ON SOME SYSTEMS OF DIFFERENCE EQUATIONS. Part 5.

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Dedicated to the memory of
Professor N.M.Korobov.

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§2.0. Foreword.

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§5.1. The matrix $W_1^{\sim}(z;\nu)$ and test $-\nu^{6+4l}E_{4+2l}=A_l^*(z;\nu)A_l^*(z;-\nu)$ in the case $l=1$.

In view of (10) of the Part 4 and resuls of the part 3,

$$V_1^{\sim}(0,\nu)=\nu^{-5}102\begin{pmatrix}0&0&0&-\nu^3&-2\nu^4&-\nu^5\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\end{pmatrix}=$$

$$\sum_{s=0}^{10}\nu^{-s}V_1^{\sim\vee}(0,s),$$

where

$$V_1^{\sim\vee}(0,0) = -102e_{6,1,6},$$
$$V_1^{\sim\vee}(0,1) = -204e_{6,1,5},$$
$$V_1^{\sim\vee}(0,2) = -102e_{6,1,4},$$

$V_1^{\sim\vee}(0,s) = 0E_6$ for $s = 3, ..., 10$,

$$V_1^{\sim}(1,\nu) = \nu^{-5}\begin{pmatrix} 0 & 0 & 306\nu^2 & 372\nu^3 & 66\nu^4 & 0 \\ 0 & 0 & 0 & 66\nu^2 & 132\nu^{\nu^3} & 66\nu^4 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} =$$

$$\sum_{s=0}^{1} 0\nu^{-s} V_1^{\sim\vee}(1,s),$$

where

$$V_1^{\sim\vee}(1,1) = 66e_{6,1,5} + 66e_{6,2,6},$$
$$V_1^{\sim\vee}(1,2) = 372e_{6,1,4} + 132e_{6,2,5},$$
$$V_1^{\sim\vee}(1,3) = 306e_{6,1,3} + 66e_{6,2,4},$$

$V_1^{\sim\vee}(1,s) = 0E_6$ for $s = 0, 4, ..., 10$,

$$V_1^{\sim}(2,\nu) = \nu^{-5}\begin{pmatrix} 0 & -306\nu & 108\nu^2 & 268\nu^3 & 0 & 0 \\ 0 & 0 & -198\nu & -236\nu^2 & -38\nu^3 & 0 \\ 0 & 0 & 0 & -38\nu^{\nu} & -76\nu^2 & -38\nu^3 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} =$$

$$\sum_{s=0}^{10} \nu^{-s} V_1^{\sim\vee}(2,s),$$

where

$$V_1^{\sim\vee}(2,2) = 268e_{6,1,4} - 38e_{6,2,5} - 38e_{6,3,6},$$
$$V_1^{\sim\vee}(2,3) = 108e_{6,1,3} - 236e_{6,2,4} - 76e_{6,3,5},$$
$$V_1^{\sim\vee}(2,4) = -306e_{6,1,2} - 198e_{6,2,3} - 38e_{6,3,4},$$

$V_1^{\sim\vee}(2,s) = 0E_6$ for $s = 0,\ 1, 5,\ ..., 10$,

$$V_1^{\sim}(3,\nu) = \nu^{-5}\begin{pmatrix} 102 & -516\nu & -180\nu^2 & 0 & 0 & 0 \\ 0 & 198 & -84\nu & -180\nu^2 & 0 & 0 \\ 0 & 0 & 114 & 132\nu & 18\nu^2 & 0 \\ 0 & 0 & 0 & 18 & 36\nu & 18\nu^2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} =$$

$$\sum_{s=0}^{1} 0\nu^{-s} V_1^{\sim\vee}(3,s),$$

where

$$V_1^{\sim\vee}(3,3) = -180e_{6,1,3} - 180e_{6,2,4} + 18e_{6,3,5} + 18e_{6,4,6},$$

$$V_1^{\sim\vee}(3,4) = -516e_{6,1,2} - 84e_{6,2,3} + 132e_{6,3,4} + 36e_{6,4,5},$$

$$V_1^{\sim\vee}(3,5) = 102e_{6,1,1} + 198e_{6,2,2} + 114e_{6,3,3} + 18e_{6,4,4},$$

