Contraction of Gorenstein polarized varieties with high nef value

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MPI / 93-62

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MSC numbers: 14E30, 14J40, 14J45

Abstract.

Let X be a normal complex projective variety with Gorenstein-terminal singularities; let L be an ample line bundle over X and let K_X denote the canonical sheaf of X. Assuming that K_X is not nef we study the contractions of extremal faces which are supported by divisors of the form $K_X + \tau L$ with $\tau \geq (n-1)$. In other words we classify the pair (X, L) which has "nef value" $= \tau(X, L) \geq (n-2)$ as well as the structure of their associate "nef value morphisms". In the case $\tau = (n-2)$ we assume also that X is factorial. We study moreover the general case in which $(K_X + rL)$ is nef and big but not ample and the dimension of the fibers of the nef value morphism is less or equal then r.

Introduction and statement of the theorems.

Let X be a normal projective variety defined over the field of complex numbers and let L be an ample line bundle over X. We assume that X has at worst **terminal singularities**, i.e. the smallest class in which Mori's program can work; by K_X let us denote the canonical sheaf of X.

Assume that K_X is not nef; the *nef value* of the pair (X, L) is a real number defined as follow

$$\tau(X, L) = \min\{t \in \mathbf{R}, (K_X + tL) \text{ is nef}\}$$

(see [B-S1]; (0.8) or [K-M-M]; (4.1)).

By the Kawamata rationality theorem τ is a rational number and by the Kawamata base point free theorem $K_X + \tau L$ is semiample; in particular there exists a projective surjective morphism $\phi : X \to W$ into a normal variety W which is given by sections of a high multiple of $K_X + \tau L$; ϕ is called the *nef value morphism*.

Applying Mori theory and adjunction theory one can classify the pairs (X, L) with $\tau > (n-1)$; more precisely they are the projective space, the hyperquadric, $\mathbf{P}^{(n-1)}$-bundles over a smooth curve, generalized cones over either a Veronese curve or a Veronese surface. See for this the papers of M. Beltrametti-A.J. Sommese ([B-S1], section 2) and of T. Fujita ([F3], section 1).

If $\tau = (n-1)$ and if the morphism ϕ is of fiber type, that is $dimW < dimX$ or, equivalently, $K_X + (n-1)L$ is not big, then X is either a singular Del Pezzo variety, or a quadric fibration over a smooth curve, or a $\mathbf{P}^{(n-2)}$-bundle over a normal surface (see again [B-S1] or [F3]; for the definitions see section 0).

In this paper we want to prove the following

Theorem 1. *Let X be a projective variety with terminal singularities and let L be an ample line bundle on X. Assume also that X has Gorenstein singularities. If the nef value of the pair (X, L) is $\tau = (n-1)$ and the nef morphism ϕ is birational then $\phi : X \longrightarrow X'$ is the simultaneous contraction to distinct smooth points of disjoint divisors $E_i \cong \mathbf{P}^{n-1}$ such that $E_i \subset reg(X)$, $\mathcal{O}_{E_i}(E_i) \cong \mathcal{O}_{\mathbf{P}^{n-1}}(-1)$ and $L_{E_i} \cong \mathcal{O}(-1)$ for $i = 1, \ldots, t$. Furthermore $L' := (\phi L)^{**}$ and $K_{X'} + (n-1)L'$ are ample and $K_X + (n-1)L \cong \phi^*(K_{X'} + (n-1)L')$*

The pair (X', L') is called the *first reduction* of the pair (X, L), using the definition given by A.J.Sommese.

The above theorem is well known in the smooth case, (see [F1] and [So]). In the singular case there are results when X is normal and factorial (see [B-S1], Theorem (3.1.4)) and when X is Gorenstein and L has a smooth surface section (see [An] and [So]). The proof contained in this paper follows strongly the line of [An] using recent results of [A-W].

