On the linear independence of certain theta-series

by

Jürg Kramer

Max-Planck-Institut
für Mathematik
Gottfried-Claren-Str. 26
D-5300 Bonn 3

Mathematisches Institut
der Universität Basel
Rheinsprung 21
CH-4051 Basel

MPI-SFB 85/40

Introduction. Let q be an odd prime and denote by $M_2(\Gamma_0(q))$ resp. $J_{2,1}(\Gamma_0(q))$ the space of modular forms of weight 2 resp. Jacobi forms of weight 2, index 1 with respect to the congruence subgroup $\Gamma_0(q)$. In [Kr] we introduced the Jacobi-theta-series to the quaternary quadratic forms of discriminant q^2 representing 2: they are defined by

$$\vartheta_{\mathcal{I}_\lambda}(\tau,z) = \sum_{\alpha \in \mathcal{I}_\lambda} q_\infty^{n(\alpha)} \zeta^{t(\alpha)} \in J_{2,1}(\Gamma_0(q))$$

$$(q_\infty := \exp(2\pi i\tau)\ ,\quad \tau \in \mathfrak{H}\ ;\quad \zeta := \exp(2\pi i z)\ ,\quad z \in \mathbb{C})\ ,$$

where $\mathcal{I}_1,\dots,\mathcal{I}_T$ is a set of representatives of the isomorphism classes of maximal orders of the (definite) quaternion algebra $K/\mathbb{Q}$, ramified over q and ∞; T denotes the type number of K, and $n(\cdot)$ resp. $t(\cdot)$ is the reduced norm resp. trace of K. We proved that the Jacobi-theta-series are linear independent, if the representation of the classical Hecke-algebra, given by $T(p) \longmapsto B^+(p)$ ($T(p)$ classical Hecke-operator, $B^+(p)$ reduced Brandt-matrix (cf. [Po3]), $p \neq 2, q$ a prime), allows only to reduce the 1-dimensional part over $\mathbb{Q}$, which corresponds to the Eisenstein series of $M_2(\Gamma_0(q))$ (cf. [Kr], Satz II.3). Then, by proving a necessary and sufficient condition for the injectivity of the map $J_{2,1}(\Gamma_0(q)) \longrightarrow M_2(\Gamma_0(q))$ $(f(\tau,z) \longmapsto f(\tau,0))$, we deduced a criterion for the linear independence of the classical theta-series $\vartheta_{\mathcal{I}_\lambda}(\tau) := \vartheta_{\mathcal{I}_\lambda}(\tau,0)$ $(\lambda = 1,\dots,T)$.

In this note we shall give similar criterions for the linear independence of the Jacobi-theta-series resp. classical theta-series attached to the quaternary quadratic forms of discriminant q representing 2 (theorem 2 resp. 3). The results are essentially obtained by translating the above mentioned theorem to certain theta-series to ternary quadratic forms, namely those studied in [Gr]. We explicitly determine the (predicted) linear relation for these theta-series in the case $q = 389$; by the way we are led to the linear relation between the classical theta-series to the quaternary quadratic forms of discriminant 389 representing 2, which was already calculated by Kitaoka (cf.[HH]).

Acknowledgement: I am thankful to M. Eichler and D. Zagier for stimulating discussions, to B. Gross for the manuscript [Gr] and to C. Tschudin (Basel) for his assistance concerning the computer-calculations. Finally I would like to express my hearty thanks to the Max-Planck-Institut für Mathematik for its hospitality during my stay at Bonn.

<u>2. The main result.</u> We restrict to primes $q \equiv 1 \bmod 4$ and consider the equivalence classes of quaternary quadratic forms of discriminant q representing 2; these are in 1-1-correspondence to the similitude classes of certain quaternary lattices of reduced discriminant q (level q), which were especially studied in [Ki]. There one also finds the formula for the corresponding class number T' .

