THE ANALOGS OF THE RIEMANN TENSOR FOR EXCEPTIONAL STRUCTURES ON SUPERMANIFOLDS

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ABSTRACT. H. Hertz called any manifold M with a given nonintegrable distribution *nonholonomic*. Vershik and Gershkovich stated and R. Montgomery proved that the space of germs of any nonholonomic distribution on M with an open and dense orbit of the diffeomorphism group is either (1) of codimension one or (2) an Engel distribution.

No analog of this statement for supermanifolds is formulated yet, we only have some examples: our list (an analog of É. Cartan's classification) of simple Lie superalgebras of vector fields with polynomial coefficients and a particular (Weisfeiler) grading contains 16 series similar to contact ones and 11 exceptional algebras preserving nonholonomic structures.

Here we compute the cohomology corresponding to the analog of the Riemann tensor for the *super*manifolds corresponding to the 15 exceptional simple vectorial Lie superalgebras, 11 of which are nonholonomic. The cohomology for analogs of the Riemann tensor for the manifolds with an exceptional Engel manifolds are computed in [L0]; for the 16 series of contact-like distributions on supermanifolds see [GLS3].

INTRODUCTION

The main result. In this paper the ground field is $\mathbb{C}$. Here, for each of the 15 exceptional simple infinite dimensional vectorial Lie superalgebras $\mathfrak{g} = \mathop{\oplus}\limits_{i\geq -d} \mathfrak{g}_i$ in their Weisfeiler grading we have computed $H^i(\mathfrak{g}_-; \mathfrak{g})$ for $i = 0, 1, 2$, where $\mathfrak{g}_- = \mathop{\oplus}\limits_{i<0} \mathfrak{g}_i$. These cohomology are especially interesting for $i = 2$. If $d = 1$, then $H^2(\mathfrak{g}_-; \mathfrak{g})$ can be interpreted as the space of values of the curvature tensor for the G-structure, where G is a Lie supergroup whose Lie superalgebra is $\mathfrak{g}_0$; if $d > 1$ we interprete $H^2(\mathfrak{g}_-; \mathfrak{g})$ as the space of values of the recently introduced *nonholonomic curvature* for the nonintegrable distribution.

To make the text of interest to a wider audience, we would like to review the list of simple vectorial Lie superalgebras and some not so known background, cf. [D], but had to delete this from this text for the lack of space; for the same reason we skipped the results of computations of $H^i(\mathfrak{g}_-; \mathfrak{g})$ for $i = 0, 1$ (these results will be preprinted; $H^1(\mathfrak{g}_-; \mathfrak{g})$, though not so interesting, perhaps, as $H^2(\mathfrak{g}_-; \mathfrak{g})$, also has an interpretation of interest: its elements represent derivations of $\mathfrak{g}_-$ into $\mathfrak{g}$, cf. [CK1].)

The results demonstrate one more range of applicability of SuperLie package. It is designed for various computations with Lie superalgebras, not only for computation of (co)homology; for other results and comparison with hand-made calculations, see, e.g., [GLS2]. SuperLie is *Mathematica*-based which facilitates its usage but imposes in-build *Mathematica* restrictions. We hope to draw attention to possibilities SuperLie (now installed at MPIM, Bonn; LPT, ENS; Department of Mathematics, University of Stockholm) reveals to its user.

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In particular, our results (as well as similar results of Poletaeva [P1] — [P3] performed by bare hands) vividly demonstrate that in the absence of complete reducibility computer-aided study is indispensable.

0.1. A result of Vershik and Gershkovich. Nonholonomic curvature. R. Montgomery [Mo] proved the following statement whose particular case was stated in [VG1]. Let W_n^k be the space of germs of k-dimensional distributions at $0 \in \mathbb{C}^n$. (Both [Mo] and [VG1] consider the real case but the result is the same.) The group Diff_n of germs of diffeomorphisms of $\mathbb{C}^n$ acts on W_n^k and it is interesting to find out the conditions for existence of the frame (i.e., pointwise values of the vector fields from a given distribution) that generates a finite dimensional (hence, nilpotent) Lie algebra. The answer:

for $1 < k < n-1$ and $(k, n) \neq (2, 4)$ any parametrization of any open subset of the space of generic orbits of Diff_n-action on W_n^k requires $\geq k(n-k)-n$ functions in n indeterminates. The exceptional case $k = 1$ is trivial. So W_n^k has an open and dense Diff_n-orbit if and only if either (1) $k = n-1$ (for n odd this is the contact structure) or (2) $(k, n) = (2, 4)$ (and then the distribution is an Engel one).

In [L0], the notion of nonholonomic curvature is introduced and the above cases (1) and (2) are considered. It turns out that the nonholonomic curvature vanishes if $k = n - 1$ (this is a reformulation of Darboux theorem on a canonical form of the contact form for n odd) whereas for $(k, n) = (2, 4)$ — the Engel distribution — $\dim H^2(\mathfrak{g}_-; \mathfrak{g}) = 2$.

Observe that the (infinite dimensional) algebra of symmetries of the exceptional (case (1) or (2)) nonholonomic distribution is simple only if the distribution is of codimension 1 and n is odd. Contemporary mathematicians are often more than necessary fixed on simple Lie algebras and this is, perhaps, an explanation why the Lie algebra preserving an Engel structure (cf. [L0]) was neglected for a long time. Differential geometers, though mildly interested in the cases the total algebra of symmetries is simple, are more interested in cases when it is of finite dimension (simple or not), and therefore their interests are orthogonal to ours as is seen from motivations and results reviewed, e.g., in [Y].

We do not know any super version of the above result of Vershik and Gershkovich [VG1] but we classified simple Lie superalgebras of vector fields ([LSh0], [LSh1], [LSh3], cf. [Ka7]) and, we see that, unlike non-super case, there are 16 series and 11 exceptional simple vectorial Lie superalgebras that preserve nonholonomic distributions. The series will be considered elsewhere.

0.2. A nonholonomic analog of the Riemann tensor. H. Hertz called any manifold with a nonintegrable distribution a *nonholonomic* one. Until 1989, there was no general definition of the analog of the Riemann tensor for nonholonomic manifolds, cf. lamentations in [VG1] and [WB], though all the ingredients had been discovered ([T], [Y]). Vershik even conjectured [V] that such a general definition does not exist, though in particular cases of small dimension the nonholonomic curvature tensor was computed. In particular, in supergravity.

Recall that SUGRA(N) is a supergravity theory (or equations thereof) on an N-extended Minkowski superspace. Whatever SUGRA(N) and Minkowski superspace are (there are several versions of the definition and, unless $N = 1$, there is no consensus among physicists which of the definitions is "it", Manin [M] suggests still other — "exotic" — versions of Minkowski superspaces, and this list of *ad hoc* superizations of Minkowski space will, clearly, be continued, see e.g., [GL3]), they are superizations of the gravity theory (or Einstein-Hilbert's equations) on the Minkowski space. So the problem whose existence Wess honestly acknowledged in his lectures [WB] "We do not know how to write the super Riemann tensor" (on $N > 2$ extended Minkowski superspaces) sounds strange: take any textbook on differential geometry (say, [St]) and superize definition of the Riemann tensor or, more generally, structure functions — analogs of the Riemannian tensor — for any G-structure according to Sign Rule. This was precisely what A. Schwarz with his colleagues and students suggested to do [S], see also [CAF].

The results of such an approach, however, seem to coincide with the equations physicists write from their mysterious physical considerations only for $N = 1$ (but actually do not even in this case, cf. [S], [CAF] with [GL1]).

In [GL1], [GL2] it was indicated that the roots of the problem Wess addressed lie not in the prefix "super" which only causes some signs in the classical definitions of the structure functions. The point is that every of numerous versions of Minkowski *super*space is *nonholonomic*, unlike Minkowski space, and since there was no general recipe for computing nonholonomic analog of the curvature tensor, to write SUGRA(N) equations or even determine what is N-extended Minkowski superspace (which of the versions satisfies some natural requirements) was a problem.

