"On the Borel-Quillen Localization and the Sullivan's Fixed Point Conjecture"

by

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Introduction:

The purpose of this paper is twofold. First, to show statements of the type conjectured by Sullivan in [Su] (known as the Sullivan's fixed point conjecture) lead to criteria for "finite dimensionality" of G-spaces, where G is a finite group. Secondly, Sullivan's fixed point conjecture and the Borel-Quillen localization theorem are closely related in the following sense. For a finite dimensional G-space K, where G is an elementary p-group, the Borel-Quillen localization theorem states that $H_G^*(K;\mathbb{F}_p) \longrightarrow H_G^*(K^G;\mathbb{F}_p)$ is an isomorphism modulo $H_G^*(\{point\};\mathbb{F}_p)$-torsion, where H_G^* is Borel's equivariant cohomology [Hw] [Q] . The above theorem is not true for infinite dimensional spaces in general. The Sullivan conjecture implies that the Borel-Quillen localization holds for the infinite dimensional G-space $Map(E_G,K)$, where $\dim K < \infty$. Conversely, if the Borel-Quillen localization holds for $Map(E_G,X)$, then $E_G \times X$ is G-homotopy equivalent to $E_G \times K$ with $\dim K < \infty$. Here E_G is the usual universal contractible free G-space. This provides an answer to a problem posed in [A2]. This question and other problems of this nature arise naturally in the geometric and differential topological aspects of transformation groups of manifolds. In particular, at present most methods of constructing group actions on a given manifold yield only infinite dimension free G-spaces. See

[A1][AB][AV][W] and their references. The validity of the Sullivan's conjecture has been announced by G . Carlson, H. Miller, and J. Lannes

While Borel-Quillen localization theorem applies to p-elementary abelian groups, and the Sullivan conjecture holds only for p-groups, we have formulated our results for all finite groups. The proof of the main topological result (Theorem 2.4) are reduced to the case of cyclic groups of prime order using an inductive argument. The main algebraic tool which provides such a local-to-global passage is the main algebraic result (Theorem 1.2) of Section 1 which is a projectivity criterion for integral and modular representations occuring as the cohomology of certain G-spaces. (See also [A2] Theorem 2.1). This theorem is the appropriate substitute for the projectivity criteria of Rim [R] and is a natural successor of the criteria of Chouinard and Dade. The proof of our projectivity criterion uses recent results of J. Carlson on rank varieties [Cj] and Avrunin-Scott's proof of the Carlson conjecture on the isomorphism between Quillen's cohomological support variety and Carlson's rank variety of a modular representation [AS]. In the process, we introduce a "rank variety" for G-spaces and compare it with the Quillen's cohomological variety [Q] as well as Carlson's rank variety associated to the total reduced cohomology of a G-space (with the induced G-module structure).

An exposition of some of these ideas and several applications to topological realizations of homotopy actions are to be found in [A2].

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Section 1. A Projectivity Criterion

Let G be a finite group, and let k be an algebraic closure of $\mathbb{F}_p$ = the field with p-elements. All modules are assumed to be finitely generated. A classical result of Rim [R] states that a $\mathbb{Z}G$-module M is $\mathbb{Z}G$-projective if and only if its restrictions $M|\mathbb{Z}P$ are $\mathbb{Z}P$-projective for all Sylow subgroups $P \subseteq G$. Chouinard has refined this result [Ch] by replacing the p-Sylow subgroups in Rim's theorem by (maximal) p-elementary abelian subgroups. Thus the projectivity of M is detected by all its restrictions to $M|\mathbb{Z}A$ for all p-elementary abelian $A \subseteq G$, i.e. $A \cong \mathbb{Z}_p \oplus \mathbb{Z}_p \oplus \ldots\ldots \oplus \mathbb{Z}_p$. To decide the projectivity of $M|\mathbb{Z}A$, it suffices to consider the kA-module $M \otimes k$. Thus, let A be a p-elementary abelian group of rank r and with $\{e_1,\ldots\ldots,e_r\}$ a set of generators, and let I be the augmentation ideal $0 \longrightarrow I \longrightarrow kA \xrightarrow{\varepsilon} k \longrightarrow 0$. It is possible to choose a k-subspace $L \subset I$ with $\dim_k L = r$ and such that $I \cong L \oplus I^2$ as k-vector spaces. Then L generates kA as a k-algebra and for each $l \in L$, $(1+l)^p = 1$. The elements $\sigma \in kA$ of the form $\sigma = 1+l$, $l \in L$ (for such an L) are called "shifted units" and the cyclic subgroups $S \equiv \langle\sigma\rangle$ of order p are called "shifted cyclic subgroup". (See [ ][ ]). In [ ] Dade has proved that a given kA-module M is kA-projective (hence kA-free since kA is local) if and only if $M|kS$