We prove also this general theorem:

Theorem 2. *Let X be a projective variety and assume it has terminal, $\mathbf{Q}$-factorial, Gorenstein singularities; let L be an ample line bundle on X. Assume that the nef value of the pair (X, L) is $\tau = r = \frac{u}{v}$, with u, v coprime positive integers; assume also that $u \geq dimF$ for every fiber F of ϕ and that the nef morphism ϕ is birational. Then $\phi : X \longrightarrow X'$ is the simultaneous contraction of disjoint prime divisors E_i to algebraic subset $B_i \subset X'$*

with $dim B_i = n - u - 1$, X' has terminal, $\mathbf{Q}$-factorial singularities and all fibers F are isomorphic to $\mathbf{P}^r$. Moreover the general fibers F' are contained in the smooth set of X and $N_{E/X|F'} \cong \mathcal{O}(-1)$.

This last theorem is proved in the smooth case in [B-S2] and in a stronger form, but always in the smooth case, in [A-W].

From now on we assume that X is a projective variety with terminal and **factorial singularities** and that L is a line bundle on it. The case in which $\tau(X,L) > (n-2)$ was studied in the sections 2 and 3 of [B-S]; in the section 2 of the present paper we consider the case $\tau(X,L) = (n-2)$. In the smooth case this was studied in [B-S], section 4, while in dimension 3 it was proved in [Mo], in the smooth case, and in [Cu] in the Gorenstein case (we apply some proofs contained in these last papers). More precisely we prove:

Theorem 3. *Let X be a projective variety and assume it has terminal and factorial singularities; let L be a line bundle on X. Assume that the nef value $\tau(X,L)$ of the pair (X,L) is $(n-2)$ and let $\phi : X \longrightarrow Y$ be the nef value morphism. Then either (for the definitions see the section 0)*

(3.1) $K_X \approx -(n-2)L$, i.e. (X,L) is a (singular) Mukai variety,

(3.2) (X,L) is a Del Pezzo fibration over a smooth curve under ϕ,

(3.3) (X,L) is a quadric fibration over a normal surface under ϕ; if moreover ϕ is an elementary contraction (i.e. the contraction of an extremal ray), then (X,L) is quadric bundle over a smooth surface under ϕ,

(3.4) (X,L) is a scroll over a normal three dimensional variety with terminal singularities under ϕ (if X is smooth then the image is also smooth),

(3.5) ϕ is a divisorial contraction and it is an isomorphism outside $\phi^{-1}(Z)$ where $Z \subset Y$ is an algebraic subset of Y such that $dim(Z) \leq 1$. Let R be an extremal ray on X such that $(K_X + (n-2)L)R = 0$ and let E be the exceptional locus of R. Then ϕ factors through $\rho = \rho_R : X \longrightarrow W$, the contraction morphism of R and we have the followig possibilities for ρ:

(i) $\rho(E) = C$ is 1-dimensional, Y is smooth near C, C is a locally complete intersection and ρ is the blown-up of the ideal sheaf I_C.

(ii) $\rho(E) = \{x\}$ is a 0-dimensional and either

(a) $(E, L_E) \cong (\mathbf{P}^{(n-1)}, \mathcal{O}(1))$, with $N_{E_X} \cong \mathcal{O}_{\mathbf{P}^{(n-2)}}(-2)$, or

(b) $(E, L_E) \cong (\mathbf{Q}, \mathcal{O}_{\mathbf{Q}}(1)$ with $N_{E_X} \cong \mathcal{O}_{\mathbf{Q}}(-1)$, $\mathbf{Q}$ (possibly singular) hyperquadric in $\mathbf{P}^n$.

Also in these two last cases ρ is the blown-up of the ideal sheaf I_p.

If $n > 3$ and ϕ is birational then all the exceptional locus of the extremal rays contracted by ϕ are disjoint, therefore ϕ is the simultaneous contraction of all the above described exceptional sets (3.5).

I would like to thank J. Wiśniewski for some remarks on a first version of the paper and Andrew J. Sommese for useful conversations. I also like to thank the Max-Planck-Institute für Mathematik in Bonn, where the last part of this work was done, for support and hospitality. I was also partially supported by MURST and GNSAGA.

0. Notation and preliminaries.

(0.1). We use the standard notations from algebraic geometry. Our language is compatible with this of [K-M-M] to which we refer for definitions of the following: **Q**-divisor, **Q**-Gorenstein, numerically effective, terminal or log terminal singularities, ...