Here we will reproduce the definition ([L0], [LP2], [GL1]) of the Riemann tensor R in terms of Lie algebra cohomology rather than in terms of Spencer cohomology (cf. [G], [St]) and give its generalization to nonholonomic case.

Namely, fix a point on the nonholonomic (super)manifold with a nonintegrable distribution D. Let D be given by a system of Pfaff equations and let $\mathfrak{G}$ be the filtered Lie superalgebra preserving this system of equations, $\mathfrak{m}$ the associated graded one. Set $\mathfrak{m}_- = \mathop{\oplus}\limits_{i<0} \mathfrak{m}_i$; by default we let $\mathfrak{g}_0 = \mathfrak{m}_0$, the Lie superalgebra of grading preserving derivations of $\mathfrak{g}_- = \mathfrak{m}_-$. Clearly, $\mathfrak{m}_-$ is nilpotent, see [VG2], [Y]. Very often an additional structure on D is given; if this is the case, we take for $\mathfrak{g}_0$ a subalgebra of $\mathfrak{m}_0$ that preserves this additional structure. Let $\mathfrak{g}$ be the defined below generalized Cartan prolong of the pair $(\mathfrak{g}_-, \mathfrak{g}_0)$. Then, by the same arguments as in [St], **the possible values of the nonholonomic Riemann tensor R at the point span the superspace $H^2(\mathfrak{g}_-; \mathfrak{g})$.**

0.3. The projective connections and their nonholonomic analogs. The projective connection on the n-dimensional manifold is the one whose group of automorphisms is locally isomorphic to $\mathfrak{sl}(n+1) = \mathfrak{g}_- \oplus \mathfrak{gl}(n) \oplus (\mathfrak{g}_-)^*$, cf. [G]. The corresponding structure functions are from $H^2(\mathfrak{g}_-; \mathfrak{sl}(n+1))$.

Similarly, for any $\mathbb{Z}$-graded Lie superalgebra $\mathfrak{g}$ of finite depth, let $\mathfrak{h} \subset \mathfrak{g}$ be a subalgebra with the same nonpositive part. Then the elements of $H^2(\mathfrak{g}_-; \mathfrak{h})$ are analogs of the projective structure functions, especially resembling them if $\dim \mathfrak{h} < \infty$. Such cohomology is considered in [GLS3].

§1. Description of simple vectorial Lie superalgebras

For the lack of space we deleted all the preliminaries. The reader willing to see them is referred to [Sh14], [Sh5] and [LSh3]. For a detailed background see a preprinted version at www.mpim-bonn.mpg.de. Observe only that Π is the shift of parity functor on superspaces, *Vol* is the space of densities with the generator *vol* in a fixed coordinate system.

1.1. The exceptional vectorial Lie subsuperalgebras. Here are the terms $\mathfrak{g}_i$ for $i \leq 0$ of 14 of the 15 exceptional algebras, the last column gives $\dim \mathfrak{g}_-$. Here $\Lambda(n)$ is the Grassmann superalgebra with n generators; id is the identity representation, $\Lambda(\mathrm{id})$ its exterior algebra; Vol_0 is the space of densities with integral 0; and $T^0_0(\vec{0}) = Vol_0/\mathrm{const}$ is well-defined only as module over $\mathfrak{svect}$:

$\mathfrak{g}$	$\mathfrak{g}_{-2}$	$\mathfrak{g}_{-1}$	$\mathfrak{g}_0$	dim $\mathfrak{g}_-$
$\mathfrak{vle}(4\|3)$	$-$	$\Pi(\Lambda(3)/\mathbb{C}1)$	$\mathfrak{c}(\mathfrak{vect}(0\|3))$	$4\|3$
$\mathfrak{vle}(4\|3;1)$	$\mathbb{C}\cdot 1$	$\mathrm{id}\otimes\Lambda(2)$	$\mathfrak{c}(\mathfrak{sl}(2)\otimes\Lambda(2)\supplus T^{1/2}(\mathfrak{vect}(0\|2))$	$5\|4$
$\mathfrak{vle}(4\|3;K)$	$\mathrm{id}_{\mathfrak{sl}(3)}$	$\mathrm{id}_{\mathfrak{sl}(3)}\otimes\mathrm{id}_{\mathfrak{sl}(2)}\otimes 1$	$\mathfrak{sl}(3)\oplus\mathfrak{sl}(2)\oplus\mathbb{C}z$	$3\|6$
$\mathfrak{vas}(4\|4)$	$-$	spin	$\mathfrak{as}$	$4\|4$
$\mathfrak{kas}$	$\mathbb{C}\cdot 1$	$\Pi(\mathrm{id})$	$\mathfrak{co}(6)$	$1\|6$
$\mathfrak{kas}(;1)$	$\Lambda(1)$	$\mathrm{id}_{\mathfrak{sl}(2)}\otimes\mathrm{id}_{\mathfrak{gl}(2)}\otimes\Lambda(1)$	$(\mathfrak{sl}(2)\oplus\mathfrak{gl}(2)\otimes\Lambda(1))\supplus\mathfrak{vect}(0\|1)$	$5\|5$
$\mathfrak{kas}(;3\xi)$	$-$	$\Lambda(3)$	$\Lambda(3)\oplus\mathfrak{sl}(1\|3)$	$4\|4$
$\mathfrak{kas}(;3\eta)$	$-$	$Vol_0(0\|3)$	$\mathfrak{c}(\mathfrak{vect}(0\|3))$	$4\|3$
$\mathfrak{mb}(4\|5)$	$\Pi(\mathbb{C}\cdot 1)$	$Vol(0\|3)$	$\mathfrak{c}(\mathfrak{vect}(0\|3))$	$4\|5$
$\mathfrak{mb}(4\|5;1)$	$\Lambda(2)/\mathbb{C}\cdot 1$	$\mathrm{id}_{\mathfrak{sl}(2)}\otimes\Lambda(2)$	$\mathfrak{c}(\mathfrak{sl}(2)\otimes\Lambda(2)\supplus T^{1/2}(\mathfrak{vect}(0\|2))$	$5\|6$
$\mathfrak{mb}(4\|5;K)$	$\mathrm{id}_{\mathfrak{sl}(3)}$	$\Pi(\mathrm{id}_{\mathfrak{sl}(3)}\otimes\mathrm{id}_{\mathfrak{sl}(2)}\otimes\mathbb{C})$	$\mathfrak{sl}(3)\oplus\mathfrak{sl}(2)\oplus\mathbb{C}z$	$3\|8$
$\mathfrak{ksle}(9\|6)$	$\mathbb{C}\cdot 1$	$\Pi(T_0^0(\vec{0}))$	$\mathfrak{svect}(0\|4)_{3,4}$	$9\|6$
$\mathfrak{ksle}(9\|6;2)$	$\mathrm{id}_{\mathfrak{sl}(3\|1)}$	$\mathrm{id}_{\mathfrak{sl}(2)}\otimes\Lambda(3)$	$(\mathfrak{sl}(2)\otimes\Lambda(3))\supplus\mathfrak{sl}(1\|3)$	$11\|9$
$\mathfrak{ksle}(9\|6;K)$	id	$\Pi(\Lambda^2(\mathrm{id}))$	$\mathfrak{sl}(5)$	$5\|10$

Observe that none of the simple W-graded vectorial Lie superalgebras is of depth >3 and only two algebras are of depth 3: one of the above, $\mathfrak{mb}(4|5;K)$, for which we have $\mathfrak{mb}(4|5;K)_{-3}\cong\Pi(\mathrm{id}_{\mathfrak{sl}(2)})$, and another one, $\mathfrak{ksle}(9|6;CK)=\mathfrak{ck}(9|11)$.