$T' = T - (h(\sqrt{-q})/4 - a/2)$, where

T is the type number of K (cf. introduction),

$h(\sqrt{-q})$ the ideal class number of $\mathbb{Q}(\sqrt{-q})$, and

$$a = \begin{cases} 0 & q \equiv 1 \bmod. 8 \\ 1 & q \equiv 5 \bmod. 8 \end{cases}$$

The lattices $K_1,\dots,K_{T'}$ representing the T' similtude classes can always be chosen such that the first T'' lattices $K_1,\dots,K_{T''}$ $(T'' := h(\sqrt{-q})/4 - a/2)$ have two inequivalent representations of 2, while for the latter ones $K_{T''+1},\dots,K_{T'}$ all representations of 2 are equivalent (cf. [Ki] , § 1.). The quadratic forms corresponding to $K_1,\dots,K_{T'}$ will be denoted by $G_1[x], \dots, G_{T'}[x]$; the matrices G_μ can always be chosen in the form

$$G_\mu = \begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & * & * & * \\ 0 & * & * & * \\ 0 & * & * & * \end{pmatrix} \qquad (\mu = 1,\dots,T') .$$

Putting $r = (1,0,0,0)^t$ and writing r_μ for a column vector with $G_\mu[r_\mu] = 2$, which is inequivalent to $r (\mu = 1,\dots,T'')$, we get the following $T = T' + T''$ Jacobi-theta-series (of Nebentype):

$$\vartheta_{G_\mu, r}(\tau,z) = \sum_{x \in \mathbb{Z}^4} q_\infty^{G_\mu[x]/2} \zeta^{r^t G_\mu x} \qquad (\mu = 1,\dots,T') ,$$

(1)

$$\vartheta_{G_\mu, r_\mu}(\tau,z) = \sum_{x \in \mathbb{Z}^4} q_\infty^{G_\mu[x]/2} \zeta^{r_\mu^t G_\mu x} \qquad (\mu = 1,\dots,T'') .$$

These are obviously elements of the space $J_{2,1}(\Gamma_0(q),\chi_q)$ of Jacobi forms of weight 2, index 1 with respect to $\Gamma_0(q)$ and with the character $\chi_q(\cdot) = \left(\frac{q}{\cdot}\right)$. In [Kr] , Corollar I.3 it is shown that this space is isomorphic to Kohnen's space $M^+_{3/2}(\Gamma_0(4q),\chi_q)$ (cf. [Ko1] , [Ko2]) , Now one easily proves (cf. [Kr], Satz II.1)

Theorem 1. The isomorphism between $J_{2,1}(\Gamma_\circ(q),\chi_q)$ and $M^+_{3/2}(\Gamma_\circ(4q),\chi_q))$ maps the Jacobi-theta-series $\vartheta_{G_{\mu},r}(\tau,z), \vartheta_{G_{\nu},r_\nu}(\tau,z)$ $(\mu = 1,\dots,T';\ \nu = 1,\dots,T'')$ to the theta-series $\vartheta_{M^*_\mu}(\tau), \vartheta_{N^*_\nu}(\tau)$ attached to ternary lattices M^*_μ, N^*_ν of reduced discriminant 32q (level 4q).

Remark 1. The lattices M^*_μ, N^*_ν $(\mu = 1,\dots,T';\ \nu = 1,\dots,T'')$ just represent the similitude classes of ternary lattices of reduced discriminant 32q (level 4q).

Remark 2. In [Kr], Satz II.1 it is proved that the isomorphism between $J_{2,1}(\Gamma_\circ(q))$ and $M^+_{3/2}(\Gamma_\circ(4q))$ maps the Jacobi-theta-series $\vartheta_{I_\lambda}(\tau,z)$ $(\lambda = 1,\dots,T)$ to the theta-series $\vartheta_{L^*_\lambda}(\tau)$ attached to ternary lattices L^*_λ of reduced discriminant $32q^2$ (level 4q). These are the theta-series considered in [Gr], where the dimension of their $\mathbb{C}$-span is determined, which leads to a necessary and sufficient condition for their linear independence.

