ABUNDANT FAMILIES OF JACOBI ELLIPTIC FUNCTION-LIKE SOLUTIONS FOR A GENERALIZED VARIABLE COEFFICIENTS 2D KDV EQUATION VIA THE EXTENDED MAPPING METHOD

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Abstract. Based on the symbolic computation system-Mathematica and an extended mapping method, we study a generalized variable coefficients 2D KdV equation and obtain new families of exact solutions, which include Jacobi-elliptic function-like, combined Jacobi elliptic function-like, rational-like, singular soliton-like, combined singular soliton-like, kink-bell soliton-like and triangular function-like solutions. The solutions of the well-known cylindrical Kadomtsev-Petviashvili equation can be recovered as special case of the 2D KdV equation.

1. Introduction

In the recent years, there has been a great amount of activity aiming to find methods for the exact solutions of nonlinear partial differential equations (NLPDEs). The exact solutions, if available can provide much physical information and more insight into the physical aspect of the problems. They can also help in the stability analysis of solutions and in the verification of numerical analysis of these NLPDEs. Looking for exact solitary wave solutions to the NLPDEs has long been a major concern for both mathematicians and physicists [1-10]. These kinds of solutions may describe various phenomena in physics and other fields, such as solitons, which propagate without spreading and have particle-like properties, represent one of the most striking aspects of nonlinear phenomena. These behaviors can be encountered in the 2D KdV-type equations [11,12] which play a great role in explaining the dynamical process in nature. The 2D KdV-type equations are encountered in many apparently unrelated phenomena in different physical systems such as in plasmas, in fluid and in lattice vibrations of a crystals at low temperature. The 2D KdV equation reads [11,12]

$$(u_t + 6uu_x + u_{xxx})_x + A(t)u_x + B(t)u_{yy} = 0, \qquad \mathbf{(1)}$$

where A and B are time-dependent coefficients. The importance of Eq.(1) is well known and there is a continuing high level of interest in the contruction of exact solutions to its special forms. Eq.(1) can degenerate to the cylindical KdV equation [11,12],

$$u_t + 6uu_x + u_{xxx} + \frac{u}{2t} = 0, \qquad \mathbf{(2)}$$

which describes solitary waves in a channel with slowly varying depth. Eq.(1) can also degenerate to the cylindrical Kadomtsev-Petviashvili (KP) equation [11,12].

$$(u_t+6uu_x+u_{xxx})_x+\tfrac{1}{2t}u_x+\tfrac{3\sigma^2}{t^2}u_{yy} = 0, \quad \sigma^2 = \pm 1, \qquad \mathbf{(3)}$$

This equation describes surface waves in a fluid with small deviation in axial symmetry. Very recently, the 2D KdV equation (1) has been investigated by Elwakil et al [13] by using homogeneous balance method. They obtained soliton-like solutions of this equation and its degeneracies (2) and (3). But the periodic solutions of Eq.(1) and their cylindrical generalizations (2) and (3) are still unknown so far. The aim of this letter is to obtain the periodic wave solutions of Eq.(1) and (3) by use of extended mapping method [14]. Specially we have found new rich Jacobi function-like, combined Jacobi function-like, kink-bell-like, rational-like, singular soliton-like, combined singular soliton-like, triangular function -like and combined triangular function-like solutions.
In the next section, we briefly describe the extended mapping method. In section 3, we

present the new rich exact solutions of the 2D KdV equation and as special cases, we deduce solutions of Eq.(3). We finally conclude this letter in the last section.

2. Brief description of extended mapping method and new solutions of the 2D KdV equation

2.1. Brief description of extended mapping method

. For a given NLPDE, say in three independent variables X= (x,y,t) and dependent variable u, we seek its solutions in the form

$$u(x,y,t) = \sum_{i=0}^{n} A_i(X)\phi^i(\xi(X)), \tag{4}$$

while ϕ satisfies the first kind elliptic equation

$$(\frac{d\phi}{d\xi})^2 = r + a\phi^2(\xi) + b/(2)\phi^4(\xi), \tag{5}$$

$$\xi = p(y,t)x + q(y,t),$$

where r, a and b, are constants. To determine u explicitly, we take the following five steps.

