & 346 & 290 & 110 & 22 & 1 \end{pmatrix},$$

$$V_1^{\wedge}(2) = \begin{pmatrix} 146 & 0 & 0 & 0 & 0 & 0 \\ 146 & 146 & 0 & 0 & 0 & 0 \\ 146 & 94 & -52 & 0 & 0 & 0 \\ 146 & 126 & -72 & -52 & 0 & 0 \\ 146 & 218 & 54 & -16 & 2 & 0 \\ 146 & 346 & 290 & 110 & 22 & 2 \end{pmatrix},$$

$$R_1^{\wedge}(3) = V_1^{\wedge}(3) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 66 & 0 & 0 & 0 & 0 & 0 \\ 104 & 66 & 0 & 0 & 0 & 0 \\ 122 & 116 & 12 & 0 & 0 & 0 \\ 128 & 202 & 92 & 12 & 0 & 0 \end{pmatrix},$$

$$R_1^{\wedge}(4) = V_1^{\wedge}(4) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 18 & 0 & 0 & 0 & 0 & 0 \\ 30 & 18 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$R_2^{\vee}(0) = \begin{pmatrix} -17 & 24 & -88 & 0 & 2872 & 6928 & 5712 & 0 \\ -16 & 15 & -32 & 0 & 1160 & 2544 & 1824 & 0 \\ -16 & 80 & -9 & 0 & 640 & 1464 & 1080 & 0 \\ -16 & -16 & -8 & -1 & 312 & 752 & 576 & 0 \\ -16 & -32 & -24 & -8 & 127 & 328 & 264 & 0 \\ -16 & -48 & -56 & -32 & 32 & 111 & 96 & 0 \\ -16 & -64 & -104 & -88 & -32 & 16 & 23 & 0 \\ -16 & -80 & -168 & -192 & -128 & -48 & -8 & -1 \end{pmatrix},$$

$$V_2^{\vee}(0) = \begin{pmatrix} -16 & 32 & -56 & 88 & 3064 & 7288 & 6320 & 1904 \\ -16 & 16 & -24 & 32 & 1248 & 2736 & 2184 & 608 \\ -16 & 0 & -8 & 8 & 672 & 1552 & 1272 & 360 \\ -16 & -16 & -8 & 0 & 320 & 784 & 664 & 192 \\ -16 & -32 & -24 & -8 & 128 & 336 & 296 & 88 \\ -16 & -48 & -56 & -32 & 32 & 112 & 104 & 32 \\ -16 & -64 & -104 & -88 & -32 & 16 & 24 & 8 \\ -16 & -80 & -168 & -192 & -128 & -48 & -8 & 0 \end{pmatrix},$$

$$R_2^{\vee}(1) = \begin{pmatrix} -215 & 448 & -1120 & -11488 & 16944 & -7616 & 0 & 0 \\ -208 & 281 & -512 & -4640 & -5008 & -992 & 0 & 0 \\ -208 & 80 & -183 & -2560 & -2000 & 1872 & 2160 & 0 \\ -208 & -128 & -96 & -1225 & -800 & 1664 & 1728 & 0 \\ -208 & -336 & -224 & -576 & -279 & 1024 & 1256 & 0 \\ -208 & -544 & -560 & -448 & -144 & 441 & 480 & 0 \\ -208 & -752 & -1104 & -880 & -368 & 48 & 137 & 0 \\ -208 & -960 & -1856 & -1952 & -1200 & -416 & -64 & -7 \end{pmatrix},$$

$$V_2^{\vee}(1) = \begin{pmatrix} -208 & 496 & -960 & -11136 & 16368 & -6896 & 0 & 0 \\ -208 & 288 & -464 & -4480 & -4656 & -416 & 720 & 0 \\ -208 & 80 & -176 & -2512 & -1840 & 2244 & 2736 & 720 \\ -288 & -128 & -96 & -1248 & -752 & 1824 & 2080 & 576 \\ -208 & -336 & -224 & -576 & -272 & 1072 & 1216 & 352 \\ -208 & -544 & -560 & -448 & -144 & 448 & 528 & 160 \\ -208 & -752 & -1104 & -880 & -368 & 48 & 144 & 48 \\ -208 & -960 & -1856 & -1952 & -1200 & -416 & -64 & 0 \end{pmatrix},$$