$V_1^{\sim\vee}(3,s) = 0E_6$ for $s = 0,\ 1,\ 2,\ 6,\ ...,10,$

$$V_1^{\sim}(4,\nu) =$$

$$\nu^{-5}\begin{pmatrix} 240 & -198\nu & 0 & 0 & 0 & 0 \\ -66\nu^{-1} & 348 & 108\nu & 0 & 0 & 0 \\ 0 & -114\nu^{-1} & 60 & 108\nu & 0 & 0 \\ 0 & 0 & -54\nu^{-1} & -60 & -6\nu & 0 \\ 0 & 0 & 0 & -6\nu^{-1} & -12 & -6\nu \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} =$$

$$\sum_{s=0}^{1} 0\nu^{-s} V_1^{\sim\vee}(4,s),$$

where

$$V_1^{\sim\vee}(4,4) = -198e_{6,1,2} + 108e_{6,2,3} + 108e_{6,3,4} - 6e_{6,4,5} +$$
$$(-6)e_{6,5,6},$$

$$V_1^{\sim\vee}(4,5) = 240e_{6,1,1} + 348e_{6,2,2} + 60e_{6,3,3} - 60e_{6,4,4} +$$
$$(-12)e_{6,5,5},$$

$$V_1^{\sim\vee}(4,6) = -66e_{6,2,1} - 114e_{6,3,2} - 54e_{6,4,3} - 6e_{6,5,4},$$

$V_1^{\sim\vee}(3,s) = 0E_6$ for $s = 0,\ ...,\ 3,\ 7,\ ...,10,$

$$V_1^{\sim}(5,\nu) = \nu^{-5}\begin{pmatrix} 146 & 0 & 0 & 0 & 0 & 0 \\ -160\nu^{-1} & 146 & 0 & 0 & 0 & 0 \\ 38\nu^{-2} & -212\nu^{-1} & -52 & 0 & 0 & 0 \\ 0 & 54\nu^{-2} & -36\nu^{-1} & -52 & 0 & 0 \\ 0 & 0 & 18\nu^{-2} & 20\nu^{-1} & 2 & 0 \\ 0 & 0 & 0 & 2\nu^{-2} & 4\nu^{-1} & 2 \end{pmatrix} =$$

$$\sum_{s=0}^{10} \nu^{-s} V_1^{\sim\vee}(5,s),$$

where

$$V_1^{\sim\vee}(5,5) = 146e_{6,1,1} + 146e_{6,2,2} - 52e_{6,3,3} - 52e_{6,4,4} +$$
$$2e_{6,5,5} + 2e_{6,6},$$

$$V_1^{\sim\vee}(5,6) = -160e_{6,2,1} - 212e_{6,3,2} - 36e_{6,4,3} + 20e_{6,5,4} +$$
$$4e_{6,6,5},$$

$$V_1^{\sim\vee}(5,7) = 38e_{6,3,1} + 54e_{6,4,2} + 18e_{6,5,3} + 2e_{6,6,4},$$

$V_1^{\sim\vee}(5,s) = 0E_6$ for $s = 0, ..., 4, 8, ..., 10,$

$$V_1^{\sim}(6,\nu) =$$

$$\nu^{-5}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ -102\nu^{-1} & 0 & 0 & 0 & 0 & 0 \\ 96\nu^{-2} & -102\nu^{-1} & 0 & 0 & 0 & 0 \\ -18\nu^{-3} & 108\nu^{-2} & 12\nu^{-1} & 0 & 0 & 0 \\ 0 & -18\nu^{-3} & 12\nu^{-2} & 12\nu^{-1} & 0 & 0 \\ 0 & 0 & -6\nu^{-3} & -12\nu^{-2} - 6\nu^{-1} & 0 & \end{pmatrix} =$$

