Tables of Spines and 3-Manifolds up to Complexity 7

Sergei V. Matveev[1]

This paper is a development of [4]. First, I reproduce from [4] the tables of minimal special spines of closed orientable irreducible 3-manifolds up to complexity 6. The reason is that the figures in [4] were prepared in the PCX format and occupy a lot of memory (more than 500 Megabytes in compressed form). This makes it difficult downloading the paper.* Now, after redrawing them in the EPS format, they take only 5 Megabytes. I thank E. Sbrodova for the help in preparing new figures.

Second, we extend the table by adding a part of M. Ovchinnikov's table of closed orientable irreducible manifolds of complexity 7. Almost all such manifolds have minimal special spines modeled on *nonclosed chains* and *triangles with one, two, or three tails*, see Fig. 1. The structure of such manifolds for every complexity is completely described in [4]. So we only list the remaining 22 manifolds by reproducing one minimal special spine for each of them.

Third, we include M. Sokolov's table of the Turaev-Viro invariants of order ≤ 7 for manifolds up to complexity 6.

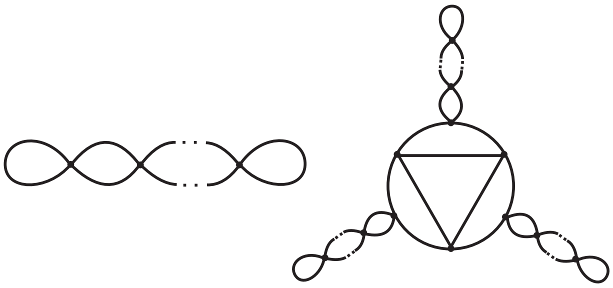


Figure 1: A nonclosed chain and a triangle with three tails

1 Spines and complexity of 3-manifolds

We recall basic notions of the complexity theory of 3-manifolds [3].

Definition 1.1 *Let M be a compact connected 3-dimensional manifold with boundary. A subpolyhedron $P \subset M$ is called a* spine *of M if $M \searrow P$, that is, M collapses to P. By a spine of a closed connected 3-manifold M we mean a spine of $M \setminus IntB^3$ where B^3 is a 3-ball in M. A spine of a disconnected 3-manifold is the union of spines of its connected components.*

[1]Chelyabinsk State University and Max-Planck-Institute für Mathematik (Bonn). The author is partially supported by grants E00-1.0-2.11, RFBR-02-01-01-013, and UR.04.01.033.

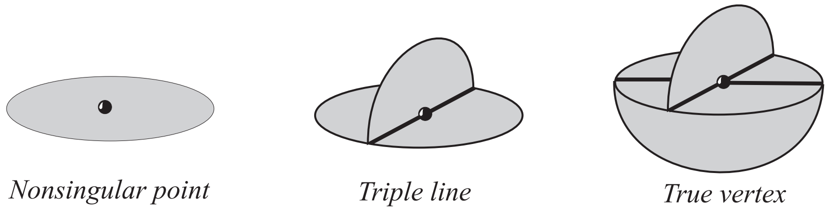


Figure 2: Allowable neighborhoods in a simple polyhedron

Definition 1.2 *A compact polyhedron P is said to be* almost simple *if the link of any of its points can be embedded into a circle with three radii. This graph, which is actually the complete graph with four vertices, will be denoted by Γ_4.*

Definition 1.3 *A compact polyhedron P is called* simple*, if the link of each point $x \in P$ is homeomorphic to one of the following 1-dimensional polyhedra:*

(a) a circle (then x is called nonsingular*);*

(b) a circle with a diameter (then x is a triple point*);*

(c) a circle with three radii (then x is a true vertex*).*

If, in addition, the set of all nonsingular points of P consists of open 2-cells and P has at least one true vertex, then P is called special.

Typical neighborhoods of points of a simple polyhedron are shown in Fig. 2.

It is known (see [1, 3]) that any homeomorphism between special spines can be extended to a homeomorphism between the corresponding manifolds. It means that a special spine P of a 3-manifold M may serve as a presentation of M. Moreover, M can be reconstructed from a regular neighborhood $N(SP)$ in P of the singular graph SP of P: starting from $N(SP)$, one can easily reconstruct P by attaching 2-cells to all the circles in $\partial N(SP)$, and then reconstruct M. If M is orientable, then $N(SP)$ can be embedded into R^3. This gives us a very convenient way for presenting 3-manifolds: we simply draw pictures.

Definition 1.4 *The* complexity *$c(M)$ of a compact 3-manifold M is equal to k if M possesses an almost simple spine with k true vertices and has no almost simple spines with a smaller number of true vertices.*

It turns out that every closed orientable irreducible 3-manifold M of complexity $k > 0$ has a special spine P with exactly k true vertices. Since M has no special spines with $< k$ true vertices, we will call P *minimal*. There are only three closed orientable irreducible 3-manifolds of complexity $k = 0$: S^3, RP^3, and $L(3,1)$. Their minimal almost simple spines are: the point, the projective plane, and the quotient space of D^2 by a free action on ∂D^2 of the group Z_3.

2 Manifolds of complexity ≤ 6

We order closed orientable irreducible 3-manifolds of each complexity $k \leq 6$ arbitrarily and write k_i for the manifold numbered i among manifolds of complexity k. This method of notation is borrowed from knot theory (for example, see [7]). Fortunately for us, each of those manifolds belongs to one of the following classes I – IV.

I. $L_{p,q}$ is the lens space with parameters p, q.

II. S^3/G is the quotient space of S^3 by a free linear action of a nonabelian finite group G. All such groups are known, see [5]. They are:

1. Finite cyclic groups;
2. Groups $Q_{4n}, n \geq 2$;
3. Groups $D_{2^k(2n+1)}, k \geq 3, n \geq 1$;
4. Groups P_{24}, P_{48}, P_{120}, and $P'_{8(3^k)}, k \geq 2$;
5. Direct products of any of these groups with a cyclic group of coprime order.

Lower indices show the orders of the groups. Presentations by generators and relations, and abelian quotients of the groups (coinciding with the first homology groups of the corresponding 3-manifolds) are the following:

1. $Q_{4n} = \langle x, y | \ x^2 = (xy)^2 = y^n \rangle; H_1 = Z_2 \oplus Z_2$ if n is even, and $H_1 = Z_4$ if n is odd.
2. $D_{2^k(2n+1)} = \langle x, y | \ x^{2^k} = 1, y^{2n+1} = 1, xyx^{-1} = y^{-1} \rangle; H_1 = Z_{2^k}$.
3. $P_{24} = \langle x, y \mid x^2 = (xy)^3 = y^3, x^4 = 1 \rangle; H_1 = Z_3$.
4. $P_{48} = \langle x, y \mid x^2 = (xy)^3 = y^4, x^4 = 1 \rangle; H_1 = Z_2$.
5. $P_{120} = \langle x, y \mid x^2 = (xy)^3 = y^5, x^4 = 1 \rangle; H_1 = 0$.
6. $P'_{8(3^k)} = \langle x, y, z \mid x^2 = (xy)^2 = y^2, zxz^{-1} = y, zyz^{-1} = xy, z^{3^k} = 1 \rangle; H_1 = Z_{3^k}$.

III. Let $h_A: T \to T$ be a self-homeomorphism of the torus $T = S^1 \times S^1$ corresponding to an unimodular integer matrix A of order 2. Then $T \times I/A$ is the Stallings manifold with fiber T and monodromy map h_A. In other words, $T \times I/A$ is obtained from $T \times I$ by identifying its boundary tori via h_A.

IV. Recall that the boundary T of the orientable I-bundle $K \tilde{\times} I$ over the Klein bottle K is a torus. Choose a coordinate system (m, l) on the torus such that m projects onto a meridian of K (i.e., a nonseparating orientation-preserving circle) and l double covers a longitude of K (an orientation-reversing circle). Let A be an integer matrix of order 2 with determinant (-1). Then $K \tilde{\times} I \cup K \tilde{\times} I/A$ denotes the manifold obtained by pasting together two copies of $K \tilde{\times} I$ via h_A.

All closed orientable irreducible 3-manifolds of complexity ≤ 6 and the majority of manifolds of complexity 7 are Seifert fibered manifolds. We use the following notation: $(F, (p_1, q_1), \ldots, (p_k, q_k))$ is the orientable Seifert manifold, where F is the base surface and $(p_i, q_i), 1 \leq i \leq k$, are nonnormalized parameters of k disjoint fibers, including all exceptional ones. If F is closed, then k must be positive. By definition, $(F, (p_1, q_1), \ldots, (p_k, q_k))$ can be constructed as follows:

1. Let $F \tilde{\times} S^1$ be a unique orientable S^1-bundle over F. We cut off from it k solid tori $D_i \times S^1 \subset F \tilde{\times} S^1$, where $D_i, 1 \leq i \leq k$ are disjoint discs in F.

2. Then we glue each solid torus $D_i \times S^1$ back along a homeomorphism taking its meridian onto a curve of the type (p_i, q_i).

We do not write Seifert structures and homology groups of lens spaces, since they are well-known: $L_{p,q} = (S^2, (q, p))$ and $H_1(L_{p,q}; Z) = Z_p$.

Table 1: Complexity 0

c_i	M
0_1	S^3
0_2	RP^3
0_3	$L_{3,1}$

Table 2: Complexity 1

c_i	M
1_1	$L_{4,1}$
1_2	$L_{5,2}$

Table 3: Complexity 2

c_i	M
2_1	$L_{5,1}$
2_2	$L_{7,2}$
2_3	$L_{8,3}$

c_i	M	Seifert structure	$H_1(M;Z)$
2_4	S^3/Q_8	$(S^2,(2,1),(2,1)(2,-1))$	$Z_2 \oplus Z_2$

Table 4: Complexity 3

c_i	M
3_1	$L_{6,1}$
3_2	$L_{9,2}$
3_3	$L_{10,3}$

c_i	M
3_4	$L_{11,3}$
3_5	$L_{12,5}$
3_6	$L_{13,5}$

c_i	M	Seifert structure	$H_1(M;Z)$
3_7	S^3/Q_{12}	$(S^2,(2,1),(2,1),(3,-2))$	Z_4

Table 5: Complexity 4

c_i	M
4_1	$L_{7,1}$
4_2	$L_{11,2}$
4_3	$L_{13,3}$
4_4	$L_{14,3}$
4_5	$L_{15,4}$

c_i	M
4_6	$L_{16,7}$
4_7	$L_{17,5}$
4_8	$L_{18,5}$
4_9	$L_{19,7}$
4_{10}	$L_{21,8}$

c_i	M	Seifert structure	$H_1(M;Z)$
4_{11}	$S^3/Q_8 \times Z_3$	$(S^2,(2,1),(2,1),(2,1))$	$Z_2 \oplus Z_6$
4_{12}	S^3/Q_{16}	$(S^2,(2,1),(2,1),(4,-3))$	$Z_2 \oplus Z_2$
4_{13}	S^3/D_{24}	$(S^2,(2,1),(2,1),(3,-1))$	Z_8
4_{14}	S^3/P_{24}	$(S^2,(2,1),(3,1),(3,-2))$	Z_3

Table 6: Complexity 5

c_i	M
5_1	$L_{8,1}$
5_2	$L_{13,2}$
5_3	$L_{16,3}$
5_4	$L_{17,3}$
5_5	$L_{17,4}$
5_6	$L_{19,4}$
5_7	$L_{20,9}$
5_8	$L_{22,5}$
5_9	$L_{23,5}$
5_{10}	$L_{23,7}$

c_i	M
5_{11}	$L_{24,7}$
5_{12}	$L_{25,7}$
5_{13}	$L_{25,9}$
5_{14}	$L_{26,7}$
5_{15}	$L_{27,8}$
5_{16}	$L_{29,8}$
5_{17}	$L_{29,12}$
5_{18}	$L_{30,11}$
5_{19}	$L_{31,12}$
5_{20}	$L_{34,13}$

c_i	M	Seifert structure	$H_1(M;Z)$
5_{21}	$S^3/Q_8 \times Z_5$	$(S^2,(2,1),(2,1),(2,3))$	$Z_2 \oplus Z_{10}$
5_{22}	$S^3/Q_{12} \times Z_5$	$(S^2,(2,1),(2,1),(3,2))$	Z_{20}
5_{23}	$S^3/Q_{16} \times Z_3$	$(S^2,(2,1),(2,1),(4,-1))$	$Z_2 \oplus Z_6$
5_{24}	S^3/Q_{20}	$(S^2,(2,1),(2,1),(5,-4))$	Z_4
5_{25}	$S^3/Q_{20} \times Z_3$	$(S^2,(2,1),(2,1),(5,-2))$	Z_{12}
5_{26}	S^3/D_{40}	$(S^2,(2,1),(2,1),(5,-3))$	Z_8
5_{27}	S^3/D_{48}	$(S^2,(2,1),(2,1),(3,1))$	Z_{16}
5_{28}	$S^3/P_{24} \times Z_5$	$(S^2,(2,1),(3,2),(3,-1))$	Z_{15}
5_{29}	S^3/P_{48}	$(S^2,(2,1),(3,1),(4,-3))$	Z_2
5_{30}	S^3/P'_{72}	$(S^2,(2,1),(3,2),(3,-2))$	Z_9
5_{31}	S^3/P_{120}	$(S^2,(2,1),(3,1),(5,-4))$	0

Table 7: Complexity 6

c_i	M	c_i	M	c_i	M	c_i	M
6_1	$L_{9,1}$	6_{10}	$L_{28,5}$	6_{19}	$L_{35,8}$	6_{28}	$L_{41,16}$
6_2	$L_{15,2}$	6_{11}	$L_{29,9}$	6_{20}	$L_{36,11}$	6_{29}	$L_{43,12}$
6_3	$L_{19,3}$	6_{12}	$L_{30,7}$	6_{21}	$L_{37,8}$	6_{30}	$L_{44,13}$
6_4	$L_{20,3}$	6_{13}	$L_{31,7}$	6_{22}	$L_{37,10}$	6_{31}	$L_{45,19}$
6_5	$L_{21,4}$	6_{14}	$L_{31,11}$	6_{23}	$L_{39,14}$	6_{32}	$L_{46,17}$
6_6	$L_{23,4}$	6_{15}	$L_{32,7}$	6_{24}	$L_{39,16}$	6_{33}	$L_{47,13}$
6_7	$L_{24,5}$	6_{16}	$L_{33,7}$	6_{25}	$L_{40,11}$	6_{34}	$L_{49,18}$
6_8	$L_{24,11}$	6_{17}	$L_{33,10}$	6_{26}	$L_{41,11}$	6_{35}	$L_{50,19}$
6_9	$L_{27,5}$	6_{18}	$L_{34,9}$	6_{27}	$L_{41,12}$	6_{36}	$L_{55,21}$