Remark 3. Dualizing the result of [Po2], theorem 1, one realizes that M^*_μ, N^*_ν can be considered as sublattices of index q in the lattices L^*_λ (eventually by taking suitable other representatives for the similitude classes of ternary lattices of reduced discriminant $32q^2$, which again are denoted by L^*_λ). If we arrange the corresponding maximal orders I_λ such that the first $2T - H$ (H = class number of K) possess principle two-sided ideal of norm q, while for the remaining $H - T$ this ideal is not principle, we have the following inclusions, in the case $q \equiv 1 \mod 8$:

$$(2)\qquad \begin{array}{lll} M^*_\nu & \subset & L^*_{2\nu-1} \quad (\nu = 1,\dots, T'') \\ N^*_\nu & \subset & L^*{}_{2\nu} \quad (\nu = 1,\dots, T'') \\ M^*_{T''+\nu} & \subset & L^*_{2T''+\nu} \quad (\nu = 1,\dots, H-T) \ . \end{array}$$

(2) reflects the correspondence between quaternary quadratic forms of discriminant q^2 and those of discriminant q (both representing 2), which is given in [Ki] and [Po1], and can be stated as follows (if $q \equiv 1$ mod. 8):

$$\{I_{2\nu-1}, I_{2\nu}\} \longleftrightarrow K_\nu \qquad (\nu = 1,\dots,T'')$$
$$\{I_{2T''+\nu}\} \longleftrightarrow K_{T''+\nu} \qquad (\nu = 1,\dots,H-T) \ ;$$

in the case $q \equiv 5$ mod. 8 there is an "exceptional" maximal order with principle two-sided ideal of norm q, which corresponds alone to a quaternary lattice of reduced discriminant q.

We are now in position to give a criterion for the linear independence of the Jacobi-theta-series (1):

<u>Theorem 2.</u> The Jacobi-theta-series (1) are linear independent, if the representation of the Hecke-algebra, given by $T(p) \longmapsto B^+(p)$, allows only to reduce the 1-dimensional part over $\mathbb{Q}$, which corresponds to the Eisenstein series of $M_2(\Gamma_0(q))$.

<u>Proof.</u> Paying attention to the operator $T(q)\colon M^+_{3/2}(\Gamma_0(4q)) \longrightarrow M^+_{3/2}(\Gamma_0(4q),\chi_q)$, defined by $\sum_{n \in \mathbb{N}_0} c(n)q^n_\infty \mid T(q) := \sum_{n \in \mathbb{N}_0} c(qn)q^n_\infty$ (this operator was introduced in [SS], § 3.) and applying remark 3 (especially (2)) we get (if $q \equiv 1$ mod.8):

$$\vartheta_{L^*_{2\nu-1}} \mid T(q)\ (\tau) = \vartheta_{M^*_\nu}(\tau) \qquad (\nu = 1,\ldots,T'')$$

$$\vartheta_{L^*_{2\nu}} \mid T(q)\ (\tau) = \vartheta_{N^*_\nu}(\tau) \qquad (\nu = 1,\ldots,T'')$$

$$\vartheta_{L^*_{2T''+\nu}} \mid T(q)\ (\tau) = \vartheta_{M^*_{T''+\nu}}(\tau) \qquad (\nu = 1,\ldots,H-T)\ .$$

From this one concludes that the operator $T(q)$ induces an isomorphism between the subspaces of $M^+_{3/2}(\Gamma_o(4q))$ resp. $M^+_{3/2}(\Gamma_o(4q),\chi_q)$ spanned by the theta-series $\vartheta_{L^*_\lambda}(\tau)$ $(\lambda = 1,\ldots,T)$ resp. $\vartheta_{M^*_\mu}(\tau)$, $\vartheta_{N^*_\nu}(\tau)$ $(\mu = 1,\ldots,T'\ ;\ \nu = 1,\ldots,T'')$. So theorem 2 follows from theorem 1 and [Kr], Satz II.3 (cf. introduction). An analogous argument applies to $q \equiv 5 \bmod 8$.