$$\sum_{s=0}^{1} 0\nu^{-s} V_1^{\sim\vee}(6,s),$$

where

$$V_1^{\sim\vee}(6,6) = -102e_{6,2,1} - 102e_{6,3,2} + 12e_{6,4,3} + 12e_{6,5,4} +$$

$$(-6)e_{6,6,5},$$

$$V_1^{\sim\vee}(6,7) = 96e_{6,3,1} + 108e_{6,4,2} + 12e_{6,5,3} - 12e_{6,6,4},$$

$$V_1^{\sim\vee}(6,8) = -18e_{6,4,1} - 18e_{6,5,2} - 6e_{6,6,3},$$

$V_1^{\sim\vee}(5,s) = 0E_6$ for $s = 0, ..., 5, 9, 10,$

$$V_1^{\sim}(7,\nu) = \nu^{-5}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 66\nu^{-2} & 0 & 0 & 0 & 0 & 0 \\ -48\nu^{-3} & 66\nu^{-2} & 0 & 0 & 0 & 0 \\ 6\nu^{-4} & -36\nu^{-3} & 12\nu^{-2} & 0 & 0 & 0 \\ 0 & 6\nu^{-4} & 12\nu^{-3} & 12\nu^{-2} & 0 & 0 \end{pmatrix} =$$

$$\sum_{s=0}^{1} 0\nu^{-s} V_1^{\sim\vee}(7,s),$$

where

$$V_1^{\sim\vee}(7,7) = 66e_{6,3,1} + 66e_{6,4,2} + 12e_{6,5,3} + 12e_{6,6,4},$$

$$V_1^{\sim\vee}(7,8) = -48e_{6,4,1} - 36e_{6,5,2} + 12e_{6,6,3},$$

$$V_1^{\sim\vee}(7,9) = 6e_{6,5,1} + 6e_{6,6,2},$$

$V_1^{\sim\vee}(5,s) = 0E_6$ for $s = 0, ..., 6, 10,$

$$V_1^{\sim}(8) = \nu^{-5}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -38\nu^{-3} & 0 & 0 & 0 & 0 & 0 \\ 16\nu^{-4} & -38\nu^{-3} & 0 & 0 & 0 & 0 \\ -2\nu^{-5} & -4\nu^{-4} & -20\nu^{-3} & 0 & 0 & 0 \end{pmatrix} =$$

$$\sum_{s=0}^{1} 0\nu^{-s} V_1^{\sim\vee}(8,s),$$

where

$$V_1^{\sim\vee}(8,8) = -38e_{6,4,1} - 38e_{6,5,2} - 20e_{6,6,3},$$

$$V_1^{\sim\vee}(8,9) = 16e_{6,5,1} - 4e_{6,6,2},$$

$$V_1^{\sim\vee}(8,10) = -2e_{6,6,1},$$

$V_1^{\sim\vee}(9,s) = 0E_6$ for $s = 0, ..., 7$,

$$V_1^{\sim}(9) = \nu^{-5} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 18\nu^{-4}8 & 0 & 0 & 0 & 0 & 0 \\ 0 & 18\nu^{-4}8 & 0 & 0 & 0 & 0 \end{pmatrix} =$$

$$\sum_{s=0}^{1} 0\nu^{-s} V_1^{\sim\vee}(9,s),$$

where

$$V_1^{\sim\vee}(9,9) = e_{6,5,1} + e_{6,6,2},$$

$V_1^{\sim\vee}(9,s) = 0E_6$ for $s = 0, ..., 8, 10$

$$V_1^{\sim}(10) = \nu^{-5} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -6\nu^{-5} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

$$\sum_{s=0}^{1} 0\nu^{-s} V_1^{\sim\vee}(10,s),$$

where

$$V_1^{\sim\vee}(10,10) = -6e_{6,6,1},$$

$V_1^{\sim\vee}(9,s) = 0E_6$ for $s = 0, ..., 9$.