is kS-projective for all such shifted cyclic subgroups of kA . (Note that almost all shifted cyclic subgroups of kA do not come from cyclic subgroups of A .)

J. Carlson has defined the rank variety of M, $V_A^r(M)$ to be the set of all $l \in L$ such that $M|kS$ is not kS-free together with 0 , where S is generated by $1 + l \in kA$. $V_A^r(M)$ is independent of L up to isomorphism and it defines an affine algebraic subvariety of k^r [Cj]. Thus Carlson's formulation of Dade's result is then: M is kA-projective if and only if $V_A^r(M) = 0$.

On the other hand, Quillen had earlier studied [Q] cohomological varieties arising from equivariant cohomology ring $H_G^*(X;k)$ for a G-space X , where $H_G^*(X;k) \equiv H^*(E_G \times_G X;k)$ is the Borel's equivariant cohomology and the fibration $X \longrightarrow E_G \times_G X \longrightarrow BG$ is the Borel construction [Hw].

Consider $H_G = \oplus H^{2i}(G;k)$ as a finitely generated commutative k-algebra and consider the associated affine variety Max H_G . The cohomological variety of $H_G(X;k)$ is defined [Q] to be the set $V_G(X)$ of ring homomorphism from $H_G(X;k)$ to k endowed with the Zariski topology. Since $H^*(G,M)$ is also an H_G-module, by analogy one defines the cohomological support variety $V_G(M)$ to be the largest support in Max H_G of $H^*(G,N \otimes M)$ where N is any kG-module [Cj][An]. For an elementary abelian group A , J. Carlson showed [Cj] that $V_A^r(M)$ injects into $V_A(M)$ and conjectured that these two varieties are isomorphic. This conjecture was proved by Avrunin-Scott [AS] and it was used to provide a piecewise description of $V_G(M)$ in terms of strata given by $V_A^r(M)$ for elementary abelian subgroup $A \subseteq G$ in a manner similar to Quillen's stratification of $H_G(X)$. Avrunin-Scott's stratification theorem suggests the search for the definition of a Carlson-type "rank variety" $V_G^r(X)$ for G-spaces with similar relationships $V_G(X)$ as in the case of kG-modules.

In [A2], we introduced the rank variety of a G-space X (with $H^*(X;k)$ finitely generated) as follows. Two G-spaces X_1 and X_2 are "freely equivalent", if there exists a G-space Y such that $X_i \subset Y$ and $Y - X_i$ are free G-spaces with cohomological dimension mod p of $(Y - X_i)/G$ (for $i = 1,2$) being finite [A2]. This defines an equivalence relation between G-spaces and one has $V_G(X_1) \cong V_G(X_2)$ if X_1 and X_2 are freely equivalent. Moreover, for any G-space X with $H^*(X;k)$ finitely generated, we may find a mod k Moore space Y with a G-action which is freely equivalent to X , and we define $V_A^r(X) \equiv V_A^r(\bar{H}^*(Y;k))$ for each p-elementary abelian subgroup $A \subseteq G$. The rank variety $V_A^r(X)$ is independent of Y up to isomorphism for each A and for a general finite group G , we define $V_G^r(X) \equiv \lim\limits_{A \in \mathcal{E}} \text{ind}\, V_A^r(X)$ where $\mathcal{E}$ is the category of p-elementary abelian subgroups of G with morphisms induced by inclusions and conjugations in G .