This $\mathfrak{ck}((9|11)$ is the 15-th exceptional simple vectorial Lie superalgebra; its non-positive terms are as follows (we assume that the $\mathfrak{sl}(2)$- and $\mathfrak{sl}(3)$-modules are purely even):

$$\mathfrak{ck}((9|11)_0\simeq(\mathfrak{sl}(2)\oplus\mathfrak{sl}(3)\otimes\Lambda(1))\supplus\mathfrak{vect}(0|1);$$
$$\mathfrak{ck}((9|11)_{-1}\simeq\mathrm{id}_{\mathfrak{sl}(2)}\otimes\left(\mathrm{id}_{\mathfrak{sl}(3)}\otimes\Lambda(1)\right);$$
$$\mathfrak{ck}((9|11)_{-2}\simeq\mathrm{id}^*_{\mathfrak{sl}(3)}\otimes\Lambda(1);$$
$$\mathfrak{ck}((9|11)_{-3}\simeq\Pi(\mathrm{id}_{\mathfrak{sl}(2)}\otimes\mathbb{C}).$$

1.2. A description of $\mathfrak{g}$ as $\mathfrak{g}_{\bar 0}$ and $\mathfrak{g}_{\bar 1}$. In [CK2] the exceptional algebras are described as $\mathfrak{g}=\mathfrak{g}_{\bar 0}\oplus\mathfrak{g}_{\bar 1}$. For several series such description is of little value because each homogeneous component $\mathfrak{g}_{\bar 0}$ and $\mathfrak{g}_{\bar 1}$ has a complicated structure. For the exceptions (and for twisted polyvector fields) the situation is totally different! Apart from being beautiful, such description (from [CK2]) is useful for the construction of simple Volichenko algebras, cf. [LS].

Recall in this relation a theorem [Gr] that completely describes bilinear differential operators acting in tensor fields and invariant under all changes of coordinates. It turned out that almost all of the first order operators determine a Lie superalgebra on its domain. Some of these superalgebras are simple or close to simple. In the constructions below we use some of these invariant operators.

$\underline{\mathfrak{g}=\mathfrak{ksle}(5|10)}$: $\mathfrak{g}_{\bar 0}=\mathfrak{svect}(5|0)\simeq d\Omega^4$, $\mathfrak{g}_{\bar 1}=\Pi(d\Omega^1)$ with the natural $\mathfrak{g}_{\bar 0}$-action on $\mathfrak{g}_{\bar 1}$ (the Lie derivative) and the bracketing of odd elements being twice their product. (We identify:

$$\partial_i=\mathrm{sign}(ijklm)dx_jdx_kdx_ldx_l\quad\text{for any permutation }(ijklm)\text{ of }(12345).$$

$\underline{\mathfrak{g}=\mathfrak{vas}(4|4)}$: $\mathfrak{g}_{\bar 0}=\mathfrak{vect}(4|0)$, and $\mathfrak{g}_{\bar 1}=\Omega^1\otimes Vol^{-1/2}$ with the natural $\mathfrak{g}_{\bar 0}$-action on $\mathfrak{g}_{\bar 1}$ and the bracketing of odd elements being

$$[\omega_1\otimes vol^{-1/2},\omega_2\otimes vol^{-1/2}]=(d\omega_1\wedge\omega_2+\omega_1\wedge d\omega_2)\otimes vol^{-1}],$$

where we identify

$$dx_i\wedge dx_j\wedge dx_k\otimes vol^{-1}=\operatorname{sign}(ijkl)\partial_l \text{ for any permutation } (ijkl) \text{ of } (1234).$$

$\underline{\mathfrak{g}=\mathfrak{vle}(3|6)}$: $\mathfrak{g}_{\bar 0}=\mathfrak{vect}(3|0)\oplus\mathfrak{sl}(2)^{(1)}_{\geq 0}$, where $\mathfrak{g}^{(1)}_{\geq 0}=\mathfrak{g}\otimes\mathbb{C}[t]$, and $\mathfrak{g}_{\bar 1}=\left(\Omega^1\otimes Vol^{-1/2}\right)\otimes \mathrm{id}_{\mathfrak{sl}(2)^{(1)}_{\geq 0}}$ with the natural $\mathfrak{g}_{\bar 0}$-action on $\mathfrak{g}_{\bar 1}$.

Recall that $\mathrm{id}_{\mathfrak{sl}(2)}$ is the irreducible $\mathfrak{sl}(2)$-module L^1 with highest weight 1; its tensor square splits into $L^2\simeq\mathfrak{sl}(2)$ and the trivial module L^0; accordingly, denote by $v_1\wedge v_2$ and $v_1\bullet v_2$ the projections of $v_1\otimes v_2\in L^1\otimes L^1$ onto the skew-symmetric and symmetric components, respectively. For $f_1,f_2\in\Omega^0$, $\omega_1,\omega_2\in\Omega^1$ and $v_1,v_2\in L^1$ we set

$$\begin{aligned}&[(\omega_1\otimes v_1)vol^{-1/2},(\omega_2\otimes v_2)vol^{-1/2}]=\\&(\omega_1\wedge\omega_2)\otimes(v_1\wedge v_2)+d\omega_1\wedge\omega_2+\omega_1\wedge d\omega_2)\otimes(v_1\bullet v_2))\,vol^{-1},\end{aligned}$$

where we identify Ω^0 with $\Omega^3\otimes_{\Omega^0}Vol^{-1}$ and $\Omega^2\otimes_{\Omega^0}Vol^{-1}$ with $\mathfrak{vect}(3|0)$ by setting

$$dx_i\wedge dx_j\otimes vol^{-1}=\operatorname{sign}(ijk)\frac{\partial}{\partial x_k} \text{ for any permutation } (ijk) \text{ of } (123).$$

$\underline{\mathfrak{g}=\mathfrak{mb}(3|8)}$: $\mathfrak{g}_{\bar 0}=\mathfrak{vect}(3|0)\oplus\mathfrak{sl}(2)^{(1)}_{\geq 0}$, and $\mathfrak{g}_{\bar 1}=\mathfrak{g}_{-1}\oplus\mathfrak{g}_1$, where

$$\mathfrak{g}_{-1}=\left(\Pi Vol^{-1/2}\right)\otimes\mathrm{id}_{\mathfrak{sl}(2)^{(1)}_{\geq 0}} \text{ and } \mathfrak{g}_1=\left(\Omega^1\otimes Vol^{-1/2}\right)\otimes\mathrm{id}_{\mathfrak{sl}(2)^{(1)}_{\geq 0}};$$

clearly, one can interchange $\mathfrak{g}_{\pm1}$.

Multiplication is similar to that of $\mathfrak{g}=\mathfrak{vle}(3|6)$. For $f_1,f_2\in\Omega^0$, $\omega_1,\omega_2\in\Omega^1$ and $v_1,v_2\in L^1$ we set

$$\begin{aligned}&[(\omega_1\otimes v_1)vol^{-1/2},(\omega_2\otimes v_2)vol^{-1/2}]=0,\\&[(f_1\otimes v_1)vol^{-1/2},(f_2\otimes v_2)vol^{-1/2}]=(df_1\wedge df_2)\otimes(v_1\wedge v_2)vol^{-1},\\&[(f_1\otimes v_1)vol^{-1/2},(\omega_1\otimes v_2)vol^{-1/2}]=(f_1\omega_1\otimes(v_1\wedge v_2)+(df_1\omega_1+f_1d\omega_1)\otimes(v_1\bullet v_2))\,vol^{-1}.\end{aligned}$$

$\underline{\mathfrak{g}=\mathfrak{kas}}$: $\mathfrak{g}_{\bar 0}=\mathfrak{vect}(1|0)\oplus\mathfrak{sl}(4)^{(1)}_{\geq 0}$, and $\mathfrak{g}_{\bar 1}=\mathfrak{g}_{-1}\oplus\mathfrak{g}_1$, where $\mathfrak{g}_{-1}=\Pi\left(\Lambda^2(\mathrm{id}_{\mathfrak{sl}(2)^{(1)}_{\geq 0}})\right)$ and $\mathfrak{g}_1=\Pi\left(S^2(\mathrm{id}_{\mathfrak{sl}(2)^{(1)}_{\geq 0}})\right)$; clearly, one can interchange $\mathfrak{g}_{\pm1}$.

§2. Main result

The above description of the exceptional algebras is nice to visualize them, but in calculations we have used sometimes the description of the elements from [Sh14], which we do not reproduce to save space.