We just explain some special definition used in the statements. Let X be a normal, r-Gorenstein variety of dimension n and L be an an ample line bundle on X. The pair (X, L) is called a scroll (respectively a quadric fibration, respectively a Del Pezzo fibration) over a normal variety Y of dimension m if there exists a surjective morphism with connected fibers $\phi : X \to Y$ such that

$$r(K_X + (n - m + 1)L \approx p^*\mathcal{L}$$

(respectively $r(K_X + (n - m)L \approx p^*\mathcal{L}$; respectively $r(K_X + (n - m - 1)L \approx p^*\mathcal{L})$ for some ample line bundle $\mathcal{L}$ on Y. A projective n-dimensional normal variety X is called a quadric bundle over a projective variety Y of dimension r if there exists a surjective morphism $\phi : X \to Y$ such that every fiber is isomorphic to a quadric in $\mathbf{P}^{(n-r+1)}$ and if there exists a vector bundle E of rank $(n - r + 2)$ on Y and an embedding of X as a subvariety of $\mathbf{P}(E)$.

(0.2). Let X be a projective normal variety of dimension n defined over the field of complex numbers and let L be an ample line bundle on X.

Assume in this section that X has at most log-terminal singularities.

(0.3) Let R be an extremal ray on X and let $\rho = \rho_R : X \longrightarrow W$ be the *contraction morphism* of R.

(0.3.1) Observe that if τ is the nef value of the pair (X, L) and R is an extremal ray such that $(K_X + \tau L)R = 0$, then the nef value morphism of (X, L) factors through ρ_R.

The following is one of the main result in the paper [A-W].

Theorem (0.4). *(see [A-W], theorem (5.1) and lemma (5.3)) Let $\phi : X \to W$ be a nef value morphism for the pair (X, L) with nef value $\tau = r$; assume also that X has log terminal singularities. Let F be a fiber of ϕ. Assume moreover that*

(5.1.1) *either* $dim F < r + 1$ *if* $dim Z < dim X$
or $dim F \leq r + 1$ *if* ϕ *is birational.*

Then there exists a divisor G from $|L|$ which does not contain any component of the fiber F and which has at worst log terminal singularities on F. Moreover the evaluation morphism $\phi^*\phi_*L \to L$ is surjective at every point of F.

Corollary (0.5). *In the hypothesis of the theorem (0.4) and in order to study the structure of the nef value morphism it is possible to assume that L is base point free.*

Proof. Observe first that we can change L with $L + m(K_X + rL)$, where m is any positive rational number such that $m(K_X + rL)$ is Cartier. If $m >> 0$ then $L + m(K_X + rL)$ is base point free; by abuse of notation this bundle will be called again L.

Lemma (0.6). *(see [F3], lemma 1.5) Let $\rho : X \longrightarrow W$ be the contraction morphism of an extremal ray R as above. Suppose that ρ is birational and that $dim\rho^{-1}(x) = k > 0$ for a point x in W. Then*

$$(K_X + (k+1)A)R > 0$$

for any ρ-ample line bundle A. Moreover if $(K_X + kA)R \leq 0$ the normalization of any k-dimensional component of $\rho^{-1}(x)$ is isomorphic to $\mathbf{P}^k$ and the pull back of A on it is $\mathcal{O}(1)$.

Lemma (0.7). *(see [B-S1], Corollary 0.6.1 and [F3],Theorem 2.4) Let (X, L) be as above and let R_1, R_2 be two distinct extremal rays of divisorial type on X. Let E_1, E_2 be the loci of R_1, R_2 respectively and assume that E_i are $\mathbf{Q}$-Cartier. Assume also that $(K_X + tL)R_i = 0$ for some rational number t, $i = 1,2$. Let $\lceil t \rceil = r$ be the smallest integer $\geq t$. If $\lceil t \rceil \geq (n+1)/2$ then E_1, E_2 are disjoint. Moreover, the same is true in the case $t = (n-2)$ and $n > 3$ (i.e. for $n = 4$).*