Step 1. Determine n which is fixed by balancing the highest order derivative terms and the highest nonlinear terms in the given NLPDE.

Step 2. Substituting (4) and (5) into the given NLPDE and collecting coefficients of $x^j\phi^k\sqrt{r + a\phi^2(\xi) + b/(2)\phi^4(\xi)}$, $x^l\phi^s$ (j=0,1,...;k=0,1,...; l=0,1,...; s=1,2,...), then setting each coefficient to zero to derive a set of partial differential equations of $A_i(i = 0, 1..., n)$ and ξ.

Step 3. Solving the system of partial differential equations obtained in step 2, yields the A_i and ξ functions.

Step 4. It is well known that the general solutions of (2) are:

$$\phi(\xi)=\begin{cases} sn\ \xi;\ cd\ \xi = cn\ \xi/dn\ \xi & : \ r=1,\ \ a=-(m^2+1),\ \ b=2m^2; \\ cn\ \xi & : \ r=1-m^2,\ \ a=2m^2-1,\ \ b=-2m^2; \\ dn\ \xi & : \ r=m^2-1,\ \ a=2-m^2,\ \ b=-2; \\ ns\ \xi=1/sn\ \xi;\ dc\ \xi=dn\ \xi/cn\ \xi & : \ r=m^2,\ \ a=-(m^2+1),\ \ b=2; \\ nc\ \xi=1/cn\ \xi & : \ r=-m^2,\ \ a=2m^2-1,\ \ b=2(1-m^2); \\ nd\ \xi=1/dn\ \xi & : \ r=-1,\ \ a=2-m^2,\ \ b=2(m^2-1); \\ cs\ \xi=cn\ \xi/sn\ \xi & : \ r=1-m^2,\ \ a=2-m^2,\ \ b=2; \\ sc\ \xi=sn\ \xi/cn\ \xi & : \ r=1,\ \ a=2-m^2,\ \ b=2(1-m^2); \\ sd\ \xi=sn\ \xi/dn\ \xi & : \ r=1,\ \ a=2m^2-1,\ \ b=2m^2(m^2-1); \\ ds\ \xi=dn\ \xi/sn\ \xi & : \ r=m^2(m^2-1),\ \ a=2m^2-1,\ \ b=2; \\ ns\ \xi\pm cs\ \xi & : \ r=1/4,\ a=(1-2m^2)/2,\ \ b=1/2; \\ nc\ \xi\pm sc\ \xi & : \ r=(1-m^2)/4,\ a=(1+m^2)/2,\ \ b=(1-m^2)/2; \\ ns\ \xi+ds\ \xi & : \ r=m^4/4,\ a=(m^2-2)/2,\ \ b=1/2; \\ sn\ \xi\pm icn\ \xi & : \ r=m^2/4,\ \ a=(m^2-2)/2,\ \ b=m^2/2,\ \ i^2=-1; \\ C/\xi & : \ r=0,\ \ a=0\ \ b=2, \end{cases} \quad \mathbf{(6)}$$

where C is a constant. The Jacobi elliptic functions $sn\ \xi = sn\ (\xi|m)$, $cn\ \xi = cn\ (\xi|m)$, and $dn\ \xi = dn\ (\xi|m)$, where $m(0 < m < 1)$ is the modulus of elliptic function, are double periodic and possess properties of triangular functions, namely

$$sn^2\xi + cn^2\xi = 1,\ \ dn^2\ \xi + m^2 sn^2\ \xi = 1,\ \ \frac{d}{d\xi}sn\ \xi = cn\ \xi dn\ \xi,$$

$$\frac{d}{d\xi}cn\ \xi = -sn\ \xi dn\ \xi,\quad \frac{d}{d\xi}dn\ \xi = -m^2 cn\ \xi sn\ \xi. \quad \mathbf{(7)}$$

In addition, we see that other solutions are obtained from solutions (6) in case of degeneracies:

When m$\rightarrow$ 0, the Jacobi elliptic functions degenerate to the trigonometric functions solutions of the given NLPDE i.e.

$$sn\ \xi \rightarrow\ \sin\ \xi,\ \ cn\ \xi \rightarrow\ \cos\ \xi,\ \ dn\ \xi \rightarrow\ 1. \quad \mathbf{(8)}$$

When m$\rightarrow$ 1, the Jacobi elliptic functions degenerate to the solitonic solutions of the given NLPDE i.e.

$$sn\ \xi \rightarrow\ \tanh\ \xi,\ \ cn\ \xi \rightarrow\ sech\ \xi,\ \ dn\ \xi \rightarrow\ sech\ \xi. \quad \mathbf{(9)}$$

Detailed explanations about Jacobi elliptic functions can be found in Refs.[15,16]. The other 9 Jacobi functions are all expressible in terms of $sn\ \xi,\ cn\ \xi,\ dn\ \xi$.