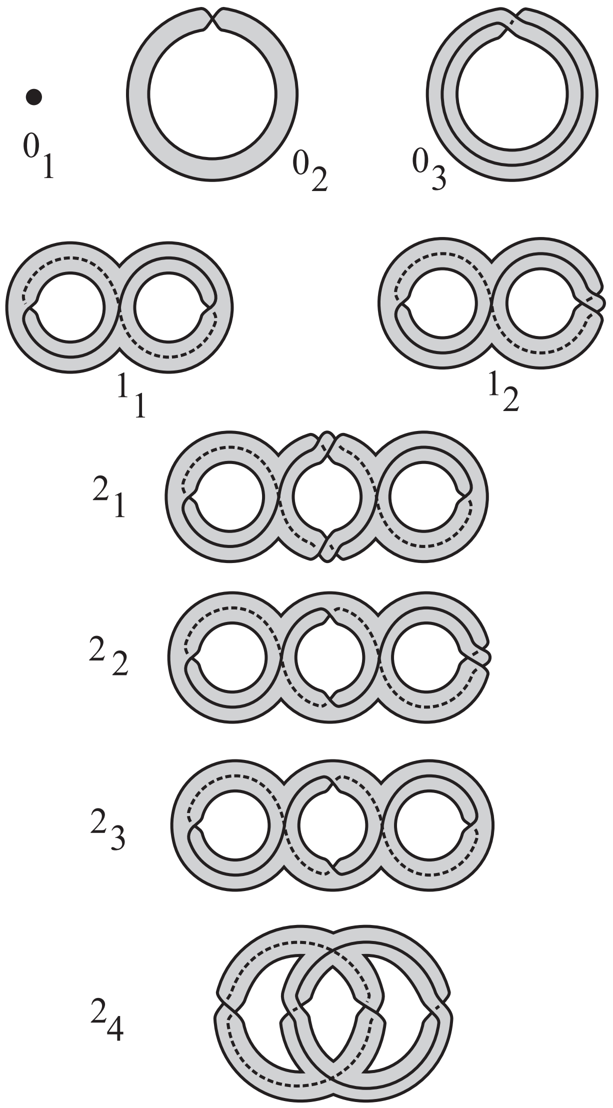
c_i	M	Seifert structure	$H_1(M;Z)$
6_{37}	$S^3/Q_8 \times Z_7$	$(S^2,(2,1),(2,1),(2,5))$	$Z_2 \oplus Z_{14}$
6_{38}	$S^3/Q_{12} \times Z_7$	$(S^2,(2,1),(2,1),(3,4))$	Z_{28}
6_{39}	$S^3/Q_{16} \times Z_5$	$(S^2,(2,1),(2,1),(4,1))$	$Z_2 \oplus Z_{10}$
6_{40}	$S^3/Q_{16} \times Z_7$	$(S^2,(2,1),(2,1),(4,3))$	$Z_2 \oplus Z_{14}$
6_{41}	$S^3/Q_{20} \times Z_7$	$(S^2,(2,1),(2,1),(5,2))$	Z_{28}
6_{42}	S^3/Q_{24}	$(S^2,(2,1),(2,1),(6,-5))$	$Z_2 \oplus Z_2$
6_{43}	$S^3/Q_{28} \times Z_3$	$(S^2,(2,1),(2,1),(7,-4))$	Z_{12}
6_{44}	$S^3/Q_{28} \times Z_5$	$(S^2,(2,1),(2,1),(7,-2))$	Z_{20}
6_{45}	$S^3/Q_{32} \times Z_3$	$(S^2,(2,1),(2,1),(8,-5))$	$Z_2 \oplus Z_6$
6_{46}	$S^3/Q_{32} \times Z_5$	$(S^2,(2,1),(2,1),(8,-3))$	$Z_2 \oplus Z_{10}$
6_{47}	S^3/D_{56}	$(S^2,(2,1),(2,1),(7,-5))$	Z_8
6_{48}	S^3/D_{80}	$(S^2,(2,1),(2,1),(5,-1))$	Z_{16}
6_{49}	S^3/D_{96}	$(S^2,(2,1),(2,1),(3,5))$	Z_{32}
6_{50}	S^3/D_{112}	$(S^2,(2,1),(2,1),(7,-3))$	Z_{16}
6_{51}	S^3/D_{160}	$(S^2,(2,1),(2,1),(5,3))$	Z_{32}
6_{52}	$S^3/P_{24} \times Z_7$	$(S^2,(2,1),(3,1),(3,1))$	Z_{21}
6_{53}	$S^3/P_{24} \times Z_{11}$	$(S^2,(2,1),(3,2),(3,2))$	Z_{33}
6_{54}	$S^3/P_{48} \times Z_5$	$(S^2,(2,1),(3,2),(4,-3))$	Z_{10}
6_{55}	$S^3/P_{48} \times Z_7$	$(S^2,(2,1),(3,1),(4,-1))$	Z_{14}
6_{56}	$S^3/P_{48} \times Z_{11}$	$(S^2,(2,1),(3,2),(4,-1))$	Z_{22}
6_{57}	$S^3/P_{120} \times Z_7$	$(S^2,(2,1),(3,1),(5,-3))$	Z_7
6_{58}	$S^3/P_{120} \times Z_{13}$	$(S^2,(2,1),(3,1),(5,-2))$	Z_{13}
6_{59}	$S^3/P_{120} \times Z_{17}$	$(S^2,(2,1),(3,2),(5,-3))$	Z_{17}
6_{60}	$S^3/P_{120} \times Z_{23}$	$(S^2,(2,1),(3,2),(5,-2))$	Z_{23}
6_{61}	S^3/P'_{216}	$(S^2,(2,1),(3,2),(3,1))$	Z_{27}
6_{62}		$(S^2,(3,2),(3,1),(3,-2))$	$Z_3 \oplus Z_3$
6_{63}		$(S^2,(3,2),(3,2),(3,-2))$	$Z_3 \oplus Z_6$
6_{64}		$(S^2,(3,2),(3,2),(3,-1))$	$Z_3 \oplus Z_9$

Table 7: Complexity 6 (continued)

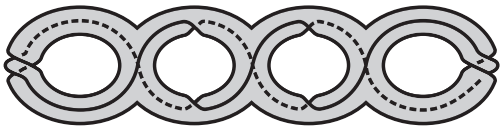
c_i	M	Seifert structure	$H_1(M;Z)$
6_{65}	$T \times I/\begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix}$	$(S^2,(2,-1),(3,1),(6,1))$	Z
6_{66}	$T \times I/\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	$(S^2,(2,-1),(4,1),(4,1))$	$Z_2 \oplus Z$
6_{67}	$T \times I/\begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix}$	$(S^2,(3,-1),(3,1),(3,1))$	$Z_3 \oplus Z$
6_{68}	$T \times I/\begin{pmatrix} -1 & 0 \\ -1 & -1 \end{pmatrix}$	$(K,(1,1))$	$Z_4 \oplus Z$
6_{69}	$T \times I/\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$	$(T,(1,1))$	$Z \oplus Z$
6_{70}	$T \times I/\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$	$(S^2,(2,1),(2,-1),(2,1),$ $(2,-1))$	$Z_2 \oplus Z_2 \oplus Z$
6_{71}	$T \times I/\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$T \times S^1$	$Z \oplus Z \oplus Z$
6_{72}	$K\tilde{\times}I \cup K\tilde{\times}I/\begin{pmatrix} -1 & 0 \\ -1 & 1 \end{pmatrix}$	$(S^2,(2,1),(2,1),(2,1),$ $(2,-1))$	$Z_2 \oplus Z_2 \oplus Z_4$
6_{73}	$K\tilde{\times}I \cup K\tilde{\times}I/\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$(RP^2,(2,1),(2,-1))$	$Z_4 \oplus Z_4$
6_{74}	$K\tilde{\times}I \cup K\tilde{\times}I/\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$	$(RP^2,(2,1),(2,1))$	$Z_4 \oplus Z_4$

3 Spines of manifolds up to complexity 6

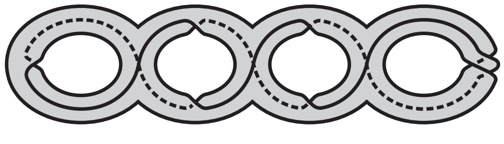
For any manifold $k_i, 1 \leq k \leq 6$, we present all its minimal special spines. Manifolds $0_1 - 0_3$ of complexity 0 are presented by their minimal almost simple spines. The spines are given by regular neighborhoods of their singular graphs.



3_1

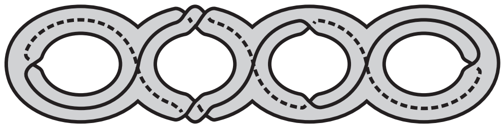


3_2

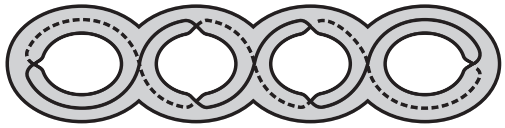


3_3

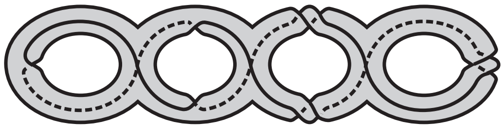
3_4



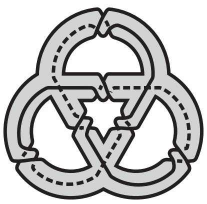
3_5



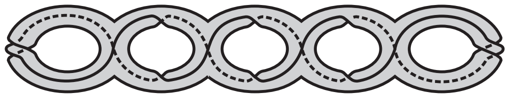
3_6



3_7



4_1



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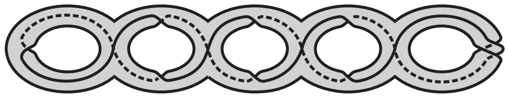

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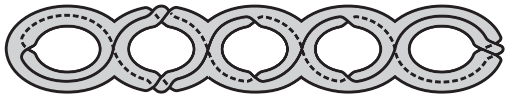

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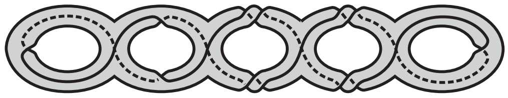

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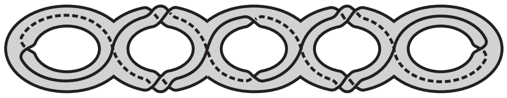
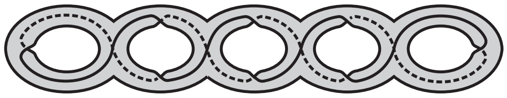
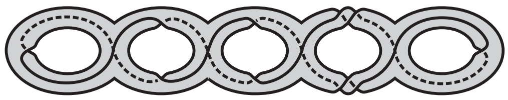


4_6

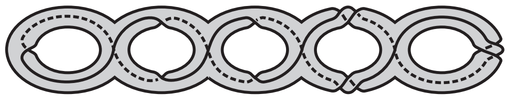


4_7

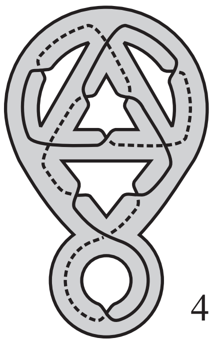


4_8

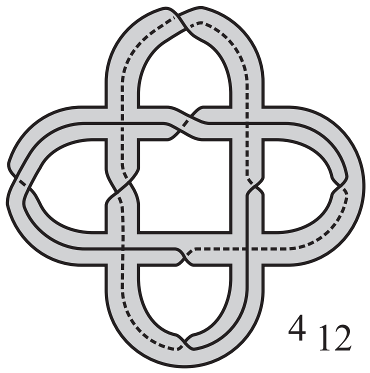
4_9



4_{10}



4_{11}



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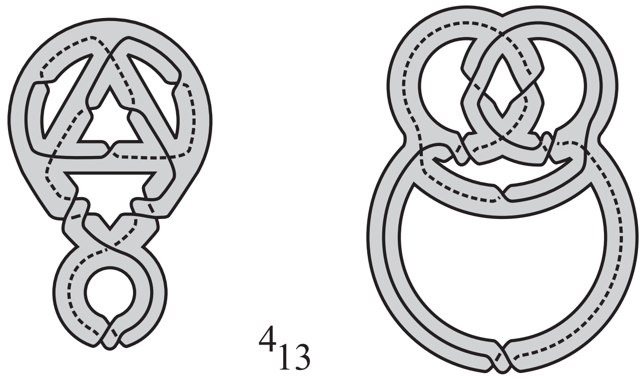

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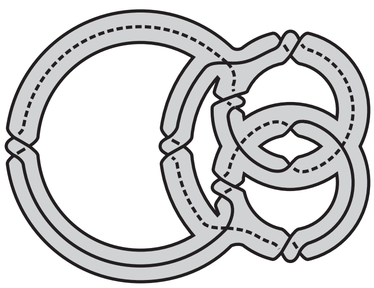

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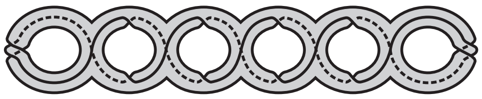

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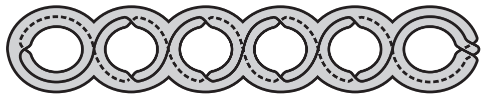

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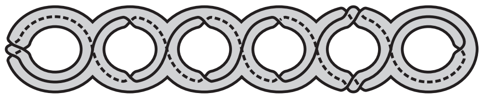

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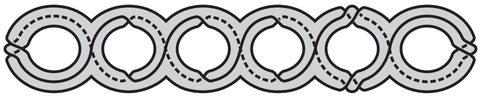

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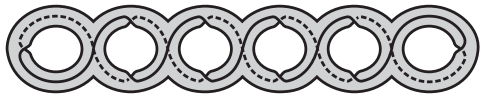

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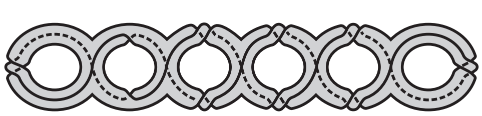

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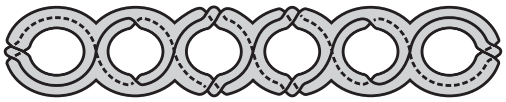

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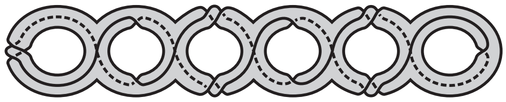

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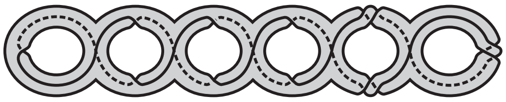

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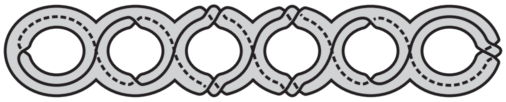

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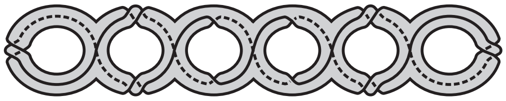

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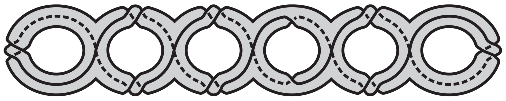

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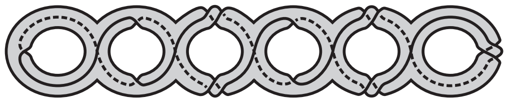

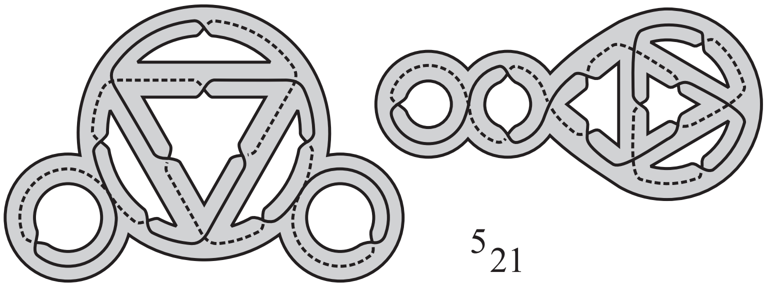

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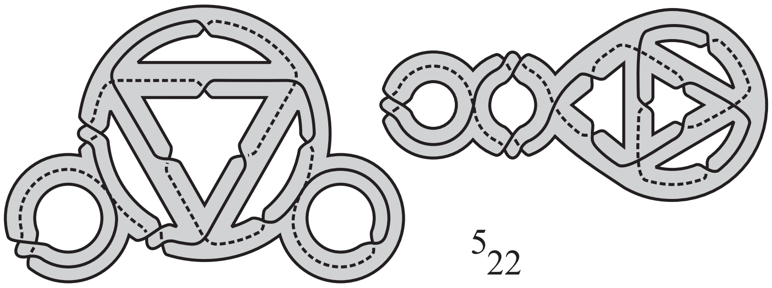

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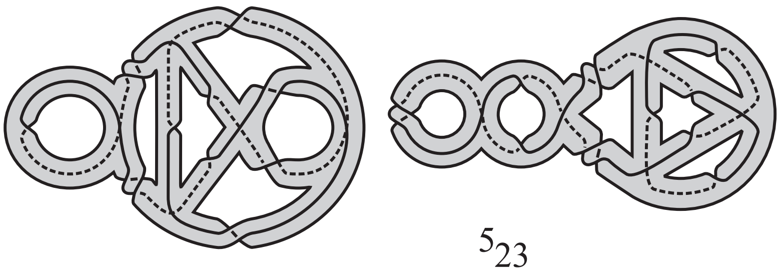

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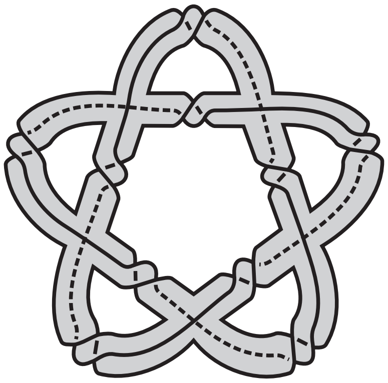

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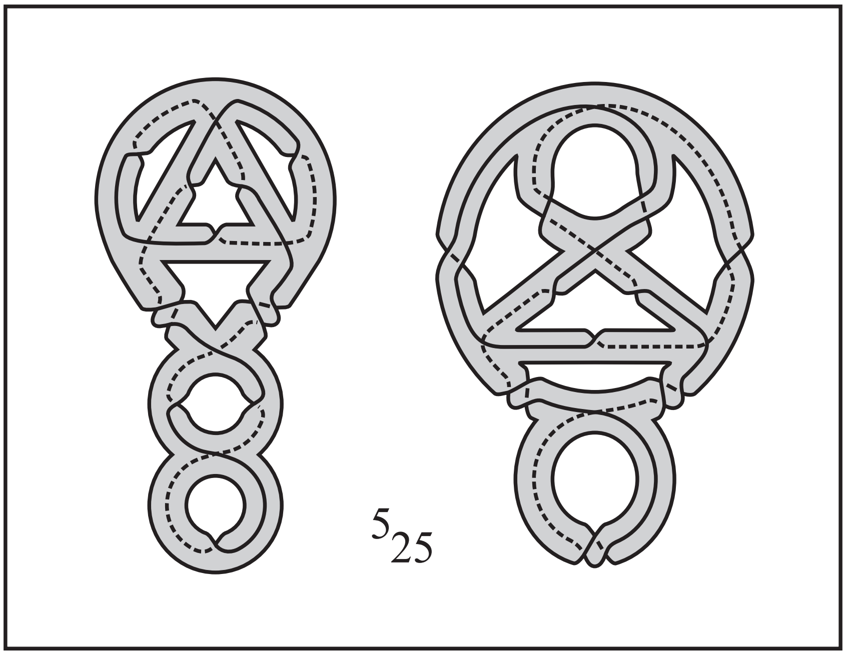