Before stating the criterion for the linear independence of the classical theta-series to the quaternary quadratic forms of discriminant q representing 2, given by

$$(3) \qquad \vartheta_{G_\mu}(\tau) := \vartheta_{G_{\mu},r}(\tau,0) \qquad (\mu = 1,\ldots,T')\ ,$$

we define by σ the linear map

$$\sigma : J_{2,1}(\Gamma_o(q),\chi_q) \longrightarrow M_2(\Gamma_o(q),\chi_q)$$

$$f(\tau,z) \longmapsto f(\tau,0)$$

(here $M_2(\Gamma_o(q),\chi_q)$ denotes the space of modular forms of weight 2 with respect to $\Gamma_o(q)$ with the character χ_q) . Evidently $\dim(\ker\sigma) \geq T''$, if the condition in theorem 2 is satisfied (the differences

(4) $\vartheta_{G_{\mu},r}(\tau,z) \quad - \quad \vartheta_{G_{\mu},r_{\mu}}(\tau,z) \quad (\mu = 1,\ldots,T")$

are T" linear independent elements of $\ker\sigma$) .

<u>Theorem 3.</u> The theta-series (3) are linear independent, if the representation of the Hecke-algebra, given by $T(p) \longmapsto B^{+}(p)$, allows only to reduce the 1-dimensional part over $\mathbb{Q}$, which corresponds to the Eisenstein series of $M_2(\Gamma_o(q))$ <u>and</u> if $\dim(\ker\sigma) = T"$.

<u>Proof.</u> It has only to be noted that, under the conditions stated, the differences (4) form a basis of $\ker\sigma$.

<u>3. The example $q = 389$.</u> The necessary and sufficient condition for the linear independence of the theta-series $\vartheta_{L^*_{\lambda}}(\tau)$ (cf. remark 2) is violated for the first time for $q = 389$ $(T = 22 , T' = 17)$. We explicitly calculated the single existing linear relation:

(5) $\sum_{\lambda=1}^{22} \alpha_{\lambda}\vartheta_{L^*_{\lambda}}(\tau) \equiv 0$, where $(\alpha_1,\ldots,\alpha_{22}) =$

$(0,0;+1,-1;+1,-1;+1,0;-1,0;0;0;0;0;+1;+1;+1;+1;-1;-1;-1;-1)$;

the corresponding 22 ternary quadratic forms of discriminant $32q^2$ are listed (in the same order) in the first column of table 1. By remark 2 the same relation is valid for the Jacobi-theta-series $\vartheta_{I_{\lambda}}(\tau,z)$; the corresponding quaternary quadratic forms of discriminant q^2 representing 2 are given in the second column of table 1. It is worth to be noted that by putting $z = 0$ the corresponding theta-series $\vartheta_{I_\lambda}(\tau)$ are also involved in only one linear relation, which is of course again of the same type.

Via the diagram

$$\begin{array}{ccccc} M^{+}_{3/2}(\Gamma_0(4q)) & \xrightarrow{\cong} & J_{2,1}(\Gamma_0(q)) & \xrightarrow{z=0} & M_2(\Gamma_0(q)) \\ T(q)\downarrow \quad \uparrow T(q) & & & & \\ M^{+}_{3/2}(\Gamma_0(4q),\chi_q) & \xrightarrow{\cong} & J_{2,1}(\Gamma_0(q),\chi_q) & \xrightarrow{z=0} & M_2(\Gamma_0(q),\chi_q) \end{array}$$

it is now easy to calculate from (5) the linear relation between the theta-series (3), which was already found by Kitoaka (cf.[HH]):

$$\sum_{\mu=1}^{17} \beta_\mu \vartheta_{G_\mu}(\tau) \equiv 0 , \quad \text{where} \quad (\beta_1,\dots,\beta_{17}) =$$
$$(0;0;0;+1;-1;0;0;0;0;+1;+1;+1;+1;-1;-1;-1;-1) \;;$$

the corresponding 17 quaternary quadratic forms of discriminant q representing 2 are listed in the third column of table 1. Our list especially shows the Kitaoka-Ponomarev-correspondence in the case $q = 389$.