Step 5. Substituting A_i, ξ and (6) or (8) or (9) into (1) to obtain Jacobi elliptic function-like, combined Jacobi elliptic function-like and rational-like solutions or triangular function-like solutions or singular soliton-like, combined singular soliton-like and kink bell soliton-like solutions respectively. The relations between values of r, a, b and the corresponding solutions $\phi(\xi)$ are given in (6).

2.2. **New exact solutions of the 2D KdV equation**

. We discuss the Eqs.(1) and (3) using the method described in the previous section. Balancing u_{xxx} with uu_x, ansatz (4) becomes

$$u = f + h\phi(\xi) + g\phi^2(\xi), \qquad (\mathbf{10})$$

$$\xi = p(y,t)x + q(y,t),$$

where f, h, g, p and q are functions of y and t to be determined. Substituting (9) and (2) into (8) and collecting coefficients of power of $x^j\phi^k\sqrt{r + a\phi^2(\xi) + b/(2)\phi^4(\xi)}$, $x^l\phi^s$ (j=0,1; k=0,1; l=0,1,2; s=1,...,4), (notice that f, h, g, p and q are independent of x) with the aid of Mathematica, then setting each coefficient to zero, we obtain the following set of partial differential equations.

$$12rfgp^2 + 6rh^2p^2 + 8argp^4 + 2rgpq_t + 2rB(t)gq_y^2 + B(t)f_{yy} = 0, \qquad (\mathbf{11.1})$$

$$6afhp^2 + 36rghp^2 + a^2hp^4 + 6rbhp^4 + ahpq_t + aB(t)hq_y^2 + B(t)h_{yy} = 0, \qquad (\mathbf{11.2})$$

$$24afgp^2 + 36rg^2p^2 + 12ah^2p^2 + 16a^2gp^4 + 36brgp^4 + 4agpq_t + 4aB(t)q_y^2 + B(t)g_{yy} = 0, \qquad (\mathbf{11.3})$$

$$6bfhp^2 + 54aghp^2 + 10abhp^4 + bhpq_t + bB(t)hq_y^2 = 0, \qquad (\mathbf{11.4})$$

$$18bfgp^2 + 48ag^2p^2 + 9bh^2p^2 + 60abgp^4 + 3bgpq_t + 3bB(t)q_y^2 = 0, \qquad (\mathbf{11.5})$$

$$6hbp^2(6g + bp^2) = 0, \qquad (\mathbf{11.6})$$

$$30bgp^2(g + bp^2), \qquad (\mathbf{11.7})$$

$$2rgB(t)p_y^2 = 0, \qquad (\mathbf{11.8})$$

$$ahB(t)p_y^2 = 0, \qquad (\mathbf{11.9})$$

$$4agB(t)p_y^2 = 0, \qquad (\mathbf{11.10})$$

$$bhB(t)p_y^2 = 0, \qquad (\mathbf{11.11})$$

$$3bgB(t)p_y^2 = 0, \qquad (\mathbf{11.12})$$

$$2rg(pp_t + 2B(t)p_yq_y) = 0, \qquad (\mathbf{11.13})$$

$$ah(pp_t + 2B(t)p_yq_y) = 0, \qquad (\mathbf{11.14})$$

$$4ag(pp_t + 2B(t)p_y q_y) = 0, \quad (\mathbf{11.15})$$

$$bh(pp_t + 2B(t)p_y q_y) = 0, \quad (\mathbf{11.16})$$

$$3bg(pp_t + 2B(t)p_y q_y) = 0, \quad (\mathbf{11.17})$$

$$B(t)(2h_y p_y + hp_{yy}) = 0, \quad (\mathbf{11.18})$$

$$2B(t)(2g_y p_y + gp_{yy}) = 0, \quad (\mathbf{11.19})$$

$$A(t)hp + ph_t + hp_t + 2B(t)h_y q_y + B(t)hq_{yy} = 0, \quad (\mathbf{11.20})$$

$$2[A(t)gp + pg_t + gp_t + 2B(t)g_y q_y + B(t)gq_{yy}] = 0, \quad (\mathbf{11.21})$$