Theorem . *The $\mathfrak{g}_0$-modules $H^2(\mathfrak{g}_-;\mathfrak{g})$ are given by the lists of modules over the semisimple part of $(\mathfrak{g}_0)_{\bar 0}$ given in §3.*

Comments. 1) **important**: observe that if the $\mathfrak{g}_0$-module from $H^2(\mathfrak{g}_-;\mathfrak{g})$ is indecomposable, then, in equations like Einstein equations or Wess-Zumino constraints, there is no need to vanish all its irreducible components: it suffices to vanish only the modules that generate all.

2) The highest weights are given with respect to the standard basis of Cartan subalgebra of the maximal semisimple part of $(\mathfrak{g}_0)_{\bar 0}$, or, if this part is $\mathfrak{sl}(n)$, with respect to the matrix units

E_{ii} of the $\mathfrak{gl}(n)$. The degree of the cocycle is given relative to the grading of $\mathfrak{g}$. The weight of vector A is denoted by $w(A)$. "mult" denotes the multiplicity of the corresponding module in the space of closed/exact forms. If it is not equal to $r/0$, the respective cohomologies are not pure (are defined up to a coboundary) and it may well happen that our choice of the representative may be beautified. The expression $d[v]^2$ means $(dv)^2$. Few more notations appear in respective places.

3) For $\mathfrak{g}$ of depth d the degrees of cohomology (interpreted as orders k of the structure functions responsible for obstructions to flattening the structure under investigation up to kth infinitesimal neighborhood, cf. [St]) may range from $2-d$, which we will indicate for $d>1$. Recall that the structure functions of order k are defined provided structure functions of lesser orders vanish. It happens sometimes (the case of Riemannian metric) that in these lesser orders there are no cohomology (torsion-free property of Levi-Civita connection). Otherwise that vanishing conditions on low-order structure functions are analogs of Wess-Zumino constraints in supergravity, cf. [WZ].

4) Due to a theorem formulated only for $d=1$ (by Serre for Lie algebras and by Serganova for superalgebras, cf. [LPS]), the order of nonzero structure functions from $H^2(\mathfrak{g}_{-1};\mathfrak{g})$ is always equal to 1 under certain conditions (*involutivity*). As we will see, the considered Lie superalgebras of depth 1 are all involutive. The yet nonexisting analog of Serre's theorem for $d>1$ is more complicated: cohomology may be non-vanishing in several degrees. If the lowest degrees are absent, we indicate this by writing "torsion-free" by analogy with the Riemannian manifolds.

§3. The $\mathfrak{g}_0$-modules $H^2(\mathfrak{g}_-;\mathfrak{g})$

$\mathfrak{g}=\mathfrak{vle}(4|3)$**.** Cohomology: a single irreducible $\mathfrak{g}_0$-module in deg $=1$, dim $=(24|24)$.

Set: $w(u_i)=(1,1,1)-\varepsilon_i$, $w(y)=(0,0,0)$, $w(\xi_i)=\varepsilon_i$ and $\partial_0=\frac{\partial}{\partial y}$, $\partial_i=\frac{\partial}{\partial u_i}$, $\delta_i=\frac{\partial}{\partial \xi_i}$. The $\mathfrak{gl}(3)$-highest vectors are:

$\mathfrak{gl}(3)$-highest vectors	$\mathfrak{gl}(3)$-weight	dim	mult
$\partial_1\, d[\partial_2]\wedge d[\partial_3]$	$(2,0,0)$	$(6\|0)$	3/2
$\partial_0\, d[\partial_0]\wedge d[\delta_1]$	$(1,0,0)$	$(0\|3)$	5/4
$\partial_1\, d[\partial_3]\wedge d[\delta_1]$	$(2,0,-1)$	$(0\|15)$	2/1
$\partial_1\, d[\partial_3]\wedge d[\partial_0]$	$(1,0,-1)$	$(8\|0)$	3/2
$\partial_1\, d[\delta_1]\wedge d[\delta_1]$	$(2,-1,-1)$	$(10\|0)$	1/0
$\partial_1\, d[\partial_0]\wedge d[\delta_1]$	$(1,-1,-1)$	$(0\|6)$	1/0

$\mathfrak{g}=\mathfrak{vle}(4|3;1)$**.** $2-d=0$.

Cohomology in deg $=1$, torsion-free, dim $=(20|20)$:

The $(\mathfrak{g}_0)_{\bar 0}$-highest weight vectors (with respect to $\mathfrak{sl}(2)\oplus\mathfrak{gl}(2)$) are as follows: (the first coordinate of the weight is given with respect to a copy of $\mathfrak{sl}(2)$ realized as $\mathfrak{o}(3)$, with half-integer weights; the last two coordinates are with respect to a copy of $\mathfrak{gl}(2)$). In this realization $w(u_1)=(1,1,1)$, $w(u_2)=(1,0,1)$, $w(u_3)=(1,1,0)$, $w(y)=(-1,0,0)$, $w(\xi_1)=(0,0,0)$, $w(\xi_2)=(0,1,0)$, $w(\xi_3)=(0,0,1)$. Denote the elements of $\mathfrak{g}_-$ as follows: $g_1=\partial_2$; $g_2=\partial_3$; $g_3=\delta_2$; $g_4=\delta_3$; $g_5=-u_2\partial_1+\xi_1\delta_2$; $g_6=-u_3\partial_1+\xi_1\delta_3$; $g_7=-y\delta_3-\xi_1\partial_2+\xi_2\partial_1$; $g_8=y\delta_2-\xi_1\partial_3+\xi_3\partial_1$; $g_9=\partial_1$. We have

N	$\mathfrak{sl}(2)\oplus\mathfrak{gl}(2)$-highest vectors	$\mathfrak{sl}(2)\oplus\mathfrak{gl}(2)$-weight	dim	mult
[1]	$2\,\partial_2 dg_1\wedge dg_5+\partial_2 dg_2\wedge dg_6+\partial_3 dg_2\wedge dg_5$	$(0,1,0)$	$(2\vert 0)$	5/4
[2]	$\partial_2 dg_2\wedge dg_5-$	$(0,2,-1)$	$(4\vert 0)$	2/1
[3]	$-\delta_3 dg_3\wedge dg_8+(y\delta_3-\xi_1\partial_2+\xi_2\partial_1)dg_8\wedge dg_8$	$(1,2,-1)$	$(0\vert 8)$	3/2
[4]	$\partial_2 dg_5\wedge dg_6$	$(0,1,0)$	$(2\vert 0)$	1/0
[5]	$2\partial_2 dg_5\wedge dg_7+\partial_2 dg_6\wedge dg_8+\partial_3 dg_5\wedge dg_8$	$(1,1,0)$	$(0\vert 4)$	2/1
[6]	$\partial_2 dg_5\wedge dg_8$	$(1,2,-1)$	$(0\vert 8)$	1/0
[7]	$-\delta_3 dg_5\wedge dg_8-\partial_2 dg_8\wedge dg_8$	$(2,2,-1)$	$(12\vert 0)$	1/0

$\mathfrak{g}_0$-modules:
$[A]=[2]+[3]+[6]+[7]$ of dim $=(16|16)$ and $[B]=[A]+[1]+[4]+[5]$ of dim $=(20|20)$; the modules $[A]$ and $[B]/[A]$ are irreducible.

$\mathfrak{g}=\mathfrak{vle}(4|3;K)$. $2-d=0$.

The $\mathfrak{g}_0$-highest weight vectors are as follows: $\mathfrak{g}_0=\mathfrak{sl}(2)\oplus\mathfrak{gl}(3)$; the first coordinate of the weight is given with respect to $\mathfrak{sl}(2)$ realized as $x_-=\partial_y$, $x_+=y^2\partial_y+y\sum_i\xi_i\delta_i+\xi_1\xi_2\partial_3-\xi_1\xi_3\partial_2+\xi_2\xi_3\partial_1$; $\mathfrak{gl}(3)$ is realized as $x^i_j=-u_j\partial_i+\xi_i\delta_j$ $(i\neq j)$ and $x^i_i=-u_i\partial_i+\xi_i\delta_i+\sum_k u_i\partial_i$. In this realization $w(u_1)=(2,0,1,1)$, $w(u_2)=(2,1,0,1)$, $w(u_3)=(2,1,1,0)$, $w(y)=(-2,0,0,0)$, $w(\xi_1)=(0,-1,0,0)$, $w(\xi_2)=(0,0,-1,0)$, $w(\xi_3)=(0,0,0,-1)$).