Table 1

Ternary quadratic forms of discriminant $32 \cdot 389^2$	Quaternary quadratic forms of discriminant 389^2, representing 2	Quaternary quadratic forms of discriminant 389, representing 2
1. $\begin{pmatrix} 1040 & 520 & 4 \\ 520 & 1038 & 2 \\ 4 & 2 & 6 \end{pmatrix}$	1. $\begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & -2 & -1 \\ 0 & -2 & 262 & 131 \\ 0 & -1 & 131 & 260 \end{pmatrix}$	1. $\begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & 66 & -65 & 64 \\ 0 & -65 & 66 & -65 \\ 0 & 64 & -65 & 66 \end{pmatrix}$
2. $\begin{pmatrix} 784 & 4 & 12 \\ 4 & 262 & 8 \\ 12 & 8 & 24 \end{pmatrix}$	2. $\begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & 66 & 2 & -1 \\ 0 & 2 & 6 & -3 \\ 0 & -1 & -3 & 196 \end{pmatrix}$	
3. $\begin{pmatrix} 814 & -72 & 10 \\ -72 & 144 & -20 \\ 10 & -20 & 46 \end{pmatrix}$	3. $\begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & 12 & 5 & -9 \\ 0 & 5 & 36 & 13 \\ 0 & -9 & 13 & 210 \end{pmatrix}$	2. $\begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & 10 & -7 & 18 \\ 0 & -7 & 16 & -13 \\ 0 & 18 & -13 & 36 \end{pmatrix}$
4. $\begin{pmatrix} 824 & -56 & -36 \\ -56 & 102 & 10 \\ -36 & 10 & 62 \end{pmatrix}$	4. $\begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & 16 & -18 & -9 \\ 0 & -18 & 46 & 23 \\ 0 & -9 & 23 & 206 \end{pmatrix}$	
5. $\begin{pmatrix} 798 & 18 & -40 \\ 18 & 94 & -36 \\ -40 & -36 & 80 \end{pmatrix}$	5. $\begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & 24 & 9 & -28 \\ 0 & 9 & 20 & -19 \\ 0 & -28 & -19 & 232 \end{pmatrix}$	3. $\begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & 6 & -5 & 10 \\ 0 & -5 & 24 & -9 \\ 0 & 10 & -9 & 20 \end{pmatrix}$
6. $\begin{pmatrix} 798 & -40 & 2 \\ -40 & 80 & -4 \\ 2 & -4 & 78 \end{pmatrix}$	6. $\begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & 20 & 1 & -19 \\ 0 & 1 & 20 & 9 \\ 0 & -19 & 9 & 218 \end{pmatrix}$	
7. $\begin{pmatrix} 832 & 108 & 8 \\ 108 & 216 & 16 \\ 8 & 16 & 30 \end{pmatrix}$	7. $\begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & 8 & 4 & 2 \\ 0 & 4 & 54 & 27 \\ 0 & 2 & 27 & 208 \end{pmatrix}$	4. $\begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & 8 & 2 & 4 \\ 0 & 2 & 14 & 27 \\ 0 & 4 & 27 & 54 \end{pmatrix}$
8. $\begin{pmatrix} 808 & -60 & 8 \\ -60 & 120 & -16 \\ 8 & -16 & 54 \end{pmatrix}$	8. $\begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & 14 & 4 & 2 \\ 0 & 4 & 30 & 15 \\ 0 & 2 & 15 & 202 \end{pmatrix}$	