From these equations, it follows that

$$p_y = 0, \qquad p_t = 0, \qquad h = 0, \quad (\mathbf{12})$$

From (12), we obtain that p is constant. And now (11.1)-(11.21) become

$$12rfgp^2 + 6rh^2p^2 + 8argp^4 + 2rgpq_t + 2rB(t)gq_y + B(t)f_{yy} = 0, \quad (\mathbf{13.1})$$

$$24afgp^2 + 36rg^2p^2 + 12ah^2p^2 + 16a^2gp^4 + 36brgp^4 + 4agpq_t + 4aB(t)q_y^2 + B(t)g_{yy} = 0, \quad (\mathbf{13.2})$$

$$18bfgp^2 + 48ag^2p^2 + 9bh^2p^2 + 60abgp^4 + 3bgpq_t + 3bB(t)q_y^2 = 0, \quad (\mathbf{13.3})$$

$$30bgp^2(g + bp^2), \quad (\mathbf{13.4})$$

$$2[A(t)gp + pg_t + gp_t + 2B(t)g_y q_y + B(t)gq_{yy}] = 0, \quad (\mathbf{13.5})$$

Solving Eqs.(13.1)-(13.5) we can obtain easily the following solutions

$$g = -bp^2, \quad (\mathbf{14.1})$$

$$q_{yy} = -p\frac{A(t)}{B(t)}, \quad (\mathbf{14.2})$$

$$f = -\frac{1}{6p^2}[4ap^4 + pq_t + B(t)q_y^2]. \quad (\mathbf{14.3})$$

Where the following constraint should be respected

$$A(t)B_t(t) - B(t)A_t(t) + 2B(t)A^2(t) = 0. \quad (\mathbf{15})$$

Integrating twice Eq.(14.2), we obtain

$$q = -\frac{p}{2}\frac{A(t)}{B(t)}y^2 + K_1(t)y + K_2(t). \quad (\mathbf{16})$$

Then, using Eqs.(16) and (15) into (14.3), we have

$$f = -\frac{1}{6p^2}\{4ap^4 - \frac{A^2(t)}{B(t)}p^2y^2 + pK_{1t}(t) + pK_{2t}(t) + B(t)[K_1(t) - p\frac{A(t)}{B(t)}y]^2\}. \quad (\mathbf{17})$$

Where K_1 and K_2 are arbitrary functions of t, while p, r, a and b are arbitrary constants. Thanks for the arbitrariness of r, a and b, we may choose them properly in Ordinary differential equation (5) such that the corresponding solution $\phi(\xi)$ of it, is one of Jacobi elliptic function-like or one of the combined Jacobi elliptic function-like or a rational-like solutions given in relation (6). it is important to note that the function coefficients A(t) and B(t) from Eq.(1) will not be chosen at random, but should satisfy the constraint condition (15).

Hence

$$u_{(r,a,b)} = -\frac{1}{6p^2}\{4ap^4 - \frac{A^2(t)}{B(t)}p^2y^2 + pK_{1t}(t) + pK_{2t}(t) + B(t)[K_1(t) - p\frac{A(t)}{B(t)}y]^2\} - bp^2\phi^2[px - \frac{p}{2}\frac{A(t)}{B(t)}y^2 + K_1(t)y + K_2(t)], \quad \mathbf{(18)}$$

where the subscript (r, a, b) gives the values of r, a and b for the corresponding solution chosen.