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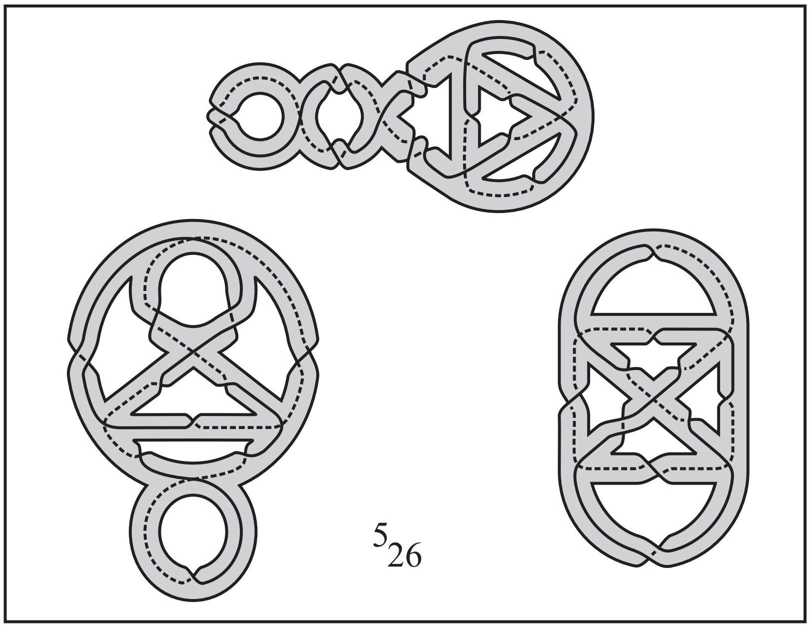

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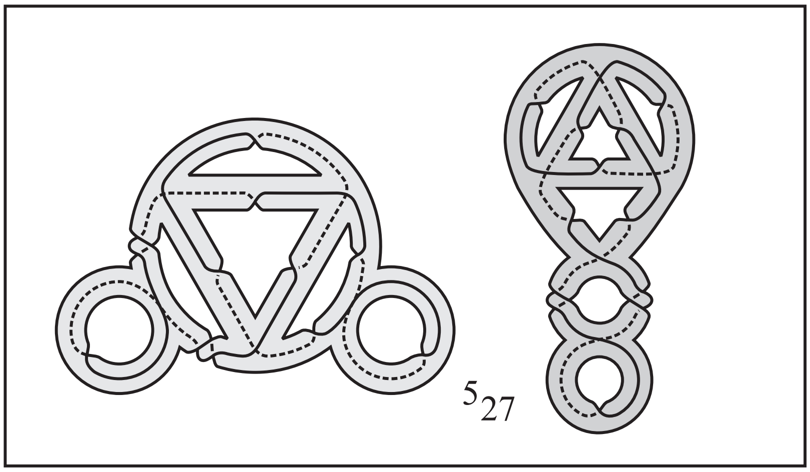

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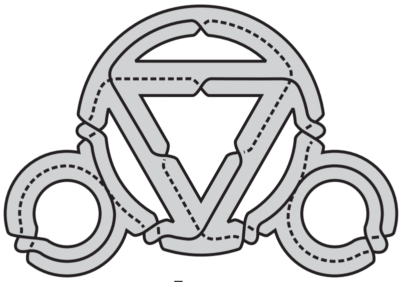

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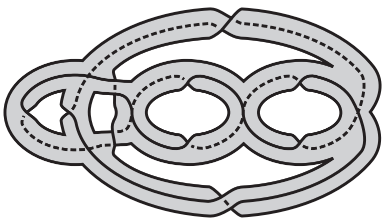

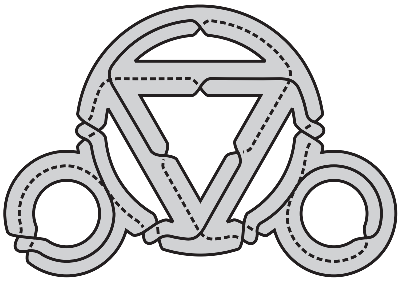

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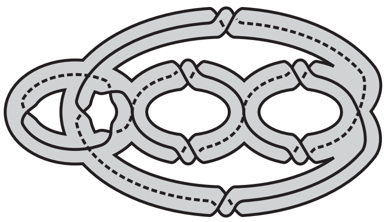

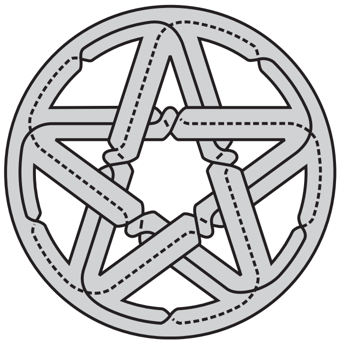

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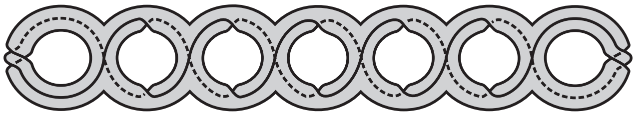
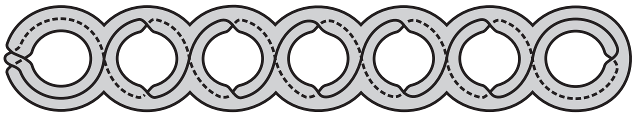
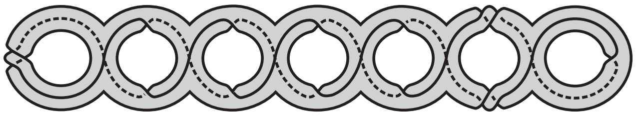
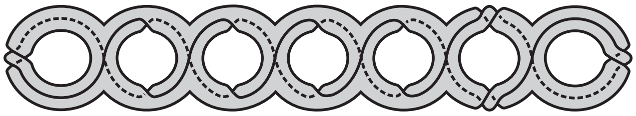
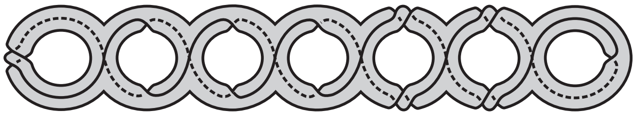
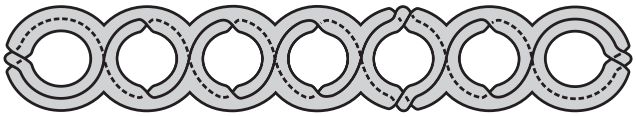
6_1 

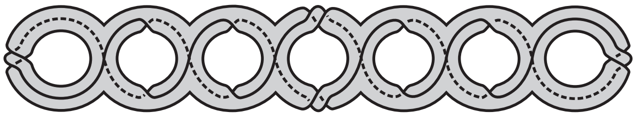
6_2 

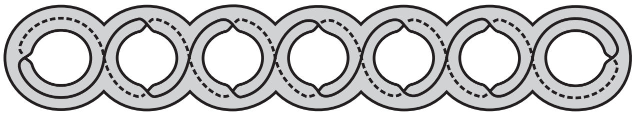
6_3 

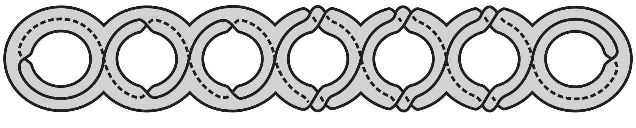
6_4 

6_5 

6_6 

6_7 

6_8 

6_9 

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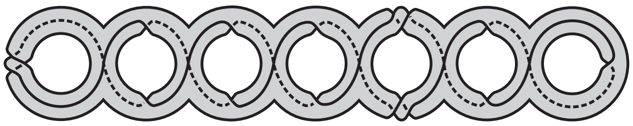

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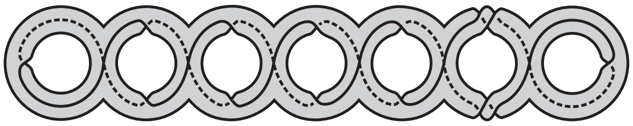

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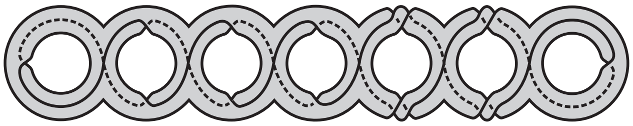

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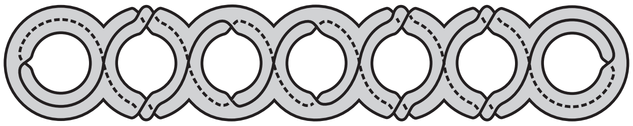

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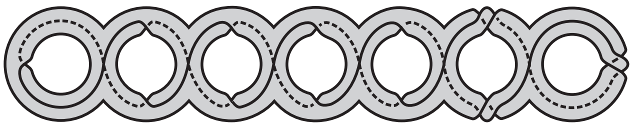

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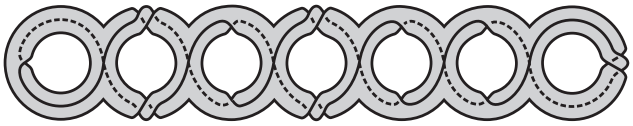

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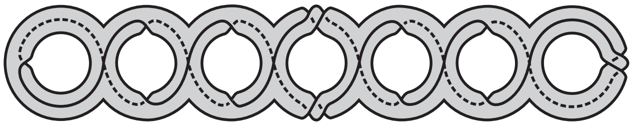

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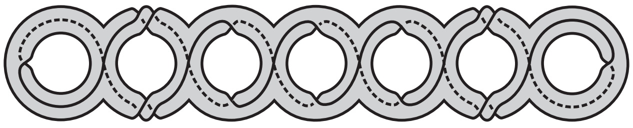

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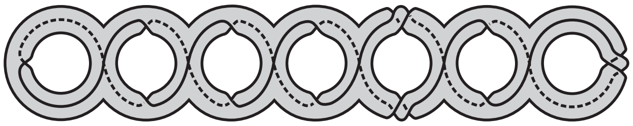

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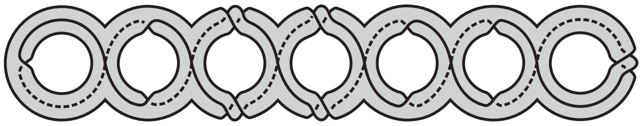

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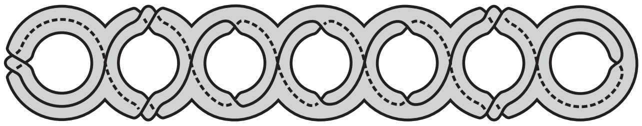

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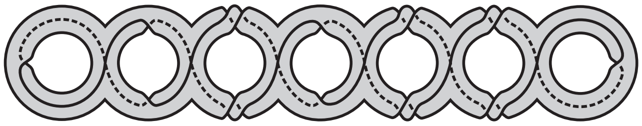

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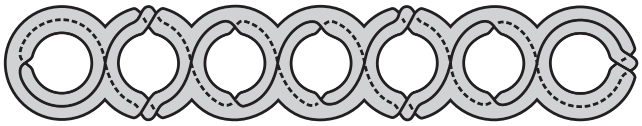

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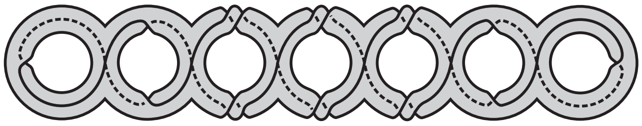

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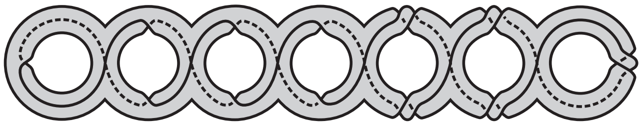

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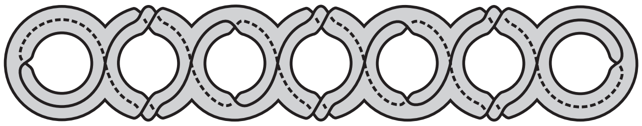

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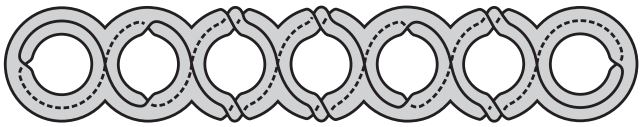

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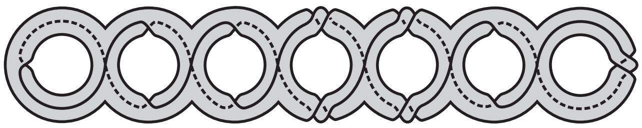

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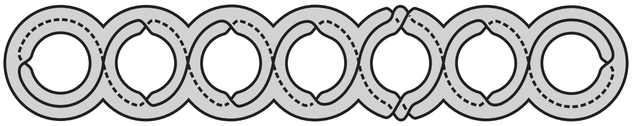

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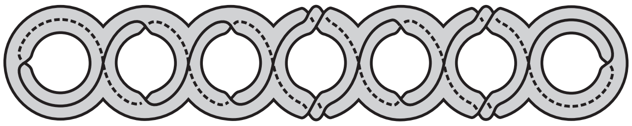

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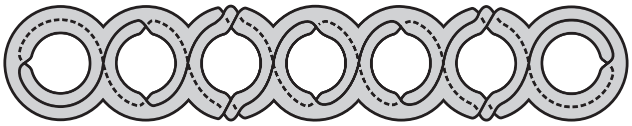

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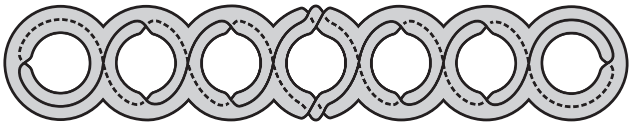

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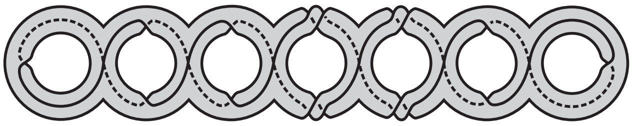

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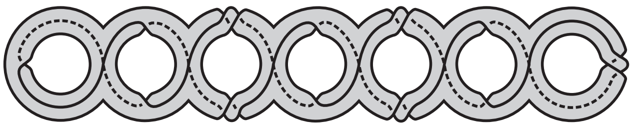

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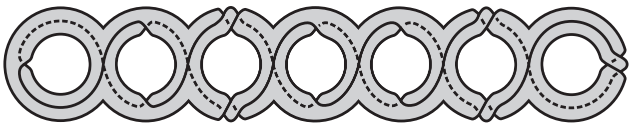

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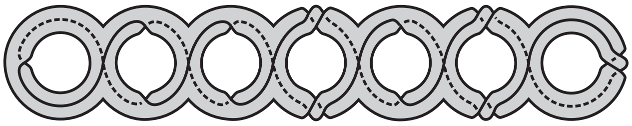

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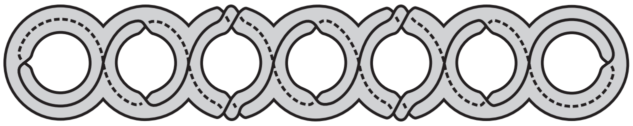

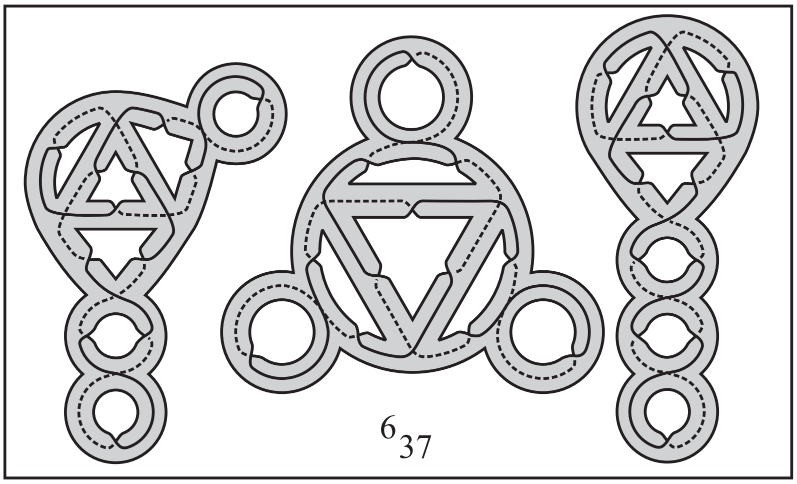

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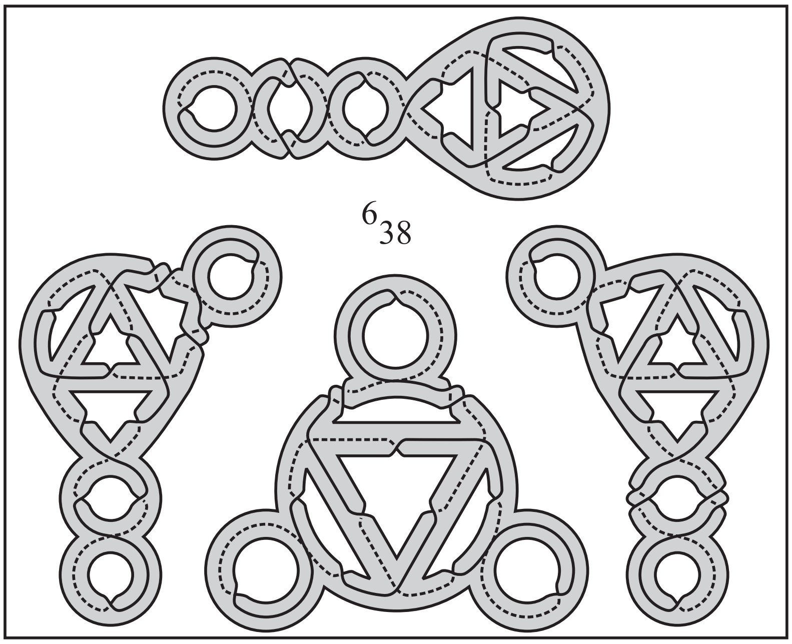