1.1 Theorem

Let X be a connected G-space such that $H^*(X;k)$ is infinitely generated, and let $\bar{H}^*(X) \equiv \bigoplus_{i>0} H^i(X;k)$ considered as a kG-module. Then there exists an isomorphism $V_G^r(X) \cong V_G(X)$ and an injection $j : V_G(X) \longrightarrow V_G^r(\bar{H}^*(X))$. Further, if the $H^*(-;k)$-spectral sequence of the Borel construction $X \longrightarrow E_A \times_A X \longrightarrow BA$ collapses for each maximal p-elementary abelian $A \subseteq G$. Then j is also an isomorphism.

This result is used to establish the following projectivity criterion:

1.2 Theorem

Suppose X is a connected G-space such that for each maximal p-elementary abelian subgroup $A \subseteq G$ the $H^*(-ik)$-spectral sequence of $X \longrightarrow E_A \times_A X \longrightarrow BA$ collapses. Then $\bigoplus_{i>0} H^i(X;k)$ is a projective kG-module if and only if it is projective as a kC-module for subgroup $C \subseteq G$ of order p. Similarly, $\bigoplus_{i>0} H^i(X;\mathbb{Z})$ is a projective $\mathbb{Z}G$-module if and only it is $\mathbb{Z}C$-projective for all cyclic subgroups C of prime order.

Note that if X is a Moore space with G-action and $X^G \neq \emptyset$, then the conditions of Theorem 1.2 are satisfied, and we get a projectivity criterion for the cohomology of Moore spaces with G-action. This has the following corollary, first proved by G. Carlsson for the case $G = \mathbb{Z}_p \times \mathbb{Z}_p$ [Cg] and for $G \supset Q_8$ by P. Vogel [V] by different methods :

1.3 Corollary

Suppose that the Sylow subgroups of G are not all cyclic. Then there exist $\mathbb{Z}G$-modules which are not isomorphic (over $\mathbb{Z}G$) to the homology of any Moore space with G-action.

These modules are constructed from considering the induced modules $kG \otimes_{kS} k$ for shifted cyclic subgroups $S \subset kG$ which do not arise from cyclic subgroups of G . Such a module is not kG-projective but it is kC-projective for all cyclic subgroups of G .

1.4. Corollary

Suppose that the Sylow subgroups of G are not all cyclic. Then there exist a connected G-space X such that the $\mathbb{Z}G$-module $\bar{H}_*(X)$ is not $\mathbb{Z}G$-isomorphic to the homology of any Moore space with G-action (with the induced $\mathbb{Z}G$-structure). Moreover, there are decomposable $\mathbb{Z}G$-modules which do not occur as homology of any G-space at all.

1.5 Corollary

Let G be as in 1.4. Then there exists topological spaces X with $\pi_1(X) = G$ such that X has no homology decomposition (twisted) in the sense Eckmann-Hilton [Hp].

This corollary is in sharp contrast with the simply-connected case where homology decompositions and Postnikov decompositions exist [Hp]. Note that there are twisted Postnikov decompositions for all non-simply-connected spaces [B].

Section 2. Criteria For Finite Dimensionality of G-spaces

Throughout this section we will be concerned with topological spaces X such that $H_i(X)$ or $H_i(X;R)$ (for some coefficient system R) are finitely generated. E always denotes the universal contractible free G-space. For any G-space X, Map (E,X) is a G-space endowed with the action $(g,f) \longmapsto f^g$, $f^g(u) = gf(g^{-1}u)$ and the set of equivariant maps $Map_G(E,X)$ is the fixed set $Map(E,X)^G$. For each $\chi \in X^G$, one has "the constant map" $f_\chi : E \longrightarrow X$ with $f(E) = \chi$ which is equivariant. In [Su] Sullivan conjectured that for any p-group G , "the map of constants" $X^G \longrightarrow Map_G(E,X)$ induces a weak homotopy equivalence after p-profinite completion. Recently G. Carlsson and H. Miller have independently proved this conjecture.