Therefore, we can consider several cases,

Case 1

$$u_{(1,-m^2-1,2m^2)} = -\frac{1}{6p^2}\{-4(m^2+1)p^4 - \frac{A^2(t)}{B(t)}p^2y^2 + pK_{1t}(t) + pK_{2t}(t) + B(t)[K_1(t) - p\frac{A(t)}{B(t)}y]^2\} - 2m^2p^2\ sn^2[px - \frac{p}{2}\frac{A(t)}{B(t)}y^2 + K_1(t)y + K_2(t)], \quad \mathbf{(19.1)}$$

Case 2

$$u'_{(1,-m^2-1,2m^2)} = -\frac{1}{6p^2}\{-4(m^2+1)p^4 - \frac{A^2(t)}{B(t)}p^2y^2 + pK_{1t}(t) + pK_{2t}(t) + B(t)[K_1(t) - p\frac{A(t)}{B(t)}y]^2\} - 2m^2p^2\ cd^2[px - \frac{p}{2}\frac{A(t)}{B(t)}y^2 + K_1(t)y + K_2(t)], \quad \mathbf{(19.2)}$$

Case 3

$$u_{(1-m^2,2m^2-1,-2m^2)} = -\frac{1}{6p^2}\{4(2m^2-1)p^4 - \frac{A^2(t)}{B(t)}p^2y^2 + pK_{1t}(t) + pK_{2t}(t) + B(t)[K_1(t) - p\frac{A(t)}{B(t)}y]^2\} + 2m^2p^2\ cn^2[px - \frac{p}{2}\frac{A(t)}{B(t)}y^2 + K_1(t)y + K_2(t)], \quad \mathbf{(19.3)}$$

Case 4

$$u_{(m^2-1,2-m^2,-2)} = -\frac{1}{6p^2}\{4(2-m^2)p^4 - \frac{A^2(t)}{B(t)}p^2y^2 + pK_{1t}(t) + pK_{2t}(t) + B(t)[K_1(t) - p\frac{A(t)}{B(t)}y]^2\} + 2p^2\ dn^2[px - \frac{p}{2}\frac{A(t)}{B(t)}y^2 + K_1(t)y + K_2(t)], \quad \mathbf{(19.4)}$$

Case 5

$$u_{(m^2,-m^2-1,2)} = -\frac{1}{6p^2}\{-4(m^2+1)p^4 - \frac{A^2(t)}{B(t)}p^2y^2 + pK_{1t}(t) + pK_{2t}(t) + B(t)[K_1(t) - p\frac{A(t)}{B(t)}y]^2\} - 2p^2\ ns^2[px - \frac{p}{2}\frac{A(t)}{B(t)}y^2 + K_1(t)y + K_2(t)], \quad \mathbf{(19.5)}$$

Case 6

$$u'_{(m^2,-m^2-1,2)} = -\frac{1}{6p^2}\{-4(m^2+1)p^4 - \frac{A^2(t)}{B(t)}p^2y^2 + pK_{1t}(t) + pK_{2t}(t) + B(t)[K_1(t) - p\frac{A(t)}{B(t)}y]^2\} - 2p^2\ dc^2[px - \frac{p}{2}\frac{A(t)}{B(t)}y^2 + K_1(t)y + K_2(t)], \quad \mathbf{(19.6)}$$

Case 7

$$u_{(-m^2,2m^2-1,2-2m^2)} = -\frac{1}{6p^2}\{4(2m^2-1)p^4 - \frac{A^2(t)}{B(t)}p^2y^2 + pK_{1t}(t) + pK_{2t}(t) + B(t)[K_1(t) - p\frac{A(t)}{B(t)}y]^2\} - 2(1-m^2)p^2\ nc^2[px - \frac{p}{2}\frac{A(t)}{B(t)}y^2 + K_1(t)y + K_2(t)], \quad \mathbf{(19.7)}$$

Case 8

$u_{(-1,2-m^2,2m^2-2)} = -\frac{1}{6p^2}\{4(2-m^2)p^4 - \frac{A^2(t)}{B(t)}p^2y^2 + pK_{1t}(t) + pK_{2t}(t) + B(t)[K_1(t) - p\frac{A(t)}{B(t)}y]^2\} + 2(1-m^2)p^2\ nd^2[px - \frac{p}{2}\frac{A(t)}{B(t)}y^2 + K_1(t)y + K_2(t)],$ (**19.8**)