$$R_2^{\vee}(2) = \begin{pmatrix} -1062 & 2912 & 12592 & 8544 & 0 & 0 & 0 & 0 \\ -1048 & 1992 & 4240 & -1536 & -2880 & 0 & 0 & 0 \\ -1048 & 888 & 2856 & -4432 & -7888 & -2880 & 0 & 0 \\ -1048 & -160 & 1328 & -3214 & -5680 & -1952 & 0 & 0 \\ -1048 & -1208 & 16 & -1872 & -3062 & -560 & 528 & 0 \\ -1048 & -2256 & -1720 & -1504 & -1400 & -30 & 480 & 0 \\ -1048 & -3304 & -4168 & -2968 & -1336 & -136 & 202 & 0 \\ -1048 & 4352 & -7520 & -7040 & -3848 & -1184 & -160 & -14 \end{pmatrix},$$

$$V_2^{\vee}(2) = \begin{pmatrix} -1048 & 2984 & 12752 & 8720 & 0 & 0 & 0 & 0 \\ -1048 & 1936 & 4312 & -1376 & -2704 & 0 & 0 & 0 \\ -1048 & 888 & 2600 & -4360 & -7728 & -2704 & 0 & 0 \\ -1048 & -160 & 1328 & -3200 & -5608 & -1792 & 176 & 0 \\ -1048 & -1208 & 16 & -1872 & -3048 & -488 & 688 & 176 \\ -1048 & -2256 & -1720 & -1504 & -1400 & -16 & 552 & 160 \\ -1048 & -3304 & -4168 & -2968 & -1336 & -136 & 216 & 72 \\ -1048 & -4352 & -7520 & -7040 & -3848 & -1184 & -160 & 0 \end{pmatrix},$$

$$R_2^{\vee}(3) =$$

$$\begin{pmatrix} -2583 & -4000 & 0 & 0 & 0 & 0 & 0 & 0 \\ -2576 & 1049 & 3616 & 0 & 0 & 0 & 0 & 0 \\ -2576 & 912 & 7113 & 3613 & 0 & 0 & 0 & 0 \\ -2576 & -224 & 5152 & 2105 & -704 & 0 & 0 & 0 \\ -2576 & -2032 & 2240 & 352 & -2039 & -704 & 0 & 0 \\ -2576 & -4256 & -1552 & -928 & -1680 & -615 & 0 & 0 \\ -2576 & -6704 & -6640 & -3760 & -1648 & -368 & 41 & 0 \\ -2576 & -9248 & -13600 & -10688 & -4848 & -1216 & -128 & -7 \end{pmatrix},$$

$$V_2^{\vee}(3) =$$

$$\begin{pmatrix} -2576 & -3984 & 0 & 0 & 0 & 0 & 0 & 0 \\ -2576 & 1056 & 3632 & 0 & 0 & 0 & 0 & 0 \\ -2576 & 912 & 7120 & 3632 & 0 & 0 & 0 & 0 \\ -2576 & -224 & 5152 & 2112 & -688 & 0 & 0 & 0 \\ -2576 & -2032 & 2240 & 352 & -2032 & -688 & 0 & 0 \\ -2576 & -4256 & -1552 & -928 & -1680 & -608 & 16 & 0 \\ -2576 & -6704 & -6640 & -3760 & -1648 & -368 & 48 & 16 \\ -2576 & -9248 & -13600 & -10688 & -4848 & -1216 & -128 & 0 \end{pmatrix},$$