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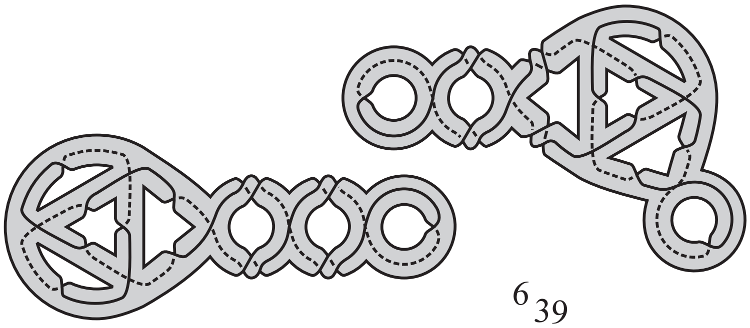

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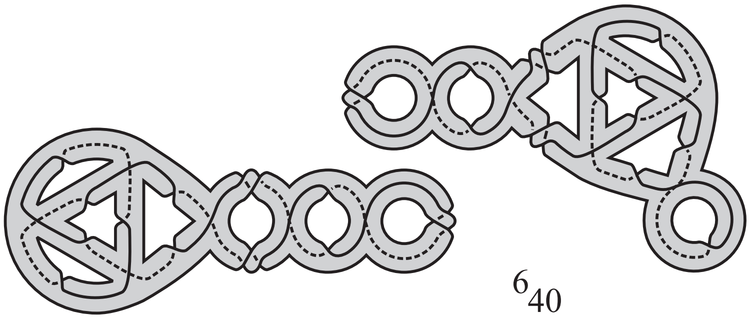

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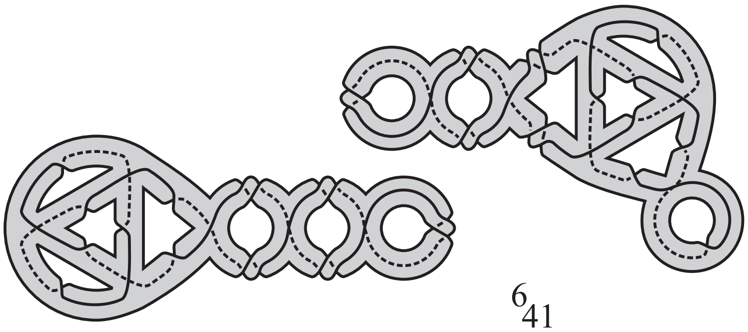

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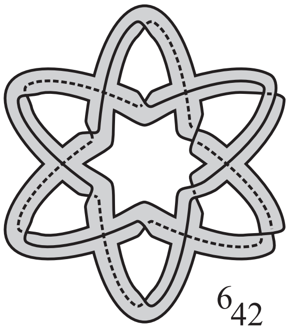

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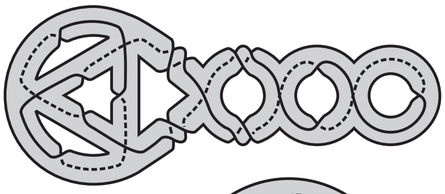

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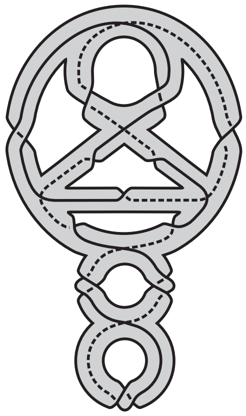

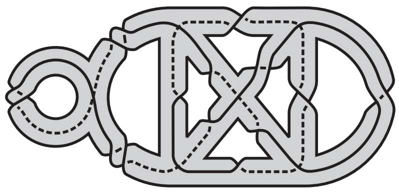

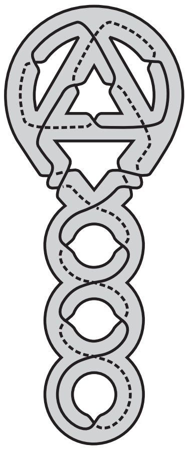

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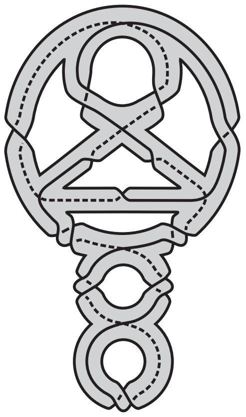

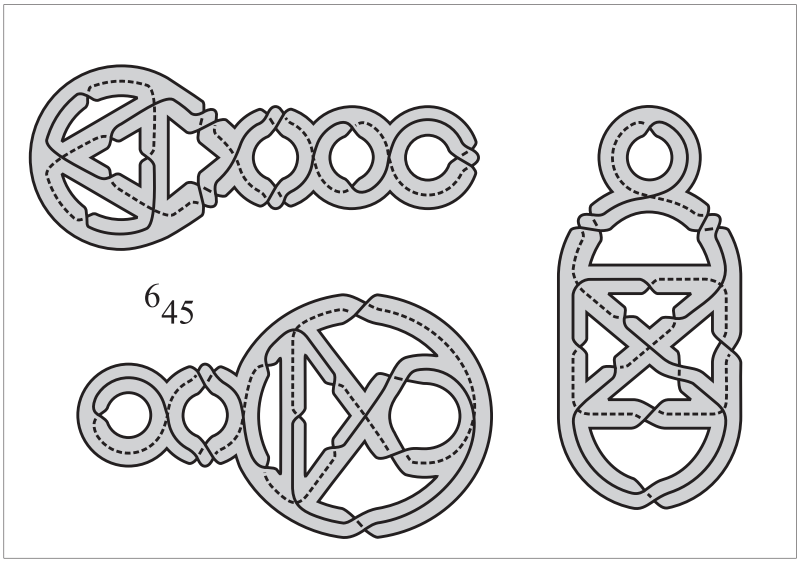

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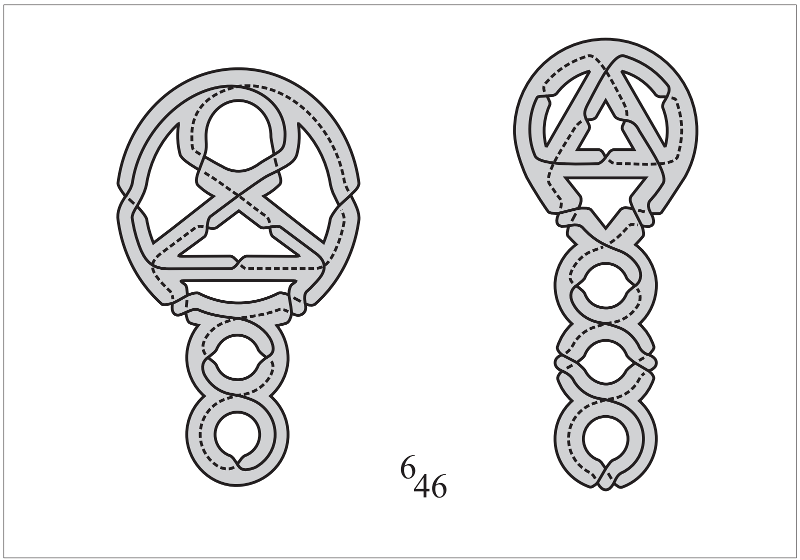

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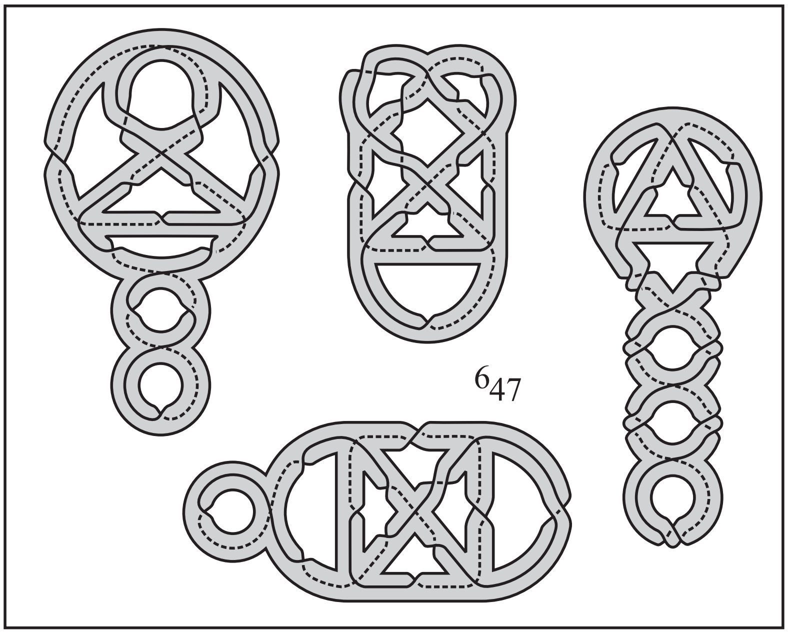
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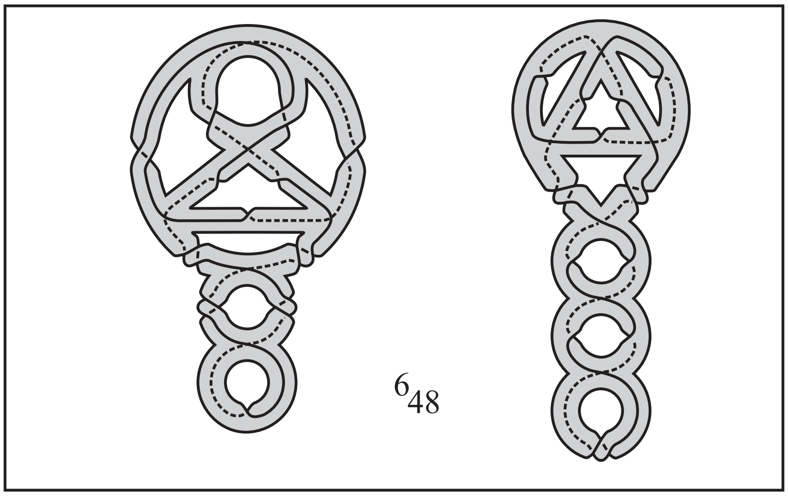
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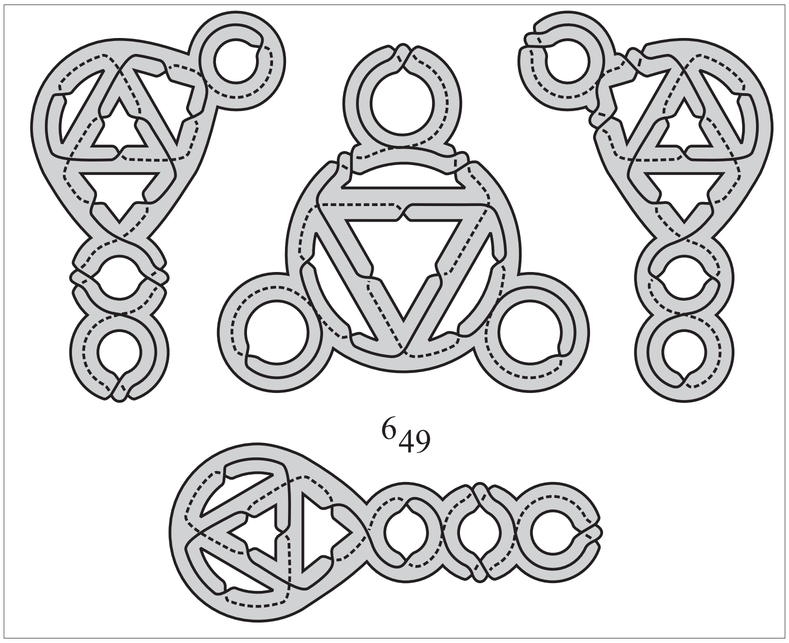

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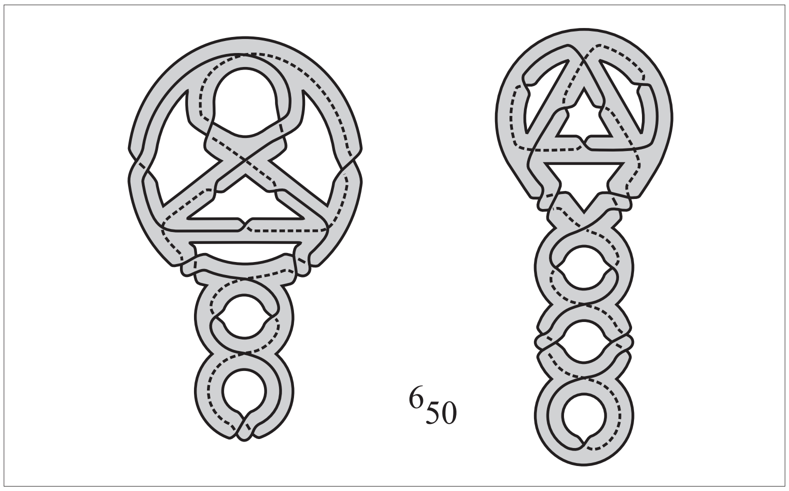

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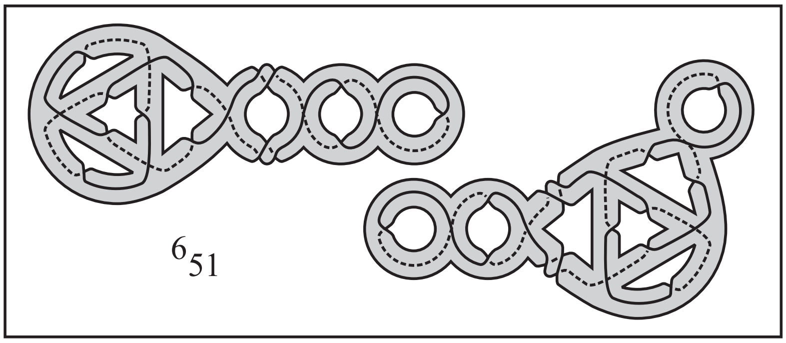

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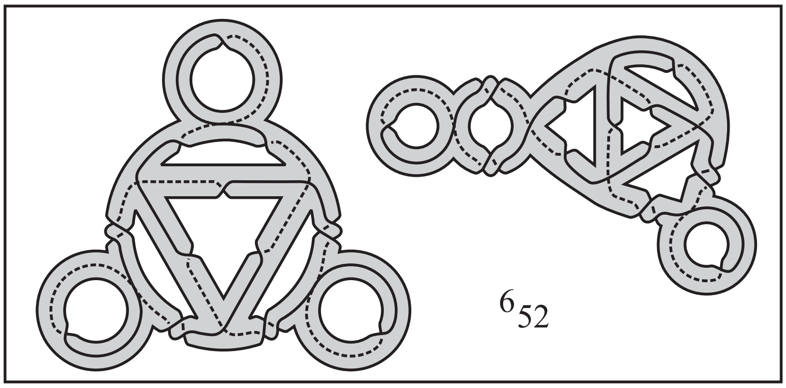

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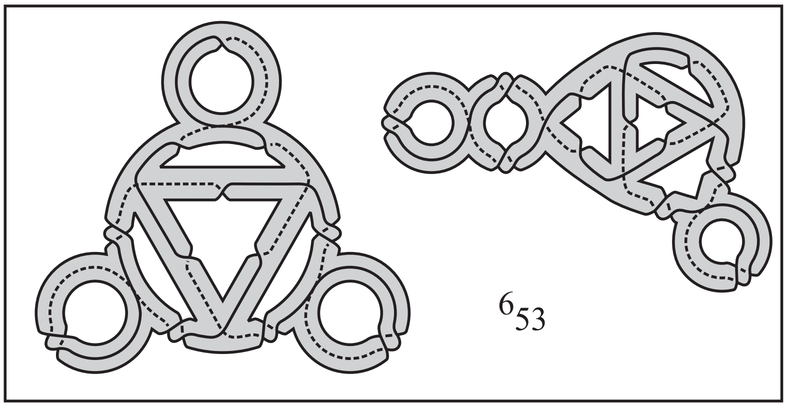

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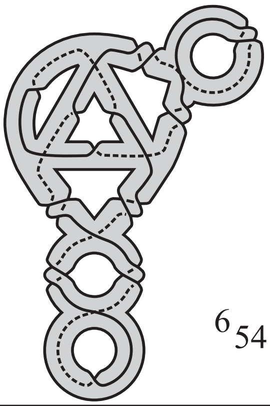