The completion functor used in the proofs of Carlsson and Miller are adaptations of the Bousfield-Kan completions [BK]. In order to clarify those properties of the completion functors used in the theorems below, we formulate the notion of a quasicompletion functor. This hopefully allows a wider domain of applicability of our results.

2.1 Definition

Let $\mathcal{R}$ be a functor from the category of topological spaces to itself. $\mathcal{R}$ is called a quasicompletion functor if the following are satisfied :

(C1) $\mathcal{R}$ commutes with arbitrary disjoint unions and finite products.

(C2) There is a coefficient system R associated to $\mathcal{R}$ such that if $f : X \longrightarrow Y$ induces an isomorphism $f_* : H_*(X;R) \longrightarrow H_*(Y;\mathbf{R})$, then $\mathcal{R}(f)_* : H_*(\mathcal{R}(X);R) \longrightarrow H_*(\mathcal{R}(Y);R)$ is also an isomorphism.

(C3) There is a full subcategory of topological spaces $Top(\mathcal{R})$ (associated to $\mathcal{R}$) and there is a natural transformation τ : identity $\longrightarrow \mathcal{R}$ which satisfy:

(i) If $\pi_1(X) = 0$, then $X \in Top(\mathcal{R})$.

(ii) If $X \in Top(\mathcal{R})$ then $\mathcal{R}(X) \in Top(\mathcal{R})$, and $\mathcal{R}(X) \longrightarrow \mathcal{R}(\mathcal{R}(X))$ is induces $H_*(-;R)$-isomorphism.

(iii) For all $X \in Top(\mathcal{R})$, the map $\tau(X) : X \longrightarrow \mathcal{R}(X)$ induces an $H_*(-;R)$-isomorphism.

2.2 Definition

Let $\mathcal{G}$ be a category of groups and $\mathcal{R}$ be a quasicompletion functor. We say that $\mathcal{R}$ is adapted to $\mathcal{G}$ if the following is satisfied:

(C4) For all $G \in \mathcal{G}$ and all finite dimensional G-spaces X such that $H_*(X;R)$ and $H_*(X^G;R)$ are finitely generated, the map of constants $X^G \longrightarrow \mathrm{Map}_G(E,X)$ induces an isomorphism
$H_*(\mathcal{R}(X^G);R) \longrightarrow H_*(\mathrm{Map}_G(E,\mathcal{R}(X));R)$.

When $R = \mathbb{F}_p$ and $\mathcal{R}_p$ is the Bousfield-Kan [BK] $\mathbb{F}_p$-completion (or the Dwyer-Miller-Neisendorfer version), then $\mathcal{R}_p$ is adapted to the category of all finite p-groups by the validity of the Sullivan's conjecture mentioned above. We remark that a quasicompletion functor may be defined only on a subcategory of topological spaces, in which case the conditions (C1)-(C4) must undergo the appropriate modifications. For the Bousfield-Kan $\mathbb{F}_p$-completion functor, $\mathrm{Top}(\mathcal{R})$ consists of $\mathbb{F}_p$-good spaces [BK].

Let A be a p-elementary abelian group, and let $f : X_1 \longrightarrow X_2$ be an A-map between A-spaces. Let e_A be the product of 2-dimensional cohomology classes in $H^2(A;\mathbb{F}_p)$. Since the Borel's equivariant cohomology groups $H_A^*(X_i;\mathbb{F}_p)$ are modules over $H^*(A;\mathbb{F}_p)$, we may localize $H_A^*(X_i;\mathbb{F}_p)$ by inverting the element $e_A \in H^*(A;\mathbb{F}_p)$ (see [Hw][Q] e.g.).