Table 1, continued

9. $\begin{pmatrix} 830 & 104 & 2 \\ 104 & 208 & 4 \\ 2 & 4 & 30 \end{pmatrix}$ 10. $\begin{pmatrix} 830 & -54 & -50 \\ -54 & 86 & 22 \\ -50 & 22 & 78 \end{pmatrix}$	9. $\begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & 8 & 1 & -7 \\ 0 & 1 & 52 & 25 \\ 0 & -7 & 25 & 214 \end{pmatrix}$ 10. $\begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & 20 & -25 & -32 \\ 0 & -25 & 52 & 51 \\ 0 & -32 & 51 & 252 \end{pmatrix}$	5. $\begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & 14 & -13 & 26 \\ 0 & -13 & 20 & -25 \\ 0 & 26 & -25 & 52 \end{pmatrix}$
11. $\begin{pmatrix} 782 & 2 & -8 \\ 2 & 390 & -4 \\ -8 & -4 & 16 \end{pmatrix}$	11. $\begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & 98 & 1 & -98 \\ 0 & 1 & 4 & -3 \\ 0 & -98 & -3 & 294 \end{pmatrix}$	6. $\begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & -1 & -1 \\ 0 & -1 & 2 & 1 \\ 0 & -1 & 1 & 98 \end{pmatrix}$
12. $\begin{pmatrix} 400 & -28 & -16 \\ -28 & 142 & -30 \\ -16 & -30 & 94 \end{pmatrix}$	12. $\begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & 24 & -16 & -4 \\ 0 & -16 & 44 & 11 \\ 0 & -4 & 11 & 100 \end{pmatrix}$	7. $\begin{pmatrix} 10 & 1 & 8 & 11 \\ 1 & 4 & 1 & 1 \\ 8 & 1 & 12 & 4 \\ 11 & 1 & 4 & 18 \end{pmatrix}$
13. $\begin{pmatrix} 312 & -104 & -36 \\ -104 & 294 & 12 \\ -36 & 12 & 64 \end{pmatrix}$	13. $\begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & 74 & -3 & 26 \\ 0 & -3 & 16 & -9 \\ 0 & 26 & -9 & 78 \end{pmatrix}$	8. $\begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & 6 & 5 & 3 \\ 0 & 5 & 6 & 3 \\ 0 & 3 & 3 & 26 \end{pmatrix}$
14. $\begin{pmatrix} 270 & 12 & -32 \\ 12 & 208 & -36 \\ -32 & -36 & 96 \end{pmatrix}$	14. $\begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & 68 & -3 & -8 \\ 0 & -3 & 52 & 9 \\ 0 & -8 & 9 & 24 \end{pmatrix}$	9. $\begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & 18 & 3 & 17 \\ 0 & 3 & 8 & 3 \\ 0 & 17 & 3 & 18 \end{pmatrix}$
15. $\begin{pmatrix} 408 & -56 & -20 \\ -56 & 206 & 18 \\ -20 & 18 & 62 \end{pmatrix}$	15. $\begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & 16 & -20 & -5 \\ 0 & -20 & 76 & 19 \\ 0 & -5 & 19 & 102 \end{pmatrix}$	10. $\begin{pmatrix} 8 & 1 & 3 & 12 \\ 1 & 4 & 1 & 1 \\ 3 & 1 & 8 & -2 \\ 12 & 1 & -2 & 26 \end{pmatrix}$
16. $\begin{pmatrix} 334 & -138 & -34 \\ -138 & 262 & 42 \\ -34 & 42 & 78 \end{pmatrix}$	16. $\begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & 20 & -30 & -28 \\ 0 & -30 & 106 & 73 \\ 0 & -28 & 73 & 120 \end{pmatrix}$	11. $\begin{pmatrix} 10 & 5 & 4 & 10 \\ 5 & 6 & 3 & 1 \\ 4 & 3 & 8 & -3 \\ 10 & 1 & -3 & 22 \end{pmatrix}$
17. $\begin{pmatrix} 302 & -76 & -52 \\ -76 & 184 & 44 \\ -52 & 44 & 112 \end{pmatrix}$	17. $\begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & 76 & 19 & -13 \\ 0 & 19 & 46 & -11 \\ 0 & -13 & -11 & 28 \end{pmatrix}$	12. $\begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & 10 & 9 & 13 \\ 0 & 9 & 10 & 13 \\ 0 & 13 & 13 & 32 \end{pmatrix}$