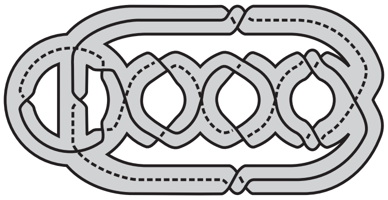

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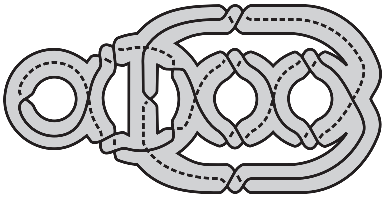

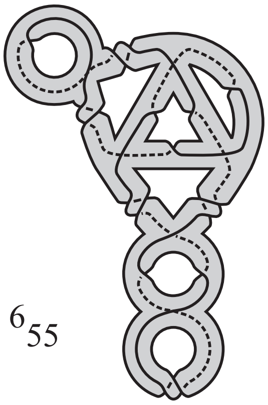

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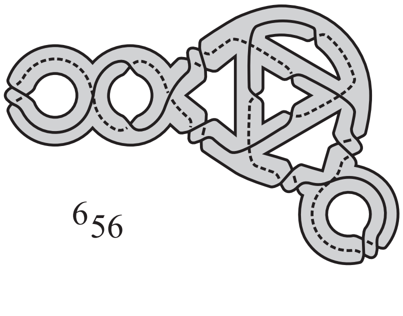

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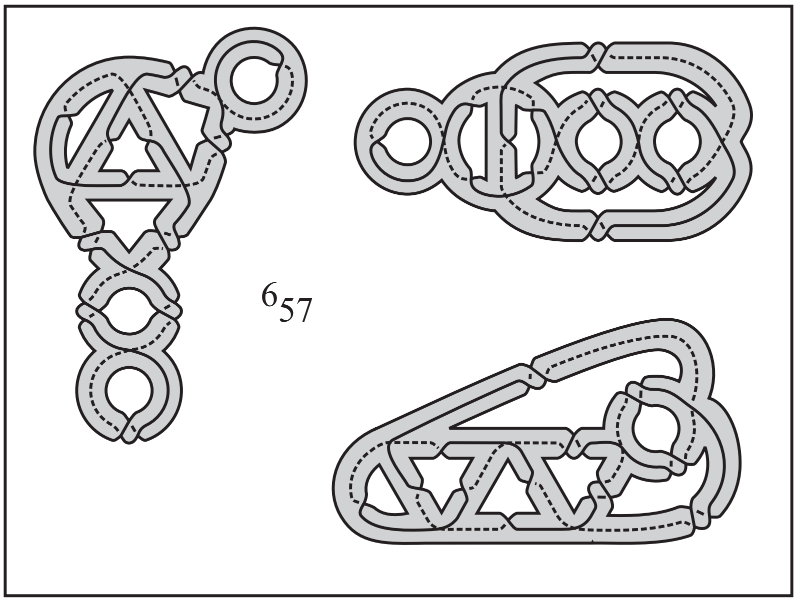

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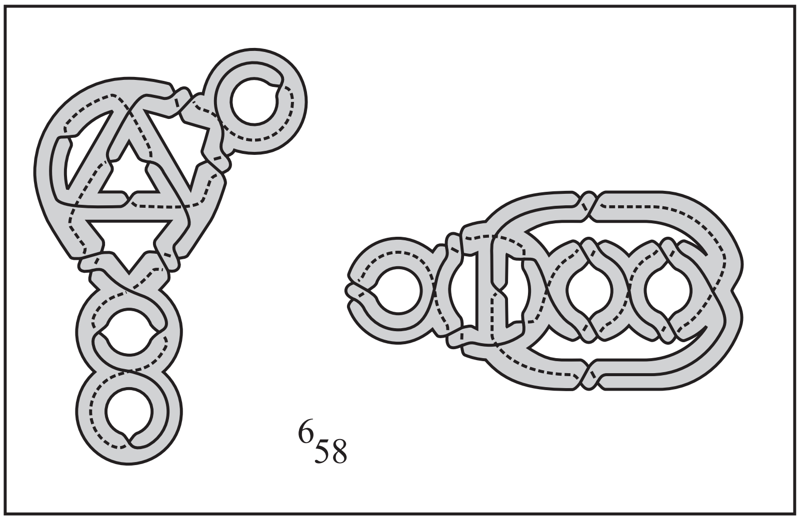

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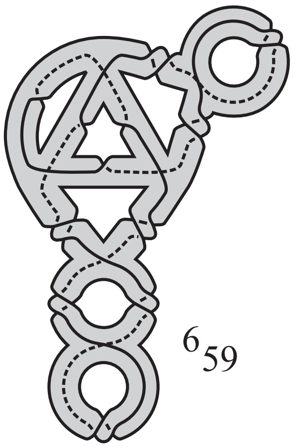

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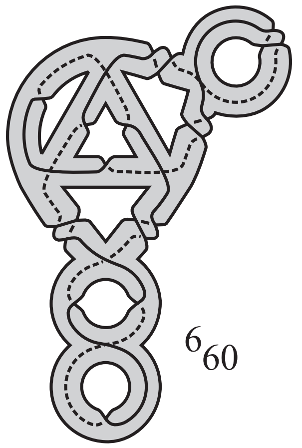

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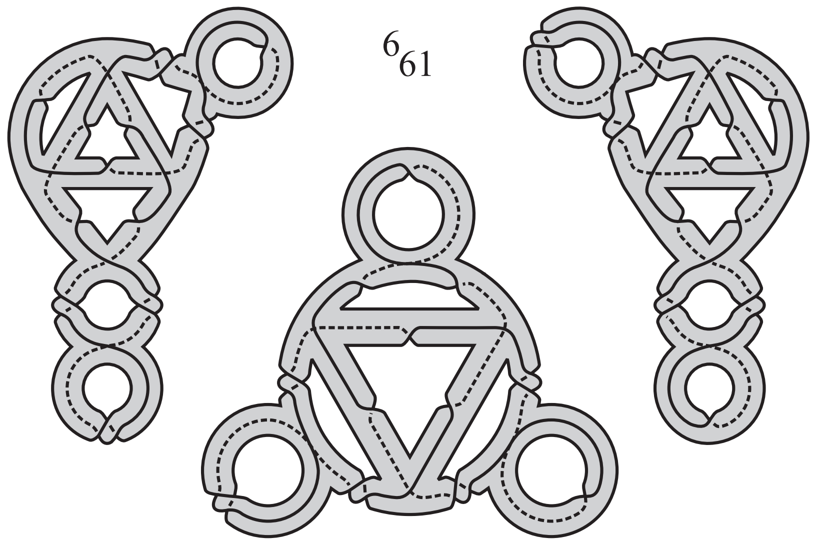

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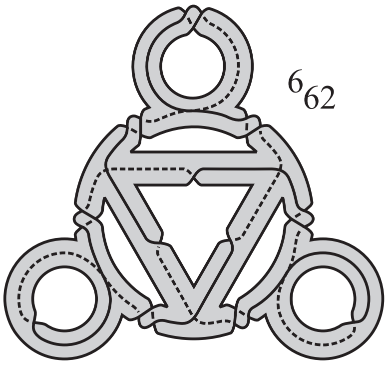

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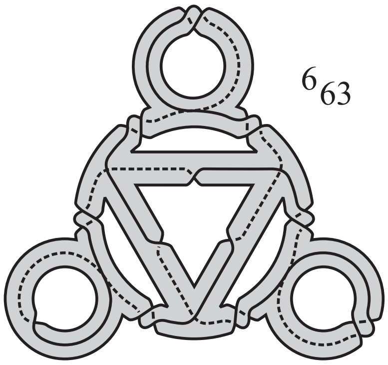

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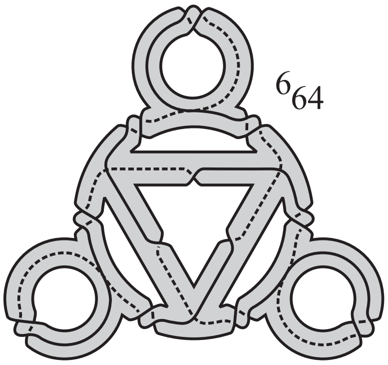

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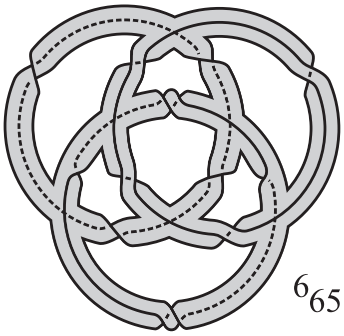

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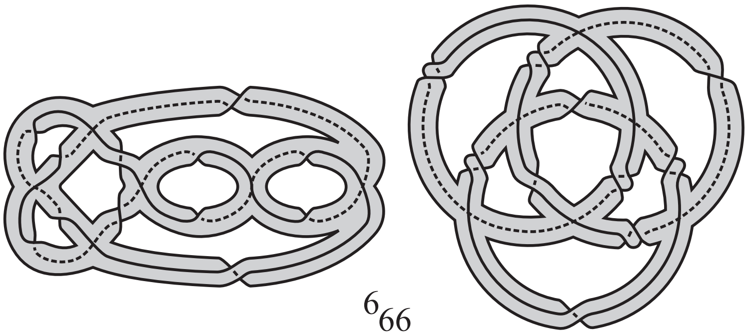

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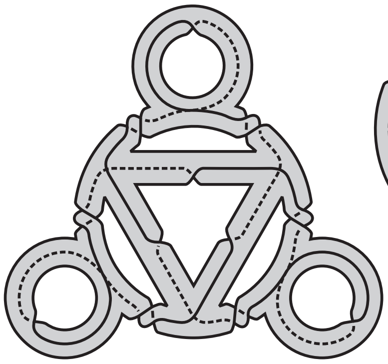

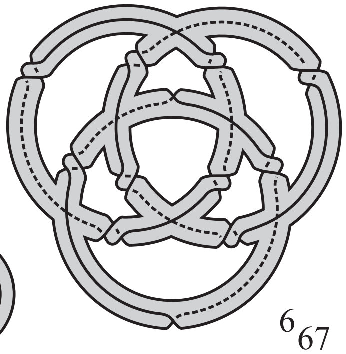

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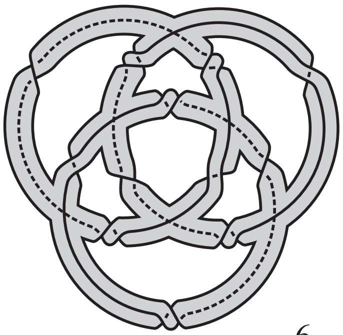

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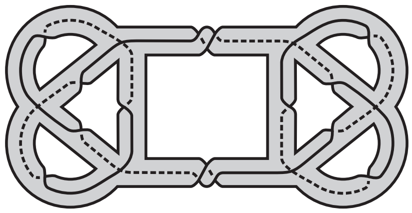

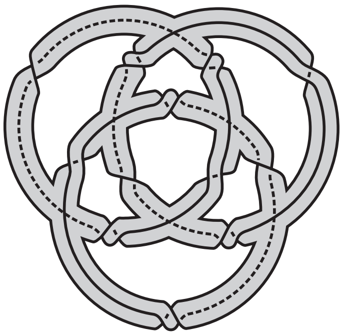

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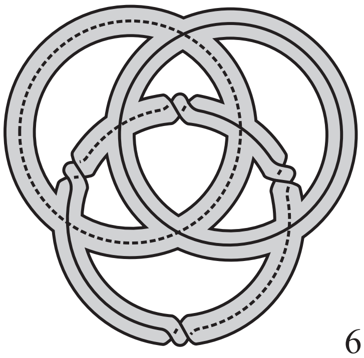

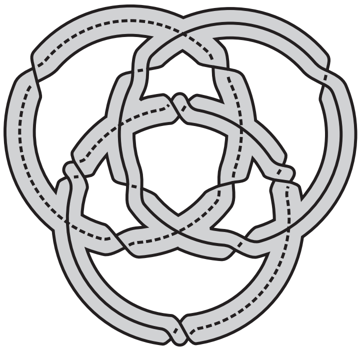

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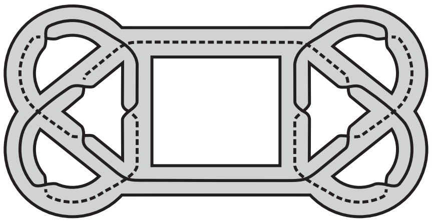

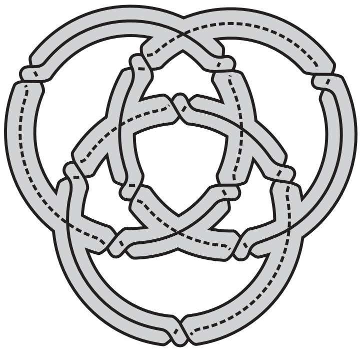

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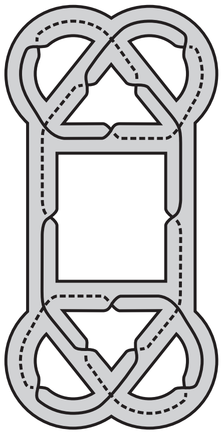

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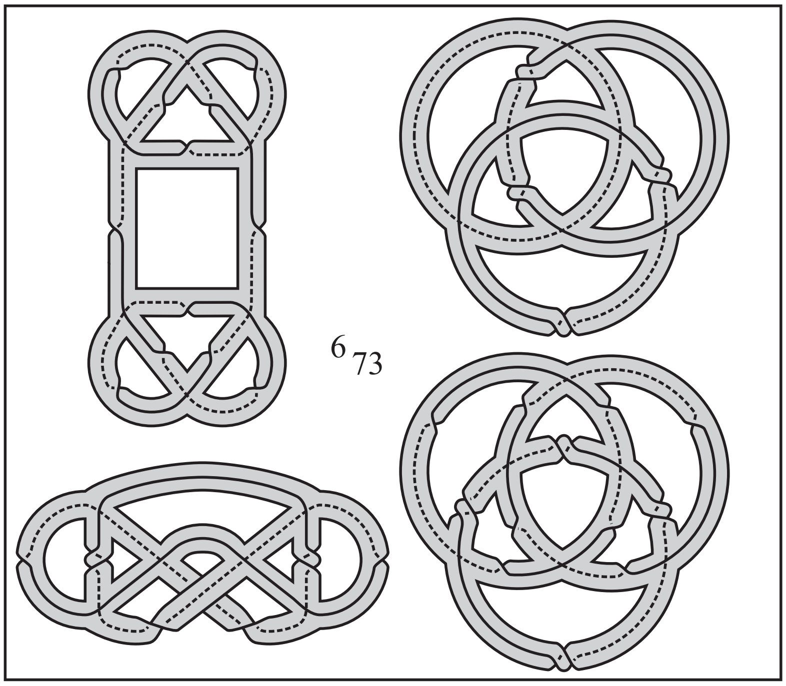

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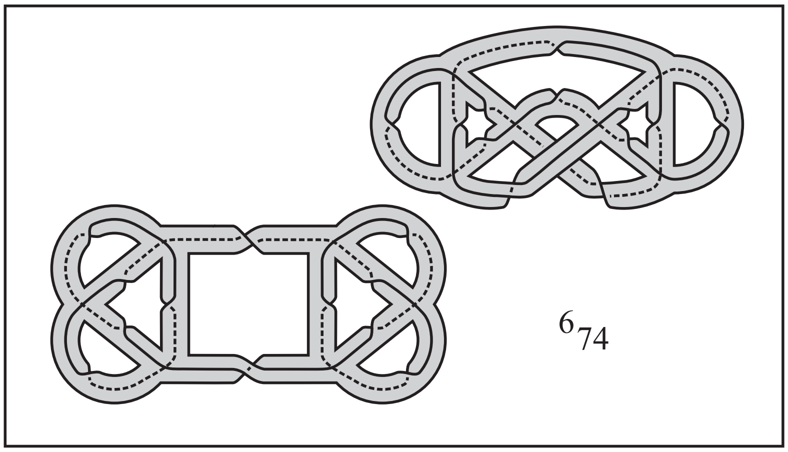


6_{74}

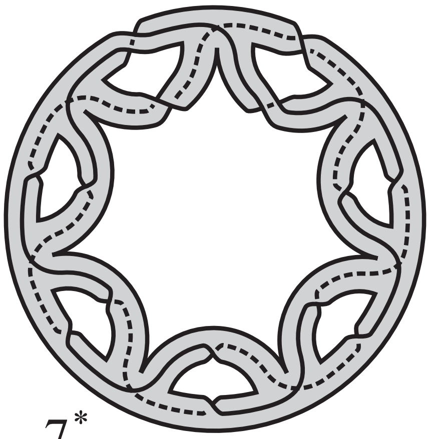
4 Minimal spines of some manifolds of complexity 7

Looking at the previous table of manifolds of complexity ≤ 6, one can see that the great majority of them have minimal spines modeled on nonclosed chains and triangles with tails. The same tendency holds for manifolds of complexity 7. In order to spare place, we reproduce only a part of Ovchinnikov's table [6] of all closed orientable irreducible 3-manifolds of complexity 7. Namely, we list below only those manifolds of complexity 7 that do not admit minimal spines of the two aforementioned types. Each manifold is represented by only one minimal spine chosen arbitrarily. We number the manifold in the same order as they appear in [6]. The star in notation 7_k^* tells us that the numbering differs from the original one (for example, manifold 7_1^* coincides with the manifold 7_{88} from [6]).