Proof. The above result is proved in [B-S1] and [F3] with slightly different hypothesis. We will follow here the proof of [F3]. Let $S = E_1 \cap E_2$; we have that $dim(S) = (n-2)$ (since the E_i are $\mathbf{Q}$-Cartier). Let then Y be a fiber of the map $\rho_{2|S} : S \rightarrow \rho_2(E_2)$. Since $(K_X + tL)R_i = 0$ by the lemma (0.6) we have that $dim(F_i) \geq r$ for all fiber F_i of ρ_i; in particular this implies that $dim\rho_i(E_i) \leq (n - r - 1)$ and that $dimY \geq (r-1)$. By our hypothesis $dimY > dim\rho_1(E_1)$; then there exists a curve in Y contracted by ρ_1 (and of course by ρ_2): this will give a contradiction. The case in which $n = 4$ and $t = (n-2)$ can be proved exactly as in the last part of [F3].

Proposition (0.8). *(Bertini-Seidenberg) Assume that X has at worst terminal (resp. canonical, resp. log terminal) singularities and that L is base point free. Then the general element of L is normal and has at worst terminal (resp. canonical, resp. log terminal) singularities.*

Proof. Let $f : Y \rightarrow X$ be a resolution of the singularities of X. Since f^*L is base point free we know by the usual Bertini theorem that a dense set of elements of f^*L are smooth. Let G be one of the dense set U of elements of L such that $\tilde{G} = f^{-1}(G)$ is smooth. It is easy to prove that G is normal (Seidenberg theorem), that $sing(G) \subset sing(X)$ and, by standard adjunction considerations, that G has at worst log terminal singularities (resp.can., term.).

1. Proof of the theorems 1 and 2.

(1.0) **Assume** from now on that X has at most terminal singularities; in particular X has rational singularities (see (0.2.7) in [K-M-M]) and $codim(Sing(X)) \geq 3$ (see (0.2.3) in [B-S1]).

Let u, v coprime positive integers as in the theorem 2. Then, if a, b are positive integers such that $av - bu = 1$, we have that the line bundle $\tilde{L} = bK_X + aL$ is ample and that u is the nef value of the pair $(X, \tilde{L})$; this is noticed and proved in [B-S2], lemma (1.2). We will from now on consider the line bundle $\tilde{L}$ instead of L and, by abuse, we will call it again L; we then consider the pair (X, L) with nef value $r = u$.

(1.1) Let $\phi : X \longrightarrow X'$ be the nef value morphism, which we assume to be birational, R be an extremal ray on X such that $(K_X + rL)R = 0$ and $\rho : X \longrightarrow Y$ the contraction of R. Then ϕ factors through ρ.

We want first to understand the structure of the map ρ; let F be a fiber and E be the exceptional locus of ρ. Note that, by (0.6), we have that $dimF \geq r$; on the other hand, since ϕ is birational, we have that $dimF = (n-1)$ in the first theorem. For the second we have the hypothesis that $dimF \leq r$ and therefore $dimF = r$. Applying again the lemma (0.6) we get that the normalization of F is $\mathbf{P}^r$ and that the pull back of L on this normalization is $\mathcal{O}(1)$. But, by the theorem (0.4), L is base point free on F and therefore $h^0(L_{|F}) \geq n$. Now it is obvious, computing for instance the delta genus of the pair (X, L) (see [F0]), that $(F, L) = (\mathbf{P}^r, \mathcal{O}(1))$.

Take now $n - 1 - r$ general very ample divisors on Z , call them H_i, and consider the intersection of their pull-back to X. The resulting variety, X'', has again terminal singularities by the Bertini theorem; call again, by abuse of notation, $L = L_{|X''}$ and let $n'' = dimX'' = r + 1$. The restriction of ρ to X'' is given by a high multiple of $K_{X''} + rL$ and contracts a general fiber F, being now a divisor in X'', to a point. (Note that this step is empty for the theorem 1)

By the theorem (0.4) there exist (an open subset of) sections of L not containing the fiber F and with at worst terminal singularities.