Cohomology: a single irreducible $\mathfrak{g}_0$-module in deg $=0$, dim $=(30|0)$:

$\mathfrak{sl}(2)\oplus\mathfrak{gl}(3)$-highest vectors	$\mathfrak{sl}(2)\oplus\mathfrak{gl}(3)$-weight	dim	mult
$\partial_1 d[\delta_1]^2$	$(2,2,-1,-1)$	$(30\vert 0)$	1/0

$\mathfrak{g}=\mathfrak{vas}(4|4)$. The weights are given relative to $\mathfrak{gl}(4)\subset\mathfrak{g}_0$. In this realization, the weight of u_i is $\varepsilon_i-\frac{1}{2}(1,1,1,1)$ and the weight of ξ_i is $-\varepsilon_i$; we set $\partial_i=\frac{\partial}{\partial u_i}$, $\delta_i=\frac{\partial}{\partial\xi_i}$ for $1\leq i\leq 4$.

Cohomology: in deg $=1$, dim $=(40|40)$:

N	$\mathfrak{gl}(4)$-highest vectors	weight	dim	mult
[1]	$\delta_1 d[\partial_2]\wedge d[\partial_3]-\delta_2 d[\partial_1]\wedge d[\partial_3]+\delta_3 d[\partial_1]\wedge d[\partial_2]$	$(0,0,0,-1)$	$(0\vert 4)$	4/3
[2]	$\partial_4 d[\partial_1]\wedge d[\partial_2]$	$(\frac{1}{2},\frac{1}{2},-\frac{1}{2},-\frac{3}{2})$	$(20\vert 0)$	3/2
[3]	$\partial_4 d[\delta_4]\wedge d[\delta_4]$	$(\frac{1}{2},\frac{1}{2},\frac{1}{1},-\frac{5}{2})$	$(20\vert 0)$	1/0
[4]	$\delta_1 d[\delta_4]\wedge d[\delta_4]$	$(1,0,0,-2)$	$(0\vert 36)$	2/1

The module is reducible but indecomposable. Vector [1] is the highest weight vector in the quotient module.

$\mathfrak{g}=\mathfrak{kas}$. $2-d=0$. For brevity, instead of K_f we write simply f and df^2 means $(df)^2$.

Cohomology: a single irreducible module in deg $=1$, hence torsion free. The $\mathfrak{o}(6)$-weights are:

$\mathfrak{o}(4)$-highest vectors	$\mathfrak{o}(6)$-weight	dim	mult
$\xi_1\xi_2 d[\xi_3]\wedge d[\eta_1]-\xi_1\eta_3 d[\eta_1]\wedge d[\eta_2]+\xi_2\eta_3 d[\eta_1]\wedge d[\eta_1]$	$(2,1,-1)$	$(45\vert 0)$	1/0

$\mathfrak{g}=\mathfrak{kas}(;1)$. $2-d=0$. Nontrivial cohomology are in degrees 0 and 1.

The weights are given relative to $\mathfrak{o}(6)$, but the corresponding vectors are heighest only with respect to $\mathfrak{o}(4)=\mathfrak{o}(6)\cap\mathfrak{g}_0$.

Cohomology in deg $=0$, dim $=(6|6)$:

N	$\mathfrak{o}(4)$-highest vectors	$\mathfrak{o}(6)$-weight	dim	mult
[1]	$d[\xi_1\xi_3]\wedge d[\xi_1\eta_2]$	$(-2,1,-1)$	$(3\vert 0)$	1/0
[2]	$d[\xi_1\eta_2]\wedge d[\xi_1\eta_3]$	$(-2,1,1)$	$(3\vert 0)$	1/0
[3]	$\xi_1 d[\xi_1\xi_3]\wedge d[\xi_1\eta_2]$	$(-1,1,-1)$	$(0\vert 3)$	2/1
[4]	$\xi_1 d[\xi_1\eta_2]\wedge d[\xi_1\eta_3]$	$(-1,1,-1)$	$(0\vert 3)$	2/1

[2] $\oplus$ [4] is $\mathfrak{g}_0$-irreducible; [1] and [3] are glued as $\mathfrak{g}_0$-modules.

Cohomology in deg = 1, dim = $(8|8)$:

N	$\mathfrak{o}(4)$-highest vectors	$\mathfrak{o}(6)$-weight	dim	mult
[1]	$\xi_1 d[\xi_1\eta_2]\wedge(\xi_2 d[\xi_3]-\eta_3 d[\eta_2])$	$(0,2,-1)$	$(0\vert 8)$	2/1
[2]	$\xi_1 d[\eta_2]\wedge(\xi_2 d[\xi_3]-\eta_3 d[\eta_2])$	$(1,2,-1)$	$(8\vert 0)$	1/0

A single irreducible $\mathfrak{g}_0$-module.

$\mathfrak{g}=\mathfrak{kas}(;3\eta)$. Cohomology in degrees 1 and 2.

deg = 1: a single irreducible $\mathfrak{g}_0$-module of dim = $(12|12)$ (the vectors are highest with respect to $\mathfrak{gl}(3)=\mathfrak{o}(6)\cap\mathfrak{g}_0$, the weights are the same as in $\mathfrak{o}(6)$):

N	$\mathfrak{gl}(3)$-highest vectors	weight	dim	mult
[1]	$d[\eta_1\eta_2]\wedge d[\eta_1\eta_3]$	$(2,1,1)$	$(3\vert 0)$	1/0
[2]	$d[\eta_1]\wedge d[\eta_2\eta_3]-d[\eta_2]\wedge d[\eta_1\eta_3]+d[\eta_3]\wedge d[\eta_1\eta_2]$	$(1,1,1)$	$(0\vert 1)$	2/1
[3]	$d[\eta_1]\wedge d[\eta_1\eta_2]$	$(2,1,0)$	$(0\vert 8)$	2/1
[4]	$d[1]\wedge d[\eta_1\eta_2]$	$(1,1,0)$	$(3\vert 0)$	3/2
[5]	$d[\eta_1]^2$	$(2,0,0)$	$(6\vert 0)$	3/2
[6]	$d[1]\wedge d[\eta_1]$	$(1,0,0)$	$(0\vert 3)$	5/4

deg = 2: dim = $(15|16)$, a single irreducible $\mathfrak{g}_0$-module:

N	$\mathfrak{gl}(3)$-highest vectors	weight	dim	mult
[1]	$\xi_1\eta_3 d[\eta_2]\wedge d[\eta_1\eta_2]-\xi_2\eta_3 d[\eta_1]\wedge d[\eta_1\eta_2]$	$(2,2,-1)$	$(0\vert 10)$	1/0
[2]	$-\xi_1\eta_3 d[\eta_1]\wedge d[\eta_2]+\xi_2\eta_3 d[\eta_1]^2+\xi_1\eta_1\eta_3 d[\eta_1]\wedge d[\eta_1\eta_2]$ $+2\xi_1\eta_2\eta_3 d[\eta_2]\wedge d[\eta_1\eta_2]-\xi_2\eta_2\eta_3 d[\eta_1]\wedge d[\eta_1\eta_2]$	$(2,1,-1)$	$(15\vert 0)$	2/1
[3]	$\eta_3(\eta_1 d[\eta_1]+\eta_2 d[\eta_2])\wedge(\xi_1 d[\eta_2]-\xi_2 d[\eta_1])$	$(1,1,-1)$	$(0\vert 6)$	3/2

$\mathfrak{g}=\mathfrak{kas}(;3\xi)$. Cohomology in deg = 1, dim = $(52|52)$ (the vectors are highest with respect to $\mathfrak{gl}(3)=\mathfrak{o}(6)\cap\mathfrak{g}_0$, the weights are given in the following table with respect to $\mathfrak{o}(6)$), where $\mathfrak{g}_0$-modules:

$[A]=[2']+[3']+[4]+[5']+[6']+[7']$ of dim = $(12|12)$, where $[2']$, $[3']$, $[5']$, $[6']$, $[7']$ are generated by $[2_1]-[2_2]$, $[3_1]-2[3_2]+[3_3]$, $2[5_1]-[5_2]+[5_3]$, $[6_1]+[6_2]$, and $[7_1]+[7_2]$, respectively;

$[B]=[8']+[9']+[10]+[11]+[12]+[13]$ of dim = $(24|24)$, where $[8']$ and $[9']$ are generated by $[8_1]+2[8_2]$, and $[9_1]+2[9_2]$, respectively;

$[C]=[B]+[3'']+[5'']+[6_1]+[7_1]+[8_1]+[9_1]$ of dim = $(36|36)$, where $[3'']$ and $[5'']$ are generated by $[3_1]-[3_2]$ and $[5_2]-3[5_1]$, respectively;

$[D]=[A]+[C]$ of dim = $(48|48)$;

$[E]=[D]+[1]+[2'']+[3''']+[5''']$ of dim = $(52|52)$.