$$R_2^{\vee}(4) = V_2^{\vee}(4) =$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1904 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -2512 & -1904 & 0 & 0 & 0 & 0 & 0 & 0 \\ -2872 & -1536 & 976 & 0 & 0 & 0 & 0 & 0 \\ -3064 & -2104 & 1744 & 976 & 0 & 0 & 0 & 0 \\ -3152 & -3760 & -8 & 608 & -80 & 0 & 0 & 0 \\ -3184 & -6272 & -4088 & -1320 & -432 & -80 & 0 & 0 \\ -3192 & -9264 & -10600 & -6208 & -1992 & -320 & -16 & 0 \end{pmatrix}$$

$$R_2^{\vee}(5) = V_2^{\vee}(5) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -720 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1296 & -720 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1648 & -1312 & -16 & 0 & 0 & 0 & 0 & 0 \\ -1808 & -2320 & -688 & -16 & 0 & 0 & 0 & 0 \\ -1856 & -3840 & -2816 & -896 & -112 & 0 & 0 & 0 \end{pmatrix},$$

$$R_2^{\vee}(6) = V_2^{\vee}(6) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -176 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -336 & -176 & 0 & 0 & 0 & 0 & 0 & 0 \\ -408 & -448 & -112 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$R_2^{\vee}(7) = V_2^{\vee}(7) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -16 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$R_2^{\wedge}(0) = \begin{pmatrix} 17 & -24 & 88 & 0 & -968 & -1216 & 0 & 0 \\ 16 & -15 & 32 & 0 & -1160 & -2544 & -1824 & 0 \\ 16 & 0 & 9 & 0 & -640 & -1464 & -1080 & 0 \\ 16 & 16 & 8 & 1 & -312 & -752 & -576 & 0 \\ 16 & 32 & 24 & 8 & -127 & -328 & -264 & 0 \\ 16 & 48 & 56 & 32 & -32 & -111 & -96 & 0 \\ 16 & 64 & 104 & 88 & 32 & -16 & -23 & 0 \\ 16 & 80 & 168 & 192 & 128 & 48 & 8 & 1 \end{pmatrix},$$

$$V_2^{\wedge}(0) = \begin{pmatrix} 16 & -32 & 56 & -88 & -1160 & -1576 & -608 & 0 \\ 16 & -16 & 24 & -32 & -1248 & -2736 & -2184 & -608 \\ 16 & 0 & 8 & -8 & -672 & -1552 & -1272 & -360 \\ 16 & 16 & 8 & 0 & -320 & -784 & -664 & -192 \\ 16 & 32 & 24 & 8 & -128 & -336 & -296 & -88 \\ 16 & 48 & 56 & 32 & -32 & -112 & -104 & -32 \\ 16 & 64 & 104 & 88 & 32 & -16 & -24 & -8 \\ 16 & 80 & 168 & 192 & 128 & 48 & 8 & 0 \end{pmatrix},$$

$$R_2^{\wedge}(1) = \begin{pmatrix} 181 & -400 & 944 & 3872 & 2432 & 0 & 0 & 0 \\ 176 & -251 & 448 & 4640 & 6112 & 2432 & 0 & 0 \\ 176 & -80 & 165 & 2560 & 3280 & 1056 & 0 & 0 \\ 176 & 96 & 80 & 1253 & 1424 & -160 & -576 & 0 \\ 176 & 272 & 176 & 560 & 533 & -368 & -528 & 0 \\ 176 & 448 & 448 & 384 & 208 & -219 & -288 & 0 \\ 176 & 624 & 896 & 704 & 304 & -16 & -91 & 0 \\ 176 & 800 & 1520 & 1568 & 944 & 320 & 48 & 5 \end{pmatrix},$$