We see that $V_1^{\sim\vee}(r,s) = 0E_6$, if $r \in [1,10] \cap \mathbb{Z}$ and $s \in [0, r-1] \cap \mathbb{Z}$. Therefore, in view in view of (26) in the Part 4, $V_1^{\sim *}(-i) = 0$ for $i = 0, ..., 10$. In view of (25) in Part 4,

$$V_1^{\sim *}(0) = V_1^{\sim\vee}(0,0) + V_1^{\sim\vee}(1,1) +$$

$$V_1^{\sim\vee}(2,2) + V_1^{\sim\vee}(3,3) + V_1^{\sim\vee}(4,4) +$$

$$V_1^{\sim\vee}(5,5) + V_1^{\sim\vee}(6,6) + V_1^{\sim\vee}(7,7) +$$

$$V_1^{\sim\vee}(8,8) + V_1^{\sim\vee}(9,9) + V_1^{\sim\vee}(10,10) =$$

$$-102e_{6,1,6} + 66e_{6,1,5} + 66e_{6,2,6} + 268e_{6,1,4} - 380e_{6,2,5} - 38e_{6,3,6} +$$

$$(-180)e_{6,1,3} - 180e_{6,2,4} + 18e_{6,3,5} + 18e_{6,4,6} + (-198)e_{6,1,2} +$$

$$108e_{6,2,3}+108e_{6,3,4}-6e_{6,4,5}+(-6)e_{6,5,6}+146e_{6,1,1}+$$
$$146e_{6,2,2}-52e_{6,3,3}-52e_{6,4,4}+2e_{6,5,5}+2e_{6,6,6}+$$
$$(-102)e_{6,2,1}-102e_{6,3,2}+12e_{6,4,3}+12e_{6,5,4}+(-6)e_{6,6,5}+$$
$$+66e_{6,3,1}+66e_{6,4,2}+12e_{6,5,3}+12e_{6,6,4}+(-38)e_{6,4,1}+$$
$$(-38)e_{6,5,2}-20e_{6,6,3}+18e_{6,5,1}+18e_{6,6,2}+(-6)e_{6,6,1}=$$
$$\begin{pmatrix} 146 & -198 & -180 & 268 & 66 & -102\\ -102 & 146 & 108 & -180 & -38 & 66\\ 66 & -102 & -52 & 108 & 18 & -38\\ -38 & 66 & 12 & -52 & -6 & 18\\ 18 & -38 & 12 & 12 & 2 & -6\\ -6 & 18 & -20 & 12 & -6 & 2 \end{pmatrix},$$
$$V_1^{\sim *}(1)=V_1^{\sim\vee}(0,1)+V_1^{\sim\vee}(1,2)+$$
$$V_1^{\sim\vee}(2,3)+V_1^{\sim\vee}(3,4)+V_1^{\sim\vee}(4,5)+$$
$$V_1^{\sim\vee}(5,6)+V_1^{\sim\vee}(6,7)+V_1^{\sim\vee}(7,8)+$$
$$V_1^{\sim\vee}(8,9)+V_1^{\sim\vee}(9,10)=$$
$$-204e_{6,1,5}+372e_{6,1,4}+132e_{6,2,5}+108e_{6,1,3}+(-236)e_{6,2,4}+$$
$$(-76)e_{6,3,5}-516e_{6,1,2}-84e_{6,2,3}+132e_{6,3,4}+36e_{6,4,5}+$$
$$240e_{6,1,1}+348e_{6,2,2}+60e_{6,3,3}-60e_{6,4,4}+(-12)e_{6,5,5}+$$
$$(-160)e_{6,2,1}-212e_{6,3,2}-36e_{6,4,3}+20e_{6,5,4}+4e_{6,6,5}+$$
$$96e_{6,3,1}+108e_{6,4,2}+12e_{6,5,3}-12e_{6,6,4}+(-48)e_{6,4,1}+$$
$$(-36)e_{6,5,2}+12e_{6,6,3}+16e_{6,5,1}-4e_{6,6,2}=$$
$$\begin{pmatrix} 240 & -516 & 108 & 372 & -204 & 0\\ -160 & 348 & -84 & -236 & 132 & 0\\ 96 & -212 & 60 & 132 & -76 & 0\\ -48 & 108 & -36 & -60 & 36 & 0\\ 16 & -36 & 12 & 20 & -12 & 0\\ 0 & -4 & 12 & -12 & 4 & 0 \end{pmatrix},$$
$$V_1^{\sim *}(2)=V_1^{\sim\vee}(0,2)+V_1^{\sim\vee}(1,3)+$$
$$V_1^{\sim\vee}(2,4)+V_1^{\sim\vee}(3,5)+V_1^{\sim\vee}(4,6)+$$
$$V_1^{\sim\vee}(5,7)+V_1^{\sim\vee}(6,8)+V_1^{\sim\vee}(7,9)+$$
$$V_1^{\sim\vee}(8,10)=$$
$$-102e_{6,1,4}+306e_{6,1,3}+66e_{6,2,4}-306e_{6,1,2}-198e_{6,2,3}+$$
$$(-38)e_{6,3,4}+102e_{6,1,1}+198e_{6,2,2}+114e_{6,3,3}+18e_{6,4,4}+$$
$$(-66)e_{6,2,1}+(-114)e_{6,3,2}-54e_{6,4,3}-6e_{6,5,4}+$$
$$38e_{6,3,1}+54e_{6,4,2}+18e_{6,5,3}+2e_{6,6,4}+$$
$$(-18)e_{6,4,1}-18e_{6,5,2}-6e_{6,6,3}+6e_{6,5,1}+6e_{6,6,2}-2e_{6,6,1}=$$