Definition.

We say that f induces a "Borel-Quillen localized isomorphism" if the induced map $f^* : H_A^*(X_2;\mathbb{F}_p)[e_A^{-1}] \longrightarrow H_A^*(X_1;\mathbb{F}_p)[e_A^{-1}]$ is an isomorphism.

The celebrated Borel-Quillen localization theorem may be translated into the statement that for all finite dimensional A-spaces X, the inclusions $X^A \longrightarrow X$ induces a Borel-Quillen localized isomorphism. In the sequel, we will need the following sets of subgroups of G : $P_p(G)$ is the set of p-subgroups of G and $A_p(G) = \{(P_1,P_2) \mid P_i \in P_p(G)\ ,\ P_2 \triangleleft P_1$ and P_1/P_2 is p-elementary abelian$\}$. This notation is used without further mention.

2.3 Proposition

For each $p \mid |G|$, suppose that R_p is a quasi-completion functor whose associated coefficient system is $\mathbb{F}_p$ and R_p is adapted to $P_p(G)$. Assume that Y is a finite dimensional G-space such that $H_*(Y^P;\mathbb{F}_p)$ are finitely generated for each $P \in P_p(G)$ and Y^P belongs to $Top(R_p)$. Let X be a G-space such that $E \times X$ and $E \times Y$ are G-homotopy equivalent. Then for each $P \in P_p(G)$ and each $(P_1,P_2) \in A_p(G)$ (for any p) the following holds :

(i) All spaces $Map_P(E, R_p(X))$ are in the image of R_p up to $H_*(-;\mathbb{F}_p)$-isomorphism.

(ii) There exist finite dimensional complexes $F(P) \in Top(R_p)$ (with $H_*(F(P);\mathbb{F}_p)$ finitely generated) and maps $\eta(P) : F(P) \longrightarrow Map_P(E,X)$ such that the induced composition of maps $\hat{\eta} : R_p(F(P)) \longrightarrow Map_P(E, R_p(X))$ are $H_*(-;\mathbb{F}_p)$-isomorphisms.

(iii) The maps $\lambda(P_1,P_2) : Map_{P_1}(E, R_p(X)) \longrightarrow Map_{P_2}(E, R_p(X))$ induce Borel-Quillen localized isomorphisms.

The converse of this proposition is the content of the theorem below which yields the criteria for finite dimensionality of certain classes of G-spaces. We may also regard this theorem as a converse to Borel-Quillen localization theorem [Hw][Q].

2.4 Theorem

Let G, P, and R_p be as in 2.3 above. Let X be a G-space which satisfies conditions (i) and (ii) of Proposition 2.3. Then there exists a finite dimensional G-space Y such that for each $P \in P_p(G)$, $H_*(Y^P;\mathbb{F}_p)$ is finitely generated, $Y^P \in Top(R_p)$, and $E \times Y$ and $E \times X$ are

G-homotopy equivalent if and only if for each $(P_1,P_2) \in A_p(G)$ with $P_1/P_2 \cong \mathbb{Z}_p$, the inclusion of the $\mathbb{Z}_p$-fixed point sets $\lambda(P_1,P_2) : \mathrm{Map}_{P_1}(E,R_p(X)) \longrightarrow \mathrm{Map}_{P_2}(E,R_p(X))$ induces Borel-Quillen localized isomorphisms.

2.5 Addendum

In 2.3 and 2.4 if $F(P)$ are finite complexes and $\pi_1(X) = 0$, then a finiteness obstruction $w(X) \in \tilde{K}_0(\mathbb{Z}G)$ may be defined such that $w(X) = 0$ if and only if Y can be taken to be G-homotopy equivalent to a finite G-complex. By passage to a quotient of $\tilde{K}_0(\mathbb{Z}G)$, $w(X)$ becomes well-defined in the sense that it will depend only on the G-homotopy type of X.