$$V_2^{\wedge}(1) = \begin{pmatrix} 176 & -432 & 848 & 3696 & 2240 & 0 & 0 & 0 \\ 176 & -256 & 416 & 4544 & 5936 & 2240 & 0 & 0 \\ 176 & -80 & 160 & 2528 & 3184 & 880 & -192 & 0 \\ 176 & 96 & 80 & 1248 & 1392 & -256 & -752 & -192 \\ 176 & 272 & 176 & 560 & 528 & -400 & -624 & -176 \\ 176 & 448 & 448 & 384 & 208 & -224 & -320 & -96 \\ 176 & 624 & 896 & 704 & 304 & -16 & -96 & -32 \\ 176 & 800 & 1520 & 1568 & 944 & 320 & 48 & 0 \end{pmatrix},$$

$$R_2^{\wedge}(2) = \begin{pmatrix} 734 & -2160 & -2880 & 0 & 0 & 0 & 0 & 0 \\ 728 & -1450 & -5072 & -2880 & 0 & 0 & 0 & 0 \\ 728 & -728 & -2898 & -688 & 768 & 0 & 0 & 0 \\ 728 & 0 & -1472 & 710 & 2208 & 768 & 0 & 0 \\ 728 & 728 & -320 & 768 & 1742 & 640 & 0 & 0 \\ 728 & 1456 & 936 & 800 & 920 & 246 & -96 & 0 \\ 728 & 2184 & 2584 & 1736 & 792 & 136 & -66 & 0 \\ 728 & 2912 & 4816 & 4288 & 2216 & 640 & 80 & 6 \end{pmatrix},$$

$$V_2^{\wedge}(2) = \begin{pmatrix} 728 & -2184 & -2912 & 0 & 0 & 0 & 0 & 0 \\ 728 & -1456 & -5096 & -2912 & 0 & 0 & 0 & 0 \\ 728 & -728 & -2904 & -712 & 736 & 0 & 0 & 0 \\ 728 & 0 & -1472 & 704 & 2184 & 736 & 0 & 0 \\ 728 & 728 & -320 & 768 & 1736 & 616 & -32 & 0 \\ 728 & 1456 & 936 & 800 & 920 & 240 & -120 & -32 \\ 728 & 2184 & 2584 & 1736 & 792 & 136 & -72 & -24 \\ 728 & 2912 & 4816 & 4288 & 2216 & 640 & 80 & 0 \end{pmatrix},$$

$$R_2^{\wedge}(3) = \begin{pmatrix} 1409 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1408 & 1409 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1408 & 384 & -1023 & 0 & 0 & 0 & 0 & 0 \\ 1408 & 352 & -2080 & -1023 & 0 & 0 & 0 & 0 \\ 1408 & 992 & -1344 & -800 & 129 & 0 & 0 & 0 \\ 1408 & 2048 & 352 & -32 & 384 & 129 & 0 & 0 \\ 1408 & 3328 & 2848 & 1344 & 544 & 128 & 1 & 0 \\ 1408 & 4704 & 6366 & 4480 & 1792 & 384 & 33 & 1 \end{pmatrix},$$

$$V_2^{\wedge}(3) = \begin{pmatrix} 1408 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1408 & 1408 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1408 & 384 & -1024 & 0 & 0 & 0 & 0 & 0 \\ 1408 & 352 & -2080 & -1024 & 0 & 0 & 0 & 0 \\ 1408 & 992 & -1344 & -800 & 128 & 0 & 0 & 0 \\ 1408 & 2048 & 352 & -32 & 384 & 128 & 0 & 0 \\ 1408 & 3328 & 2848 & 1344 & 544 & 128 & 0 & 0 \\ 1408 & 4704 & 6366 & 4480 & 1792 & 384 & 32 & 0 \end{pmatrix},$$

$$R_2^{\wedge}(4) = V_2^{\wedge}(4) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 608 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 968 & 608 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1160 & 808 & -160 & 0 & 0 & 0 & 0 & 0 \\ 1248 & 1264 & -56 & -160 & 0 & 0 & 0 & 0 \\ 1280 & 2128 & 1016 & 168 & 32 & 0 & 0 & 0 \\ 1288 & 3280 & 3160 & 1472 & 344 & 32 & 0 & 0 \end{pmatrix},$$

$$R_2^{\wedge}(5) = V_2^{\wedge}(5) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 192 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 368 & 192 & 0 & 0 & 0 & 0 & 0 & 0 \\ 464 & 432 & 64 & 0 & 0 & 0 & 0 & 0 \\ 496 & 800 & 400 & 64 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$R_2^{\wedge}(6) = V_2^{\wedge}(6) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 32 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 56 & 32 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