$$\begin{pmatrix} 102 & -306 & 306 & -102 & 0 & 0 \\ -66 & 198 & -198 & 66 & 0 & 0 \\ 38 & -114 & 114 & -38 & 0 & 0 \\ -18 & 54 & -54 & 18 & 0 & 0 \\ 6 & -18 & 18 & -6 & 0 & 0 \\ -2 & 6 & -6 & 2 & 0 & 0 \end{pmatrix}.$$

Since $V_1^{\sim\vee}(r,s) = 0E_8$, if $r = 0, ..., 7$, $s \in [k+3,\, 10] \cap \mathbb{Z}$, it follows that $V_1^{\sim *}(i) = 0$ for $i = s - r \geq 3$. Clearly,

$$V_1^{\sim *}(0)T_{6,-1} = \begin{pmatrix} 146 & 198 & -180 & -268 & 66 & 102 \\ -102 & -146 & 108 & 180 & -380 & -66 \\ 66 & 102 & -52 & -108 & 18 & 38 \\ -38 & -66 & 12 & 52 & -6 & -18 \\ 18 & 38 & 12 & -12 & 2 & 6 \\ -6 & -18 & -20 & -12 & -6 & -2 \end{pmatrix},$$

$$V_1^{\sim *}(1)T_{6,-1} = \begin{pmatrix} 240 & 516 & 108 & -372 & -204 & 0 \\ -160 & -348 & -84 & 236 & 132 & 0 \\ 96 & 212 & 60 & -132 & -76 & 0 \\ -48 & -108 & -36 & 60 & 36 & 0 \\ 16 & 36 & 12 & -20 & -12 & 0 \\ 0 & 4 & 12 & 12 & 4 & 0 \end{pmatrix},$$

$$V_1^{\sim *}(2)T_{6,-1} = \begin{pmatrix} 102 & 306 & 306 & 102 & 0 & 0 \\ -66 & -198 & -198 & -66 & 0 & 0 \\ 38 & 114 & 114 & 38 & 0 & 0 \\ -18 & -54 & -54 & -18 & 0 & 0 \\ 6 & 18 & 18 & 6 & 0 & 0 \\ -2 & -6 & -6 & -2 & 0 & 0 \end{pmatrix},$$

and, In view of (21) in the Part 4,

$$S_1^{\sim}T_{6,-1} = \begin{pmatrix} -1 & -6 & -18 & -38 & -66 & -102 \\ 0 & 1 & 6 & 18 & 38 & 66 \\ 0 & 0 & -1 & -6 & -18 & -38 \\ 0 & 0 & 0 & 1 & 6 & 18 \\ 0 & 0 & 0 & 0 & -1 & -6 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