The modules $[A]$, $[B]$, $[C]/[B]$ and $[E]/[D]$ are irreducible, $\dim([C]/[B]) = (12|12)$ and $\dim([E]/[D]) = (4|4)$.

N	$\mathfrak{gl}(3)$-highest vectors	weight	dim	mult
$[1]$	$d[\xi_1\xi_2\xi_3]^2$	$(-2,-2,-2)$	$(1\|0)$	$1/0$
$[2_1]$	$d[\xi_2\xi_3]\wedge d[\xi_1\xi_2\xi_3]$	$(-1,-2,-2)$	$2\cdot(0\|3)$	$2/0$
$[2_2]$	$\xi_1 d[\xi_1\xi_2\xi_3]^2$			
$[3_1]$	$d[\xi_3]\wedge d[\xi_1\xi_2\xi_3]$	$(-1,-1,-2)$	$3\cdot(3\|0)$	$4/1$
$[3_2]$	$d[\xi_1\xi_3]\wedge d[\xi_2\xi_3]$			
$[3_3]$	$\xi_1\xi_2 d[\xi_1\xi_2\xi_3]^2$			
$[4]$	$\xi_1 d[\xi_2\xi_3]\wedge d[\xi_1\xi_2\xi_3]$	$(0,-2,-2)$	$(6\|0)$	$1/0$
$[5_1]$	$d[1]\wedge d[\xi_1\xi_2\xi_3]$	$(-1,-1,-1)$	$3\cdot(0\|1)$	$6/3$
$[5_2]$	$d[\xi_1]\wedge d[\xi_2\xi_3]-d[\xi_2]\wedge d[\xi_1\xi_3]+d[\xi_3]\wedge d[\xi_1\xi_2]$			
$[5_3]$	$\xi_1\xi_2\xi_3 d[\xi_1\xi_2\xi_3]^2$			
$[6_1]$	$d[\xi_3]\wedge d[\xi_2\xi_3]$	$(0,-1,-2)$	$2\cdot(0\|8)$	$4/2$
$[6_2]$	$\xi_1\xi_2 d[\xi_2\xi_3]\wedge d[\xi_1\xi_2\xi_3]$			
$[7_1]$	$d[1]\wedge d[\xi_2\xi_3]$	$(0,-1,-1)$	$2\cdot(3\|0)$	$7/5$
$[7_2]$	$\xi_1 d[1]\wedge d[\xi_1\xi_2\xi_3]$			
$[8_1]$	$d[\xi_3]^2$	$(0,0,-2)$	$2\cdot(6\|0)$	$4/2$
$[8_2]$	$\xi_1\xi_2 d[\xi_1\xi_3]\wedge d[\xi_2\xi_3]$			
$[9_1]$	$d[1]\wedge d[\xi_3]$	$(0,0,-1)$	$2\cdot(0\|3)$	$8/6$
$[9_2]$	$\xi_1\xi_2\xi_3 d[\xi_1\xi_3]\wedge d[\xi_2\xi_3]$			
$[10]$	$\xi_1 d[\xi_3]^2$	$(1,0,-2)$	$(0\|15)$	$2/1$
$[11]$	$\xi_1 d[1]\wedge d[\xi_3]$	$(1,0,-1)$	$(8\|0)$	$4/3$
$[12]$	$\xi_1\xi_2 d[\xi_3]^2$	$(1,1,-2)$	$(10\|0)$	$1/0$
$[13]$	$\xi_1\xi_2 d[1]\wedge d[\xi_3]$	$(1,1,-1)$	$(0\|6)$	$1/0$

$\mathfrak{g}=\mathfrak{mb}(4|5)$. $2-d=0$. Here $w(u_0)=0$, $w(u_1)=(1,0)$, $w(u_2)=(-1,1)$, $w(u_3)=(0,-1)$, $w(\xi_i)=-w(u_i)$.

Cohomology: in deg $=1$, hence, torsion free. A single irreducible $\mathfrak{g}_0$-module of dim $=(12|12)$ glued of the following $\mathfrak{sl}(3)$-modules:

N	$\mathfrak{gl}(3)$-highest vectors	weight	dim	mult
$[1]$	$u_0 d[u_0]\wedge d[\xi_0]$	$(0,0)$	$(0\|1)$	$4/3$
$[2]$	$u_0 d[u_0]\wedge d[u_3]-\xi_0 d[u_3]\wedge d[\xi_0]$	$(0,1)$	$(3\|0)$	$6/5$
$[3]$	$u_0 d[u_3]\wedge d[\xi_0]$	$(0,1)$	$(0\|3)$	$3/2$
$[4]$	$u_0 d[\xi_0]\wedge d[\xi_1]$	$(1,0)$	$(3\|0)$	$3/2$
$[5]$	$u_0 d[u_3]\wedge d[\xi_1]+u_1 d[u_3]\wedge d[\xi_0]$	$(1,1)$	$(0\|8)$	$3/2$
$[6]$	$u_0 d[u_3]^2-\xi_3 d[u_3]\wedge d[\xi_0]$	$(0,2)$	$(6\|0)$	$1/0$

$\mathfrak{g}=\mathfrak{mb}(4|5;1)$. $2-d=0$.

Cohomology in deg $= 0$, dim $= (22|22)$:

N	$\mathfrak{sl}(2)\oplus\mathfrak{sl}(2)$-highest vectors	weight	dim	mult
[1]	$\xi_1 d[u_2\xi_1]\wedge d[u_3\xi_1]$	$(0,0)$	$(0\|1)$	6/5
[2_1]	$d[u_2]\wedge d[u_0u_3-\xi_1\xi_2]-d[u_3]\wedge d[u_0u_2+\xi_1\xi_3]$	$(0,1)$	$2\times(2\|0)$	6/4
[2_2]	$d[u_2\xi_1]\wedge d[u_3\xi_1]$			
[3]	$d[u_0u_2+\xi_1\xi_3]\wedge d[u_3\xi_1]-d[u_0u_3-\xi_1\xi_2]\wedge d[u_2\xi_1]$	$(0,2)$	$(0\|3)$	2/1
[4]	$d[\xi_2]\wedge d[u_3\xi_1]$	$(2,1)$	$(6\|0)$	4/3
[5_1]	$d[\xi_2]\wedge d[u_0u_3-\xi_1\xi_2]$	$(2,2)$	$2\times(0\|9)$	3/1
[5_2]	$d[u_0u_3-\xi_1\xi_2]\wedge d[u_3\xi_1]$			
[6]	$d[u_0u_3-\xi_1\xi_2]^2$	$(2,3)$	$(12\|0)$	1/0

$\mathfrak{g}_0$-modules:
$[A]=[4]+[5_1]+[5_2]+[6]$ of dim $=18|18$;
$[B]=[2_2]+[3]+[A]$ of dim $=20|21$;
$[C]=[1]+[2_1]+[B]$ of dim $=22|22$.
Irreducible modules: $[A]$, $[B]/[A]$ (of dim $=2|3$) and $[C]/[B]$ (of dim $=2|1$).