Comparing the above

$$a^*_{l,i,k}(z;\nu),\ a^{\vee}_{l,i,k}(z;\nu),\ a^{\wedge}_{l,i,k}(z;\nu)$$

where $l = 0, 1, 2$, $i = 1$, ..., $4 + 2l$, $k = 1$, ..., $4 + 2l$, with the elements of the matrices respectively $U_l^{\vee}(z, \mu)$, $U_l^{\wedge}(z, \mu)$, we see that

$$A_l^{\vee}(z;\nu) = (1/2)U_l^{\vee}(z;\nu),\ A_l^{\wedge}(z;\nu) = U_l^{\wedge}(z;\nu),$$

and, in view of (27) ,

$$A_l^*(z;\nu) = (1/2)U_l^{\vee}(z;\nu) + U_l^{\wedge}(z;\nu)\tau,$$

for $l = 0, 1, 2$. In view of (21), (26),

$$\nu^{3+2l}X_{l,k}(z;\nu-1) = A_l^*(z;\nu)X_{l,k}(z;\nu), \tag{32}$$

where $l = 0$, 1, 2 and $k \in \mathfrak{K}_l$, $|z| > 1, \nu \in \mathbb{Z}$. Replacing in (32) ν by $-\nu$ and taking in the account (25), we see that

$$(-\nu)^{3+2l}X_{l,k}(z;\nu) = A_l^*(z;-\nu)X_{l,k}(z;\nu+1), \tag{33}$$

Therefore the following equality must be fufilled:

$$-\nu^{6+4l}E_{3+2l} = A_l^*(z;\nu)A_l^*(z;-\nu), \tag{34}$$

where E_{3+2l} is the $(3 + 2l) \times (3 + 2l)$ unit matrix, $l = 0, 1, 2$, $z \in \mathbb{C}$, $\nu \in \mathbb{C}$. This equality opens the possibility for us to check independently the previous calculations.

§2.3. Corrections in the first part of this paper.

The equality (40) in the first part must have the same form, as (14) here.

References.