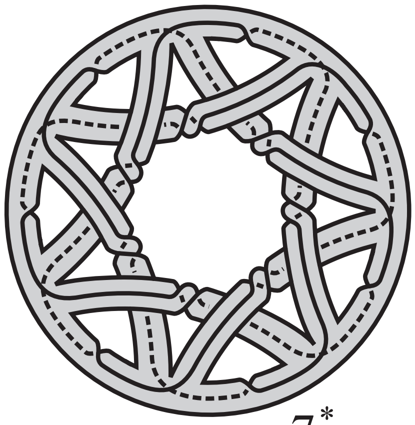
We present 3-manifolds as follows. Let $D_k, k = 1, 2$, be an oriented Seifert manifold of the type $(D^2, (2,1), (3,k))$. In other words, D_k is obtained from $D^2 \times S^1$ by cutting off two disjoint fibered solid tori and gluing them back via homeomorphisms taking their meridians onto curves of the types $(2,1)$ and $(3,k)$, respectively. We equip the boundary torus of each D_k with the coordinate system (μ, λ), where μ is a meridian of $D^2 \times S^1$ and λ is a fiber. Let A be an integer matrix of order 2 with determinant (-1). Then $D_k \cup K\tilde{\times}I/A$ denotes the manifold obtained by attaching $K\tilde{\times}I$ to D_k via a homeomorphism $h_A\colon \partial(K\tilde{\times}I) \to \partial D_k$ corresponding to A.

Table 8: Some manifolds of complexity 7

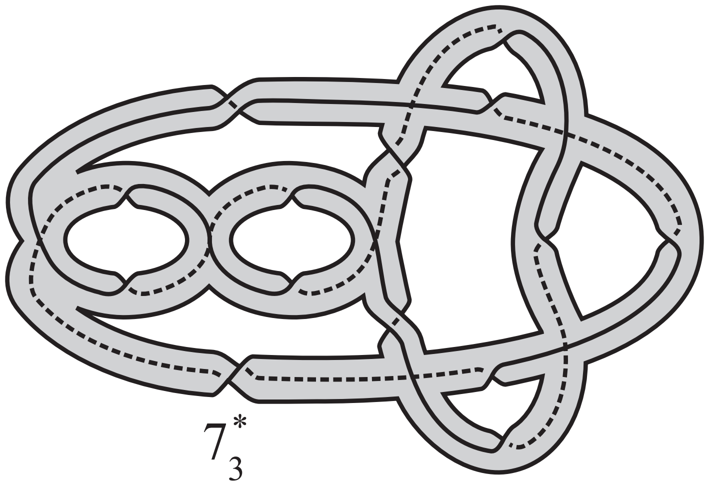
c_i	M	Seifert structure	$H_1(M;Z)$
7_1^*	S^3/Q_{28}	$(S^2,(2,-1),(2,1),(7,1))$	Z_4
7_2^*		$(S^2,(2,-1),(3,1),(7,1))$	0
7_3^*		$(S^2,(2,-1),(4,1),(4,1))$	Z_2
7_4^*	$T\times I/\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$	$(T^2,(1,2))$	$Z\oplus Z\oplus Z_2$
7_5^*	$T\times I/\begin{pmatrix} -1 & 0 \\ -2 & -1 \end{pmatrix}$	$(K^2,(1,2))$	$Z_2\oplus Z_2\oplus Z$
7_6^*	$T\times I/\begin{pmatrix} -2 & -1 \\ -1 & -1 \end{pmatrix}$	–	$Z\oplus Z_5$
7_7^*	$T\times I/\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$	–	Z
7_8^*	$K\tilde{\times}I\cup K\tilde{\times}I/\begin{pmatrix} 1 & 0 \\ -2 & -1 \end{pmatrix}$	$(T^2,(2,1),(2,1),(2,1),(2,1))$	$Z_2\oplus Z_2\oplus Z_8$
7_9^*	$K\tilde{\times}I\cup K\tilde{\times}I/\begin{pmatrix} -1 & -1 \\ 1 & 2 \end{pmatrix}$	–	$Z_4\oplus Z_4$
7_{10}^*	$K\tilde{\times}I\cup K\tilde{\times}I/\begin{pmatrix} 1 & -1 \\ -2 & 1 \end{pmatrix}$	–	$Z_4\oplus Z_8$
7_{11}^*	$K\tilde{\times}I\cup K\tilde{\times}I/\begin{pmatrix} 0 & 1 \\ 1 & -2 \end{pmatrix}$	$(RP^2,(2,1),(2,1))$	$Z_4\oplus Z_4$
7_{12}^*	$K\tilde{\times}I\cup K\tilde{\times}I/\begin{pmatrix} 1 & -2 \\ -1 & 1 \end{pmatrix}$	–	$Z_2\oplus Z_2\oplus Z_4$
7_{13}^*	$D_2\cup K\tilde{\times}I/\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$(RP^2,(2,1),(3,2))$	Z_{24}
7_{14}^*	$D_2\cup K\tilde{\times}I/\begin{pmatrix} 1 & 0 \\ -1 & -1 \end{pmatrix}$	$(RP^2,(2,1),(2,1),(2,1)(3,-1))$	$Z_2\oplus Z_{14}$
7_{15}^*	$D_1\cup K\tilde{\times}I/\begin{pmatrix} 1 & 0 \\ -1 & -1 \end{pmatrix}$	$(S^2,(2,1),(2,1),(2,1)(3,-2))$	$Z_2\oplus Z_{10}$
7_{16}^*	$D_2\cup K\tilde{\times}I/\begin{pmatrix} -1 & -1 \\ 0 & 1 \end{pmatrix}$	–	Z_4
7_{17}^*	$D_1\cup K\tilde{\times}I/\begin{pmatrix} -1 & -1 \\ 0 & 1 \end{pmatrix}$	–	Z_4
7_{18}^*	$D_2\cup K\tilde{\times}I/\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$	$(S^2,(2,1),(2,1),(2,1)(3,-4))$	$Z_2\oplus Z_2$
7_{19}^*	$D_2\cup K\tilde{\times}I/\begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix}$	$(RP^2,(2,1),(3,2))$	Z_{24}
7_{20}^*	$D_1\cup K\tilde{\times}I/\begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix}$	$(RP^2,(2,1),(3,1))$	Z_{24}
7_{21}^*	$D_2\cup K\tilde{\times}I/\begin{pmatrix} 1 & -1 \\ -1 & 0 \end{pmatrix}$	–	Z_{28}
7_{22}^*	$D_1\cup K\tilde{\times}I/\begin{pmatrix} 1 & -1 \\ -1 & 0 \end{pmatrix}$	–	Z_{20}



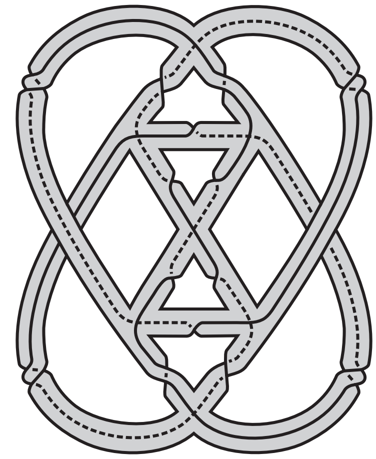
7_1^*



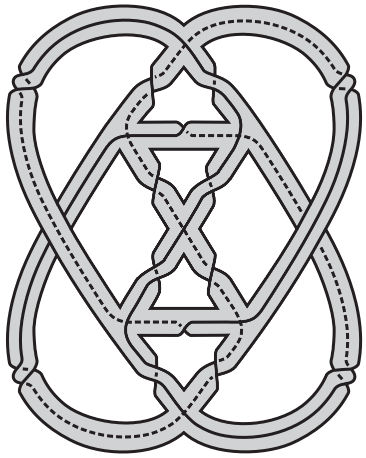
7_2^*



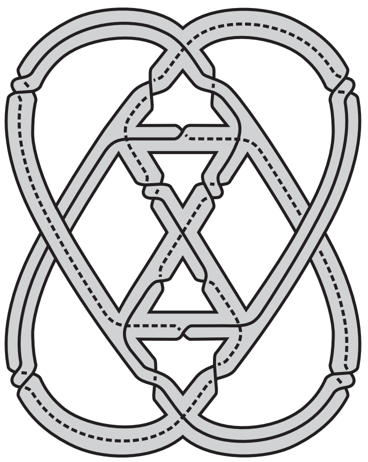
7_3^*



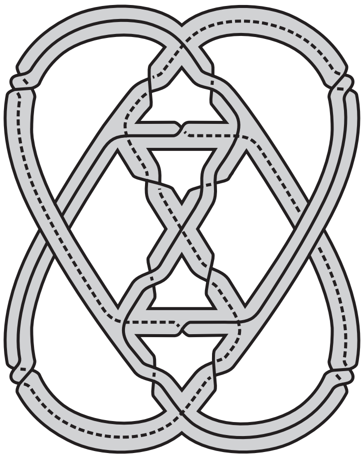
7_4^*



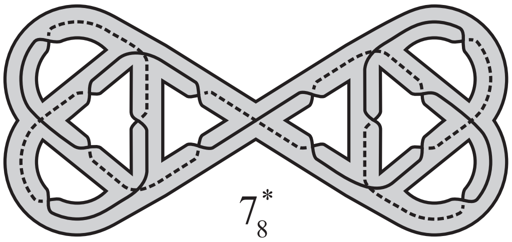
7_5^*



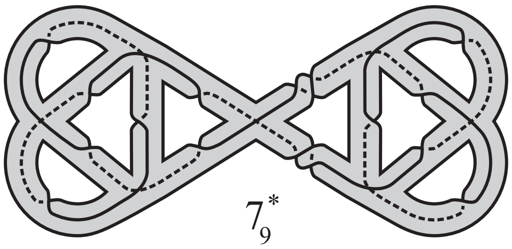
7_6^*



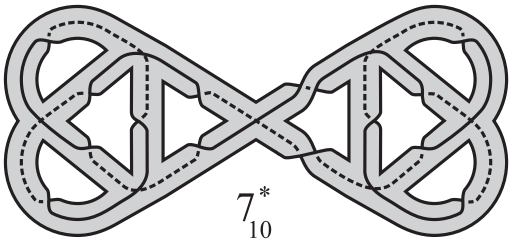
7_7^*



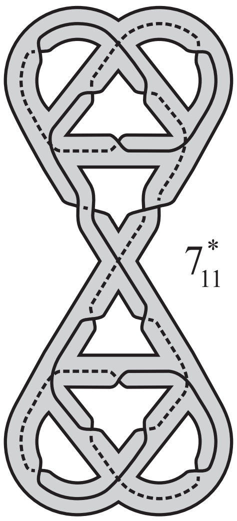
7_8^*



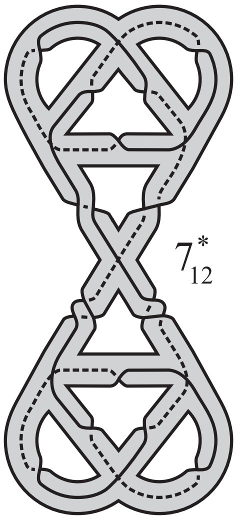
7_9^*



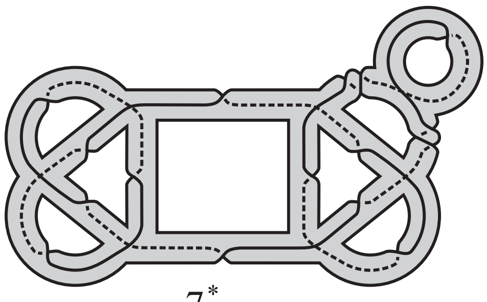
7_{10}^*



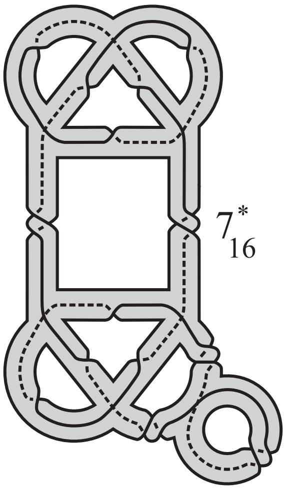
7_{11}^*



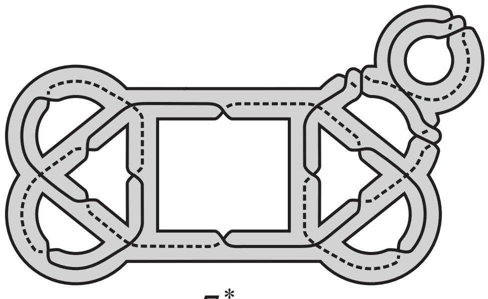
7_{12}^*



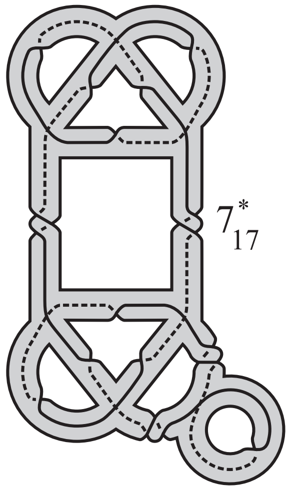
7^*_{13}



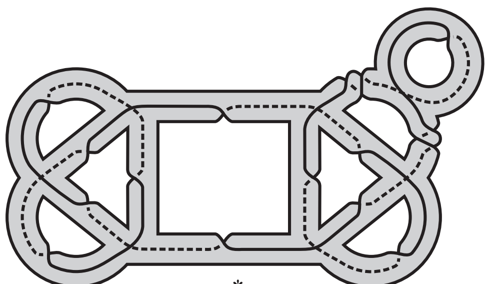
7^*_{16}



7^*_{14}



7^*_{17}



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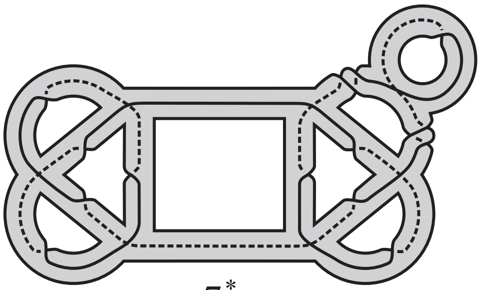

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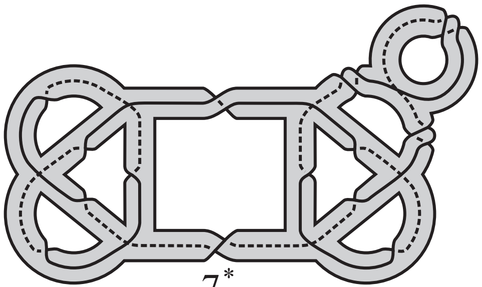

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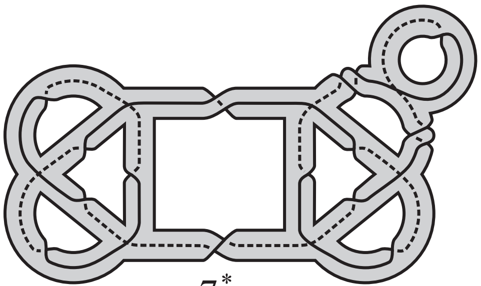
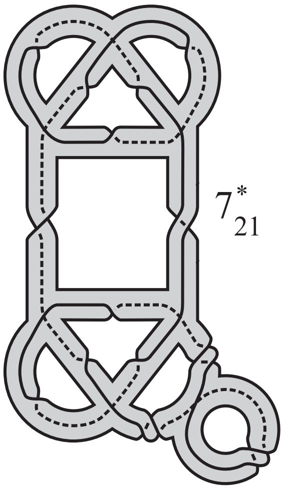
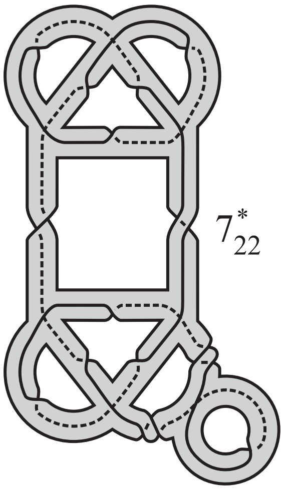


7^*_{20}



7^*_{21}



7^*_{22}

5 Tables of Turaev-Viro invariants

The tables were composed by my former student M. Sokolov. They contain the values of Turaev-Viro invariants of order ≤ 7 and their summands for all closed orientable irreducible 3-manifolds up to complexity 6. The invariants are presented by polynomials of q, where q is a primitive root of unity of degree $2r$. For the sake of compactness of notation, we write σ_k instead of $q^k + q^{-k}$.