$\mathfrak{g}=\mathfrak{mb}(4|5;K)$. $2-d=-1$.

The $\mathfrak{g}_0$-highest weight vectors are as follows: $\mathfrak{g}_0=\mathfrak{sl}(2)\oplus\mathfrak{gl}(3)$; the first coordinate of the weight is given with respect to $\mathfrak{sl}(2)$ realized as $x_+=q_0(\tau+q_0\xi_0-\sum_{i=1}^3 q_i\xi_i)+2\xi_1\xi_2\xi_3$, $x_-=\xi_0$; $\mathfrak{gl}(3)$ is realized as $x^i_j=q_i\xi_j$ $(i\neq j)$ and $x^i_i=\tau+q_i\xi_i-q_0\xi_0$. In this realization $w(q_0)=(1,-1,-1,-1)$, $w(u_1)=(-1,1,0,0)$, $w(u_2)=(-1,0,1,0)$, $w(u_3)=(-1,0,0,1)$, $w(\xi_0)=(-2,1,1,1)$, $w(\xi_1)=(0,-1,0,0)$, $w(\xi_2)=(0,0,-1,0)$, $w(\xi_3)=(0,0,0,-1)$).

Cohomology: a single irreducible module in deg $=-1$:

$\mathfrak{sl}(2)\oplus\mathfrak{gl}(3)$-highest vectors	weight	dim	mult
$u_0dq_3\wedge dq_3$	$(3,0,0,-2)$	$(24\|0)$	1/0

$\mathfrak{g}=\mathfrak{ksle}(9|6;K)$. $2-d=0$. Cohomology are in degrees 0 and 1 and constitute irreducible modules. (The weights are given in $\mathfrak{gl}$-basis of matrix diagonal units.)

deg	$\mathfrak{gl}(5)$-highest vectors	$\mathfrak{gl}(5)$-weight	dim	mult
0	$\partial_5 d[\pi dx_4dx_5]^2$	$(0,0,0,-2,-3)$	$(175\|0)$	1/0
1	$\sum\partial_i d[\pi dx_4dx_5]\wedge d[\partial_i]+\sum\pi dx_idx_j(d[\pi dx_idx_4]\wedge d[\pi dx_jdx_5]-$ $d[\pi dx_idx_5]\wedge d[\pi dx_jdx_4]-d[\pi dx_idx_j]\wedge d[\pi dx_4dx_5])$	$(0,0,0,-1,-1)$	$(0\|10)$	3/2

$\mathfrak{g}=\mathfrak{ksle}(9|6;CK)$. $2-d=-1$. Cohomology are in degrees -1, 0, and 1.
Cohomology in deg $=-1$, dim $=(36|36)$:

N	$\mathfrak{sl}(2)\oplus\mathfrak{sl}(3)$-highest vectors	weight	dim	mult
[1]	$\partial_2 d[\pi dx_2dx_5]^2$	$(3,0,2)$	$(24\|0)$	1/0
[2]	$\partial_2 d[\pi dx_2dx_5]\wedge d[x_5\partial_1]$	$(3,0,2)$	$(0\|24)$	1/0
[3]	$\partial_2 d[x_4\partial_1]\wedge d[x_5\partial_1]$	$(3,1,0)$	$(12\|0)$	1/0
[4]	$\partial_2 d[\pi dx_2dx_4]\wedge d[x_5\partial_1]-\partial_2 d[\pi dx_2dx_5]\wedge d[x_4\partial_1]$	$(3,1,0)$	$(0\|12)$	1/0

$\mathfrak{g}_0$-modules:
$[A]=[1]+[2]$ of dim $=24|24$;

$[B]=[A]+[3]+[4]$ of $\dim = 36|36$.
Irreducible modules: [A] and [B]/[A] (of $\dim = 12|12$).

Cohomology in $\deg = 0$, single $\mathfrak{g}_0$-module, $\dim = 10|10$:

N	$\mathfrak{sl}(2)\oplus\mathfrak{sl}(3)$-highest vectors	weight	dim	mult
[1]	$\partial_5 d[x_5\partial_1]\wedge d[x_5\partial_2]$	$(0,0,3)$	$(10\|0)$	1/0
[2]	$\partial_5 d[\pi dx_1dx_5]\wedge d[x_5\partial_1]+\partial_5 d[\pi dx_2dx_5]\wedge d[x_5\partial_2]-$ $\pi dx_3dx_4 d[x_5\partial_1]\wedge d[x_5\partial_2]$	$(0,0,3)$	$(0\|10)$	1/0

Cohomology: in $\deg = 1$, a single $\mathfrak{g}_0$-module, $\dim = (6|6)$:

N	$\mathfrak{sl}(2)\oplus\mathfrak{sl}(3)$-highest vectors	weight	dim	mult
[1]	$-\partial_2 d[x_5\partial_2]\wedge d[\partial_1]+\sum\partial_i d[x_5\partial_1]\wedge d[\partial_i]+$ $\sum\big(\pi dx_1dx_j d\big[\pi dx_idx_j\big]\wedge d[x_5\partial_i]+x_j\partial_i d\big[x_j\partial_1\big]\wedge d[x_5\partial_i]\big)$	$(1,0,1)$	$(6\|0)$	8/7
[2]	$-\sum\partial_i d[\pi dx_2dx_5]\wedge d[\partial_i]+$ $\sum\pi dx_idx_j d[\pi dx_2dx_5]\wedge d\big[\pi dx_idx_j\big]-$ $\sum_{1\leq i\leq 2}\sum_{3\leq j\leq 5}x_j\partial_i d[\pi dx_2dx_5]\wedge d\big[x_j\partial_i\big]$	$(1,0,1)$	$(0\|6)$	8/7

$\mathfrak{g}=\mathfrak{ksle}(9|6)$. $2-d=0$. Cohomology in $\deg = 1$, hence, torsion-free. $\dim = (168|167)$