We then take $(r+1-2)$ general sections of L not containing F and intersecting scheme theoretically with X'' in a surface with terminal singularities. Since terminal singularities in dimension two are smooth, this surface is smooth. Being L an ample Cartier divisor this implies in particular that $dim(SingX'' \cap F) < n'' - 2$.

Assume that X'' has hypersurface singularities; we can now apply the main theorem of [L-S], namely the theorem (2.1), to our map $\rho_{|X''}$: this says that either $F \cap Sing(X'')$ is empty or of pure dimension $n'' - 2$. Therefore, for what above, F is contained in the smooth locus of X'' and $\rho_{|X''}$ is the blow-down of $F \cong \mathbf{P}^r$ to a smooth point on Y and $N_{F/X''} \cong \mathcal{O}(-1)$. Since X'' is the intersection of Cartier divisors, then X itself is smooth in a neighborhood of F. We can therefore apply the theorem (4.1.iii) of [A-W] and conclude in particular that $dimE = (n-1)$. Therefore ρ is a contraction of divisorial type, E is a prime divisor on X and X' has terminal, $\mathbf{Q}$-factorial singularities (see [K-M-M], proposition (5.1.6)).

We will prove now that if X is Gorenstein then every singular point x is locally a hypersurface (that is if R is the local ring $\mathcal{O}_{X,x}$ of x on X, then R is isomorphic to $\frac{S}{fS}$, where S is a regular local ring of dimension (n+1)). Note first that if X is Gorenstein the same is for X''.

Claim (1.2). *If X'' is Gorenstein then every singular point x is locally a hypersurface.*

Remark (1.2.1). *If the dimension of X'' is three the claim is proved in [L-S]; the following is the proof of [L-S] adapted in higher dimension. It is on the other hand well known that a rational Gorenstein 3-fold singularity is terminal iff it is cDV (compound Du Val; see Corollary 3.12 in [Re]) and therefore, in particular, it is locally a hypersurface.*

Proof. Since L is base point free and ample for every point $x \in X''$ we have that the linear system $|L - x|$ has finite base point. In particular there exists a general divisor, D, of L passing through x and with singularities in codimension two. Since X'' is Gorenstein the same is for D which, by Serre criterion, is therefore also normal. By induction we have $(n-2)$-divisors in the linear system $|L - x|$ which intersect scheme theoretically in a Gorenstein surface, S, containing x. It is easy to see, using the adjunction formula, that $F \cap S$ is a rational curve P, that ρ_S contracts P to a point and that $K_S P = -1$.

We use now the theorem (0.1) in [L-S]: we have that x is an A_n-type rational singularity for some $n \geq 1$ on S and therefore it is a hypersurface singularity on S. Since the divisors in L are locally principal and S is a surface section of L, we have that X'' is a hypersurface at x (and therefore also X).

(1.3) Let us go back to the birational nef value morphism $\phi : X \longrightarrow X'$ and let R_i for i in a finite set of indexes be extremal rays on X such that $(K_X + (n-1)L)R_i = 0$. Let E_i be the loci of the R_i. By the theorem (0.7) and what we have proved above we have that the E_i are pairwise disjoint. The structure of each $\rho_{R_i} : X \longrightarrow Y$, the contraction of R_i, is given above. Therefore ϕ is the simultaneous contraction of all the E_i, and the theorems are proved (see for instance the last part of the proof of the theorem (3.1) in [B-S1]).

2. Proof of the theorem 3.

(2.1) Let $\tau = (n-2)$ be the nef value of the pair (X, L) and let $\phi : X \to Y$ be the nef value morphism.

(2.2) If $dimY < dimX$ then for every fiber F we have $dim(F) \geq (n-3)$ (see for instance the remark (3.1.2) in [A-W]); then it follows easily, by definition, that we are in one of the cases (3.1)-(3.4). It remains to prove the second part of the point (3.3): assume therefore that ϕ is an elementary contraction and that $dim(Y) = 2$; in particular ϕ is equidimensional. Take now an arbitrary point $p \in Y$ and we will show that Y is smooth at p. By the corollary (0.5) we can take $(n-2)$ general sections of L intersecting transversally in a smooth surface S and intersecting $\phi^{-1}(p)$ in a finite numbers of points. Replacing Y with an affine neighborhood of p, we can assume that S and Y are affine and that $S \to Y$ is a finite, generically 2-1 map. The proof of the smoothness of p is now exactly as in [Cu], p. 524, lines 9-17. The rest of the statement follows similarly to [Cu], p. 524, using Grauert criterion (see also [A-B-W]).