Case 9

$u_{(1-m^2,2-m^2,2)} = -\frac{1}{6p^2}\{4(2-m^2)p^4 - \frac{A^2(t)}{B(t)}p^2y^2 + pK_{1t}(t) + pK_{2t}(t) + B(t)[K_1(t) - p\frac{A(t)}{B(t)}y]^2\} - 2p^2\ cs^2[px - \frac{p}{2}\frac{A(t)}{B(t)}y^2 + K_1(t)y + K_2(t)],$ (**19.9**)

Case 10

$u_{(1,2-m^2,2-2m^2)} = -\frac{1}{6p^2}\{4(2-m^2)p^4 - \frac{A^2(t)}{B(t)}p^2y^2 + pK_{1t}(t) + pK_{2t}(t) + B(t)[K_1(t) - p\frac{A(t)}{B(t)}y]^2\} - 2(1-m^2)p^2\ sc^2[px - \frac{p}{2}\frac{A(t)}{B(t)}y^2 + K_1(t)y + K_2(t)],$ (**19.10**)

Case 11

$u_{(1,2m^2-1,2m^4-2m^2)} = -\frac{1}{6p^2}\{4(2m^2-1)p^4 - \frac{A^2(t)}{B(t)}p^2y^2 + pK_{1t}(t) + pK_{2t}(t) + B(t)[K_1(t) - p\frac{A(t)}{B(t)}y]^2\} - 2m^2(m^2-1)p^2\ sd^2[px - \frac{p}{2}\frac{A(t)}{B(t)}y^2 + K_1(t)y + K_2(t)],$ (**19.11**)

Case 12

$u_{(m^4-m^2,2m^2-1,2)} = -\frac{1}{6p^2}\{4(2m^2-1)p^4 - \frac{A^2(t)}{B(t)}p^2y^2 + pK_{1t}(t) + pK_{2t}(t) + B(t)[K_1(t) - p\frac{A(t)}{B(t)}y]^2\} - 2p^2\ ds^2[px - \frac{p}{2}\frac{A(t)}{B(t)}y^2 + K_1(t)y + K_2(t)],$ (**19.12**)

Case 13

$u_{(1/4,1/2-m^2,1/2)} = -\frac{1}{6p^2}\{2(1-2m^2)p^4 - \frac{A^2(t)}{B(t)}p^2y^2 + pK_{1t}(t) + pK_{2t}(t) + B(t)[K_1(t) - p\frac{A(t)}{B(t)}y]^2\} - \frac{p^2}{2}\ \{ns[px - \frac{p}{2}\frac{A(t)}{B(t)}y^2 + K_1(t)y + K_2(t)] \pm\ cs[px - \frac{p}{2}\frac{A(t)}{B(t)}y^2 + K_1(t)y + K_2(t)]\}^2,$ (**19.13**)

Case 14

$u_{(1/4-m^2/4,1/2+m^2/2,1/2-m^2/2)} = -\frac{1}{6p^2}\{2(1+m^2)p^4 - \frac{A^2(t)}{B(t)}p^2y^2 + pK_{1t}(t) + pK_{2t}(t) + B(t)[K_1(t) - p\frac{A(t)}{B(t)}y]^2\} - \frac{1-m^2}{2}p^2\ \{nc[px - \frac{p}{2}\frac{A(t)}{B(t)}y^2 + K_1(t)y + K_2(t)] \pm\ sc[px - \frac{p}{2}\frac{A(t)}{B(t)}y^2 + K_1(t)y + K_2(t)]\}^2,$ (**19.14**)

Case 15

$u_{(m^2/4,m^2/2-1,1/2)} = -\frac{1}{6p^2}\{2(m^2-2)p^4 - \frac{A^2(t)}{B(t)}p^2y^2 + pK_{1t}(t) + pK_{2t}(t) + B(t)[K_1(t) - p\frac{A(t)}{B(t)}y]^2\} - \frac{p^2}{2}\ \{ns[px - \frac{p}{2}\frac{A(t)}{B(t)}y^2 + K_1(t)y + K_2(t)] +\ ds[px - \frac{p}{2}\frac{A(t)}{B(t)}y^2 + K_1(t)y + K_2(t)]\}^2,$ (**19.15**)