[1] R.Apéry, Interpolation des fractions continues
et irrationalite de certaines constantes,
Bulletin de la section des sciences du C.T.H., 1981, No 3, 37 – 53;
[2] F.Beukers, A note on the irrationality of $\zeta(2)$ and $\zeta(3)$,
Bull. London Math. Soc., 1979, 11, 268 – 272;
[3] A.van der Porten, A proof that Euler missed...Apéry's proof of the irrationality of $\zeta(3)$,
Math Intellegencer, 1979, 1, 195 – 203;
[4] W. Maier, Potenzreihen irrationalen Grenzwertes,
J.reine angew. Math., 156, 1927, 93 – 148;
[5] E.M. Nikišin, On irrationality of the values of the functions F(x,s) (in Russian),
Mat.Sb. 109 (1979), 410 – 417;
English transl. in Math. USSR Sb. 37 (1980), 381 – 388;
[6] G.V. Chudnovsky, Pade approximations to the generalized hyper-geometric functions
I,J.Math.Pures Appl., 58, 1979, 445 – 476;
[7] ______________, Transcendental numbers, Number Theory,Carbondale,
Lecture Notes in Math, Springer-Verlag, 1979, 751, 45 – 69;
[8] ______________, Approximations rationelles des logarithmes de nombres rationelles
C.R.Acad.Sc. Paris, Série A, 1979, 288, 607 – 609;
[9] ______________, Formules d'Hermite pour les approximants de Padé de logarithmes
et de fonctions binômes, et mesures d'irrationalité,
C.R.Acad.Sc. Paris, Série A, 1979, t.288, 965 – 967;
[10] ______________,Un systḿe explicite d'approximants de Padé
pour les fonctions hypérgéometriques généraliesées,
avec applications a l'arithmétique,
C.R.Acad.Sc. Paris, Série A, 1979, t.288, 1001 – 1004;
[11] ______________, Recurrenses defining Rational Approximations
to the irrational numbers, Proceedings
of the Japan Academie, Ser. A, 1982, 58, 129 – 133;
[12] ______________, On the method of Thue-Siegel,
Annals of Mathematics, 117 (1983), 325 – 382;
[13] K.Alladi and M. Robinson, Legendre polinomials and irrationality,
J. Reine Angew.Math., 1980, 318, 137 – 155;
[14] A. Dubitskas, An approximation of logarithms of some numbers,
Diophantine approximations II,Moscow, 1986, 20 – 34;
[15] ___________, On approximation of $\pi/\sqrt{3}$ by rational fractions,
Vestnik MGU, series 1, 1987, 6, 73 – 76;
[16] S.Eckmann, Über die lineare Unaqbhangigkeit der Werte gewisser Reihen,
Results in Mathematics, 11, 1987, 7 – 43;
[17] M.Hata, Legendre type polinomials and irrationality mesures,
J. Reine Angew. Math., 1990, 407, 99 – 125;
[18] A.O. Gelfond, Transcendental and algebraic numbers (in Russian),
GIT-TL, Moscow, 1952;
[19] H.Bateman and A.Erdélyi, Higher transcendental functions,1953,
New-York – Toronto – London, Mc. Grow-Hill Book Company, Inc.;
[20] E.C.Titchmarsh, The Theory of Functions, 1939, Oxford University Press;
[21] E.T.Whittaker and G.N. Watson, A course of modern analysis,
1927, Cambridge University Press;
[22] O.Perron, Über die Poincaresche Differenzengleichumg,
Journal für die reine und angewandte mathematik,
1910, 137, 6 – 64;
[23] A.O.Gelfond, Differenzenrechnung (in Russian), 1967, Nauka, Moscow.
[24] A.O.Gelfond and I.M.Kubenskaya, On the theorem of Perron
in the theory of differrence equations (in Russian),
IAN USSR, math. ser., 1953, 17, 2, 83 – 86.
[25] M.A.Evgrafov, New proof of the theorem of Perron
(in Russian),IAN USSR, math. ser., 1953, 17, 2, 77 – 82;
[26] G.A.Frejman, On theorems of of Poincaré and Perron
(in Russian), UMN, 1957, 12, 3 (75), 243 – 245;
[27] N.E.Nörlund, Differenzenrechnung, Berlin, Springer Verlag, 1924;

[28] I.M.Vinogradov, Foundtions of the Number Theory, (in Russian), 1952, GIT-TL;
[29] J. Diedonne, Foundations of modern analysis,
Institut des Hautes Études Scientifiques, Paris,
Academic Press, New York and London, 1960
[30] CH.-J. de la Vallée Poussin, Course d'analyse infinitésimale,
Russian translation by G.M.Fikhtengolts,
GT-TI, 1933;
[31] H.WEYL, Algebraic theory of numbers,
1940,
Russian translation by L.I.Kopejkina,