Table 9: TV-invariants for manifolds of complexity 0 – 2

c_i	M	ν	3	4	5	6	7
0_1	S^3	0	1	1	1	1	1
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	1	1	1
0_2	RP^3	0	1	2	σ_2+2	4	$-\sigma_3+2\sigma_2+3$
		1	-1	$-\sigma_1$	$-\sigma_2-2$	$-2\sigma_1$	$\sigma_3-2\sigma_2-3$
		2	0	0	0	0	0
		Σ	0	$-\sigma_1+2$	0	$-2\sigma_1+4$	0
0_3	$L_{3,1}$	0	1	1	σ_2+2	3	σ_2+2
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	σ_2+2	3	σ_2+2
1_1	$L_{4,1}$	0	1	2	1	4	σ_2+2
		1	0	0	0	0	0
		2	1	0	1	0	σ_2+2
		Σ	2	2	2	4	$2\sigma_2+4$
1_2	$L_{5,2}$	0	1	1	0	1	$-\sigma_3+2\sigma_2+3$
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	0	1	$-\sigma_3+2\sigma_2+3$
2_1	$L_{5,1}$	0	1	1	σ_2+3	1	$-\sigma_3+2\sigma_2+3$
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	σ_2+3	1	$-\sigma_3+2\sigma_2+3$
2_2	$L_{7,2}$	0	1	1	σ_2+2	1	0
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	σ_2+2	1	0
2_3	$L_{8,3}$	0	1	2	σ_2+2	4	1
		1	0	0	0	0	0
		2	1	2	σ_2+2	0	1
		Σ	2	4	$2\sigma_2+4$	4	2
2_4	Q_8	0	1	4	σ_2+4	10	$2\sigma_2+7$
		1	0	0	0	0	0
		2	3	6	$3\sigma_2+12$	18	$6\sigma_2+21$
		Σ	4	10	$4\sigma_2+16$	28	$8\sigma_2+28$

Table 10: TV-invariants for manifolds of complexity 3

c_i	M	ν	3	4	5	6	7
3_1	$L_{6,1}$	0	1	2	1	6	1
		1	-1	σ_1	-1	0	-1
		2	0	0	0	0	0
		Σ	0	σ_1+2	0	6	0
3_2	$L_{9,2}$	0	1	1	1	3	$-\sigma_3+2\sigma_2+3$
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	1	3	$-\sigma_3+2\sigma_2+3$
3_3	$L_{10,3}$	0	1	2	0	4	σ_2+2
		1	-1	σ_1	0	$2\sigma_1$	$-\sigma_2-2$
		2	0	0	0	0	0
		Σ	0	σ_1+2	0	$2\sigma_1+4$	0
3_4	$L_{11,3}$	0	1	1	1	1	σ_2+2
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	1	1	σ_2+2
3_5	$L_{12,5}$	0	1	2	σ_2+2	6	$-\sigma_3+2\sigma_2+3$
		1	0	0	0	0	0
		2	1	0	σ_2+2	6	$-\sigma_3+2\sigma_2+3$
		Σ	2	2	$2\sigma_2+4$	12	$-2\sigma_3+4\sigma_2+6$
3_6	$L_{13,5}$	0	1	1	σ_2+2	1	1
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	σ_2+2	1	1
3_7	Q_{12}	0	1	2	σ_2+4	10	$-\sigma_3+2\sigma_2+5$
		1	0	0	0	0	0
		2	1	0	σ_2+4	6	$-\sigma_3+2\sigma_2+5$
		Σ	2	2	$2\sigma_2+8$	16	$-2\sigma_3+4\sigma_2+10$
4_1	$L_{7,1}$	0	1	1	σ_2+2	1	$-\sigma_3+3\sigma_2+6$
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	σ_2+2	1	$-\sigma_3+3\sigma_2+6$
4_2	$L_{11,2}$	0	1	1	1	1	σ_2+2
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	1	1	σ_2+2

Table 11: TV-invariants for manifolds of complexity 4

c_i	M	ν	3	4	5	6	7
4_3	$L_{13,3}$	0	1	1	σ_2+2	1	1
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	σ_2+2	1	1
4_4	$L_{14,3}$	0	1	2	1	4	0
		1	-1	$-\sigma_1$	-1	$2\sigma_1$	0
		2	0	0	0	0	0
		Σ	0	$-\sigma_1+2$	0	$2\sigma_1+4$	0
4_5	$L_{15,4}$	0	1	1	σ_2+3	3	1
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	σ_2+3	3	1
4_6	$L_{16,7}$	0	1	2	1	4	$-\sigma_3+2\sigma_2+3$
		1	0	0	0	0	0
		2	1	2	1	0	$-\sigma_3+2\sigma_2+3$
		Σ	2	4	2	4	$-2\sigma_3+4\sigma_2+6$
4_7	$L_{17,5}$	0	1	1	σ_2+2	1	σ_2+2
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	σ_2+2	1	σ_2+2
4_8	$L_{18,5}$	0	1	2	σ_2+2	6	σ_2+2
		1	-1	$-\sigma_1$	$-\sigma_2-2$	0	$-\sigma_2-2$
		2	0	0	0	0	0
		Σ	0	$-\sigma_1+2$	0	6	0
4_9	$L_{19,7}$	0	1	1	1	1	$-\sigma_3+2\sigma_2+3$
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	1	1	$-\sigma_3+2\sigma_2+3$
4_{10}	$L_{21,8}$	0	1	1	1	3	$-\sigma_3+3\sigma_2+6$
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	1	3	$-\sigma_3+3\sigma_2+6$
4_{11}	$Q_8\times Z_3$	0	1	4	$2\sigma_2+5$	12	$-2\sigma_3+7\sigma_2+12$
		1	0	0	0	0	0
		2	3	6	$6\sigma_2+15$	36	$-6\sigma_3+21\sigma_2+36$
		Σ	4	10	$8\sigma_2+20$	48	$-8\sigma_3+28\sigma_2+48$
4_{12}	Q_{16}	0	1	4	1	10	$-\sigma_3+2\sigma_2+5$
		1	-2	0	-2	$4\sigma_1$	$2\sigma_3-4\sigma_2-10$
		2	1	2	1	6	$-\sigma_3+2\sigma_2+5$
		Σ	0	6	0	$4\sigma_1+16$	0

Table 12: TV-invariants for manifolds of complexity 4

c_i	M	ν	3	4	5	6	7
4_{13}	D_{24}	0	1	2	$2\sigma_2+5$	10	$-2\sigma_3+5\sigma_2+8$
		1	0	0	0	0	0
		2	1	2	$2\sigma_2+5$	6	$-2\sigma_3+5\sigma_2+8$
		Σ	2	4	$4\sigma_2+10$	16	$-4\sigma_3+10\sigma_2+16$
4_{14}	P_{24}	0	1	1	$2\sigma_2+5$	3	$-2\sigma_3+4\sigma_2+6$
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	$2\sigma_2+5$	3	$-2\sigma_3+4\sigma_2+6$
5_1	$L_{8,1}$	0	1	2	σ_2+2	4	1
		1	0	0	0	0	0
		2	1	-2	σ_2+2	0	1
		Σ	2	0	$2\sigma_2+4$	4	2
5_2	$L_{13,2}$	0	1	1	σ_2+2	1	1
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	σ_2+2	1	1
5_3	$L_{16,3}$	0	1	2	1	4	$-\sigma_3+2\sigma_2+3$
		1	0	0	0	0	0
		2	1	-2	1	0	$-\sigma_3+2\sigma_2+3$
		Σ	2	0	2	4	$-2\sigma_3+4\sigma_2+6$
5_4	$L_{17,3}$	0	1	1	σ_2+2	1	σ_2+2
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	σ_2+2	1	σ_2+2
5_5	$L_{17,4}$	0	1	1	σ_2+2	1	σ_2+2
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	σ_2+2	1	σ_2+2
5_6	$L_{19,4}$	0	1	1	1	1	$-\sigma_3+2\sigma_2+3$
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	1	1	$-\sigma_3+2\sigma_2+3$
5_7	$L_{20,9}$	0	1	2	σ_2+3	4	1
		1	0	0	0	0	0
		2	1	0	σ_2+3	0	1
		Σ	2	2	$2\sigma_2+6$	4	2

Table 13: TV-invariants for manifolds of complexity 4

c_i	M	ν	3	4	5	6	7
5_8	$L_{22,5}$	0	1	2	σ_2+2	4	1
		1	-1	σ_1	$-\sigma_2-2$	$-2\sigma_1$	-1
		2	0	0	0	0	0
		Σ	0	σ_1+2	0	$-2\sigma_2+4$	0
5_9	$L_{23,5}$	0	1	1	σ_2+2	1	$-\sigma_3+2\sigma_2+3$
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	σ_2+2	1	$-\sigma_3+2\sigma_2+3$
5_{10}	$L_{23,7}$	0	1	1	σ_2+2	1	$-\sigma_3+2\sigma_2+3$
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	σ_2+2	1	$-\sigma_3+2\sigma_2+3$
5_{11}	$L_{24,7}$	0	1	2	1	6	σ_2+2
		1	0	0	0	0	0
		2	1	-2	1	-6	σ_2+2
		Σ	2	0	2	0	$2\sigma_2+4$
5_{12}	$L_{25,7}$	0	1	1	0	1	σ_2+2
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	0	1	σ_2+2
5_{13}	$L_{25,9}$	0	1	1	σ_2+3	1	σ_2+2
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	σ_2+3	1	σ_2+2
5_{14}	$L_{26,7}$	0	1	2	1	4	$-\sigma_3+2\sigma_2+3$
		1	-1	σ_1	-1	$-2\sigma_1$	$\sigma_3-2\sigma_2-3$
		2	0	0	0	0	0
		Σ	0	σ_1+2	0	$-2\sigma_1+4$	0
5_{15}	$L_{27,8}$	0	1	1	σ_2+2	3	1
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	σ_2+2	3	1
5_{16}	$L_{29,8}$	0	1	1	1	1	1
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	1	1	1

Table 14: TV-invariants for manifolds of complexity 5 (continued)

c_i	M	ν	3	4	5	6	7
		0	1	1	1	1	1
5_{17}	$L_{29,12}$	1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	1	1	1
		0	1	2	σ_2+3	6	$-\sigma_3+2\sigma_2+3$
5_{18}	$L_{30,11}$	1	-1	$-\sigma_1$	$-\sigma_2-3$	0	$\sigma_3-2\sigma_2-3$
		2	0	0	0	0	0
		Σ	0	$-\sigma_1+2$	0	6	0
		0	1	1	1	1	σ_2+2
5_{19}	$L_{31,12}$	1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	1	1	σ_2+2
		0	1	2	1	4	1
5_{20}	$L_{34,13}$	1	-1	$-\sigma_1$	-1	$2\sigma_1$	-1
		2	0	0	0	0	0
		Σ	0	$-\sigma_1+2$	0	$2\sigma_1+4$	0
5_{21}	$Q_8\times Z_5$						
		0	1	4	σ_2+3	10	$-3\sigma_3+6\sigma_2+11$
		1	0	0	0	0	0
		2	3	6	$3\sigma_2+9$	18	$-9\sigma_3+18\sigma_2+33$
		Σ	4	10	$4\sigma_2+12$	28	$-12\sigma_3+24\sigma_2+44$
5_{22}	$Q_{12}\times Z_5$						
		0	1	2	σ_2+3	10	$-2\sigma_3+5\sigma_2+8$
		1	0	0	0	0	0
		2	1	0	σ_2+3	6	$-2\sigma_3+5\sigma_2+8$
		Σ	2	2	$2\sigma_2+6$	16	$-4\sigma_3+10\sigma_2+16$
5_{23}	$Q_{16}\times Z_3$						
		0	1	4	σ_2+2	12	$2\sigma_2+5$
		1	-2	0	$-2\sigma_2-4$	0	$-4\sigma_2-10$
		2	1	2	σ_2+2	12	$2\sigma_2+5$
		Σ	0	6	0	24	0

Table 15: TV-invariants for manifolds of complexity 5

c_i	M	ν	3	4	5	6	7
5_{24}	Q_{20}	0	1	2	$3\sigma_2+5$	4	$2\sigma_2+7$
		1	0	0	0	0	0
		2	1	0	$3\sigma_2+5$	0	$2\sigma_2+7$
		Σ	2	2	$6\sigma_2+10$	4	$4\sigma_2+14$
5_{25}	$Q_{20}\times Z_3$	0	1	2	$-\sigma_2+1$	6	$-2\sigma_3+7\sigma_2+12$
		1	0	0	0	0	0
		2	1	0	$-\sigma_2+1$	6	$-2\sigma_3+7\sigma_2+12$
		Σ	2	2	$-2\sigma_2+2$	12	$-4\sigma_3+14\sigma_2+24$
5_{26}	D_{40}	0	1	2	$-\sigma_2+1$	4	$-3\sigma_3+6\sigma_2+11$
		1	0	0	0	0	0
		2	1	2	$-\sigma_2+1$	0	$-3\sigma_3+6\sigma_2+11$
		Σ	2	4	$-2\sigma_2+2$	4	$-6\sigma_3+12\sigma_2+22$
5_{27}	D_{48}	0	1	2	σ_2+4	10	$2\sigma_2+5$
		1	0	0	0	0	0
		2	1	2	σ_2+4	6	$2\sigma_2+5$
		Σ	2	4	$2\sigma_2+8$	16	$4\sigma_2+10$
5_{28}	$P_{24}\times Z_5$	0	1	1	σ_2+3	3	$2\sigma_2+4$
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	σ_2+3	3	$2\sigma_2+4$
5_{29}	P_{48}	0	1	2	σ_2+2	10	2
		1	-1	σ_1	$-\sigma_2-2$	0	-2
		2	0	0	0	0	0
		Σ	0	σ_1+2	0	10	0
5_{30}	P'_{72}	0	1	1	σ_2+4	3	2
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	σ_2+4	3	2
5_{31}	P_{120}	0	1	1	$3\sigma_2+5$	1	$-2\sigma_3+5\sigma_2+8$
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	$3\sigma_2+5$	1	$-2\sigma_3+5\sigma_2+8$

Table 16: TV-invariants for manifolds of complexity 6

c_i	M	ν	3	4	5	6	7
6_1	$L_{9,1}$	0	1	1	1	3	$-\sigma_3+2\sigma_2+3$
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	1	3	$-\sigma_3+2\sigma_2+3$
6_2	$L_{15,2}$	0	1	1	0	3	1
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	0	3	1
6_3	$L_{19,3}$	0	1	1	1	1	$-\sigma_3+2\sigma_2+3$
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	1	1	$-\sigma_3+2\sigma_2+3$
6_4	$L_{20,3}$	0	1	2	0	4	1
		1	0	0	0	0	0
		2	1	0	0	0	1
		Σ	2	2	0	4	2
6_5	$L_{21,4}$	0	1	1	1	3	0
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	1	3	0
6_6	$L_{23,4}$	0	1	1	σ_2+2	1	$-\sigma_3+2\sigma_2+3$
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	σ_2+2	1	$-\sigma_3+2\sigma_2+3$
6_7	$L_{24,5}$	0	1	2	1	6	σ_2+2
		1	0	0	0	0	0
		2	1	2	1	-6	σ_2+2
		Σ	2	4	2	0	$2\sigma_2+4$
6_8	$L_{24,11}$	0	1	2	1	6	σ_2+2
		1	0	0	0	0	0
		2	1	2	1	6	σ_2+2
		Σ	2	4	2	12	$2\sigma_2+4$
6_9	$L_{27,5}$	0	1	1	σ_2+2	3	1
		2	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	σ_2+2	3	1

Table 17: TV-invariants for manifolds of complexity 6 (continued)

c_i	M	ν	3	4	5	6	7
6_{10}	$L_{28,5}$	0	1	2	σ_2+2	4	0
		1	0	0	0	0	0
		2	1	0	σ_2+2	0	0
		Σ	2	2	$2\sigma_2+4$	4	0
6_{11}	$L_{29,9}$	0	1	1	1	1	1
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	1	1	1
6_{12}	$L_{30,7}$	0	1	2	0	6	$-\sigma_3+2\sigma_2+3$
		1	-1	$-\sigma_1$	0	0	$\sigma_3-2\sigma_2-3$
		2	0	0	0	0	0
		Σ	0	$-\sigma_1+2$	0	6	0
6_{13}	$L_{31,7}$	0	1	1	1	1	σ_2+2
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	1	1	σ_2+2
6_{14}	$L_{31,11}$	0	1	1	1	1	σ_2+2
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	1	1	σ_2+2
6_{15}	$L_{32,7}$	0	1	2	σ_2+2	4	σ_2+2
		1	0	0	0	0	0
		2	1	2	σ_2+2	0	σ_2+2
		Σ	2	4	$2\sigma_2+4$	4	$2\sigma_2+4$
6_{16}	$L_{33,7}$	0	1	1	σ_2+2	3	$-\sigma_3+2\sigma_2+3$
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	σ_2+2	3	$-\sigma_3+2\sigma_2+3$
6_{17}	$L_{33,10}$	0	1	1	σ_2+2	3	$-\sigma_3+2\sigma_2+3$
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	σ_2+2	3	$-\sigma_3+2\sigma_2+3$

Table 18: TV-invariants for manifolds of complexity 6 (continued)

c_i	M	ν	3	4	5	6	7
6_{18}	$L_{34,9}$	0	1	2	1	4	1
		1	-1	$-\sigma_1$	-1	$2\sigma_2$	-1
		2	0	0	0	0	0
		Σ	0	$-\sigma_1+2$	0	$2\sigma_1+4$	0
6_{19}	$L_{35,8}$	0	1	1	0	1	$-\sigma_3+3\sigma_2+6$
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	0	1	$-\sigma_3+3\sigma_2+6$
6_{20}	$L_{36,11}$	0	1	2	1	6	1
		1	0	0	0	0	0
		2	1	0	1	-6	1
		Σ	2	2	2	0	2
6_{21}	$L_{37,8}$	0	1	1	σ_2+2	1	$-\sigma_3+2\sigma_2+3$
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	σ_2+2	1	$-\sigma_3+2\sigma_2+3$
6_{22}	$L_{37,10}$	0	1	1	σ_2+2	1	$-\sigma_3+2\sigma_2+3$
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	σ_2+2	1	$-\sigma_3+2\sigma_2+3$
6_{23}	$L_{39,14}$	0	1	1	1	3	σ_2+2
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	1	3	σ_2+2
6_{24}	$L_{39,16}$	0	1	1	1	3	σ_2+2
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	1	3	σ_2+2
6_{25}	$L_{40,11}$	0	1	2	σ_2+3	4	$-\sigma_3+2\sigma_2+3$
		1	0	0	0	0	0
		2	1	2	σ_2+3	0	$-\sigma_3+2\sigma_2+3$
		Σ	2	4	$2\sigma_2+6$	4	$-2\sigma_3+4\sigma_2+6$
6_{26}	$L_{41,11}$	0	1	1	1	1	1
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	1	1	1