(2.3) Assume then that $dimY = dimX$, i.e. ϕ is birational. Let R be an extremal ray on X such that $(K_X + (n-2)L)R = 0$ and $\rho : X \longrightarrow Y$ the contraction of R. We want to understand the structure of the map ρ; let F be a fiber and E be the exceptional locus of ρ. Note that, by (0.6), we have $dimF \geq (n-2)$.

Lemma (2.3.1). *The dimension of the exceptional locus, E, is bigger or equal then (n-1), that is ρ is not a small contraction (see [K-M-M]).*

Proof. Assume for absurd that $dim(E) = dim(F) = (n-2)$. Then we can take (n-3) general sections of L whose intersection is a 3-dimensional, normal, Gorenstein variety with terminal singularities, X', such that $\rho_{|X'}$ is a small contraction. This is in contradiction with the theorem 0 of [Be].

(2.3.2) Assume that $dim(F) = (n-2)$; then we are in the situation of the theorem 2, $\rho(E)$ is an irreducible curve C and all the fiber of ρ have the same dimension. Since we are assuming that X is factorial then Y is k-factorial with $k = E \cdot C$, C an extremal rational curve such that $[C] = R$ (see [B-S], (0.4.4.2)). In our case is immediate to see that $k = 1$, therefore Y is factorial. Take now a point $q \in C$ and $(n-2)$ general sections of L, $\mathcal{D}_1, \ldots, \mathcal{D}_{n-2}$, intersecting transversally in a smooth surface S and intersecting the fiber ρ^{-1} in a finite number of points. Replacing Y with an affine neighborhood of q, we can assume to be in the "affine set-up" described in the section 2 of [A-W]. In particular by the Lemma (2.6.3) in [A-W] we have that the map $\rho_{|S}$ has connected fibers, therefore it is an isomorphim with its image $S' = \rho(S)$. Therefore $S' \subset Y$ is smooth; since S' is an irreducible component of $\rho(\mathcal{D}_1) \cap \ldots \cap \rho(\mathcal{D}_{n-2})$ and Y is factorial, Y is smooth in a neighborhood of C. Moreover C is a local complete intersection since it is a curve lying on a smooth surface. X is clearly the blown up of $I_C = \rho_* \mathcal{O}(-E)$, since $\mathcal{O}(-nE)$ is ρ very ample for $n >> 0$ and $\rho_* \mathcal{O}(-nE) = I_C^n$, since C is a complete intersection.

(2.3.3). Finally we assume that $dim(F) = dim(E) = (n-1)$; we want in this case to compute the Hilbert polynomial of the polarized pair $(E, L_{|E}$ (we refer to [F0] for more details). We can take $(n-3)$ general sections of L and reduce to the case in which X has dimension 3 in order to compute the invariants: $\chi_n(E, L_{|E}) = d(E, L_{|E})$ and $g(E, L_{|E}) = 1-\chi_{n-1}(E, L_{|E})$; in this case is easy to prove that $d(E, L_{|E}) = 1$ or 2 and that $g(E, L_{|E}) = 0$ (see for instance the first part of the proof of the theorem 5. in [Cu]). Then, since $H^i(E, tL_{|E}) = 0$ for $t \geq -(n-3)$, we easily compute the remaining coefficients of the Hilbert polynomial. Using [F0] we conclude then that $(E, L_{|E})$ is as described in (3.5.ii).

To prove that ρ is the blown -up of the ideal sheaf I_p in Y one proceed as in [Mo] in the case in which E is a smooth quadric or the projective space (since in this case, being X factorial, $E \subset reg(X)$). If E is a singular quadric then one conclude exactly as done in [Cu] for the 3-dimensional case (last part of the proof of Theorem 5 in [Cu]).

(2.4) To conclude we apply the lemma (0.7) as in (1.3).

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