[32] L.A.Gutnik, On the decomposition of the difference operators of Poincaré type
(in Russian), VINITI, Moscow, 1992, 2468 – 92, 1 – 55;
[33] ___________, On the decomposition of the difference operators
of Poincaré type in Banach algebras
(in Russian), VINITI, Moscow, 1992, 3443 – 92, 1 – 36;
[34] ___________, On the difference equations of Poincaré type
(in Russian), VINITI, Moscow 1993, 443 – B93, 1 – 41;
[35] ___________, On the difference equations of Poincaré type in normed algebras
(in Russian), VINITI, Moscow, 1994, 668 – B94, 1 – 44;
[36] ___________, On the decomposition of the difference equations of Poincaré type
(in Russian), VINITI, Moscow, 1997, 2062 – B97, 1 – 41;
[37] ___________, The difference equations of Poincaré type
with characteristic polynomial having roots equal to zero
(in Russian), VINITI, Moscow, 1997, 2418 – 97, 1 – 20;
[38] ___________, On the behavior of solutions
of difference equations of Poincaré type
(in Russian), VINITI, Moscow, 1997, 3384 – B97, 1 - 41;
[39] ___________, On the variability of solutions of difference equations of Poincaré type
(in Russian), VINITI, Moscow, 1999, 361 – B99, 1 – 9;
[40] ___________, To the question of the variability of solutions
of difference equations of Poincaré type (in Russian),
VINITI, Moscow, 2000, 2416 – B00, 1 – 22;
[41] ___________, On linear forms with coefficients in $\mathbb{N}\zeta(1+\mathbb{N})$,
Max-Plank-Institut für Mathematik,
Bonn, Preprint Series, 2000, 3, 1 – 13;
[42] ___________, On the Irrationality of Some Quantyties Containing $\zeta(3)$ (in Russian),
Uspekhi Mat. Nauk, 1979, 34, 3(207), 190;
[43] ___________, On the Irrationality of Some Quantities Containing $\zeta(3)$,
Eleven papers translated from the Russian,
American Mathematical Society, 1988, 140, 45 - 56;
[44] ___________, Linear independence over $\mathbb{Q}$ of dilogarithms at rational points
(in Russian), UMN, 37 (1982), 179-180;
english transl. in Russ. Math. surveys 37 (1982), 176-177;
[45] ___________, On a measure of the irrationality of dilogarithms at rational points
(in Russian), VINITI, 1984, 4345-84, 1 – 74;
[46] ___________, To the question of the smallness of some linear forms
(in Russian), VINITI, 1993, 2413-B93, 1 – 94;
[47] ___________, On linear forms, whose coefficients are logarithms
of algebraic numbers (in Russian),
VINITI, 1995, 135-B95, 1 – 149;
[48] ___________, On systems of vectors, whose coordinates
are linear combinations of logarithms of algebraic numbers
with algebraic coefficients (in Russian),
VINITI, 1994, 3122-B94, 1 – 158;

[49] ___________, On the linear forms, whose
coefficients are $\mathbb{A}$ - linear combinations
of logarithms of $\mathbb{A}$ - numbers,

VINITI, 1996, 1617-B96, pp. 1 – 23.
[50] ____________, On systems of linear forms, whose coefficients are $\mathbb{A}$ - linear combinations of logarithms of $\mathbb{A}$ - numbers, VINITI, 1996, 2663-B96, pp. 1 – 18.