In general, the above mentioned obstruction $w(X)$ is non-zero even in very simple situations, as the example below shows.

2.6 Example

Let G be the cyclic group of order 23 acting on $M \cong \mathbb{Z}/47\mathbb{Z}$ via the inclusion $G \subset \mathrm{Aut}(\mathbb{Z}/47\mathbb{Z}) \cong \mathbb{Z}/46\mathbb{Z}$. Using the calculations of Swan in [Sr], one can show that for any G-space X with $\bar{H}_*(X) \cong M$ as $\mathbb{Z}G$-modules, there are no finite G-complex K such that $E \times X$ is G-homotopy equivalent to $E \times K$. Of course, there are finite dimensional G-complexes Y such that $\bar{H}_*(Y) \cong M$ as $\mathbb{Z}G$-modules. For any such Y, $\bar{H}(Y^G;\mathbb{F}_{23}) = 0$ and $Y^G \in \mathrm{Top}(R)$ where R is the Bousfield-Kan $\mathbb{F}_{23}$-completion functor.

Section 3. Finite Complexed and p-Groups

We will consider a special case where G is a p-group and the quasi-completion functor satisfies seemingly simpler hypotheses.

3.1 Definition

Let $\mathcal{R}$ be a functor from the category of topological spaces to topological spaces, together with a natural transformation τ : identity $\longrightarrow \mathcal{R}$. Let $Top(\mathcal{R})$ be a full-subcategory of topological spaces, and let R be a coefficient system. Assume that $\mathcal{R}$ commutes with disjoint unions and finite products. Then:

(1) $\mathcal{R}$ is called a "compact" functor if for every compact space $V \in Top(\mathcal{R})$, $\mathcal{R}(V)$ is also compact.

(2) $\mathcal{R}$ is called "2-type dependent" if:

(i) the 2-type of $\mathcal{R}(V)$ depends on the 2-type of V for each $V \in Top(\mathcal{R})$

(ii) If $\Phi(V)$ is the fibre of $\tau(V) : V \longrightarrow \mathcal{R}(V)$ then $H_*(\Phi(V);R)$ depends only on the 2-type of V .

Note that the conditions on the objects of $Top(\mathcal{R})$ are more relaxed than the ones imposed in the previous section.

3.2 <u>Theorem</u>

Let G be a p-group, and let $\mathcal{R}$ be a functor with coefficient system $\mathbb{F}_p$ which is compact, 2-type dependent, and adapted to $\mathcal{P}(G)$. Let X be a G-space such that $Map_P(E,\mathcal{R}(X))$ belongs to $Top(\mathcal{R})$ for all $P \in \mathcal{P}(G)$. Then there exists a finite G-complex Y and an invariant map $f : E \times Y \longrightarrow E \times X$ which induces $H_*(-;\mathbb{F}_p)$-isomorphism if and only if:

(i) For each $P \in \mathcal{P}(G)$, there exists a finite complex $F(P)$ and a map $\eta : F(P) \longrightarrow Map_P(E,X)$ such that

$$\hat{\eta}_* : H_*(\mathcal{R}(F(P));\mathbb{F}_p) \longrightarrow H_*(Map_P(E,\mathcal{R}(X));\mathbb{F}_p)$$

is an isomorphism.

(ii) For each pair of subgroups $P_2 \triangleleft P_1$ such that P_1/P_2 is p-elementary abelian the map $\lambda(P_1,P_2) : Map_{P_1}(E,X) \longrightarrow Map_{P_2}(E,X)$ induces a Borel-Quillen localized homology isomorphism.

To obtain sharper results which would include all finite groups and replace homology equivalence f in the above by a G-homotopy equivalence, one should take into account the finiteness obstructions. The above theorem in its present form will be used for each p-subgroup of G, if G is not a p-group.

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