<u>Table 1, continued</u>

18. $\begin{pmatrix} 264 & -28 & 12 \\ -28 & 168 & -72 \\ 12 & -72 & 142 \end{pmatrix}$	18. $\begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & 36 & 18 & 3 \\ 0 & 18 & 42 & 7 \\ 0 & 3 & 7 & 66 \end{pmatrix}$	13. $\begin{pmatrix} 4 & 0 & 3 & 7 \\ 0 & 6 & 1 & 0 \\ 3 & 1 & 12 & 6 \\ 7 & 0 & 6 & 14 \end{pmatrix}$
19. $\begin{pmatrix} 406 & -68 & 2 \\ -68 & 272 & -8 \\ 2 & -8 & 46 \end{pmatrix}$	19. $\begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & 12 & 2 & -11 \\ 0 & 2 & 68 & 15 \\ 0 & -11 & 15 & 112 \end{pmatrix}$	14. $\begin{pmatrix} 10 & 2 & 1 & 17 \\ 2 & 4 & 1 & 0 \\ 1 & 1 & 6 & 1 \\ 17 & 0 & 1 & 34 \end{pmatrix}$
20. $\begin{pmatrix} 270 & -42 & -22 \\ -42 & 214 & 38 \\ -22 & 38 & 94 \end{pmatrix}$	20. $\begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & 24 & -33 & -29 \\ 0 & -33 & 96 & 49 \\ 0 & -29 & 49 & 102 \end{pmatrix}$	15. $\begin{pmatrix} 8 & 4 & 3 & 8 \\ 4 & 6 & 1 & 1 \\ 3 & 1 & 8 & -3 \\ 8 & 1 & -3 & 18 \end{pmatrix}$
21. $\begin{pmatrix} 278 & -56 & 4 \\ -56 & 168 & -12 \\ 4 & -12 & 112 \end{pmatrix}$	21. $\begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & 70 & 14 & 1 \\ 0 & 14 & 42 & 3 \\ 0 & 1 & 3 & 28 \end{pmatrix}$	16. $\begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & 10 & 9 & 1 \\ 0 & 9 & 10 & 1 \\ 0 & 1 & 1 & 14 \end{pmatrix}$
22. $\begin{pmatrix} 262 & -102 & -78 \\ -102 & 206 & 66 \\ -78 & 66 & 142 \end{pmatrix}$	22. $\begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & 36 & -52 & -55 \\ 0 & -52 & 120 & 97 \\ 0 & -55 & 97 & 140 \end{pmatrix}$	17. $\begin{pmatrix} 10 & 7 & 5 & 7 \\ 7 & 8 & 3 & 3 \\ 5 & 3 & 10 & -3 \\ 7 & 3 & -3 & 14 \end{pmatrix}$

Literature

[Gr] Gross, B.: Heights and the special values of L-series, to appear in Sém. Math. Sup., Presses Univ. Montréal.

[HH] Hsia, J.S. and Hung, D.C.: Theta series of ternary and quaternary quadratic forms, Inv. math. 73 (1983), 151-156.

[Ki] Kitaoka, Y.: Quaternary even positive definite quadratic forms of prime discriminant, Nagoya Math.J. 52 (1973), 147-161.

[Ko1] Kohnen, W.: Modular forms of half-integral weight on $\Gamma_0(4)$, Math. Ann. 248 (1980), 249-266.

[Ko2] Kohnen, W.: Newforms of half-integral weight, J. reine angew. Math. 333 (1982), 32-72.

[Kr] Kramer, J.: Jacobiformen und Thetareihen, to appear in Manuscripta Math..

[Po1] Ponomarev, P.: A correspondence between quaternary quadratic forms, Nagoya Math. J. 62 (1976), 125-140.

[Po2] Ponomarev, P.: Ternary quadratic forms and an explicit quaternary correspondence, Proceedings of the Conference on Quadratic Forms, Queen's Papers in Pure and Applied Mathematics, No. 46 (1977), 582-594.

[Po3] Ponomarev, P.: Ternary quadratic forms and Shimura's correspondence, Nagoya Math. J. 81 (1981), 123-151.

[SS] Serre, J.-P. and Stark, H.M.: Modular forms of weight 1/2, in: Modular Functions of One Variable VI, Lecture Notes in Math. 627, Springer-Verlag Berlin-Heidelberg-New York 1977, 27-67.