[51] ____________, On linear forms, whose coefficients are $\mathbb{Q}$-proportional to the number $\log 2$, and the values of $\zeta(s)$ for integer s (in Russian), VINITI, 1996, 3258-B96, 1 – 70;
[52] ____________, The lower estimate for some linear forms, coefficients of which are proportional to the values of $\zeta(s)$ for integer s (in Russian), VINITI, 1997, 3072-B97, 1 – 77;
[53] ____________, On linear forms with coefficients in $\mathbb{N}\zeta(1+\mathbb{N})$ Max-Plank-Institut für Mathematik, Bonn, Preprint Series, 2000, 3, 1 – 13;
[54] ____________, On linear forms with coefficients in $\mathbb{N}\zeta(1+\mathbb{N})$ (the detailed version,part 1), Max-Plank-Institut für Mathematik, Bonn, Preprint Series, 2001, 15, 1 – 20;
[55] ____________, On linear forms with coefficients in $\mathbb{N}\zeta(1+\mathbb{N})$ (the detailed version,part 2), Max-Plank-Institut für Mathematik, Bonn, Preprint Series,2001, 104, 1 – 36;
[56] ____________, On linear forms with coefficients in $\mathbb{N}\zeta(1+\mathbb{N})$ (the detailed version,part 3), Max-Plank-Institut für Mathematik, Bonn, Preprint Series, 2002, 57, 1 – 33;
[57] ____________, On the rank over $\mathbb{Q}$ of some real matrices (in Russian), VINITI, 1984, 5736-84; 1 – 29;
[58] ____________, On the rank over $\mathbb{Q}$ of some real matrices, Max-Plank-Institut für Mathematik, Bonn, Preprint Series, 2002, 27, 1 – 32;
[59] ____________, On the linear forms with coefficients proportional to zeta values(in Russian), VINITI, 1999, 3207-B99; 1 – 71;
[60] ____________, On linear forms with coefficients in $\mathbb{N}\zeta(1+\mathbb{N})$ (the detailed version, part 4), Max-Plank-Institut für Mathematik, Bonn, Preprint Series, 2002, 142, 1 – 27;
[61] ____________, On the dimension of some linear spaces over finite extension of $\mathbb{Q}$ (part 2), Max-Plank-Institut für Mathematik, Bonn, Preprint Series, 2002, 107, 1 – 37;
[62] ____________, On the dimension of some linear spaces over $\mathbb{Q}$ (part 3), Max-Plank-Institut für Mathematik, Bonn, Preprint Series, 2003, 16, 1 – 45.
[63] ____________,On the difference equation of Poincaré type (Part 1). Max-Plank-Institut für Mathematik, Bonn, Preprint Series, 2003, 52, 1 – 44.
[64] ____________, On the dimension of some linear spaces over $\mathbb{Q}$, (part 4) Max-Plank-Institut für Mathematik, Bonn, Preprint Series, 2003, 73, 1 – 38.
[65] ____________, On linear forms with coefficients in $\mathbb{N}\zeta(1+\mathbb{N})$ (the detailed version, part 5), Max-Plank-Institut für Mathematik, Bonn, Preprint Series, 2003, 83, 1 – 13.
[66] ____________, On linear forms with coefficients in $\mathbb{N}\zeta(1+\mathbb{N})$ (the detailed version, part 6), Max-Plank-Institut für Mathematik, Bonn, Preprint Series, 2003, 99, 1 – 33.
[67] ____________,On the difference equation of Poincaré type (Part 2).

Max-Plank-Institut für Mathematik, Bonn,
Preprint Series, 2003, 107, 1 − 25.

[68] ____________, On the asymptotic behavior of solutions of difference equation (in English). Chebyshevskij sbornik, 2003, v.4, issue 2, 142 − 153.

[69] ____________, On linear forms with coefficients in $\mathbb{N}\zeta(1+\mathbb{N})$, Bonner Mathematishe Schriften Nr. 360, Bonn, 2003, 360.

[70] ____________, On the dimension of some linear spaces over $\mathbb{Q}$, (part 5) Max-Plank-Institut für Mathematik, Bonn, Preprint Series, 2004, 46, 1 − 42.

[71] ____________,On the difference equation of Poincaré type (Part 3). Max-Plank-Institut für Mathematik, Bonn, Preprint Series, 2004, 9, 1 − 33.

[72] ____________, On linear forms with coefficients in $\mathbb{N}\zeta(1+\mathbb{N})$ (the detailed version, part 7), Max-Plank-Institut für Mathematik, Bonn, Preprint Series, 2004, 88, 1 − 27.

[73] ____________,On the difference equation of Poincaré type (Part 4). Max-Plank-Institut für Mathematik, Bonn, Preprint Series, 2004, 129, 1 − 28.

[74] ____________, On the measure of discreteness of some modules, Max-Plank-Institut für Mathematik, Bonn, Preprint Series, 2005, 32, 1 − 51.

[75] ____________, On some systems of difference equations. Part 1. Max-Plank-Institut für Mathematik, Bonn, Preprint Series, 2006, 23, 1 − 37.

E-mail: gutnik@gutnik.mccme.ru