Case 16

$u_{(m^2/4,m^2/2-1,m^2/2)} = -\frac{1}{6p^2}\{2(m^2-2)p^4 - \frac{A^2(t)}{B(t)}p^2y^2 + pK_{1t}(t) + pK_{2t}(t) + B(t)[K_1(t) - p\frac{A(t)}{B(t)}y]^2\} - \frac{m^2}{2}p^2\ \{ns[px - \frac{p}{2}\frac{A(t)}{B(t)}y^2 + K_1(t)y + K_2(t)] \pm\ cs[px - \frac{p}{2}\frac{A(t)}{B(t)}y^2 + K_1(t)y + K_2(t)]\}^2,$ (**19.16**)

Case 17

$u_{(0,0,2)} = -\frac{1}{6p^2}\{-\frac{A^2(t)}{B(t)}p^2y^2 + pK_{1t}(t) + pK_{2t}(t) + B(t)[K_1(t) - p\frac{A(t)}{B(t)}y]^2\} - \frac{2p^2C^2}{[px - \frac{p}{2}\frac{A(t)}{B(t)}y^2 + K_1(t)y + K_2(t)]^2}.$ (**19.17**)

Remark 1

When m→ 0, these solutions (19.1)-(19.16) degenerate as triangular function-like solutions of Eq.(1), namely

$u_{(0,-1,2)} = -\frac{1}{6p^2}\{-4p^4-\frac{A^2(t)}{B(t)}p^2y^2+pK_{1t}(t)+pK_{2t}(t)+B(t)[K_1(t)-p\frac{A(t)}{B(t)}y]^2\}-2p^2\ csc^2[px-\frac{p}{2}\frac{A(t)}{B(t)}y^2+K_1(t)y+K_2(t)],$ (**20.1**)

$u'_{(0,-1,2)} = -\frac{1}{6p^2}\{-4p^4-\frac{A^2(t)}{B(t)}p^2y^2+pK_{1t}(t)+pK_{2t}(t)+B(t)[K_1(t)-p\frac{A(t)}{B(t)}y]^2\}-2p^2\ sec^2[px-\frac{p}{2}\frac{A(t)}{B(t)}y^2+K_1(t)y+K_2(t)],$ (**20.2**)

$u_{(1,2,2)} = -\frac{1}{6p^2}\{8p^4-\frac{A^2(t)}{B(t)}p^2y^2+pK_{1t}(t)+pK_{2t}(t)+B(t)[K_1(t)-p\frac{A(t)}{B(t)}y]^2\}-2p^2\cot^2[px-\frac{p}{2}\frac{A(t)}{B(t)}y^2+K_1(t)y+K_2(t)],$ (**20.3**)

$u'_{(1,2,2)} = -\frac{1}{6p^2}\{8p^4-\frac{A^2(t)}{B(t)}p^2y^2+pK_{1t}(t)+pK_{2t}(t)+B(t)[K_1(t)-p\frac{A(t)}{B(t)}y]^2\}-2p^2\tan^2[px-\frac{p}{2}\frac{A(t)}{B(t)}y^2+K_1(t)y+K_2(t)],$ (**20.4**)

$u_{(1/4,1/2,1/2)} = -\frac{1}{6p^2}\{2p^4-\frac{A^2(t)}{B(t)}p^2y^2+pK_{1t}(t)+pK_{2t}(t)+B(t)[K_1(t)-p\frac{A(t)}{B(t)}y]^2\}-\frac{p^2}{2}\ \{csc[px-\frac{p}{2}\frac{A(t)}{B(t)}y^2+K_1(t)y+K_2(t)]\pm\cot[px-\frac{p}{2}\frac{A(t)}{B(t)}y^2+K_1(t)y+K_2(t)]\}^2,$ (**20.5**)

$u'_{(1/4,1/2,1/2)} = -\frac{1}{6p^2}\{2p^4-\frac{A^2(t)}{B(t)}p^2y^2+pK_{1t}(t)+pK_{2t}(t)+B(t)[K_1(t)-p\frac{A(t)}{B(t)}y]^2\}-\frac{p^2}{2}\ \{sec[px-\frac{p}{2}\frac{A(t)}{B(t)}y^2+K_1(t)y+K_2(t)]\pm\tan[px-\frac{p}{2}\frac{A(t)}{B(t)}y^2+K_1(t)y+K_2(t)]\}^2,$ (**20.6**)