Therefore direct calculations show that

(1) $$(S_1^{\sim}T_{6,-1})^2 = E_6,$$

,

(2)

$$S_1^{\sim}T_{6,-1}V_1^{\sim *}(0)T_{6,-1} = \begin{pmatrix} 146 & 678 & 1260 & 1172 & 546 & 102 \\ -102 & -466 & -852 & -780 & -358 & -66 \\ 66 & 294 & 524 & 468 & 210 & 38 \\ -38 & -162 & -276 & -236 & -102 & -18 \\ 18 & 70 & 108 & 84 & 34 & 6 \\ -6 & -18 & -20 & -12 & -6 & -2 \end{pmatrix} =$$

$$-V_1^{\sim *}(0)T_{6,-1}S_1^{\sim}T_{6,-1},$$

(3)

$$S_1^{\sim}T_{6,-1}V_1^{\sim *}(1)T_{6,-1} = \begin{pmatrix} -240 & -924 & -1332 & -852 & -204 & 0 \\ 160 & 612 & 876 & 556 & 132 & 0 \\ -96 & -364 & -516 & -324 & -76 & 0 \\ 48 & 180 & 252 & 156 & 36 & 0 \\ -16 & -60 & -84 & -52 & -12 & 0 \\ 0 & 4 & 12 & 12 & 4 & 0 \end{pmatrix} =$$

$$V_1^{\sim *}(1)T_{6,-1}S_1^{\sim}T_{6,-1},$$

(4) $$S_1^{\sim}T_{6,-1}V_1^{\sim *}(2)T_{6,-1} = \begin{pmatrix} 102 & 306 & 306 & 102 & 0 & 0 \\ -66 & -198 & -198 & -66 & 0 & 0 \\ 38 & 114 & 114 & 38 & 0 & 0 \\ -18 & -54 & -54 & -18 & 0 & 0 \\ 6 & 18 & 18 & 6 & 0 & 0 \\ -2 & -6 & -6 & -2 & 0 & 0 \end{pmatrix} =$$

$$-V_1^{\sim *}(2)T_{6,-1}S_1^{\sim}T_{6,-1},$$

(5) $$V_1^{\sim *}(i)T_{6,-1}V_1^{\sim *}(k)T_{6,-1} = 0E_6$$

for $i = 0,\ 1,\ 2$ and $k = 0,\ 1,\ 2$. So the equalities (16) – (18) from the Part 4 hold for l=1, and we had made full test of the equality $A_1(z,\nu)A_1(z,-\nu) = -\nu^{10}E_6$. The equality (18) from the Part 4 substantinates the test, which was made for the eaquality $A_1(z,\nu)A_1(z,-\nu) = -\nu^{10}E_6$ in the Part 3.

Arithmetical applications will be given in the next parts.

§5.2. Corrections in the previous parts of this paper.

On the page 8, first line from the top, in the Part 4 must stand

"and, in view of (20),"

instead of

"and, in view of (26),"

The equality

$$V_1^*(2) = \begin{pmatrix} 0 & -306 & 108 & 268 & 0 & 0 \\ 0 & 0 & -198 & -236 & -38 & 0 \\ 0 & 0 & 0 & -38 & -76 & -38 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

must stand in Part 3 instead of

$$V_1^*(2) = \begin{pmatrix} 0 & -306 & 108 & 268 & 0 & 0 \\ 0 & 0 & -198 & -236 & -380 & 0 \\ 0 & 0 & 0 & -38 & -76 & -38 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

In four last lines of the section 4.4 of the Part 4 must stand E_8 instead of E_4.

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