Table 19: TV-invariants for manifolds of complexity 6 (continued)

c_i	M	ν	3	4	5	6	7
6_{27}	$L_{41,12}$	0	1	1	1	1	1
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	1	1	1
6_{28}	$L_{41,16}$	0	1	1	1	1	1
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	1	1	1
6_{29}	$L_{43,12}$	0	1	1	$\sigma_2 + 2$	1	1
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	$\sigma_2 + 2$	1	1
6_{30}	$L_{44,13}$	0	1	2	1	4	$-\sigma_3 + 2\sigma_2 + 3$
		1	0	0	0	0	0
		2	1	0	1	0	$-\sigma_3 + 2\sigma_2 + 3$
		Σ	2	2	2	4	$-2\sigma_3 + 4\sigma_2 + 6$
6_{31}	$L_{45,19}$	0	1	1	$\sigma_2 + 3$	3	$\sigma_2 + 2$
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	$\sigma_2 + 3$	3	$\sigma_2 + 2$
6_{32}	$L_{46,17}$	0	1	2	1	4	$\sigma_2 + 2$
		1	-1	$-\sigma_1$	-1	$-2\sigma_1$	$-\sigma_2 - 2$
		2	0	0	0	0	0
		Σ	0	$-\sigma_1 + 2$	0	$-2\sigma_1 + 4$	0
6_{33}	$L_{47,13}$	0	1	1	$\sigma_2 + 2$	1	$-\sigma_3 + 2\sigma_2 + 3$
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	$\sigma_2 + 2$	1	$-\sigma_3 + 2\sigma_2 + 3$
6_{34}	$L_{49,18}$	0	1	1	1	1	0
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	1	1	0
6_{35}	$L_{50,19}$	0	1	2	$\sigma_2 + 3$	4	1
		1	-1	$-\sigma_1$	$-\sigma_2 - 3$	$-2\sigma_1$	-1
		2	0	0	0	0	0
		Σ	0	$-\sigma_1 + 2$	0	$-2\sigma_1 + 4$	0

Table 20: TV-invariants for manifolds of complexity 6 (continued)

c_i	M	ν	3	4	5	6	7
6_{36}	$L_{55,21}$	0	1	1	σ_2+3	1	1
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	σ_2+3	1	1
6_{37}	$Q_8\times Z_7$						
		0	1	4	$2\sigma_2+5$	10	$-\sigma_3+3\sigma_2+6$
		1	0	0	0	0	0
		2	3	6	$6\sigma_2+15$	18	$-3\sigma_3+9\sigma_2+18$
		Σ	4	10	$8\sigma_2+20$	28	$-4\sigma_3+12\sigma_2+24$
6_{38}	$Q_{12}\times Z_7$						
		0	1	2	$2\sigma_2+5$	10	$-\sigma_3+3\sigma_2+6$
		1	0	0	0	0	0
		2	1	0	$2\sigma_2+5$	6	$-\sigma_3+3\sigma_2+6$
		Σ	2	2	$4\sigma_2+10$	16	$-2\sigma_3+6\sigma_2+12$
6_{39}	$Q_{16}\times Z_5$						
		0	1	4	σ_2+3	10	$-2\sigma_3+5\sigma_2+8$
		1	-2	0	$-2\sigma_2-6$	$-4\sigma_1$	$4\sigma_3-10\sigma_2-16$
		2	1	2	σ_2+3	6	$-2\sigma_3+5\sigma_2+8$
		Σ	0	6	0	$-4\sigma_1+16$	0
6_{40}	$Q_{16}\times Z_7$						
		0	1	4	σ_2+2	10	$-\sigma_3+3\sigma_2+6$
		1	-2	0	$-2\sigma_2-4$	$-4\sigma_1$	$2\sigma_3-6\sigma_2-12$
		2	1	2	σ_2+2	6	$-\sigma_3+3\sigma_2+6$
		Σ	0	6	0	$-4\sigma_1+16$	0
6_{41}	$Q_{20}\times Z_7$						
		0	1	2	$-\sigma_2+1$	4	$-\sigma_3+3\sigma_2+6$
		1	0	0	0	0	0
		2	1	0	$-\sigma_2+1$	0	$-\sigma_3+3\sigma_2+6$
		Σ	2	2	$-2\sigma_2+2$	4	$-2\sigma_3+6\sigma_2+12$

Table 21: TV-invariants for manifolds of complexity 6 (continued)

c_i	M	ν	3	4	5	6	7
6_{42}	Q_{24}	0	1	4	1	16	$\sigma_2 + 2$
		1	0	0	0	0	0
		2	3	-2	3	12	$3\sigma_+ 6$
		Σ	4	2	4	28	$4\sigma_2 + 8$
6_{43}	$Q_{28} \times Z_3$						
		0	1	2	$2\sigma_2 + 5$	6	$-\sigma_3 - \sigma_2 + 2$
		1	0	0	0	0	0
		2	1	0	$2\sigma_2 + 5$	6	$-\sigma_3 - \sigma_2 + 2$
		Σ	2	2	$4\sigma_2 + 10$	12	$-2\sigma_3 - 2\sigma_2 + 4$
6_{44}	$Q_{28} \times Z_5$						
		0	1	2	$\sigma_2 + 3$	4	$3\sigma_3 - \sigma_2 + 2$
		1	0	0	0	0	0
		2	1	0	$\sigma_2 + 3$	0	$3\sigma_3 - \sigma_2 + 2$
		Σ	2	2	$2\sigma_2 + 6$	4	$6\sigma_3 - 2\sigma_2 + 4$
6_{45}	$Q_{32} \times Z_3$						
		0	1	4	$2\sigma_2 + 5$	12	$-\sigma_3 + 2\sigma_2 + 3$
		1	-2	$-4\sigma_1$	$-4\sigma_2 - 10$	0	$2\sigma_3 - 4\sigma_2 - 6$
		2	1	2	$2\sigma_2 + 5$	12	$-\sigma_3 + 2\sigma_2 + 3$
		Σ	0	$-4\sigma_1 + 6$	0	24	0
6_{46}	$Q_{32} \times Z_5$						
		0	1	4	$\sigma_2 + 3$	10	1
		1	-2	$-4\sigma_1$	$-2\sigma_2 - 6$	$4\sigma_1$	-2
		2	1	2	$\sigma_2 + 3$	6	1
		Σ	0	$-4\sigma_1 + 6$	0	$4\sigma_1 + 16$	0
6_{47}	D_{56}	0	1	2	$2\sigma_2 + 5$	4	$3\sigma_3 - \sigma_2 + 2$
		1	0	0	0	0	0
		2	1	2	$2\sigma_2 + 5$	0	$3\sigma_3 - \sigma_2 + 2$
		Σ	2	4	$4\sigma_2 + 10$	4	$6\sigma_3 - 2\sigma_2 + 4$

Table 22: TV-invariants for manifolds of complexity 6 (continued)

c_i	M	ν	3	4	5	6	7
6_{48}	D_{80}	0	1	2	$3\sigma_2+5$	4	$-2\sigma_3+7\sigma_2+12$
		1	0	0	0	0	0
		2	1	2	$3\sigma_2+5$	0	$-2\sigma_3+7\sigma_2+12$
		Σ	2	4	$6\sigma_2+10$	4	$-4\sigma_3+14\sigma_2+24$
6_{49}	D_{96}	0	1	2	$2\sigma_2+5$	10	$-\sigma_3+2\sigma_2+5$
		1	0	0	0	0	0
		2	1	2	$2\sigma_2+5$	6	$-\sigma_3+2\sigma_2+5$
		Σ	2	4	$4\sigma_2+10$	16	$-2\sigma_3+4\sigma_2+10$
6_{50}	D_{112}	0	1	2	σ_2+4	4	$-\sigma_3-\sigma_2+2$
		1	0	0	0	0	0
		2	1	2	σ_2+4	0	$-\sigma_3-\sigma_2+2$
		Σ	2	4	$2\sigma_2+8$	4	$-2\sigma_3-2\sigma_2+4$
6_{51}	D_{160}	0	1	2	$-\sigma_2+1$	4	$2\sigma_2+7$
		1	0	0	0	0	0
		2	1	2	$-\sigma_2+1$	0	$2\sigma_2+7$
		Σ	2	4	$-2\sigma_1+2$	4	$4\sigma_2+14$
6_{52}	$P_{24}\times Z_7$						
		0	1	1	σ_2+4	3	$-\sigma_3+3\sigma_2+6$
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	σ_2+4	3	$-\sigma_3+3\sigma_2+6$
6_{53}	$P_{24}\times Z_{11}$						
		0	1	1	$2\sigma_2+5$	3	2
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	$2\sigma_2+5$	3	2
6_{54}	$P_{48}\times Z_5$						
		0	1	2	σ_2+3	10	$-2\sigma_3+4\sigma_2+6$
		1	-1	$-\sigma_1$	$-\sigma_2-3$	0	$2\sigma_3-4\sigma_2-6$
		2	0	0	0	0	0
		Σ	0	$-\sigma_1+2$	0	10	0

Table 23: TV-invariants for manifolds of complexity 6 (continued)

c_i	M	ν	3	4	5	6	7
6_{55}	$P_{48} \times Z_7$						
		0	1	2	1	10	$-\sigma_3 + 3\sigma_2 + 6$
		1	-1	σ_1	-1	0	$\sigma_3 - 3\sigma_2 - 6$
		2	0	0	0	0	0
		Σ	0	$\sigma_1 + 2$	0	10	0
6_{56}	$P_{48} \times Z_{11}$						
		0	1	2	$\sigma_2 + 2$	10	$2\sigma_2 + 4$
		1	-1	$-\sigma_1$	$-\sigma_2 - 2$	0	$-2\sigma_2 - 4$
		2	0	0	0	0	0
		Σ	0	$-\sigma_1 + 2$	0	10	0
6_{57}	$P_{120} \times Z_7$						
		0	1	1	$-\sigma_2 + 1$	1	$-\sigma_3 + 3\sigma_2 + 6$
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	$-\sigma_2 + 1$	1	$-\sigma_3 + 3\sigma_2 + 6$
6_{58}	$P_{120} \times Z_{13}$						
		0	1	1	$-\sigma_2 + 1$	1	$-2\sigma_3 + 5\sigma_2 + 8$
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	$-\sigma_2 + 1$	1	$-2\sigma_3 + 5\sigma_2 + 8$
6_{59}	$P_{120} \times Z_{17}$						
		0	1	1	$-\sigma_2 + 1$	1	$-\sigma_3 + 2\sigma_2 + 5$
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	$-\sigma_2 + 1$	1	$-\sigma_3 + 2\sigma_2 + 5$
6_{60}	$P_{120} \times Z_{23}$						
		0	1	1	$-\sigma_2 + 1$	1	$2\sigma_2 + 5$
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	$-\sigma_2 + 1$	1	$2\sigma_2 + 5$

Table 24: TV-invariants for manifolds of complexity 6 (continued)

c_i	M	ν	3	4	5	6	7
6_{61}	P'_{216}	0	1	1	$2\sigma_2+5$	3	$2\sigma_2+4$
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	$2\sigma_2+5$	3	$2\sigma_2+4$
6_{62}	$M(S^2,(3,2),(3,1),(3,-2))$						
		0	1	2	$2\sigma_2+5$	12	$-2\sigma_3+6\sigma_2+9$
		1	-1	$-\sigma_1$	$-2\sigma_2-5$	$-6\sigma_1$	$2\sigma_3-6\sigma_2-9$
		2	0	0	0	0	0
		Σ	0	$-\sigma_1+2$	0	$-6\sigma_1+12$	0
6_{63}	$M(S^2,(3,2),(3,2),(3,-2))$						
		0	1	1	σ_2+4	9	$-2\sigma_3+3\sigma_2+6$
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	σ_2+4	9	$-2\sigma_3+3\sigma_2+6$
6_{64}	$M(S^2,(3,2),(3,2),(3,-1))$						
		0	1	1	$2\sigma_2+5$	9	σ_3+3
		1	0	0	0	0	0
		2	0	0	0	0	0
		Σ	1	1	$2\sigma_2+5$	9	σ_3+3
6_{65}	$T\times I/\begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix}$						
		0	1	2	σ_2+3	12	0
		1	0	0	0	0	0
		2	1	-2	σ_2+3	0	0
		Σ	2	0	$2\sigma_2+6$	12	0

Table 25: TV-invariants for manifolds of complexity 6 (continued)

c_i	M	ν	3	4	5	6	7
6_{66}	$T \times I / \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$						
		0	1	4	0	12	$-\sigma_3 + 3\sigma_2 + 6$
		1	-2	0	0	0	$2\sigma_3 - 6\sigma_2 - 12$
		2	1	0	0	0	$-\sigma_3 + 3\sigma_2 + 6$
		Σ	0	4	0	12	0
6_{67}	$T \times I / \begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix}$						
		0	1	2	$\sigma_2 + 3$	12	0
		1	0	0	0	0	0
		2	1	-2	$\sigma_2 + 3$	0	0
		Σ	2	0	$2\sigma_2 + 6$	12	0
6_{68}	$T \times I / \begin{pmatrix} -1 & 0 \\ -1 & -1 \end{pmatrix}$						
		0	1	4	$-\sigma_2 + 2$	18	$-2\sigma_3 + 6\sigma_2 + 12$
		1	0	0	0	0	0
		2	3	0	$-3\sigma_2 + 6$	18	$-6\sigma_3 + 18\sigma_2 + 36$
		Σ	4	4	$-4\sigma_2 + 8$	36	$-8\sigma_3 + 24\sigma_2 + 48$
6_{69}	$T \times I / \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$						
		0	1	4	$-\sigma_2 + 2$	18	$-2\sigma_3 + 6\sigma_2 + 12$
		1	0	0	0	0	0
		2	3	0	$-3\sigma_2 + 6$	18	$-6\sigma_3 + 18\sigma_2 + 36$
		Σ	4	4	$-4\sigma_2 + 8$	36	$-8\sigma_3 + 24\sigma_2 + 48$

Table 26: TV-invariants for manifolds of complexity 6 (continued)

c_i	M	ν	3	4	5	6	7
6_{70}	$T \times I / \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$						
		0	1	8	$4\sigma_2 + 12$	48	$-9\sigma_3 + 27\sigma_2 + 54$
		1	0	0	0	0	0
		2	7	28	$28\sigma_2 + 84$	252	$-63\sigma_3 + 189\sigma_2 + 378$
		Σ	8	36	$32\sigma_2 + 96$	300	$-72\sigma_3 + 216\sigma_2 + 432$
6_{71}	$T \times I / \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$						
		0	1	8	$4\sigma_2 + 12$	48	$-9\sigma_3 + 27\sigma_2 + 54$
		1	0	0	0	0	0
		2	7	28	$28\sigma_2 + 84$	252	$-63\sigma_3 + 189\sigma_2 + 378$
		Σ	8	36	$32\sigma_2 + 96$	300	$-72\sigma_3 + 216\sigma_2 + 432$
6_{72}	$K\tilde{\times}I \cup K\tilde{\times}I / \begin{pmatrix} -1 & 0 \\ -1 & 1 \end{pmatrix}$						
		0	1	4	$4\sigma_2 + 8$	28	$-9\sigma_3 + 19\sigma_2 + 30$
		1	0	0	0	0	0
		2	3	12	$12\sigma_2 + 24$	72	$-27\sigma_3 + 57\sigma_2 + 90$
		Σ	4	16	$16\sigma_2 + 32$	100	$-36\sigma_3 + 76\sigma_2 + 120$

Table 27: TV-invariants for manifolds of complexity 6

c_i	M	ν	3	4	5	6	7
6_{73}	$K\tilde{\times}I \cup K\tilde{\times}I/\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$						
		0	1	8	$3\sigma_2+10$	46	$-6\sigma_3+22\sigma_2+44$
		1	0	0	0	0	0
		2	7	24	$21\sigma_2+70$	198	$-42\sigma_3+154\sigma_2+308$
		Σ	8	32	$24\sigma_2+80$	244	$-48\sigma_3+176\sigma_2+352$
6_{74}	$K\tilde{\times}I \cup K\tilde{\times}I/\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$						
		0	1	4	σ_2+4	22	$-4\sigma_3+10\sigma_2+16$
		1	0	0	0	0	0
		2	3	4	$3\sigma_2+12$	30	$-12\sigma_3+30\sigma_2+48$
		Σ	4	8	$4\sigma_2+16$	52	$-16\sigma_3+40\sigma_2+64$

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Chelyabinsk State University
e-mail: *matveev@csu.ru*