Remark 2

When m→ 1, these solutions (19.1)-(19.16) degenerate as singular soliton-like, combined singular soliton-like, and kink bell-like solutions of Eq.(1), namely

$u_{(1,-2,2)} = -\frac{1}{6p^2}\{-8p^4-\frac{A^2(t)}{B(t)}p^2y^2+pK_{1t}(t)+pK_{2t}(t)+B(t)[K_1(t)-p\frac{A(t)}{B(t)}y]^2\}-2p^2\tanh^2[px-\frac{p}{2}\frac{A(t)}{B(t)}y^2+K_1(t)y+K_2(t)],$ (**21.1**)

$u_{(0,1,-2)} = -\frac{1}{6p^2}\{4p^4-\frac{A^2(t)}{B(t)}p^2y^2+pK_{1t}(t)+pK_{2t}(t)+B(t)[K_1(t)-p\frac{A(t)}{B(t)}y]^2\}+2p^2\ sech^2[px-\frac{p}{2}\frac{A(t)}{B(t)}y^2+K_1(t)y+K_2(t)],$ (**21.2**)

$u'_{(1,-2,2)} = -\frac{1}{6p^2}\{-8p^4-\frac{A^2(t)}{B(t)}p^2y^2+pK_{1t}(t)+pK_{2t}(t)+B(t)[K_1(t)-p\frac{A(t)}{B(t)}y]^2\}-2p^2\coth^2[px-\frac{p}{2}\frac{A(t)}{B(t)}y^2+K_1(t)y+K_2(t)],$ (**21.3**)

$u_{(0,1,2)} = -\frac{1}{6p^2}\{4p^4-\frac{A^2(t)}{B(t)}p^2y^2+pK_{1t}(t)+pK_{2t}(t)+B(t)[K_1(t)-p\frac{A(t)}{B(t)}y]^2\}-2p^2\ csch^2[px-\frac{p}{2}\frac{A(t)}{B(t)}y^2+K_1(t)y+K_2(t)],$ (**21.4**)

$u_{(1/4,-1/2,1/2)} = -\frac{1}{6p^2}\{-2p^4-\frac{A^2(t)}{B(t)}p^2y^2+pK_{1t}(t)+pK_{2t}(t)+B(t)[K_1(t)-p\frac{A(t)}{B(t)}y]^2\}-\frac{p^2}{2}\ \{\coth[px-\frac{p}{2}\frac{A(t)}{B(t)}y^2+K_1(t)y+K_2(t)]\pm\ csch[px-\frac{p}{2}\frac{A(t)}{B(t)}y^2+K_1(t)y+K_2(t)]\}^2,$ (**21.5**)

$u'_{(1/4,-1/2,1/2)} = -\frac{1}{6p^2}\{-2p^4-\frac{A^2(t)}{B(t)}p^2y^2+pK_{1t}(t)+pK_{2t}(t)+B(t)[K_1(t)-p\frac{A(t)}{B(t)}y]^2\}-\frac{p^2}{2}\ \{\tanh[px-\frac{p}{2}\frac{A(t)}{B(t)}y^2+K_1(t)y+K_2(t)]\pm i\ sech[px-\frac{p}{2}\frac{A(t)}{B(t)}y^2+K_1(t)y+K_2(t)]\}^2,$ (**21.6**)

By setting

$A(t)=\frac{1}{2t},\quad B(t)=\frac{3\sigma^2}{t^2},\quad \sigma^2=\pm1,$ (**22**)

we can verify that Eq.(15) holds in the condition (22), then the solutions (19.1)-(19.17), (20.1)-(20.6) and (21.1)-(21.6) are reduced to the solutions of the cylindrical Kadomtsev-Petviashvili Eq.(3).

3. Conclusion

We have used the extended mapping method and derived abundant families of solutions for a generalized variable coefficients 2D KdV equation. In addition, the variable coefficients KP equation as the special case of 2D KdV equation, also possesses the corresponding Jacobi elliptic function-like, combined Jacobi function-like, rational-like, singular soliton-like, combined singular-like, kink-bell soliton-like, triangular function-like solutions. The extended mapping method can also apply to other NLPDEs.

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