N	$\mathfrak{gl}(3)$-highest vectors	weight	dim	mult
[1]	$\sum(-1)^{p(\sigma)}\partial_{\sigma(1)}d[\pi dx_{\sigma(2)}dx_{\sigma(3)}]\wedge d[x_{\sigma(4)}\partial_5]-$ $2\sum\pi dx_idx_j d[x_i\partial_5]\wedge d\big[x_j\partial_5\big]$	$(0,0,0)$	$(0\|1)$	2/1
$[2_1]$	$\sum\partial_i d[\partial_i]\wedge d[x_4\partial_5]+\partial_4 d[\partial_i]\wedge d[x_i\partial_5]$	$(0,0,1)$	$2\times(4\|0)$	5/3
$[2_2]$	$2\partial_4(-d[\pi dx_1dx_2]\wedge d[\pi dx_3dx_4]+d[\pi dx_1dx_3]\wedge d[\pi dx_2dx_4]-$ $d[\pi dx_1dx_4]\wedge d[\pi dx_2dx_3])+\sum\pi dx_idx_j d\big[\pi dx_idx_j\big]\wedge d[x_4\partial_5]$			
[3]	$\sum\partial_4 d[\partial_i]\wedge d[\pi dx_idx_4]-\sum\pi dx_{\sigma(1)}dx_{\sigma(2)}d[\partial_{\sigma(3)}]\wedge d[x_4\partial_5]$	$(0,0,2)$	$(0\|10)$	2/1
$[4_1]$	$\sum_{1\leq i\leq 4}(\partial_i d[\partial_i]\wedge d[\pi dx_3dx_4]+\pi dx_1dx_2 d[\partial_i]\wedge d[x_i\partial_5])$	$(0,1,0)$	$2\times(0\|6)$	6/4
$[4_2]$	$\sum\pi dx_idx_i d\big[\pi dx_idx_j\big]\wedge d[\pi dx_3dx_4]-$ $2\sum(-1)^{\mathrm{sign}(\sigma)}\pi dx_1dx_2+d[\pi dx_1dx_{\sigma(2)}]\wedge d[\pi dx_{\sigma(3)\sigma(4)}]$			
$[5_1]$	$\partial_4 d[x_3\partial_5]\wedge d[x_4\partial_5]$	$(0,1,1)$	$2\times(20\|0)$	4/2
$[5_2]$	$-2\partial_4 d[\partial_1]\wedge d[\partial_2]-x_1\partial_5 d[\partial_2]\wedge d[x_4\partial_5]+x_2\partial_5 d[\partial_1]\wedge d[x_4\partial_5]$			
[6]	$\partial_4 d[\pi dx_3dx_4]\wedge d[x_4\partial_5]$	$(0,1,2)$	$(0\|45)$	1/0
[7]	$\pi dx_1dx_2 d[x_3\partial_5]\wedge d[x_4\partial_5]-\partial_3 d[\pi dx_3dx_4]\wedge d[x_4\partial_5]+$ $\partial_4 d[\pi dx_3dx_4]\wedge d[x_3\partial_5]$	$(0,2,0)$	$(0\|20)$	2/1
[8]	$\pi dx_1dx_2 d[\pi dx_3dx_4]\wedge d[x_4\partial_5]-\partial_4 d[\pi dx_3dx_4]^2$	$(0,2,1)$	$(60\|0)$	1/0
[9]	$\sum(\partial_i d[\partial_1]\wedge d[\partial_i]+x_1\partial_5 d[\partial_i]\wedge d[x_i\partial_5])$	$(1,0,0)$	$(4\|0)$	5/4
[10]	$\sum(-1)^{p(\sigma)}(\partial_{\sigma(2)}d[\pi dx_{\sigma(3)}dx_{\sigma(4)}]\wedge d[x_4\partial_5]+$ $\partial_4 d[\pi dx_{\sigma(2)}dx_{\sigma(3)}]\wedge d[x_{\sigma(3)}\partial_5])$	$(1,0,1)$	$(0\|15)$	4/3
[11]	$\partial_4 d[\partial_1]\wedge d[x_4\partial_5]$	$(1,0,2)$	$(36\|0)$	4/3
[12]	$2x_1\partial_5 d[x_3\partial_5]\wedge d[x_4\partial_5]-\partial_3 d[\partial_1]\wedge d[x_4\partial_5]+\partial_4 d[\partial_1]\wedge d[x_3\partial_5]$	$(1,1,0)$	$(20\|0)$	4/3
[13]	$\pi dx_1dx_2 d[\partial_1]\wedge d[x_4\partial_5]+\partial_4 d[\partial_1]\wedge d[\pi dx_3dx_4]$	$(1,1,1)$	$(0\|64)$	3/2

$\mathfrak{g}_0$-modules:
$[A]=[2']+[4']+[9]$, $\dim = (8|6)$, where $[2']$ and $[4']$ are generated by $2[2_1]-[2_2]$ and $2[4_1]-[4_2]$, respectively;
[B]=[A]+[1], $\dim = (8|7)$;
$[C]=[1]+\cdots+[13]$, $\dim = (168|167)$.

The modules [A], [B]/[A] (of dim $= (0|1)$), and [C]/[B] (of dim $= (160|160)$) are irreducible.

$\mathfrak{g} = \mathfrak{ksle}(9|6;2)$**.** $2-d=0$. Cohomology are in degrees 0 and 1.
Cohomology in deg $= 0$, dim $= (140|140)$:

N	$\mathfrak{sl}(3)\oplus\mathfrak{sl}(2)$-highest vectors	weight	dim	mult
[1]	$\pi dx_4dx_5d[\pi x_5dx_4dx_5]^2$	$(0,0,2)$	$(0\|3)$	$2/1$
[2]	$\partial_3d[\partial_4]\wedge d[\pi x_4dx_4dx_5]+\partial_3d[\partial_5]\wedge d[\pi x_5dx_4dx_5]$	$(0,1,0)$	$(0\|3)$	$4/3$
[3_1]	$\partial_3d[\partial_4]\wedge d[\pi x_5dx_4dx_5]$	$(0,1,2)$	$2\cdot(0\|9)$	$5/3$
[3_2]	$\pi dx_4dx_5d[x_5\partial_1]\wedge d[x_5\partial_2]$			
[3_3]	$\partial_3d[\pi x_5dx_4dx_5]^2$	$(0,1,2)$	$(9\|0)$	$2/1$
[4]	$\partial_3d[\pi dx_3dx_4]\wedge d[\pi x_5dx_4dx_5]-$ $\partial_3d[\pi dx_3dx_5]\wedge d[\pi x_4dx_4dx_5]$	$(0,2,0)$	$(6\|0)$	$1/0$
[5_1]	$\partial_3d[\partial_4]\wedge d[\pi dx_3dx_5]$	$(0,2,2)$	$(0\|18)$	$2/0$
[5_2]	$\partial_3d[\pi dx_3dx_5]\wedge d[\pi x_5dx_4dx_5]$	$(0,2,2)$	$2\cdot(18\|0)$	$2/1$
[5_3]	$\partial_3d[x_5\partial_1]\wedge d[x_5\partial_2]$			
[6]	$\partial_3d[\pi dx_3dx_5]^2$	$(0,3,2)$	$(30\|0)$	$1/0$
[7]	$\pi dx_4dx_5d[x_4\partial_1]\wedge d[\pi x_5dx_4dx_5]-$ $\pi dx_4dx_5d[x_5\partial_1]\wedge d[\pi x_4dx_4dx_5]$	$(1,0,0)$	$(3\|0)$	$3/2$
[8]	$\pi dx_4dx_5d[x_5\partial_1]\wedge d[\pi x_5dx_4dx_5]$	$(1,0,2)$	$(9\|0)$	$3/2$
[9_1]	$\partial_3d[\partial_4]\wedge d[x_4\partial_1]+\partial_3d[\partial_5]\wedge d[x_5\partial_1]$	$(1,1,0)$	$(8\|0)$	$3/2$
[9_2]	$\partial_3d[x_4\partial_1]\wedge d[\pi x_5dx_4dx_5]-\partial_3d[x_5\partial_1]\wedge d[\pi x_4dx_4dx_5]$	$(1,1,0)$	$(0\|8)$	$1/0$
[10_1]	$\partial_3d[\partial_4]\wedge d[x_5\partial_1]$	$(1,1,2)$	$(24\|0)$	$3/2$
[10_2]	$\partial_3d[x_5\partial_1]\wedge d[\pi x_5dx_4dx_5]$	$(1,1,2)$	$(0\|24)$	$1/0$
[11]	$\partial_3d[\pi dx_3dx_4]\wedge d[x_5\partial_1]-\partial_3d[\pi dx_3dx_5]\wedge d[x_4\partial_1]$	$(1,2,0)$	$(0\|15)$	$1/0$
[12]	$\partial_3d[\pi dx_3dx_5]\wedge d[x_5\partial_1]$	$(1,2,2)$	$(0\|45)$	$1/0$
[13]	$\pi dx_4dx_5d[x_4\partial_1]\wedge d[x_5\partial_1]$	$(2,0,0)$	$(0\|6)$	$2/1$
[14]	$\partial_3d[x_4\partial_1]\wedge d[x_5\partial_1]$	$(2,1,0)$	$(15\|0)$	$1/0$

The column "mult" shows the multiplicity of the homogeneous (even or odd) component of given weight.

$\mathfrak{g}_0$-modules:
$[A]=[3_1-3_2]+[5_1]+[5_2+5_3]+[6]+[10_1]+[12]$ of dim $=72|72$;
$[B]=[A]+[2]+[4]+[7]+[9_1]+[9_2]+[11]+[13]+[14]$ of dim $=104|104$;
$[C]=$ all.
The modules [A], [B]/[A] (of dim $= (32|32)$), and [C]/[B] (of dim $= (36|36)$) are irreducible.

Cohomology in deg $= 1$, dim $= (8|8)$: the $\mathfrak{sl}(3)\oplus\mathfrak{sl}(2)$-highest weights are as follows: even ones are (0, 0, 1), (1, 0, 1); odd ones are (0, 0, 1), (0, 1, 1). The corresponding highest weight vectors are too complicated to be included here.

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